Brogaard and Salerno on antirealism and the conditional fallacy

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ABSTRACT

Brogaard and Salerno 2005 have argued that antirealism resting on a counterfactual analysis of truth is flawed because it commits a conditional fallacy by entailing the absurdity that there is necessarily an epistemic agent. Brogaard and Salerno's argument relies on a formal proof built upon the criticism of two parallel proofs given by Plantinga 1982 and Rea 2000. If this argument were conclusive, antirealism resting on a counterfactual analysis of truth should probably be abandoned. I argue however that the antirealist is not committed to a controversial reading of counterfactuals presupposed in Brogaard and Salerno's proof, and that the antirealist can in principle adopt alternative readings that makes this proof invalid. My conclusion is that no reductio of antirealism resting on a counterfactual analysis of truth has yet been provided.

1. Alethic antirealism and the conditional fallacy: the state of play

Global alethic antirealists analyse the notion of truth in terms of the biconditionals instantiated by the following schema:

(AR) $\Box(T[P] \leftrightarrow (Q(P) \Box\!\!\rightarrow R(P)))$.

Where P is a placeholder for statements, $T[P]$ says that it is true that P, $Q(P)$ means that epistemic conditions are ideal (or sufficiently good) for determining whether P, $R(P)$ means that it is rationally believed that P. Furthermore, $\Box$ is the necessity operator, $\leftrightarrow$ is the material biconditional and $\Box\!\!\rightarrow$ is the counterfactual conditional. AR says that, necessarily, it is true that P if and only if, if

epistemic conditions were ideal (or sufficiently good) for determining whether *P*, it would rationally be believed that *P*.

Brogaard and Salerno 2005 have argued that antirealism resting on AR is flawed because it commits a conditional fallacy by entailing the absurdity that, necessarily, epistemic conditions are ideal to determine whether some statement is true (which implies that there is necessarily an epistemic agent). Brogaard and Salerno's argument relies on a formal proof built upon the criticism of two parallel proofs given by Plantinga 1982 and Rea 2000.[1] Precisely, both demonstrations are argued to be questionable because appealing to principles valid only in classical logic that the antirealist may reject in favour of intuitionistic principles (cf. Brogaard and Salerno 2005: 126 and 130). Additionally, Plantinga's original proof and an intuitionistically respectable reformulation of it (cf.: 124-126) are found to lack in generality. For they can successfully apply only to *Peircean* versions of antirealism, which require a commitment to the existence of *one* single epistemic situation (e.g. "the end of inquiry") appropriate to establish the truth of *any* statements. Such positions are argued to be unattractive and implausible on their own (cf.: 128-129).

In contrast, Brogaard and Salerno's novel argument against antirealism is quite general, as it targets indifferently Peircean antirealism and non-Peircean antirealism, which allows statements to have *individual* truth conditions (cf.: 135-136). Brogaard and Salerno also emphasize that their proof relies on only 'exceedingly weak logical resources' (137) that the antirealist cannot but accept. Finally, Brogaard and Salerno make a case that 'an [antirealist] attempt to interpret $\Box\!\!\rightarrow$ in a way that blocks the conditional fallacy will … carry with it unacceptable consequences of its own' (ibid.). Brogaard and Salerno are thus persuaded that 'the counterfactual analysis of truth as epistemic cannot avoid the conditional fallacy, not without a massive revisionism of our logical

[1] Plantinga 1982 and Rea 2000 take the conclusions of their proofs to show that the alethic antirealist is committed to some form of theism (i.e., roughly, to the thesis that there is necessary an omniscient epistemic agent/community). Brogaard and Salerno 2005 contend that this is a misguided interpretation of those alleged results, which should instead be seen as instances of the general problem of the conditional fallacy that plagues counterfactual analyses. (The same point is made in Wright 2000). I will not consider this issue in my paper.

resources' (ibid.) They believe that 'the counterfactual analysis of truth must fall prey to some counterintuitive consequence – viz., that an epistemic agent must exist' (ibid.).

I find Brogaard and Salerno's criticism of Plantinga's and Rea's proofs quite persuasive and I will not question it, while I will take issue with Brogaard and Salerno's new argument. If this argument were conclusive, antirealism resting on a counterfactual analysis of truth should probably be abandoned. I argue however that the antirealist is not committed to the controversial reading of counterfactuals presupposed in Brogaard and Salerno's proof, according to which counterfactuals with impossible antecedent are vacuously true. I show that, despite Brogaard and Salerno's conviction, the antirealist can in principle interpret $\Box\!\!\rightarrow$ in two alternative ways that block the conditional fallacy without making any unacceptable assumption. The antirealist can thus reject Brogaard and Salerno's proof as invalid.

In more detail, in Sect. 2, I introduce the notion of a conditional fallacy and present Brogaard and Salerno's argument against alethic antirealism. In Sect. 3, I argue that the antirealist can reject Brogaard and Salerno's proof by appealing to either of two alternative analyses of counterfactuals, and I outline Brogaard and Salerno's anticipated reply to this argument. In Sect 4, I show that Brogaard and Salerno's reply to counteract the appeal to the first alternative analysis of counterfactuals is incorrect. In Sect 5, I show that Brogaard and Salerno's reply to counteract the appeal to the second alternative analysis of counterfactuals is also incorrect. In Sect 6, I draw the conclusions of the paper.

2. The conditional fallacy and Brogaard and Salerno's proof

Suppose a given notion is analysed in the terms of the biconditionals instantiated by the following schema:

$$\Box(P \leftrightarrow (Q \Box\!\!\rightarrow R)),$$

where P, Q and R are placeholders for statements, P stands for the analysandum and $(Q \Box\!\!\rightarrow R)$ for

the analysans. The problem of the conditional fallacy arises if some substitutions for *P* and *Q* are not logically independent. In such cases, counterintuitive modal consequences can be derived: for instance, statements that are obviously contingent may turn out to be necessarily true or even necessarily false (cf.: Brogaard and Salerno 2005: 134-135). In particular, Brogaard and Salerno focus on the following meta-statement that – they contend – proves true on very weak logical assumptions (where → is the material conditional):

(CF) $\{\Box(P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R)), \Box(Q \rightarrow P)\}$ entails $\Box P$.

They argue that each instance of AR produces the fulfilment of $\Box(P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R))$, and that, for some of these instances, $\Box(Q \rightarrow P)$ will also be satisfied. Therefore, the antirealist ought to deduce, in certain cases, instances of $\Box P$. This would commit the antirealist to absurdity. (Cf.: 131).

Brogaard and Salerno give just one example of this (cf.: 135-136). Consider the instance of $\Box(P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R))$ in which:

P = 'epistemic conditions are ideal for determining whether some statement is true' (135);
Q = 'epistemic conditions are ideal for determining whether [epistemic conditions are ideal for determining whether some statement is true]'
R = 'it is rationally believed that [epistemic conditions are ideal for determining whether some statement is true]'.

This instance of $\Box(P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R))$ logically follows from the instance of AR with *P* = 'epistemic conditions are ideal for determining whether some statement is true' and the necessitation of the Equivalence Schema.[2] The antirealist is thus committed to it. But the antirealist is also committed to the correlated instance of $\Box(Q \rightarrow P)$. As, in this interpretation of *P* and *Q*, *Q* entails *P* through the

[2] Namely, $\Box(T[P] \leftrightarrow P)$.

necessitation of the Equivalence Schema.[3] Given CF, the antirealist must then derive that $\Box P$ – namely, that, necessarily, epistemic conditions are ideal for determining whether some statement is true. These epistemic conditions 'include the existence of a properly placed epistemic agent. So, necessarily, there is an epistemic agent' (135).

