

# Scalar Implicature in Discourse Representation Theory

Michael Freenor

June 1, 2009



To Geraldine



## Acknowledgements

There are countless people I'd like to thank for support during the development of this thesis. This work, as it currently stands, is the culmination of my academic career so far. Thus, everybody who has furthered my academic development to this point shares a hand in what I've done.

For help and guidance throughout my stay at Carnegie Mellon, I'd like to thank my adviser Mandy Simons and my second-reader David Danks. Both were always available and helpful to me, and molded me from an overambitious undergraduate into a more capable, level-headed researcher. Without them I wouldn't be doing the work I love, and so I'm thankful for the time and energy they put into educating me.

I'd also like to thank all of my friends at Carnegie Mellon who, without their friendship and support, this would have been impossible. I would like to acknowledge, in particular, Edward Dean, Ruth Poproski, David Gilbert, Savitar Sundaresan, Dan Malkiel, Conor Mayo-Wilson, Mark Whitehouse, Spencer Breiner, Einam Livnat, Karin Howe, and Patricia Rich for helpful conversations involving my work. These conversations were incredibly important in formulating my ideas. To all of my friends in general: thanks for being there with whiskey and poker chips.

Amongst other people at Carnegie Mellon I am indebted to are Kevin Zollman, Teddy Seidenfeld, and Clark Glymour for their thoughtful observations and critiques. Jeremy Avigad and Steve Awodey should be acknowledged for being excellent logic teachers and even better logicians.

More than anybody, however, I'd like to thank my family for their emotional support. My trek back into academia has been a difficult one, but I am lucky to have a mother and father who truly support what I want to do. I'd like to dedicate this work to my late grandmother, Geraldine Regan, who unfortunately never got to see me graduate, but who is an essential foundation for the majority of my life.



# Contents

<b>1</b>	<b>Introduction</b>	<b>9</b>
<b>2</b>	<b>Scalar Implicature</b>	<b>11</b>
2.1	Quantity Implicature . . . . .	11
2.2	A Subclass of Quantity Implicature: Scalars . . . . .	13
2.2.1	Quantitative Scales . . . . .	13
2.2.2	Formally-Defined Alternatives . . . . .	15
2.2.3	Gazdar 1979 . . . . .	16
2.2.4	Contextual Factors and Residual Issues . . . . .	18
<b>3</b>	<b>Embedded Scalar Implicature</b>	<b>25</b>
3.1	Gazdar’s Unfinished Business . . . . .	25
3.2	Cases of Embedding and Scale Reversal . . . . .	26
3.3	Embedding is Problematic for Grice . . . . .	29
<b>4</b>	<b>Localism versus Globalism</b>	<b>31</b>
4.1	Tasks for Theories: The “What” and the “When” . . . . .	31
4.2	Localism/Defaultism . . . . .	31
4.3	Globalism/Griceanism . . . . .	35
4.4	The Excluded Middle . . . . .	37
<b>5</b>	<b>A Very Short Primer on DRT</b>	<b>39</b>
5.1	What is DRT? . . . . .	39
5.2	Syntax . . . . .	39
5.3	Semantics . . . . .	41
<b>6</b>	<b>The Account</b>	<b>43</b>
6.1	Goals . . . . .	43
6.2	Scalar Entailment ( $\models_S$ ) . . . . .	43
6.3	A Defaultist Theory . . . . .	46
6.3.1	Layered DRT . . . . .	46
6.3.2	The Machinery . . . . .	47
6.4	Cutting into “When”: Cancellation . . . . .	52
6.5	A Gricean Extension . . . . .	56
<b>7</b>	<b>Discussion of the Account</b>	<b>59</b>
7.1	Computational Advantages of DRT . . . . .	59
7.2	Psychological Approaches to “When” . . . . .	60
7.3	Interactions with Deductive Closures . . . . .	60
<b>8</b>	<b>Conclusion</b>	<b>61</b>
<b>A</b>	<b>Scale Reversal</b>	<b>63</b>
<b>B</b>	<b>Definitions and Notation</b>	<b>63</b>

<b>C</b>	<b>Autonomy, Determinacy, and Indeterminacy</b>	<b>65</b>
C.1	Autonomy . . . . .	65
C.2	Determinacy and Intuition . . . . .	66
C.3	Non-autonomous, Indeterminate Theories . . . . .	67
C.4	Virtues of Autonomy . . . . .	68

## 1 Introduction

In performing utterances, speakers convey many different types of meaning. One type in particular is *conversational implicature*, the norm-governed inferences identified by Paul Grice in the late 60's.

Implicatures are, under Grice's view, inferences that depend upon a presumption of cooperation. In other words, unless we have reason to believe otherwise, we presume other people to be cooperative in conversation. Based upon this expectation, we are licensed to draw certain (normally fallacious) inferences.

Consider the following conversation between Paul and Mary:

Paul: "Do you know the score of the Penguins game?"

Mary: "I think it's 5-3 Penguins."

Paul's question only admits of a "yes" or "no" answer, literally speaking. Mary offered up a response that does not directly address Paul's question. However, under the assumption that Mary is being cooperative, she was in fact offering a relevant response to his question. Given this assumption, Paul can begin to make sense of what she said.

Mary's response, however, does not address whether or not she knows the score directly. Rather, she uses the word "think", which lessens her commitment to being correct. Assuming she's being cooperative, Paul figures that she doesn't *know* the score. If she did, she would express her certainty to him in the spirit of being cooperative. As Mary knowing something entails that she also thinks it, simply saying she thinks something when she also *knows* it is withholding information, which is non-cooperative.

This is the gist of conversational implicature. If we assume that speakers are being cooperative (and if they know we are making that assumption), communication is streamlined significantly. That is to say, speakers can exploit the expectations we have for cooperative behavior to convey more with their utterances.

This thesis has two goals. The first is to provide an "ideal description" of a sub-class of conversational implicatures, the "scalar implicatures". The endeavor is neither purely descriptive nor normative. It's not intended to be a theory of what does happen or what should happen, but a theory of what *normally* happens (or what happens in "ideal circumstances").

The second goal is a refinement of the first one. The theory (which is to serve as an ideal description of the phenomenon) offered in this thesis is a *computational* one. Once its parameters are fixed, it serves to generate predictions automatically, without the intervention of a human interpreter. The sets and procedures it employs are fully reducible, employing only constructive and bounded methods.