The success of this supposed reductio of alethic antirealism crucially depends on whether the antirealist is committed to CF. Brogaard and Salerno show CF in the modal system **S4** extended with logical resources for counterfactuals. **S4** is characterized by axiom schema T, i.e. $\Box Q \rightarrow Q$, and axiom schema 4, i.e. $\Box Q \rightarrow \Box\Box Q$. Axioms T and 4 allow for the formal characterization of features of modal notions that appear quite uncontroversial. The antirealist is plausibly committed to both axiom T and axiom 4. The proof of CF requires the following two additional axiom schemata for counterfactuals (where $\wedge$ is the logical conjunction):

(A1) $(Q \wedge (Q \mathrel{\Box\!\!\rightarrow} R)) \rightarrow R$;

(A2) $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$.

(Strictly speaking, Brogaard and Salerno formulate A1 and A2 as inference rules, but turning these rules into axiom schemata will simplify my discussion). Axiom A1, which can be formulated as a special modus ponens for counterfactuals, has intuitive appeal. The antirealist could reject A1 with difficulty.[4] Axiom A2 corresponds to the rule that a counterfactual can logically be inferred from the corresponding strict conditional. A2 is much more dubious and controversial than A1. The antirealist can straightforwardly question its validity (more on this in the next section).

[3] By the necessitation of the Equivalence Schema, (Q) 'epistemic conditions are ideal for determining whether [epistemic conditions are ideal for determining whether some statement is true]' entails 'epistemic conditions are ideal for determining whether it is true that [epistemic conditions are ideal for determining whether some statement is true]'. From the latter, it follows: (P) 'epistemic conditions are ideal for determining whether some statement is true'.

[4] This does not mean that A1 cannot be questioned, but I will not dispute its validity in this paper.

Here is Brogaard and Salerno's proof (where the inference rules corresponding to A1 and A2 have been replaced with these two axiom schemata) (cf.: 131):

1. $\Box(P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R)), \Box(Q \rightarrow P)$	(Assumptions)
2. $P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R), Q \rightarrow P$	(From 1 and axiom T, by modus ponens)
3. Q	(Assumption, for →introduction)
4. $Q \mathrel{\Box\!\!\rightarrow} R$	(From 2 and 3, by modus ponens)
5. R	(From 3, 4, and A1, by ∧introduction and modus ponens)
6. $Q \rightarrow R$	(From 3-5, by →introduction)
7. $\Box(Q \rightarrow R)$	(From 1 and 2-6, by closure)
8. $Q \mathrel{\Box\!\!\rightarrow} R$	(From 7 and A2, by modus ponens)
9. P	(From 2 and 8, by modus ponens)
10. $\Box\Box(Q \rightarrow R)$	(From 7 and axiom 4, by modus ponens)
11. $\Box(Q \mathrel{\Box\!\!\rightarrow} R)$	(From 7, 8 and 10, by closure)
12. $\Box P$	(From 1, 2, 8, 9 and 11, by closure)
QED.	

$Q \mathrel{\Box\!\!\rightarrow} R$, on line 4, follows from $P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ and $Q \rightarrow P$, on line 2, and from Q, on line 3, because Q and $Q \rightarrow P$ entail P by modus ponens, and P and $P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ entail $Q \mathrel{\Box\!\!\rightarrow} R$ by modus ponens.[5] On line 7, we can necessitate $Q \rightarrow R$ by closure (i.e. the rule that necessities entail necessities) because $Q \rightarrow R$ follows from $P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ and $Q \rightarrow P$, which are assumed on line 1 to be both necessary.[6] On line 11, we can necessitate $Q \mathrel{\Box\!\!\rightarrow} R$ by closure because $Q \mathrel{\Box\!\!\rightarrow} R$ follows

[5] Modus ponens is here defined in a broadened sense to apply to both → and ↔.

[6] Rigorously speaking, $Q \rightarrow R$ follows from $P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$, $Q \rightarrow P$ *and axiom* A1, used to derive line 5 from lines 3 and 4. The rule of closure correctly applies here because also A1, as an axiom schema, can be shown to be necessary by

from $\Box(Q \rightarrow R)$, which is shown to be necessary on line 10. On line 12, we can necessitate P by closure because P follows from $P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$, which is assumed to be necessary on line 1, and from $Q \mathrel{\Box\!\!\rightarrow} R$, which is shown to be necessary on line 11.

Brogaard and Salerno concede that 'the antirealist's commitment to classical logic is doubtful' (130) but at the same time stress that they have proven CF 'without principles that are exclusively classical' (136), so that 'the proof is valid in intuitionistic (and even minimal) logic' (ibid.). What Brogaard and Salerno plausibly mean is that their proof is valid in a suitable intuitionistic *modal* logic extended with axioms A1 and A2 for counterfactuals. To make my criticism more precise, I will make this non-classical logic explicit.

Intuitionistic modal logic has recently flourished into a family of logics each of which corresponds to a member of a parallel family of classical modal logics (see, for instance, Fisher Servi 1980 and 1984, Simpson 1994: §3.2-3.4, and Gabbay et al. 2003: §3.11 and §10).[7] **IK** is the weakest intuitionistic modal logic of the family. **IK** is conservative over intuitionistic propositional logic, contains all substitution instances of theorems of the latter logic and is closed under modus ponens.[8] Furthermore, the mere addition of the axiom schema $P \vee \neg P$ to **IK** produces the weakest classical modal system **K**. Each member of the ordered triple of intuitionistic modal logics $\mathscr{I} =$

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necessitation, i.e. the rule that if a statement is a theorem of a given modal logic, it can be necessitated in that logic. A1 represents a theorem of the logic **S4** ∪ {A1, A2}. As any axiom schema is necessary, for the sake of simplicity, I will never mention any of the axioms implicitly appealed to in the applications of the rule of closure made this paper.

[7] These papers provide slightly different axiomatizations of the *same* intuitionistic modal logics.

[8] Following Simpson (1994: 52), **IK** can be axiomatized as follows (where $\Diamond$ is the possibility operator, $\vee$ is the logical disjunction and $\perp$ is any logical contradiction):

Axioms.

(i) All substitution instances of theorems of intuitionistic propositional logic;
(ii) $\Box(Q \rightarrow R) \rightarrow (\Box Q \rightarrow \Box R)$;
(iii) $\Box(Q \rightarrow R) \rightarrow (\Diamond Q \rightarrow \Diamond R)$;
(iv) $\neg\Diamond\perp$;
(v) $\Diamond(Q \vee R) \rightarrow (\Diamond Q \vee \Diamond R)$;
(vi) $(\Diamond Q \rightarrow \Box R) \rightarrow \Box(Q \rightarrow R)$.

Rules.

(Modus Ponens) From $Q \rightarrow R$ and Q, deduce R;
(Necessitation) If Q is a theorem, then so is $\Box Q$.

<**IT**, **IS4**, **IS5**>, resulting from adding further axiom schemata to **IK**, corresponds, in the same order, to a member of the ordered triple of classical modal logics $\mathscr{C}$ = <**T**, **S4**, **S5**>, resulting from adding the same axiom schemata to **K**. Each member of $\mathscr{I}$ is thus characterized by exactly the same additional axiom schemata that characterize its counterpart in $\mathscr{C}$, and the addition of $P \vee \neg P$ to any member of $\mathscr{I}$ yields its classical counterpart in $\mathscr{C}$. Finally, the necessity operator $\Box$ and the possibility operator $\Diamond$ used in the intuitionistic logics of $\mathscr{I} \cup \{\mathbf{IK}\}$ reflect the behaviour of, respectively, the *intuitionistic* quantifiers $\forall$ and $\exists$, in the same way as the modal operator $\Box$ and $\Diamond$ used in the classical logics of $\mathscr{C} \cup \{\mathbf{K}\}$ simulate the *classical* quantifiers $\forall$ and $\exists$ (cf.: Gabbay et al. 2003: 188).[9] An expected consequence is that the intuitionistic $\Box$ and $\Diamond$ are not interdefinable (in the sense it does not hold valid that $\Diamond P \leftrightarrow \neg\Box\neg P$).[10]

The logics of $\mathscr{I} \cup \{\mathbf{IK}\}$ are provided with a possible world semantics. Precisely, these logics are sound and complete with respect to certain classes of frames of the form $<W, \leq, R>$, where W is a set of possible worlds partially ordered by $\leq$, and R is a binary relation (the accessibility relation) defined on W. These frames combine aspects of Kripke's frames for classical modal logic and Kripke's frames for intuitionistic predicate logic (see Simpson 1994: §3.3 and Gabbay et al. 2003: §3.11 and §10).

As **IS4** is just characterized by Axioms T and 4, which are necessary to Brogaard and Salerno's demonstration,[11] **IS4** $\cup$ {A1, A2} appears suitable to carrying out this proof.[12] Indeed, the logic **IS4**

[9] The following two schemata hold valid in the logics of $\mathscr{I} \cup \{\mathbf{IK}\}$: $\neg\Diamond P \leftrightarrow \Box\neg P$ and $\Diamond\neg P \rightarrow \neg\Box P$. Yet $\neg\Box P \rightarrow \Diamond\neg P$ does not hold valid in them.

[10] Although this simple and powerful systematisation of intuitionistic modal logic is today accepted by most logicians and computer scientists, weaker and interesting logics have been also studied, often called *constructive* rather than intuitionistic. See, for instance, de Paiva and Mendler 2005 and Alechina et al. 2001.

[11] As the intuitionistic $\Box$ and $\Diamond$ are not dual, the relevant axioms T and 4 are formulated as, respectively, $(\Box P \rightarrow P) \wedge (P \rightarrow \Diamond P)$ and $(\Box P \rightarrow \Box\Box P) \wedge (\Diamond\Diamond P \rightarrow \Diamond P)$ (cf. Gabbay et al. 2003: 190 and Simpson 1994: 55-56).

[12] To obtain the logic **IS4** $\cup$ {A1, A2}, we first expand the alphabet of intuitionistic modal logic with the connective $\Box\rightarrow$, and add to the formation rules of the language of intuitionistic modal logic the one such that, if P and Q are well-

∪ {A1, A2} does validate this proof. Since **IS4** is standardly (or, at least, widely) accepted among intuitionistic modal logicians, showing that the antirealist relying on a counterfactual analysis of truth cannot accept **IS4** without falling afoul of a conditional fallacy would surely yield a devastating blow to the antirealist perspective. I will however show that this is not the case: the antirealist can accept **IS4,** and even **IS4** ∪ {A1}, while rejecting the dubious A2. Without A2, no conditional fallacy will probably result. Assuming – more implausibly – that the antirealist endorses *classical* modal logic, all my arguments can easily be reformulated, with only negligible changes, to show that the antirealist can accept **S4**, and even **S4** ∪ {A1}, while dropping the dubious A2. Brogaard and Salerno 2005 claim that the counterfactual analysis of truth as epistemic cannot avoid the conditional fallacy, 'not without a massive revisionism of our logical resources' (137). This claim appears simply false.

3. The antirealist's rejoinder and Brogaard and Salerno's reply

The inference of line 8 from line 7 consists in deducing the counterfactual $Q \mathbin{\Box\!\!\rightarrow} R$ from the strict conditional $\Box(Q \rightarrow R)$. This inference is licensed by modus ponens and A2. If A2 were dropped, line 8 could not be inferred from line 7, and Brogaard and Salerno's proof of CF would be incorrect. Can the antirealist reject A2? Notice that **IS4** ∪ {A2} validates the schema $\neg\Diamond Q \rightarrow (Q \mathbin{\Box\!\!\rightarrow} R)$, which expresses the thesis that *counterpossibles* (i.e. counterfactuals with impossible antecedent)

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formed formulae, so is $P \mathbin{\Box\!\!\rightarrow} Q$. Then, we extend the set of axioms of **IS4** with A1 and A2. It would be easy to show that the logics of $\mathscr{F}$∪{**IK**} are closed under all elementary inference rules additional to modus ponens used in the proof of CF, and under all those I will use in this paper. These elementary rules are: →introduction, ∧introduction, ∧elimination and closure. When **IS4** is extended to **IS4** ∪ {A1, A2}, the latter logic will prove closed under the same rules. Notice that, since **IS4** is logically consistent, **IS4** ∪ {A1, A2} is also logically consistent. Proof. Let us reinterpret $\mathbin{\Box\!\!\rightarrow}$ as →. As a result, A1 and A2 are turned into, respectively, $(Q \wedge (Q \rightarrow R)) \rightarrow R$ and $\Box(Q \rightarrow R) \rightarrow (Q \rightarrow R)$, while there is no change in the inference rules. As both the latter schemata are valid in **IS4** (this can easily be shown by using →introduction, ∧introduction, Axiom T and modus ponens) and no formula with the form $P \wedge \neg P$ can be deduced from **IS4**, no such a formula can be deduced from **IS4** ∪ {A1, A2} either. QED.

are vacuously true.[13] This schema is not validated by **IS4** alone, neither by **IS4** ∪ {A1}. As the antirealist is plausibly committed to **IS4**, if the antirealist rejected $\neg\Diamond Q \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ as invalid, she should also reject A2 as invalid. This would make the proof of CF incorrect. The crucial question is, therefore, whether the antirealist can reject $\neg\Diamond Q \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$, that is, the claim that counterpossibles are vacuously true.

Lewis 1973 proposes that a counterfactual $Q \mathrel{\Box\!\!\rightarrow} R$ is true at a possible world *w* if and only if (a) *Q* is impossible at *w* or (b) in the possible worlds where *Q* is true and which are *closest* to *w* (i.e. in those worlds that resemble *w* as much as *Q*'s truth permits it), *R* is true (cf.: 16). Accepting clause (a) is the same as accepting that counterpossibles are vacuously true. While clause (b) has intuitive appeal and is generally accepted as a sufficient condition for the truth of a counterfactual, this is hardly the case with clause (a), which is very contentious (see, among its numerous critics, Yagisawa 1988, Zagzebski 1990, Read 1995: 90-91, Mares 1997, Nolan 1997, Priest 1997 and Goodman 2004). Lewis 1973 himself has admitted that his 'reasons [to accept (a)] are less than decisive' (25), and he seems to oscillate between the view that counterpossibles are vacuously true and the view that counterpossibles are all false (cf.: ibid.).