Chapters 2 and 3 serve to introduce the phenomenon of scalar implicatures and a more complicated subset, the embedded scalar implicatures. In chapter 2, the original formal account of scalar implicature (Gazdar 1979) is presented. Though incomplete, it is fully computational and serves as inspiration to my

theory in this respect. Chapter 3 introduces embedded scalar implicatures, the cases that Gazdar's theory doesn't handle.

Chapter 4 presents some of the contemporary debate on the subject from a variety of different camps. The theories discussed here are brought up for their historical significance but are ultimately abandoned for a new approach using discourse representation theory.

Chapter 5 gives a short introduction to discourse representation theory, which is used in chapter 6 to develop my computational account. Chapter 7 contains a brief discussion of the advantages and shortcomings of my approach, roping off some areas for further research. In chapter 8 I conclude, locating my theory with respect to the others presented in this thesis.

## 2 Scalar Implicature

Consider the following example of implicature:

Alice: How much of the candy did you eat?

Jane: I ate some of it.

Alice is interested in learning how much of the candy Jane consumed. If Jane is being a cooperative speaker, she'll have to do better than merely telling the truth; she should say just how much she ate.

Whenever Jane eats all of the candy, saying that she ate *some* of it is also true. This is because on the way to eating it all, she ate some of it as well (nobody can eat the whole without eating the parts). However, it is deceptive; if she knows that she ate all of it, then she should say so. Being cooperative means producing as accurate an utterance as she can. Given that she chooses to only say "some", *Jane implicates that she did not eat all of it*.

Such an implicature is called a *scalar implicature*; scalar implicatures are a sub-class of a type of implicature called *quantity implicatures*. This chapter will present quantity implicatures and then scalar implicatures as a special case thereof. Alongside a presentation of the phenomenon, I will present some traditional Gricean explanation. The section will conclude with the first well-known computational theory for their calculation (Gazdar 1979).

Due to some well-behaved properties of scalar implicatures, they are more amenable to formalization than their non-scalar cousins. Implicatures such as this are called "generalized" by Grice, as they do not rely on particular aspects of the context.

### 2.1 Quantity Implicature

Cooperation imposes constraints not only on what we say, but *how much* we say. Cooperative speakers are both honest and informative. When a speaker is cooperating with her listener, she will contribute as much reliable information as she has provided that it is appropriate and relevant to the purposes of the conversation. Such a speaker is said to be following the cooperative maxim of quantity, which we will define momentarily.

Suppose there are two zookeepers, Bob and Larry. Larry is a new hire and Bob is training him. Larry is about to enter a cage to clean it and before entering he asks: "What's in this cage?" Bob replies, "Birds." Larry enters the cage to clean it and is immediately devoured by a ferocious lion.

In fact, there *were* birds in the cage. Unfortunately for Larry, there were *also* man-eating lions. However, Bob did not lie; he answered Larry's question truthfully insofar as there were birds in the cage. Bob knew, however, that there were dangerous lions in the cage. Given that he knew this, he *should have said so*, and if he were cooperative then he *would have said so*.

Larry, having no reason to doubt that Bob was being cooperative (Bob is training him), interprets Bob's utterance as follows: Bob said there are birds

in the cage. He could have said any number of things, but he chose only to say that there were birds. Given that Bob is cooperative and knows the zoo, if he knows that this cage has other things in it (especially dangerous things), he should say so. Thus, Bob is saying that there are birds in that cage, and furthermore, *nothing else*. Bob's failure to communicate the presence of another type of animal, especially something like a lion, is a clear failure to cooperate.

In general, we expect cooperative speakers to be maximally informative given what they know and what is appropriate to the conversation. This expectation is captured by Grice's "maxim of quantity":

Make your contribution as informative as is required (for the current purposes of the exchange).<sup>1</sup> [10]

If the hearer assumes that the speaker is, in fact, cooperative, then this means that she expects the speaker to be respecting the maxim. If this is the case, then the hearer expects the speaker to be maximally informative given what she knows; this means that the speaker presumably doesn't know any more. This produces what is known as an *exhaustivity* effect. If we assume a speaker is being maximally informative, then we assume they don't know any more relevant information than what they have provided.

In the case of Bob, he was being particularly malicious. He knew there were lions in the cage but didn't say so. Larry, knowing that Bob knows about the zoo, drew the implication that Bob knew there wasn't anything dangerous in the cage. Larry's opinion of Bob's epistemic state was critical for such a strong inference to go through. If Bob were also new to the job, perhaps he wouldn't know if there were lions in the cage or not. Saying "there are birds in the cage" in this circumstance would simply implicate that he doesn't know whether there's anything else in there, *not* that he knows there isn't.

The context of utterance and the purposes of the conversation help to fix the background of possible utterances against which "informativeness" is judged. Bob saying "birds" implicates, at the very least, that he doesn't know there's anything other than birds in there. This entails the negation of practically every alternative proposition: "there are no lions", "there are no bears", et cetera. This is because given the purposes of the exchange and the context of utterance, Larry requires information about the presence of dangerous animals. If there were common bugs in the cage, however, Larry would not consider Bob to have been less than maximally informative. This is because exhaustivity effects range over the present purposes of the exchange, and not over all possibilities; Larry is unlikely to care whether or not there are flies and a few gnats in the cage if he's just stepping in to clean it.

---

<sup>1</sup>There is a second part to this maxim in Grice's formulation, saying "Do not make your contribution more informative than is required". Grice's expresses doubts that this is an appropriate component of the maxim (over-informative speakers may be wasting time by being over-informative, but still being cooperative). This part of the original maxim is not relevant to our purposes here, so we will exclude it.

## 2.2 A Subclass of Quantity Implicature: Scalars

Quantity implicatures that exhibit exhaustivity effects entail a large, potentially infinite number of propositions. In the cases where they do not entail an infinity of propositions, this is because the context suggests a finite (yet typically large) set of relevant alternatives. There are some cases of quantity implicature, however, that do not rely so heavily on particular aspects of context. These implicatures typically entail a small set of propositions that are predictable across contexts. Consider the following example. Some, but not all, of Larry’s birds have escaped from his cage. Bob is aware that Larry has been spending the day trying to retrieve them and makes friendly conversation with Larry:

Bob: “Are they back yet?”  
Larry: “Some of them are.”