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[13] To demonstrate that **IS4** ∪ {A2} validates $\neg\Diamond Q \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$, let us first prove that $\neg\Diamond Q \rightarrow \Box(Q \rightarrow R)$ is valid in **IK** (and so in **IS4)**. Proof. As **IK** is conservative over intuitionistic propositional logic, **IK** validates the *law of contradiction*, i.e. $\neg Q \rightarrow (Q \rightarrow R)$. By necessitation (see above, note 8), we turn $\neg Q \rightarrow (Q \rightarrow R)$ into $\Box(\neg Q \rightarrow (Q \rightarrow R))$. This schema and axiom (ii), i.e. $\Box(Q \rightarrow R) \rightarrow (\Box Q \rightarrow \Box R)$ (see ibid.), entail $\Box\neg Q \rightarrow \Box(Q \rightarrow R)$ by modus ponens. **IK** validates the schema $\neg\Diamond Q \rightarrow \Box\neg Q$ (see above, note 9). Assume $\neg\Diamond Q$ for →introduction. $\neg\Diamond Q$ and $\neg\Diamond Q \rightarrow \Box\neg Q$ entail $\Box\neg Q$ by modus ponens. $\Box\neg Q$ and $\Box\neg Q \rightarrow \Box(Q \rightarrow R)$, derived before, entail $\Box(Q \rightarrow R)$ by modus ponens. Finally, $\neg\Diamond Q$, assumed above, and $\Box(Q \rightarrow R)$ entail $\neg\Diamond Q \rightarrow \Box(Q \rightarrow R)$ by →introduction. QED. Let us now demonstrate that **IS4** ∪ {A2} validates $\neg\Diamond Q \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$. Proof. Assume $\neg\Diamond Q$ for →introduction. This assumption and the schema $\neg\Diamond Q \rightarrow \Box(Q \rightarrow R)$, which has been shown to hold valid in **IS4**, entail $\Box(Q \rightarrow R)$ by modus ponens. $\Box(Q \rightarrow R)$ and A2 – i.e. $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ – entail $Q \mathrel{\Box\!\!\rightarrow} R$ by modus ponens. Finally, $\neg\Diamond Q$, assumed initially, and $Q \mathrel{\Box\!\!\rightarrow} R$ entail $\neg\Diamond Q \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ by →introduction. QED.

Indeed, there is no general agreement on the correct semantic analysis of counterpossibles (two alternatives are considered below), and it is not evident that the alethic antirealist as such is committed to any specific view in this controversial issue. But if the antirealist is not committed to (a), she can reject it together with $\neg\Diamond Q \rightarrow (Q \mathrel{\square\!\!\rightarrow} R)$. The consequence is that A2 will also be dropped. The antirealist can thus reject Brogaard and Salerno's proof of CF as incorrect by accepting **IS4**, and even **IS4** $\cup$ {A1}, while dropping A2.

Brogaard and Salerno are well aware of this possible reply and take it seriously. If the antirealist drops (a), the antirealist should endorse an alternative analysis of counterpossibles. Brogaard and Salerno examine two possibilities. The first semantical analysis, considered also by Lewis (1973: 25), 'treats a counterfactual as false when it embeds an impossible antecedent' (Brogaard and Salerno 2005: 126). 'Alternatively, and more plausibly, counterfactuals with necessary false antecedents are to be treated as sometimes true, sometimes false' (127). Brogaard and Salerno believe that should the antirealist drop (a) and A2 to accept either alternative reading of counterpossibles, she would not escape the conditional fallacy. They are persuaded that to block the conditional fallacy, the antirealist must also show that $\neg\Diamond$Q(P) – namely, that it is impossible that conditions are ideal for determining whether P, where P means 'epistemic conditions are ideal for determining whether some statement is true'. Brogaard and Salerno seem to believe that if this additional condition is not satisfied, a reductio of antirealism will still go through even if (a) and A2 are dropped; and they argue that this additional condition *cannot* be satisfied by the antirealist independently of the semantics for counterpossibles she will accept. For $\neg\Diamond$Q(P) will not be true if $\neg\Diamond$Q(P) has the meaning just specified.

I do agree with Brogaard and Salerno that $\neg\Diamond$Q(P) is probably not true. In the next sections, I will however contend that one of Brogaard and Salerno's arguments to support this claim is inconclusive. More importantly, I will show that dropping (a) and A2 to endorse either alternative reading of counterpossibles does appear in itself *sufficient* to block the conditional fallacy – despite Brogaard and Salerno's conviction, no additional condition must be satisfied by the antirealist.

4. If counterpossibles are all false, Brogaard and Salerno's reductio is ineffective

Brogaard and Salerno 2005 try to defuse the antirealist's possible appeal to the view that all counterpossibles are false by arguing that if both this view and AR are accepted, the antecedent Q(P) of the counterfactual Q(P) $\Box\!\!\rightarrow$ R(P), instantiated in AR and used in their reductio, *cannot* be impossible. (Here, P means 'epistemic conditions are ideal for determining whether some statement is true', Q(P) means 'epistemic condition are ideal for determining whether P', and R(P) means 'it is rationally believed that P'). From this, they conclude that the appeal to the view that all counterpossibles are false is quite harmless. I quote their succinct argument in full. Brogaard and Salerno focus on counterfactuals with impossible antecedent and argue as follows:

> If such counterfactuals have always the same truth-value, then Q(P) $\Box\!\!\rightarrow$ R(P) and Q(P) $\Box\!\!\rightarrow$ R(¬P) will have the same truth-value *when* Q(P) *is impossible*. By [AR], this will give a contradiction. By reductio, Q(P) is not impossible. But then $\Box$(Q(P) → R(P)) will give us Q(P) $\Box\!\!\rightarrow$ R(P) straightforwardly (138, note 13).[14]

This passage is problematic for at least two reasons. To begin with, notice that if Q(P) $\Box\!\!\rightarrow$ R(P) and Q(P) $\Box\!\!\rightarrow$ R(¬P) have the same truth-value when Q(P) is impossible, we *cannot* obtain any contradiction by simply appealing to AR. We obtain a contradiction if the second counterfactual is replaced with Q(¬P) $\Box\!\!\rightarrow$ R(¬P); in that case, we will derive ¬P ∧ ¬¬P if both Q (P) $\Box\!\!\rightarrow$ R(P) and Q(¬P) $\Box\!\!\rightarrow$ R(¬P) are false. Brogaard and Salerno presuppose, therefore, that if Q(P) is impossible, also Q(¬P) is impossible (namely, if it is impossible that conditions are ideal for determining whether P, then it is impossible that conditions are ideal for determining whether ¬P). This will be the case if Q(¬P) *entails* Q(P) (or, more generally, if Q(¬P) and Q(P) are logically equivalent). Brogaard and Salerno seem to assume that this entailment holds true, but they give no justification

[14] I have modified the original notations.

for it. If this assumption is false, they cannot conclude that Q(P) is not impossible – shortly, that $\neg\neg\Diamond Q(P)$ – as one of the steps of their reasoning is not justified.

Brogaard and Salerno's assumption would seem to be true if Q(P) refers to a Peircean epistemic situation. For such a situation is meant to be the one appropriate to evaluate *any* statement whatsoever.[15] Brogaard and Salerno emphasize however that the notion of a Peircean epistemic situation is problematic, as 'it is difficult to imagine what such a situation would be like' (128). Other authors have forcefully argued that the notion of a Peircean epistemic situation is actually incomprehensible, and thus of no use in philosophy (for instance, Wright 1994: 44-46). It is thus dubious that the antirealist will appeal to such an interpretation of Q(P). Indeed, prominent antirealists, such as Putnam (1983: xvii-xviii) and Dummett 2005, explicitly appeal to *non*-Peircean epistemic conditions in their analysis of truth. Therefore, if Brogaard and Salerno's reasoning is to have actual relevance, it should prove correct when Q(P) refers to non-Peircean epistemic conditions. The problem is that, in this case, it is not apparent that $Q(\neg P)$ entails Q(P). And it may well be that $Q(\neg P)$ does not entail Q(P), as this happens for other interpretations of P. Let me give a simple example of it.

Following Putnam (1983: xvii), a way to specify the ideal (or sufficiently good) epistemic conditions for determining whether a statement P is true is that of laying down a list of desiderata for the attributes of the epistemic agent and (if P is an empirical statement) for the spatio-temporal location of the epistemic agent and for the features of her surroundings. Suppose P means 'this CD is scratched'. The ideal conditions for determining whether P may include desiderata such as: there is a rational agent provided with organs of sight; the rational agent is completely awake; she can directly observe the relevant CD; the CD is neat and in daylight, etc. Suppose now that a subject is hypnotized so that she becomes unable to entertain any definite belief about whether any given CD is scratched or not on the grounds of her immediate perceptions if the CD is actually scratched.

[15] This situation can roughly be characterized by the presence of an *ideally* rational agent who possesses *all possible* evidence.

Thus, if a scratched CD is shown to her, she will neither believe that the CD is scratched nor that the CD is not so. Suppose also that, if a CD is not scratched, the subject can instead have the correct belief about it. Assume finally that the subject is induced not to remember that she has been hypnotized. Let us call this specific hypnotic state, characterized by these three conditions, a S-state. The hypothesis that a S-state can be induced in a subject is not absurd or implausible.

The desideratum that the rational agent is in no S-state is very probably part of the ideal conditions for determining whether P (i.e. 'this CD is scratched'). As, if the rational agent were is in a S-state, she would be unable to determine correctly, by direct observation, that a CD is scratched when it is so. However, it does not appear necessary to include this desideratum in the ideal conditions for determining whether ¬P (i.e. 'this CD is not scratched'). For, if the rational agent is in a S-state, this will not hamper her ability to determine correctly, by direct observation, that a CD is not scratched when it is not so. In this interpretation of P, Q(¬P) does not entail Q(P). The same result very plausibly obtains for other interpretations of P.[16] And it may well be that Q(¬P) will turn out not to entail Q(P) when P means 'epistemic conditions are ideal for determining whether some statement is true', once Q(¬P) and Q(P) have been thoroughly dissected. Brogaard and Salerno have given no reason to exclude this possibility. Consequently, it is not guaranteed that if Q(P) is impossible, Q(¬P) is also impossible. In conclusion, Brogaard and Salerno have made no convincing case to the effect that, when *P* is interpreted as above, and both AR and the view that all counterpossibles are false are accepted, Q(P) is not impossible.

[16] Consider a similar case in which P means 'this golf-ball is uniformly red'. Suppose that a rational agent can be hypnotized so that she becomes unable to have any definite belief about the colour of any golf-ball on the grounds of her immediate perceptions if the ball has the colour normally recognized as red and it is uniformly so. The desideratum that the rational agent has not been hypnotized in the way specified is part of the ideal conditions for determining whether P. However, it does not seem necessary to include this desideratum in the ideal conditions for determining whether ¬P. For, if the rational agent has been hypnotized in the specified way, this will not hamper her ability to establish correctly by direct observation that a golf-ball is, for example, uniformly blue or half red and half white.

Brogaard and Salerno have however a more persuasive argument to the effect that the alethic antirealist, whatever reading of counterpossibles she might endorse, should accept that $\neg\neg\Diamond$Q(P), and even that $\Diamond$Q(P), which is intuitionistically stronger.[17] (I will come to this argument shortly). Let us concede, therefore, that the antirealist who accepts AR and the view that all counterpossibles are false is committed to $\Diamond$Q(P). The objection to CF we are considering in this section says that the antirealist could drop (a) and A2 by endorsing the view that all counterpossibles are false, and that this move would render Brogaard and Salerno's proof of CF invalid, as line 8 would no longer be deducible from line 7. How would the antirealist's commitment to $\Diamond$Q(P) help, *precisely*, to counter this objection? The second reason why the short passage quoted above is problematic is that Brogaard and Salerno are not clear enough on this point. I will try to cast light on this issue by presenting what I think it is the most plausible interpretation of Brogaard and Salerno's thought. I will then argue that Brogaard and Salerno are mistaken: even if the antirealist should probably accept that $\Diamond$Q(P), this does *not* suffice to make their reductio work. I will also show that the very same negative upshot obtains on alternative interpretations of Brogaard and Salerno's thought.

Let us proceed gradually. Brogaard and Salerno believe that if the antirealist accepts that $\Diamond$Q(P), 'then $\Box$(Q(P) $\rightarrow$ R(P)) will give [the antirealist] Q(P) $\Box\!\!\rightarrow$ R(P) straightforwardly'. Why should this be the case? A reasonable explanation is the following: all instances of the schema,

(A3) $(\Box(Q \rightarrow R) \wedge \Diamond Q) \rightarrow (Q \Box\!\!\rightarrow R)$,

would seem to be necessarily true, and the intuitive validity of A3 would seem to depend on the sole clause (b) of Lewis' analysis of counterfactuals,[18] which has strong intuitive appeal. The

[17] In the logics of $\mathscr{S} \cup \{\mathbf{IK}\}$, as expected, though $\Diamond$Q(P) entails $\neg\neg\Diamond$Q(P), the converse does not hold true.

[18] It is trivial to show that, for any given possible world *w*, if both $\Box(Q \rightarrow R)$ and $\Diamond Q$ are true at *w*, $Q \Box\!\!\rightarrow R$ is also true at *w*, and that this follows from the satisfaction of the sole clause (b) but not of the clause (a). Proof. Consider a world *w*. If $\Diamond Q$ is true at *w*, clause (a) is not satisfied. Furthermore, if $\Box(Q \rightarrow R)$ is true at *w*, in all possible worlds in which *Q*

antirealist is thus plausibly committed to A3. Consequently, if the antirealist accepts ◊Q(P), □(Q(P) → R(P)) will give her Q(P) □→ R(P) straightforwardly, by A3, ∧introduction and modus ponens. This is probably Brogaard and Salerno's thought.

But this reasoning is not sufficient to conclude that a reductio of antirealism is now operative. What Brogaard and Salerno need is a valid proof showing that the antirealist is committed to the paradoxical claim that □Q(P) – the claim that, necessarily, epistemic conditions are ideal for determining whether [epistemic conditions are ideal for determining whether some statement is true]. To achieve this goal, Brogaard and Salerno may think of appealing to a reformulation of CF that includes the additional premise ◊Q. This is the most natural reformulation:

(CF1) $\{\Box(P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R)), \Box(Q \rightarrow P), \Diamond Q\}$ entails $\Box P$.

As the antirealist is probably committed to A3, it would seem reasonable to use this axiom schema, in alternative to A2, in the proof of CF1. The problem is that it will not work. Indeed, no proof of CF1 appears at hand in **IS4** ∪ {A1, A3}.