Larry could have been more informative. He could have said “All of them are.” Bob, however, assumes that Larry is being cooperative (and he is). Under this assumption, Larry is being maximally informative given what he knows. Thus, Larry implicates that some *but not all* of his birds have been gathered back into the cage.

Furthermore, aspects of the context do not alter the content of this implicature<sup>2</sup>. The same inference would be drawn in the petting zoo, a pet store, or even if Larry were a casual bird collector. In each case, Larry’s choice of “some”, under the assumption of his cooperation, licenses an inference to “not all”.

Naturally, there’s nothing special about birds licensing implicatures from “some” to “not all”. The terms “some” and “all” appear related in the sense that, regardless of the context, they seem to be appropriate substitutions for each other in sentences. Wherever a speaker says “some” she *could have said* “all”. In the examples involving Larry and Bob, on the other hand, “there are lions” is only a relevant alternative to “there are birds” in certain contexts.

Generalized quantifiers such as “some” and “all” are not the only terms that exhibit the property of being cross-contextual relevant alternatives. Other terms such as “okay” and “great” exhibit the same behavior. If Larry says that his wife’s cooking is “okay”, it’s safe to say that he’s saying it’s *not great*. In any circumstance where a speaker can describe an item as “okay”, she could have said it was “great” instead. Due to some relation between these terms, they always present themselves as possible substitutions for the other.

### 2.2.1 Quantitative Scales

Horn 1972 observes that certain terms, appropriately grouped by the concepts they express, can be organized into *quantitative scales* ordered by entailment<sup>3</sup>.

<sup>2</sup>Insofar as the relevant alternatives remain the same. We will see in 2.2.4 that some aspects of context *do* change the implicature generated, albeit only slightly and in a regularized way.

<sup>3</sup>Saying that quantitative scales are ordered by entailment is an abuse of terminology, seeing as how terms do not stand in entailment relations. Scales are ordered by a “counter-

Terms that stand in these quantitative scales are precisely the terms triggering scalar implicatures, mentioned above. This aside will present the notion of quantitative scales. Inclusion in a scale will be what determines the set of cross-contextual, appropriate substitutions discussed above.

Horn 1989 offers the following informal definition:

**DEFINITION 1.**  $P_j$  outranks  $P_i$  on a given scale iff a statement containing an instance of the former unilaterally entails the corresponding statement containing the latter. [12]

We will rework Horn’s definition more formally:

**DEFINITION 2.** An ordered  $n$ -tuple  $H$  is a *quantitative scale* if and only if for any two terms  $t_i, t_j \in H$  such that  $j < i$ :

For any sentence  $\varphi$  containing  $t_i$  such that  $t_i$  is not under the scope of an operator<sup>4</sup>,  $\varphi[t_j/t_i] \models^A \varphi$ .

For the purposes of illustration, Horn 1989 provides a few examples of scales, some of which I will reproduce here for the same purpose.

⟨all, most, many, some⟩  
 ⟨and, or⟩  
 ⟨must, should, may⟩  
 ⟨adore, love, like⟩  
 ⟨excellent, good, OK⟩

A brief example first: let  $\varphi =$  “I ate some of the candy”, and let  $\varphi$ ’s language include a scale  $H = \langle \text{all, some} \rangle$ .  $\varphi$  contains the term “some”  $\in H$ , and its occurrence in  $\varphi$  does not occur under the scope of a logical operator. Thus, substituting any member in  $H$  stronger than “some” for its occurrence in  $\varphi$  should produce a sentence that asymmetrically entails  $\varphi$ :

$\varphi[\text{“all”}/\text{“some”}] =$  “I ate all of the candy.”  
 “I ate all of the candy”  $\models$  “I ate some of the candy”  
 “I ate some of the candy”  $\not\models$  “I ate some of the candy.”  
 Thus, “I ate all of the candy”  $\models^A$  “I ate some of the candy.”

These scales group together relevant possible substitutions of the type needed for scalar implicatures<sup>5</sup>. They provide a background against which alternative expressions can be considered.

---

“factual” notion of entailment, or the entailment relations between two sentences if you were to successfully substitute one term for the other.

<sup>4</sup>This notion is important and will be brought up again in a moment.

<sup>5</sup>Later we will see that contextual concerns change the granularity of these scales. Furthermore, as Hirschberg 1985 (and others) point out, some “scales” are contextually given. We will put these concerns aside for now, as they needlessly complicate what is intended to be an initial presentation.

### 2.2.2 Formally-Defined Alternatives

In the zookeeper example, we were able to contextually define a set of alternative statements loosely and informally. The set of alternatives includes anything Larry could reasonably care about. Suppose the zoo includes birds, lions, tigers, and bears. If Bob utters “there are birds in the cage” as a response to Larry’s question, the following are contextually appropriate, possible replies:

{ “There are birds in the cage”, “There are lions in the cage”, “There are tigers in the cage”, “There are bears in the cage”, ..., “There are baboons in the cage” }

Any combination thereof is a possible reply as well, as there could be a combination of animals in the cage (though it is unlikely). This set is not technically infinite (as there are only a finite number of possible things that could be in there), but it’s quite large.

Quantity implicatures can be characterized as a negation of alternatives whose content is not included in what was uttered<sup>6</sup>. Thus, Bob’s utterance “there are birds in the cage” implicates “there aren’t tigers” in the cage, along with the negation of the other alternatives. Though “there are birds in the cage” is an alternative of itself, it is excluded since its content is clearly included in itself.

Using quantitative scales, however, we can formally define a set of alternative expressions that does not depend upon the context as the cases above do. Suppose we have a sentence  $\varphi$ . We can define its set of alternatives with respect to scalar terms, or *scalar alternatives*.

**DEFINITION 3.** (INFORMAL) Let  $\varphi$  and  $\psi$  be sentences. We say  $\psi$  is a *scalar alternative* of  $\varphi$  if and only if the following two conditions hold:

- (1)  $\psi$  and  $\varphi$  only differ in positions where  $\varphi$  contains an occurrence of a scalar term.
- (2) For those terms where  $\psi$  and  $\varphi$  differ,  $\psi$ ’s version of the term is a member of the same quantitative scale as  $\varphi$ ’s version.