Here is an *attempted* proof of CF1 consisting in a variation of CF's original proof that makes use of A3 rather than A2:

13. $\Box(P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R)), \Box(Q \rightarrow P), \Diamond Q$	(Assumptions)
14. $P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R), Q \rightarrow P$	(From 13 and axiom T, by modus ponens)
15. Q	(Assumption, for →introduction)
16. $Q \mathrel{\Box\!\!\rightarrow} R$	(From 14 and 15, by modus ponens)

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is true that are accessible from *w* – and thus in the worlds closest to *w* – *R* is true. This satisfies clause (b), with the effect that $Q \mathrel{\Box\!\!\rightarrow} R$ also proves true at *w*. Therefore, for any given world *w*, if both $\Box(Q \rightarrow R)$ and $\Diamond Q$ are true at *w*, $Q \mathrel{\Box\!\!\rightarrow} R$ is also true at *w* on the sole clause (b). QED.

17. R	(From 15, 16, and A1, by ∧introduction and modus ponens)
18. $Q \rightarrow R$	(From 15-17, by →introduction)
19. $\Box(Q \rightarrow R)$	(From 13 and 14-18, by closure)
20. $Q \mathrel{\Box\!\!\rightarrow} R$	(From 13 and 19, by ∧-introduction and A3)
21. P	(From 14 and 20, by modus ponens)
22. $\Box\Box(Q \rightarrow R)$	(From 19 and axiom 4, by modus ponens)
23. $\Box(Q \mathrel{\Box\!\!\rightarrow} R)$	(From 13, 19 and 22, by closure)
24. $\Box P$	(From 13, 14, 20, 21 and 23, by closure)

QED.

This proof is *invalid*. On line 23, we cannot necessitate $Q \mathrel{\Box\!\!\rightarrow} R$ by closure because $Q \mathrel{\Box\!\!\rightarrow} R$ follows now from *two* schemata: $\Box(Q \rightarrow R)$, on line 19, and $\Diamond Q$, on line 13. Of these two schemata, only $\Box(Q \rightarrow R)$, but not $\Diamond Q$, is necessary ($\Box(Q \rightarrow R)$ is shown to be necessary on line 22). It is not apparent that a correct proof of CF1 is possible at all in **IS4** ∪ {A1, A3}.

Replacing the premise $\Diamond Q$, on line 13, with the stronger $\Box\Diamond Q$, will produce a valid proof. But it is not evident that the antirealist is committed to $\Box\Diamond Q$, at least when Q means 'epistemic conditions are ideal for determining whether [epistemic conditions are ideal for determining whether some statement is true]'. Consequently, the antirealist can reject the proof including the stronger premise $\Box\Diamond Q$ as unsound when the proof is used in a reductio of antirealism. The antirealist will indeed derive $\Box\Diamond Q$ from the original premise $\Diamond Q$ in a modal logic at least as strong as **IS5**, as the latter validates axiom 5, i.e. $\Diamond Q \rightarrow \Box\Diamond Q$.[19] Therefore, if the antirealist accepted **IS5**, A1 and A3, she would be committed to a proof of CF1 carried out in **IS5** ∪ {A1, A3}. But why should the

[19] More precisely, as the intuitionistic $\Box$ and $\Diamond$ are not dual, axiom 5 is to be formulated as follows: $(\Diamond Q \rightarrow \Box\Diamond Q) \wedge (\Diamond\Box Q \rightarrow \Box Q)$ (cf. Gabbay et al. 2003: 190 and Simpson 1994: 55-56).

antirealist accept **IS5**, which embeds the dubious and controversial axiom 5, to fall afoul of Brogaard and Salerno's reductio?

It might be thought that the difficulties we are facing depend on relying on A3, while Brogaard and Salerno would appeal to a more suitable axiom schema (or to a corresponding inference rule) in a revised version of CF.[20] For instance this:

(A4) $(\Box(Q \rightarrow R) \wedge \Diamond Q) \rightarrow \Box(Q \,\Box\!\!\rightarrow R)$.

Appealing to A4, rather than to A3, does result in a correct demonstration of CF1 in **IS4** ∪ {A1, A4}. For, via A4, we can directly derive $\Box(Q \,\Box\!\!\rightarrow R)$, instead than $Q \,\Box\!\!\rightarrow P$, on line 20. $\Box(Q \,\Box\!\!\rightarrow R)$ correctly follows from $\Box(Q \rightarrow R)$, on line 19, from $\Diamond Q$, on line 13, and from A4, by ∧introduction and modus ponens.[21] The problem is that there is no apparent reason why the antirealist should be committed to A4, which does *not* appear validated by the sole clause (b) of Lewis' analysis of counterfactuals. Indeed, A4 is valid in **IS5** ∪ {A3} (and so in **IS5** ∪ {A1, A3}).[22] But this does not help. For the antirealist's commitment to a modal logic as strong as **IS5** is dubious.

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[20] It can easily be shown that, in **IS4** ∪ {A3} (and so in **IS4** ∪ {A1, A3}), A3 is equivalent to (A3*) $\Diamond Q \rightarrow (\Box(Q \rightarrow R) \rightarrow (Q \,\Box\!\!\rightarrow R))$. Appealing to A3* rather than A3 in the derivation of $Q \,\Box\!\!\rightarrow R$, on line 20, produces no improvement. In this case, $\Box(Q \,\Box\!\!\rightarrow R)$ should be derived from $\Box\Box(Q \rightarrow R)$ and $\Box(Q \rightarrow R) \rightarrow (Q \,\Box\!\!\rightarrow R)$, by closure. The problem is that only the first schema but not the second is necessary.

[21] Arif Ahmed has also suggested the following axiom schema: (A4*) $\Diamond Q \rightarrow \Box(\Box(Q \rightarrow R) \rightarrow (Q \,\Box\!\!\rightarrow R))$. It would however be easy to show that, in **IS4** ∪ {A4} (and so in **IS4** ∪ {A1, A4}), A4* is equivalent to A4.

[22] Proof. Assume $\Box(Q \rightarrow R) \wedge \Diamond Q$ for →introduction. This conjunction entails $\Box(Q \rightarrow R)$ and $\Diamond Q$ by ∧elimination. By axiom 4 and modus ponens, we turn $\Box(Q \rightarrow R)$ into $\Box\Box(Q \rightarrow R)$. By axiom 5 and modus ponens, we turn $\Diamond Q$ into $\Box\Diamond Q$. $\Box(Q \rightarrow R)$, $\Diamond Q$ and A3 entail $Q \,\Box\!\!\rightarrow R$ by ∧introduction and modus ponens. As $\Box(Q \rightarrow R)$ and $\Diamond Q$ entail $Q \,\Box\!\!\rightarrow R$ and they are both necessary, by closure, we turn $Q \,\Box\!\!\rightarrow R$ into $\Box(Q \,\Box\!\!\rightarrow R)$. Finally, $\Box(Q \rightarrow R) \wedge \Diamond Q$, assumed initially for →introduction, and $\Box(Q \,\Box\!\!\rightarrow R)$ entail $(\Box(Q \rightarrow R) \wedge \Diamond Q) \rightarrow \Box(Q \,\Box\!\!\rightarrow R)$ by →introduction. QED.