**DEFINITION 4.** (FORMAL) Suppose  $\varphi$  contains  $n$  occurrences of scalar terms,  $t_1, \dots, t_k \subset H_1 \cup \dots \cup H_n$  where  $H_1, \dots, H_n$  are quantitative scales. Expression  $\psi$  is a *scalar alternative* of  $\varphi$  if and only if  $\psi = \varphi[t'_1/t_1, \dots, t'_k/t_k]$  where, for  $1 \leq i \leq n$ , there is a  $j_i \leq n$  such that  $t'_i, t_i \in H_{j_i}$ <sup>7</sup>. Write this  $ScalAlt(\varphi, \psi)$ .

**DEFINITION 5.** Let  $S$  be a set of sentences. We say that  $S$  is the *set of scalar alternatives* of an expression  $\varphi$ , written  $ScalAlt(\varphi)$ , if and only if  $S = \{\psi \mid ScalAlt(\varphi, \psi)\}$ .

<sup>6</sup>These implicatures are all entailed by the exhaustivity implicature explained above.

<sup>7</sup> $j$  is a function from indices of  $k$  to an index of  $n$ .

This notion of scalar alternative starts in Gazdar 1979 where he defines expression alternatives with respect to Horn’s quantitative scales. Similar definitions have been proposed over the years and have been refined to catch cases with multiple scalar terms, such as the formulation above. The definitions provided above are functionally equivalent to Sauerland’s 2004 presentation.

We’re now able to work an example. Suppose Bob utters “Some of your birds are in the cage” (call this  $\varphi$ ). Let  $H = \langle \text{all, most, some} \rangle$ . Thus,  $ScalAlt(\varphi)$  works out to the following:

{“Some of your birds are in the cage”, “Most of your birds are in the cage”, “All of your birds are in the cage”}

Clearly, this set of alternatives does not depend on the context in the same way as the man-eating lions example. Due to the fact that the alternatives are given due to the choice of “some” in the original utterance, we can develop a cross-contextual strategy to calculating these inferences<sup>8</sup>. Gazdar 1979 is the first well-known attempt at this, and we will step through his account now.

### 2.2.3 Gazdar 1979

Recall our informal strategy for the calculation of quantity implicatures. First, we drew the implication that the speaker doesn’t know any more than what he said. Thus, we negate any alternative not contained within the content of the utterance. For “there are birds” in the zookeeper example, we implicate “there are not lions” but not “there are not birds.”

For scalar implicatures, we will take a similar approach. Statements containing scalar expressions have an interesting property, however, that allows us to generalize in a clean way.

Consider the statement “I ate most of the candy” (call it  $\varphi$ ). Let our scale,  $H$ , be  $\langle \text{all, most, some} \rangle$ . Then:

$ScalAlt(\varphi) = \{ \text{“I ate some of the candy”, “I ate most of the candy”, “I ate all of the candy”} \}$

Negating “I ate all of the candy” gets us our implicature. Clearly, negating “I ate most of the candy” would create a contradiction. Negating “I ate some of the candy” also creates a contradiction, however; in order to eat most of the candy, I must have eaten some along the way.

Thus, negating any scalar alternative that uses a “weaker” scalar term than that used in the utterance creates a contradiction. We only wish to pick out the set of stronger alternatives, where “stronger than” is interpreted as “asymmetrically entails”. This is precisely Gazdar’s strategy.

Gazdar, in his 1979 work, presents a function for the calculation of what he calls *im-plicatures*, which are just “potential implicatures”<sup>9</sup>. While Gazdar’s

<sup>8</sup>In those contexts where the inference is licensed, that is.

<sup>9</sup>We’ll go with “potential implicatures” as the first either is phonetically ambiguous or imposes a stutter.

work deals with “clausal implicatures” and even presupposition, we will only focus on the aspect of his account dealing with scalars. This function employs the general strategy sketched above; first, scalar alternatives are calculated. Then, the “stronger” ones (those that asymmetrically entail the proposition expressed by the utterance) are negated.

Gazdar only deals with cases where scalar terms do not fall under the scope of a logical operator, as he takes operators to suspend implicatures. This includes operators such as “negation, quantifiers, connectives, and modal operators” [5] but is not taken to be exhausted by this list. Thus, his notion of scalar alternative is relativized to a particular term in a sentence, as this restriction limits his domain to cases where only a single scalar term is possible.

**DEFINITION 6.** Let  $\varphi$  be any sentence containing a single scalar term,  $\alpha$ , such that  $\alpha$  is not under the scope of a logical operator.  $\psi$  is a scalar alternative to  $\varphi$  with relation to term  $\alpha$  (written  $ScalAlt_{Gaz}(\varphi, \psi)$ ) if and only if  $\psi = \varphi[\beta/\alpha]$ , where  $\alpha, \beta$  are members of the same quantitative scale.

Using this notion, we can define the set of Gazdar’s scalar alternatives<sup>10</sup>.

**DEFINITION 7.** Let  $\varphi$  be any sentence containing a single scalar term,  $\alpha$ , such that  $\alpha$  is not under the scope of a logical operator. The set of *Gazdarian scalar alternatives* of  $\varphi$ , or  $ScalAlt_{Gaz}(\varphi)$ , is the set

$$\{\psi \mid ScalAlt_{Gaz}(\varphi, \psi)\}$$

Now that we have a precise definition of scalar alternatives in Gazdar’s restricted case, we will define the notion of “stronger scalar alternative” and then use this to define Gazdar’s set of potential scalar implicatures.

**DEFINITION 8.** Let  $\varphi$  be any sentence containing a single scalar term,  $\alpha$ , such that  $\alpha$  is not under the scope of a logical operator. The set of *stronger Gazdarian alternatives* of  $\varphi$ , or  $Strong_{Gaz}(\varphi)$ , is the set

$$\{\psi \mid \psi \in ScalAlt_{Gaz}(\varphi) \wedge \psi \models^A \varphi\}$$

This cashes out “stronger than” as “asymmetrically entails”.

**DEFINITION 9.** Let  $\varphi$  be any sentence containing a single scalar term,  $\alpha$ , such that  $\alpha$  is not under the scope of a logical operator. The set of *Gazdarian scalar implicatures* of  $\varphi$ , or  $ScalImp_{Gaz}(\varphi)$ , is the set

$$\{K\neg\psi \mid \psi \in Strong_{Gaz}(\varphi)\}$$

where  $K$  is the knowledge operator relativized to the speaker.

Gazdar takes scalar implicatures to be assertive as opposed to non-committal. This means that saying “John drank some of the beer” implicates “I know John didn’t drink all of the beer” rather than something weaker such as “It’s possible that John didn’t drink all of the beer, but I don’t know either way.” We

<sup>10</sup>My presentation differs from Gazdar’s notationally, but is functionally equivalent.

will discuss this in the upcoming section, which discusses the role of context in scalar implicatures. It's important to note now, however, that Gazdar takes "I ate some of the candy" to implicate "I know I didn't eat most of the candy" (given our scale above). His claim that scalar implicatures are always of the "know-that-not" variety is not quite accurate.

Either way, the general strategy of his account has been considered more or less accurate. In general, scalar implicatures are the negation of the stronger elements of an expression's scalar alternatives. This principle can be derived from the maxim of quantity, which mandates that the speaker should communicate as much relevant knowledge as she has. Gazdar, for the case of scalar terms that do not appear under the scope of a logical operator, has provided a good start. The real trick, as we will see in the next chapter, is dealing with the cases Gazdar sets aside.

#### 2.2.4 Contextual Factors and Residual Issues

When separating out scalar implicatures from their quantitative brothers and sisters, I said that scalar implicatures are generated based on alternatives that are *not* contextually given. This is only half true. This section will discuss how context influences scalar implicatures. The line between scalar implicatures and regular quantity implicatures will still stand, but it will be slightly blurred<sup>11</sup>.

As scalar implicatures are still conversational implicatures, context can affect them in a variety of ways. First of all, they can still be contextually or linguistically cancelled. Consider the following case:

Larry: "Most, in fact all, of the birds have returned to the cage."

Due to reconsideration mid-utterance, Larry adds in the clause "in fact all", which cancels the "but not all" reading that usually comes about due to the use of "most". Nothing surprising as of yet.

Though the relevant alternatives are determined by membership in a quantitative scale (which are cross-contextual), researchers such as Chierchia have observed that scales can be contextually truncated. As he says:

Clearly, not all the alternatives in a scale are always relevant. For example, in talking of age, we do not typically consider months (unless it's babies); and sometimes we talk in terms of decades. [2]

The "granularity" of the scale, in other words, can change with the context. Sometimes we're dealing with the scale ⟨all, most, some⟩ and sometimes we're dealing with its truncated version, ⟨all, some⟩.

Bob: "Most of your birds came back."

Larry: "*Some* of them did."

---

<sup>11</sup>An analysis of scalar implicature qua scalar implicature will still stand. The examples and points discussed in this section will not discount a treatment of scalars separate from quantity implicatures in general, but are discussed just to present the controversy and keep my presentation honest.

vs.

Bob: “Did all of the birds come back?”

Larry: “Some of them did.”

In the first example, Larry is implicating that some, but not most, of the birds have returned. In the second example, he’s saying that some, but not all, have returned. The granularity of the scale can depend upon what has been said, but it can also depend upon other contextual factors such as social convention, as mentioned by Chierchia in the above selection.

Thus the scale in play is not solely determined by the scalar term used, but can be *adjusted* by the contextual factors. Still there is a component which is robust cross-context (namely, which scale is being adjusted), and it should be stressed that this is still different from quantity implicature at large, whose relevant alternatives vary drastically between contexts. The theory presented in this thesis will not attempt to adjust scales’ granularity; it will assume that this process has already been completed before implicature calculation begins<sup>12</sup>.

There also exists a slew of examples discussed by Hirschberg 1985 involving scales that are contextually given.

A: Did you manage to read that section I gave you?

B: I read the first couple of pages. [11]

What Hirschberg observes is a scalar implicature-like effect in the sense that the first couple of pages of a section and the whole of the section comprise a part/whole relationship. These implicatures are likely licensed by the recognition of this part/whole relationship, where the particular part/whole ordering is contextually determined.

Hirschberg presents a slew of examples challenging the conventional view that scalar implicature is an entailment-based phenomenon. This involves a variety of “scales” that are not ordered by entailment, but which are partially-ordered sets under various orderings. These examples are meant to show that scalar implicatures based on Horn scales only cover a small range of a much broader phenomenon, and to show that a theory of this broader phenomenon would preclude the need to postulate Horn scales at all. In other words, Hirschberg thinks that the correct theory of scalar implicature is one based upon the notion of a *partially ordered set*, not one based upon entailment-based quantitative scales.

Here is another example presented in Hirschberg’s dissertation:

A: So, is she married?

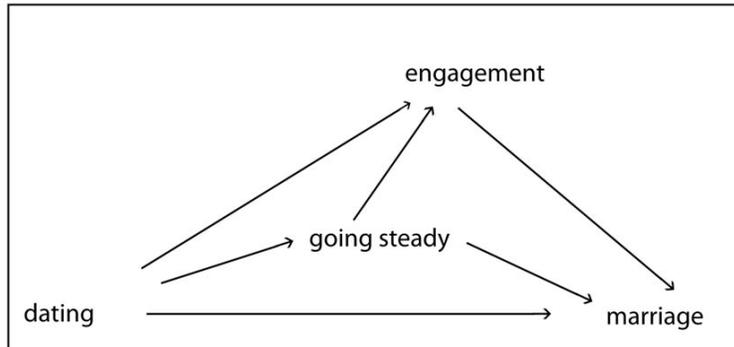
B: She’s engaged.

B’s utterance is taken to implicate that the salient female in question is engaged, *but not married*. However, B’s utterance doesn’t commit B to the truth of the female in question *dating* or *going steady* with anyone, as people

---

<sup>12</sup>This is ultimately unsatisfactory, but such a task is a project of its own.

can be engaged without having dated or having gone steady first (though they are “less developed” stages of romantic relationships). People can get married without being engaged first as well, but asserting that one is engaged as B has suggests that the “higher up” values in the prerequisite ordering are to be denied.



Such is an example of a “non-entailment based scalar implicature”, as being married says nothing about whether or not you’ve dated (but asserting “they’re dating” may implicate “they’re not married”).

While initially compelling, Hirschberg’s examples are peculiar under closer scrutiny. First of all, her examples all follow the same format; they are brief exchanges between two speakers in which the first speaker sets up the context in a particular way by asking a question. Consider the following examples as well:

A: Can you sing a Motels song right now?  
 B: Now?  
 A: Yeah.  
 B: My cousin can.

A: Have you ever knitted before?  
 B: I’ve done a lot of crocheting.