At this point, it might be suggested that all our difficulties arise because we want to justify the possibility of inferring a counterfactual from a strict conditional (i.e. the move from line 7 to line 8 in CF, and from line 19 to line 20 in CF1) by appealing to axiom schemata like A2, A3 or A4, while Brogaard and Salerno would appeal to no such principles. They would rather provide a reformulation of CF more sophisticated than CF1, which would allow inferring a counterfactual from the relative strict conditional even in **IS4** ∪ {A1}. I believe however that this alternative strategy would not fare any better. Consider for instance a reformulation of CF that makes use of the additional premise $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ rather than of $\Diamond Q$. Precisely:

(CF2) $\{\Box(P \leftrightarrow (Q \mathrel{\Box\!\!\rightarrow} R)), \Box(Q \rightarrow P), \Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)\}$ entails $\Box P$.

Notice that if the antirealist is committed to $\Diamond Q$ and to the intuitively plausible clause (b) of Lewis' analysis of counterfactuals, she will very probably be committed to the additional premise of CF2, $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$.[23] Notice also that, in a proof of CF2, we can deduce $Q \mathrel{\Box\!\!\rightarrow} R$ from $\Box(Q \rightarrow R)$ by the mere appeal to $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ and modus ponens. No additional inference rule or any axiom schema, such as A2, A3 or A4, is necessary for this deduction.

The problem with CF2 is that no proof of it appears at hand in **IS4** ∪ {A1}, and it is not obvious that such a demonstration is possible at all in this logic. Consider for instance an attempted proof of CF2 consisting in a variation of the invalid proof of CF1 produced before. Precisely, the variation is such that, to move from $\Box(Q \rightarrow R)$, on line 19, to $Q \mathrel{\Box\!\!\rightarrow} R$, on line 20, we made use of the new premise $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ and modus ponens, as explained before, rather than of $\Diamond Q$, ∧introduction, A3 and modus ponens. Although this inference is correct in **IS4** ∪ {A1}, the

[23] Consider that if $\Diamond Q$ is true, there are accessible possible worlds where Q is true. Furthermore, if $\Box(Q \rightarrow R)$ is true, in all such possible worlds, R is true. Thus, in the closest possible worlds where Q is true, R is true. This satisfies clause (b), with the effect that $Q \mathrel{\Box\!\!\rightarrow} R$ is true. Given that, if $\Box(Q \rightarrow R)$ is true, $Q \mathrel{\Box\!\!\rightarrow} R$ is also true, it follows that $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ is true.

step from $\Box\Box(Q \rightarrow R)$, on line 22, to $\Box(Q \mathrel{\Box\!\!\rightarrow} R)$, on line 23, still appears invalid, and for the very same reason as before. On line 23, we cannot necessitate $Q \mathrel{\Box\!\!\rightarrow} R$ by closure because $Q \mathrel{\Box\!\!\rightarrow} R$ follows, by modus ponens, from two schemata: $\Box(Q \rightarrow R)$, on line 19, and the new premise $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$. The problem is that the latter premise has not been assumed to be necessarily true.

If $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ were turned into $\Box(\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R))$, a proof of CF2 would go through in **IS4** $\cup$ {A1}. But it is unclear why the antirealist should be committed to such a strong premise when Q and R are interpreted as, respectively, Q(P) and R(P) – namely, as the statements that allegedly elicit a conditional fallacy. If the antirealist is committed to $\Diamond Q$ and accepts clause (b) of Lewis' analysis of counterfactuals, it does *not* seem to follow from this that she is committed to $\Box(\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R))$ too. Finally, it can easy be seen that replacing $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$, in CF2, with $\Box(Q \rightarrow R) \rightarrow \Box(Q \mathrel{\Box\!\!\rightarrow} R)$ would result in a correct demonstration in **IS4** $\cup$ {A1}. But, again, the antirealist's commitment to $\Box(Q \rightarrow R) \rightarrow \Box(Q \mathrel{\Box\!\!\rightarrow} R)$ appears unjustified when Q and R are interpreted as Q(P) and R(P).

The moral to be learned is that Brogaard and Salerno have *not* shown that their reductio (or any variation of it) will succeed if the antirealist who accepts a counterfactual analysis of truth also endorses the semantical view that all counterpossibles are false. Probably enough, if the antirealist accepts this semantical view, Brogaard and Salerno's reductio will not go through.

5. If some counterpossibles are true and some false, Brogaard and Salerno's reductio is ineffective

Brogaard and Salerno 2005 try to defuse the antirealist's possible appeal to the view that some counterpossibles are true and some false by arguing that accepting this view alone will not suffice to block their reductio and by making a case that the additional condition necessary for this purpose cannot be satisfied by the antirealist.

Brogaard and Salerno focus on counterpossibles featuring as instances of Q(P) $\mathrel{\Box\!\!\rightarrow}$ R(P) in AR. They argue that, on the new alternative analysis of counterpossibles, AR is of no use in reducing

¬◊Q(P) to absurdity. For, if Q(P) is impossible, on the sometimes-true-sometimes-false analysis, 'one cannot presuppose in advance that Q(P) □→ R(P) and Q(P) □→ R(¬P) will have the same truth value' (138, note 9). Thus, if □(Q(P) → R(P)) is true in virtue of the impossibility of Q(P), 'it is an open question whether Q(P) □→ R(P) is true. And … the move from the strict conditional to the corresponding counterfactual is not justified' (127). I find this argument persuasive (and even more persuasive considering that it is not guaranteed that should Q(P) be impossible, then Q(¬P) would also be impossible). Even so, Brogaard and Salerno insist that on this alternative analysis of counterpossibles, their reductio is operative. The reason would be that, to block it, it is not sufficient to deny that counterpossibles have always the same truth-value:

> It must be also argued in a principled way that Q(P) is impossible whenever P is a statement that triggers a conditional fallacy. For example, when P is the statement 'epistemic condition are ideal for some statement', it must be argued that it is *impossible* for conditions to be ideal for the proper evaluation of 'conditions are ideal for some statements'. It is difficult to see how such an argument would go, since often enough it is the case that conditions are in fact ideal/sufficiently good for the evaluations of this statement. Conditions are currently sufficiently good for the evaluation of whether conditions are sufficiently good for some statement. To show that Q(P) is impossible will then be to show too much. (137).

Although Brogaard and Salerno do not spell out the ideal conditions for determining whether 'epistemic condition are ideal for determining whether some statement is true', it seems plausible that an acceptable analysis of them should entail that such conditions are often fulfilled. But if Q(P) is true, given axiom T, Q(P) is also possible. Therefore, Brogaard and Salerno are probably correct in asserting that ◊Q(P). Yet they are mistaken in believing that, to block their reductio, the antirealist who claims that counterpossibles are sometimes true and sometimes false has to argue that ¬◊Q(P). Why do Brogaard and Salerno believe so? Presumably, because they assume that there

is a correct proof of CF1 or of CF2, and they believe that, if it were true that $\neg\Diamond$Q(P), the antirealist could reject such a proof as *unsound* when applied to produce a conditional fallacy.[24] The problem, which Brogaard and Salerno seem not to perceive, is that it is very dubious that the antirealist who claims that counterpossibles are sometimes true and sometimes false will accept any proof of CF1 or of CF2 as *correct*.