A: Do you need this?  
 B: I want it.

There are two reasons why these cases seem bizarre as examples of “scalar implicature”, other than the fact that they are not based upon Horn scales. First of all, in traditional examples of scalar implicature, the context does not play as large of a role as it does in Hirschberg-style examples. To see this, consider any of B’s implicature-generating utterances in isolation from each discourse. Without A’s question to “set up” the context properly, B’s utterances do not carry Hirschberg’s implicatures.

“My cousin can sing a Motels song.”  $\not\leftrightarrow$  “I can’t sing a Motels song.”  
“I’ve done a lot of crocheting.”  $\not\leftrightarrow$  “I’ve never knitted.”  
“I want this.”  $\not\leftrightarrow$  “I don’t need this.”

The reason why these sentences *do* generate implicatures in Hirschberg’s examples is what leads us to the second peculiarity: each of B’s replies in the conversations above contain a suppressed “no, but ...” at the beginning.

A: Can you sing a Motels song right now?  
B: Now?  
A: Yeah.  
B: *No, but* my cousin can.

A: Have you ever knitted before?  
B: *No, but* I’ve done a lot of crocheting.

A: Do you need this?  
B: *No, but* I want it.

This is not peculiar to just Hirschberg’s examples. In fact, many traditional cases of scalar implicature can be read as if they had a suppressed “no, but ...” as well, if found in the same conversational circumstances as those above.

A: Did you eat all of the chips?  
B: I ate some of them.

A: Did you eat all of the chips?  
B: *No, but* I ate some of them.

The difference, however, is that traditional cases of scalar implicature do not require this hidden “no, but ...”. Hirschberg’s examples require a lot of interaction with the context to even get off of the ground, which feels much different than traditional cases of scalar implicature.

“I ate some of them.”  $\rightsquigarrow$  “I ate some *but not all* of them.”

A particularly important quality of traditional scalar implicatures is that they are *generalized* in Grice’s terminology, or context-independent. Hirschberg’s cases are all *particularized*, that is to say they are only derivable in certain contexts.

It may be the case that a theory flexible enough to explain Hirschberg’s data could preclude the need for Horn scale based theories of scalar implicature. Hirschberg’s cases are straight-forward despite being hard to formalize; a question is denied by the assertion of other items in a set of relevant alternatives. What we see is an exhaustivity effect in scale-like structures that crop up in highly context-dependent ways. Surely entailment-based scales could be explained under a similar mechanism.

However, the full set of information and understanding required for a Hirschberg-styled theory to be implemented is massive and not well-delineated. The advantage Horn-based theories have over Hirschberg is that the type of information required is straight-forward, well-understood, and plausibly possessed by competent speakers of a language (knowledge of “entailment relations” between certain terms). This advantage is pragmatic rather than theoretical, but it is an advantage none-the-less.

It’s hard to say whether or not Hirschberg’s examples are genuine scalar implicatures, or if they are simply examples of quantity implicatures at large. Either way, she seems to have described a more general phenomenon than traditional scalar implicatures; just as a general theory of quantity implicature would likewise predict and explain scalar implicatures as a matter of course, a theory of Hirschberg implicatures should explain them as well. As current theory and understanding stands, however, Horn-based approaches seem much more tidy and practical to pursue; solving Hirschberg’s cases in a general way would require an amazingly broad understanding of both general and linguistic reasoning. Thus, my account can be considered a theory of a sub-problem. Even if Horn-based theories are simply scaffolding to be removed by Hirschberg theories in the future, they are still an essential step towards understanding of the phenomenon and worth pursuing.

A final note about the role of context: common knowledge about the speaker’s epistemic state determines the nature of the implicature drawn. If it’s common knowledge that the speaker is an expert about the subject at hand, then it seems that Gazdarian “know-that-not” implicatures are drawn. Consider the case of a teacher, discussing the performance of her students on a recent exam.

Jane: “Some of the students failed.”

It’s a reasonable assumption that Jane knows whether or not all of her students failed. Given this, the implicature licensed is something along the lines of “I know that not all of my students failed.” There are cases, however, where the implicature licensed is an “ignorance inference” instead<sup>13</sup>. Imagine Jane’s TA’s, Mary and Jim, only saw the top of the stack of exams (which happens to be a fail).

Mary: “Some of the students failed.”

In such a case, Mary doesn’t *know* that all of the students didn’t fail. Jim knows this as well, and so what Mary said implicates “I don’t know that all of the students failed.” Whether or not the negation rests on the inside or the outside of the knowledge operator seems to depend on beliefs about the speaker. Sometimes a speaker is willing to commit themselves to a Gazdarian implicature; other times, they are non-committal and wish to simply express their ignorance. Both implicatures conform to the maxim of quantity, but working them out requires a bit of reasoning and some beliefs about the speaker.

---

<sup>13</sup>Horn 1989 observes this.

Hirschberg 1985 provides an in-depth discussion of these issues. The main question being addressed is: what, if anything, is the epistemic status of scalar implicatures? Gazdar’s answer is that speakers are committed to knowing that a stronger value (call it  $p_j$ ) is false. As we’ve seen, this is incorrect in a variety of contexts. Soames attempts to amend Gazdar, claiming that implicatures take the form of a disjunction: either the speaker knows that not  $p_j$ , or for all the speaker knows,  $p_j$ . This is equivalent to saying “either the speaker knows  $p_j$  is not the case, or he doesn’t know either way.”

Hirschberg herself takes a different view: implicatures are characterized as “ $S$  believes higher  $p_j$  are false or  $S$  does not know whether higher  $p_j$  are true or false”. This change from Soames’ use of knowledge to simple belief is based on the observation that the hearer need not decide if the speaker *knows* not  $p_j$ , but merely whether or not she believes it. This is all the hearer can be expected to work out, in many cases. As the speaker must be confident the hearer is able to work out the implicature, it must be limited to mere belief<sup>14</sup>.

These views stand in contrast to the view that implicatures have no epistemic status at all (Chierchia 2004, for example). Such views see the implicature drawn as a simple negation of the higher values, calculated as if it were part of the conventional content of the sentence.

A full-fledged debate over the epistemic status of scalar implicatures could constitute its own book. For the purposes of this thesis, Hirschberg’s line will be adopted, as the disjunction she has provided seems to properly characterize both ignorance inferences and strong implicatures of Gazdar’s type. The account developed in full detail in section 5 calculates implicatures without any epistemic status (the negation of the next higher value). These epistemically neutral implicatures, however, can be transformed to fit any of the disjuncts that Hirschberg proposes ( $B\neg p_j$  or  $\neg K p_j \vee \neg K \neg p_j$ )<sup>15</sup>.

---

<sup>14</sup>Expecting the hearer to secure knowledge is too stringent.

<sup>15</sup>Performing the transformation properly would require a decision procedure for determining whether or not speakers intend to assert knowledge or ignorance.



### 3 Embedded Scalar Implicature

The cases that Gazdar didn't deal with have generated major controversy in the past thirty years. Examples purporting to show that apparent implicatures can be generated under the scope of logical operators were presented immediately after Grice's original account by Cohen 1971. While Gazdar prohibits these "embedded" implicatures, their robustness is too clear to simply ignore.

The issue of embedded implicatures and how they should be explained and interpreted has caused a split from Grice's original explanatory framework. In the next chapter we will examine the arguments between Gricean defenders and those deviating from Grice; in this chapter, however, we will establish the phenomenon at issue.

#### 3.1 Gazdar's Unfinished Business

Gazdar does not consider cases where a scalar term falls under the scope of a logical operator. Implicature in these cases is taken to be "suspended", meaning that the implicature normally present is not calculated. We will call the class of examples Gazdar considers "unembedded" cases; cases where scalar terms fall under the scope of a logical operator will be called "embedded".

Consider the following unembedded case:

Mary: "I ate some of the candy."

Normally (as Gazdar predicts), this implicates "I didn't eat all of the candy"<sup>16</sup>. If we take the sentence that Mary uttered and embed it under negation, for instance, things change:

Mary: "I didn't eat some of the candy."

"I didn't eat some of the candy"  $\equiv$  "It's not the case I ate some of the candy."

Tense issues aside, once we embed Mary's original assertion under negation, the original implicature should not be calculated. If Gazdar's algorithm admitted these examples into its domain, it would predict that Mary would implicate the following:

Mary: "I didn't eat some of the candy"  $\rightsquigarrow$  "It's not the case that I didn't eat all of the candy"

"It's not the case that I didn't eat all of the candy"  $\equiv$  "I ate all of the candy."

The above prediction is clearly incorrect, as the putatively implicated content contradicts the asserted content from which it was generated. Due to these observations, Gazdar makes an empirical claim that the implicatures are suspended and leaves it at that.

---

<sup>16</sup>Technically it implicates "I know I didn't eat all of the candy", but Mary asserting this has the same force as a knowledge claim.

### 3.2 Cases of Embedding and Scale Reversal

What of other logical and intensional operators? Above it appeared as if negation suspended standard scalar implicatures. Consider, however, the following case from Chierchia 2004:

John: “Some students are waiting for me.” [2]

According to the standard story, John implicates that not every student is waiting for him. If Mary reports this utterance, she might say:

Mary: “John believes that some students are waiting for him.”

as John’s utterance is good evidence he holds this belief. If Gazdar’s theory applied here, it would generate the following prediction:

“John believes that some students are waiting for him.”  $\rightsquigarrow$  “John doesn’t believe that every student is waiting for him.”

When John says “Some students are waiting for me”, he implicates that not every student is waiting for him. This means that he *believes* that not every student is waiting for him. If we label his utterance  $\varphi$  and the belief operator as  $B$ , then he has implicated  $B\neg\psi$  where  $\psi \in Strong_{Gaz}(\varphi)$ . Gazdar predicts  $\neg B\psi$ , which while consistent with the implicature, is not strong enough to capture it.

Intuitively, it makes sense that  $\varphi$  would generate the same implicature unembedded and under the belief operator. There are a slew of other intensional operators that behave the same way, including “think”, “say”, and “know”. Consider the following example:

Mary: “You told me all of the students would be there.”

Jane: “No, I said that some of the students would be there.”

In such a case, we see an embedded implicature. Namely, Jane implicates: “No, I said that some (but not all) of the students would be there.” Again, Gazdar and other traditional Griceans would predict no implicature at all.

It appears as if implicatures are suspended under the scope of certain operators, but not others. Horn 1989 observes the following trend:

Gazdar’s version is too restricted. For the implicatum to go through, the scalar expression in question need not be logically unembedded, so long as it is not embedded under certain operators. . . . Rather, we seem to need to restrict the set of logical functors in question to those which include negation and other scale reversing operators (cf. Fauconnier 1976), that is, to the set of downward-entailing operators in Ladusow 1979. [12]

Gazdar claims that implicatures are suspended under any logical operator; however, Horn suggests that only “scale reversing” downward-entailing operators have this effect. Downward-entailing operators are precisely the set of “monotone decreasing” operators defined by Barwise and Cooper 1981. When viewed as set-theoretic objects, monotone decreasing ( $\text{mon}\downarrow$ ) operators are closed under the subset operation<sup>17</sup>. Their converse, the monotone increasing ( $\text{mon}\uparrow$ ) operators, is closed under the superset operation.

Before proceeding, we will examine a brief example of the contrast between  $\text{mon}\downarrow$  and  $\text{mon}\uparrow$ . Consider negation, which is  $\text{mon}(\downarrow)$ , and the following cases of entailment:

“Mike plays El Grande<sup>18</sup>.”  $\models$  “Mike plays board games.”  
 “Mike doesn’t play board games.”  $\models$  “Mike doesn’t play El Grande.”

The set of people who play El Grande is a proper subset of the set of people who play board games in general. Thus, anyone with the property of playing El Grande always has the property of playing a board game (closed under superset). On the other hand, negation is  $\text{mon}\downarrow$  and works in the opposite direction, as it is closed under subset. Thus, not playing board games at all entails that one does not play any particular board game.

It’s mentioned by Horn that  $\text{mon}\downarrow$  operators “reverse” scales. We will now explain precisely what this reversal effect is, and what effect this has on the embedded implicatures that we should expect under  $\text{mon}\downarrow$  operators.

In the formalization of Horn’s quantitative scales in section 2, it was important to stipulate that the occurrences of scalar terms in our arbitrary sentences do not occur under the scope of a logical operator. Let our scale be  $\langle \text{all}, \text{some} \rangle$ . Let  $\psi =$  “I didn’t eat some of the candy.” Then:

$\psi[\text{“all”}/\text{“some”}] =$  “I didn’t eat all of the candy.”  
*However* “I didn’t eat all of the candy”  $\not\models$  “I didn’t eat some of the candy.”