If counterpossibles are sometimes true and sometimes false, schema A3 is still intuitively valid. For A3 appears validated by the sole clause (b) of Lewis' analysis of counterfactuals, independently of the reading of counterpossibles presupposed by the antirealist. However, as we have seen in the former section, it is unclear how, and even whether, A3 could be used to carry out a proof of CF1 (when no framework stronger than **IS4** $\cup$ {A1} is also assumed). The analysis of counterpossibles we are considering in this section does not make up for the logical problems encountered before, which hinge on the inapplicability of the rule of closure. Notice furthermore that the antirealist still appears uncommitted to $\Box\Diamond$Q(P), axiom 5 and axiom A4. Therefore, no help would come from any appeal to this statement and these two axiom schemata.

On the sometimes-true-sometimes-false analysis of counterpossibles, the antirealist is plausibly committed to the premise $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ of CF2 if she is committed to $\Diamond Q$. As this conditional commitment appears to depend on the sole clause (b) of Lewis' analysis of counterfactuals, which remains unquestioned. As I have argued before, it is however dubious that a proof of CF2 could be carried out in **IS4** $\cup$ {A1}. The new analysis of counterpossibles does not settle the logical problems we have found. Furthermore, the replacement of $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$, in CF2, with $\Box(\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R))$ or with $\Box(Q \rightarrow R) \rightarrow \Box(Q \mathrel{\Box\!\!\rightarrow} R)$ would still produce a valid proof in **IS4** $\cup$ {A1}. But it is unclear why the antirealist who claims that counterpossibles

[24] Precisely, the antirealist who maintains that some counterpossibles are true and some false could try to reject a possible proof of CF2 by arguing that if $\neg\Diamond$Q(P), then it is not clear in advance whether the premise $\Box(Q \rightarrow R) \rightarrow (Q \mathrel{\Box\!\!\rightarrow} R)$ of CF2 is true when interpreted as $\Box$(Q(P) $\rightarrow$ R(P)) $\rightarrow$ (Q(P) $\Box\!\!\rightarrow$ R(P)). This conditional would in fact be false if Q(P) $\Box\!\!\rightarrow$ R(P) proved false.

are sometimes true and sometimes false should be committed to any of these two strong premises when *Q* and *R* are interpreted as Q(P) and R(P). Brogaard and Salerno have given no argument to support this thesis.

To counter Brogaard and Salerno's reductio, the antirealist who accepts the view that some counterpossibles are true and some false does not need to argue that ¬◊Q(P). Instead, she should merely emphasize that, on her reading of counterpossibles, Brogaard and Salerno's proof of CF is invalid, for the inference from line 7 to line 8 is unjustified, and that no alternative proof which the antirealist is committed to appears available.

6. Conclusion: no proof that alethic antirealism commits a conditional fallacy has yet been delivered

Brogaard and Salerno 2005 have made a case that global alethic antirealism resting on a counterfactual analysis of truth is flawed because commits a conditional fallacy by entailing the absurdity that there is necessarily an epistemic agent. This argument is general, as it targets different forms of antirealism all at once, and it does not depends on principles of classical logic that the antirealist can reject. In these respects, Brogaard and Salerno's case is less questionable than parallel arguments made by Plantinga and Rea.

I have argued however that the antirealist can block Brogaard and Salerno's reductio with a simple move that involves no revision of intuitionistic modal logic (or of classical modal logic, should the antirealist endorse it). The crucial demonstration which Brogaard and Salerno's argument relies on takes for granted the controversial thesis that counterfactuals with impossible antecedent are vacuously true. I have argued that the antirealist can in principle reject this thesis by embracing one of these alternative views: that counterfactuals with impossible antecedent are all false or that counterfactuals with impossible antecedent are in certain cases true and in other false. I have shown that both these interpretations of counterfactuals make Brogaard and Salerno's demonstration invalid and that no obvious amendment appears at hand.

Importantly, the same method used to block Brogaard and Salerno's proof can be employed to reject any of the proofs available today in the literature that aim to show (or that can be read as aiming to show) that global antirealism resting on a counterfactual analysis of truth commits a conditional fallacy. For all these demonstrations require a strict conditional to entail the corresponding counterfactual, and so presuppose that all counterfactuals with impossible antecedent are vacuously true. I refer to Rea (2000: 296)'s proof, Brogaard and Salerno (2005: 125)'s version of Plantinga's proof, Plantinga (1982: 65)'s original proof and a weaker version of the latter due to Wright (2000: 342). It is perhaps true that global alethic antirealism resting on a counterfactual analysis of truth commits a conditional fallacy, but no proof of it has yet been delivered.

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References

Alechina, N. and V. De Paiva, M. Mendler, E. Ritter. 2001. 'Categorical and Kripke Semantics for Constructive S4 Modal Logic'. In L. Fribourg (ed.), *Proceeding of Computer Science Logic (CSL'01), LNCS 2142*.

Brogaard, B. and J. Salerno, 2005. 'Anti-realism, Theism and the Conditional Fallacy'. *Nous* 39: 123-139.

De Paiva, V. and M. Mendler. 2005. 'Constructive CK for contexts'. *Proceeding of the Workshop on Context Representation and Reasoning, Paris, France, July 2005*.

Dummett, M. 2005. 'The Justificationist's Response to a Realist'. *Mind* 114: 671-688.

Fisher Servi, G. 1980. 'Semantics for a class of intuitionistic modal calculi'. In M. L. Dalla Chiara (ed.), *Italian studies in the Philosophy of Science*: 59-72. Dordrecht: Reidel.

Fisher Servi, G. 1984. 'Axiomatizations for some intuitionistic modal logics'. *Rendiconti di Matematica di Torino* 42: 179-195.

Gabbay, D. and A. Kurucz, F. Wolter, M. Zakharyaschev. 2003. *Many-Dimensional Modal Logics: Theory and Applications*. Studies in Logic. Amsterdam: Elsevier.

Goodman, J. 2004. 'An Extended Lewis/Stalnaker Semantics and the New Problem of Counterpossibles'. *Philosophical Papers* 33: 35-66.

Lewis, D. K. 1973. *Counterfactuals*. Cambridge: Harvard University Press.

Mares, E. D. 1997. 'Who's Afraid of Impossible Worlds?'. *Notre Dame Journal of Formal Logic* 38: 516-526.

Nolan, D. 1997. 'Impossible Worlds: A Modest Approach'. *Notre Dame Journal of Formal Logic* 38: 535-572.

Plantinga, A. 1982. 'How to be an Anti-realist'. *Proceedings and Addresses of the American Philosophical Association* 56: 47-70.

Priest, G. 1997. 'Sylvan's Box'. *Notre Dame Journal of Formal Logic* 38: 573-582.

Putnam, H. 1983. *Realism and Reason: Philosophical Papers*. Vol. 3. Cambridge: Cambridge University Press.

Rea, M. 2000. 'Theism and Epistemic Truth-Equivalences'. *Nous* 34: 291-301.

Read, S. 1995. *Thinking about Logic*. Oxford: Oxford University Press.

Simpson, A. K. 1994. *The Proof Theory and Semantics of Intuitionistic Modal Logic*. PhD Thesis.

Yagisawa, T. 1988. 'Beyond Possible Worlds'. *Philosophical Studies* 53: 175-204.

Wright, C. 1994. *Truth and Objectivity*. Harvard University Press. Cambridge, MA.

Wright, C. 2000. 'Truth as Sort of Epistemic: Putnam's Peregrinations'. *Journal of Philosophy* 97: 335-364.

Zagzebski, L. 1990. 'What if the Impossible Had Been Actual?' In M. Beatty (ed.), *Christian Theism and the Problems of Philosophy*. Notre Dame: University of Notre Dame Press, 1990.