Horn 1981 reports Fauconnier (1975)’s observations [12]:

As Fauconnier also observes, it is a characteristic property of negation and other polarity triggers that they “reverse” the ordering of elements on a scale. ...

- a. ...  $\rightarrow$ <sup>19</sup> Odette has three children  $\rightarrow$  Odette has two children  $\rightarrow$  Odette has one child
- b. Odette doesn’t have a child  $\rightarrow$  Odette doesn’t have two children  $\rightarrow$  Odette doesn’t have three children  $\rightarrow$  ...

<sup>17</sup>For a precise definition of  $\text{mon}\downarrow$  and  $\text{mon}\uparrow$ , refer to Appendix B.

<sup>18</sup>El Grande is a popular board game.

<sup>19</sup>Fauconnier uses “ $\rightarrow$ ” to denote entailment.

This is precisely why the proviso in our definition of quantitative scale is so important. As Fauconnier and Horn note, scales are reversed under polarity triggers. In the above example, our scale  $\langle \text{all}, \text{some} \rangle$  no longer predicts the relations, but rather  $\langle \text{some}, \text{all} \rangle$  does.

Let our scale be  $H' = \langle \text{some}, \text{all} \rangle$ , which is the reversed form of  $H$ . Now “all” is no longer the strongest member of our scale, but “some” is. Thus we proceed as follows.

$\psi[\text{“all”}/\text{“some”}] = \text{“I didn’t eat all of the candy.”}$   
 $\text{“I didn’t eat some of the candy”} \models^A \text{“I didn’t eat all of the candy”}.$   
 This is what we expect given the structure of our “reversed” quantitative scale.

The standard presentation given above is a bit confusing. It’s mentioned that polarity triggers reverse scales, or “reverse” entailment. Suppose we have two sentences  $\varphi$  and  $\psi$  such that  $\psi = \varphi[t'/t]$  and  $t, t'$  are both elements of some quantitative scale,  $H$ . If  $\psi \models^A \varphi$ , then  $\neg\varphi \models^A \neg\psi$ . This is what is meant by negation and other polarity items “reversing” the entailment relation<sup>20</sup>.

Since scales are reversed under  $\text{mon}\downarrow$  operators, it’s no surprise that the cases Gazdar examined under negation resulted in implicature suspension.

Mary: “I didn’t eat some of the candy.”

If the scale we’re working with is  $\langle \text{all}, \text{some} \rangle$ , then the negation operator present in the above sentence reverses this scale to  $\langle \text{some}, \text{all} \rangle$ . In this case, “some” becomes the maximally informative option. Thus, we don’t expect an implicature in this case anymore than we do in the case where Mary says “I ate all of the candy.”

What of cases where, after we have reversed our scale, the term used is not the maximally informative one? Chierchia 2004 examines a slew of examples, which we will enumerate. I’ll spoil the ending; under  $\text{mon}\downarrow$  operators, we see the same sorts of implicature effects once we’re working with the appropriate, reversed scale. Each A item below will exhibit a scalar term embedded under a  $\text{mon}\downarrow$  operator such that once the relevant scale is reversed, this term is not maximally informative. Each corresponding B element will be the intuitive implicature generated by the A item, as observed by Chierchia.

- (1A) John doesn’t eat and smoke.
- (1B) John either eats or smokes.

- (2A) I doubt that most students will show up.
- (2B) I think that some student will show up.

---

<sup>20</sup>Precisely speaking,  $\neg\varphi$  and  $\neg\psi$  are not the same sentences as  $\varphi$  and  $\psi$ , respectively. Therefore, to claim that the negation reversed the entailment relation present between  $\varphi$  and  $\psi$  is a bit strange and perhaps misleading. A more precise explication of this concept can be found in Appendix A.

- (3A) It won't happen that every student will complain.  
(3B) Some student will complain.

- (4A) Not every friend of mine has a car.  
(4B) Some friend of mine has a car.

In each case, once we have reversed the relevant scale in play, the scalar items are not maximal. Thus, as we would expect in  $\text{mon}\uparrow$  cases, the intuitive implicature is the negation of the next strongest alternative. The next strongest alternative is now on the reversed scale, which gives the “reversal” effect for scalar implicatures under  $\text{mon}\downarrow$  contexts.

### 3.3 Embedding is Problematic for Grice

The issue of embedding causes difficulties for computational accounts in general<sup>21</sup>, and also for Grice's framework. This section will discuss Grice's commitments and why embedded implicatures don't fit in.

In the cases of embedded implicatures above, we appealed to roughly Gricean intuitions when explaining each case. Whenever an item was not the maximally informative item on its scale (or reversed scale) in an embedded position, the speaker *could have been more informative*. Proceeding along the typical Gricean line, we explained the occurrence of the scalar implicature at the embedded level.

However, Grice's commitments, prior to revision<sup>22</sup>, seem to block this sort of explanation from taking place. For Grice, there are no embedded implicatures; this has to do with the general picture of interpretation that he builds into his theory of conversational implicature.

Grice's model of interpretation leaves no room for embedded implicatures, as he leaves no room for pragmatics at the sub-sentential level. “What is implicated” is calculated on the basis of “what is said”, or the conventional content of an utterance. Assertions, under this model, are the “contributions” that people make to a conversation. Thus, the sentence is taken to be the appropriate level of analysis. Pragmatics, under this view, accounts for aspects of meaning *after* the conventional meaning of the sentence is worked out.

Consider Grice's definition of conversational implicature:

A man who, by (in, when) saying (or making as if to say) that  $p$  has implicated that  $q$ , may be said to have conversationally implicated that  $q$ , provided that (1) he is presumed to be observing the conversational maxims, or at least the Cooperative Principle; (2) the supposition that he is aware that, or thinks that,  $q$  is required in order to make his saying or making as if to say  $p$  (or doing so in *those*

---

<sup>21</sup>These computational issues will be addressed in section 4.

<sup>22</sup>Levinson 2001 and Russell 2006 make good arguments that Gricean explanation can, in fact, pick up these cases. However, these arguments are needed to make this point clear. Sauerland 2004, on the other hand, provides a technical solution to the problem (which, under Russell's view, is not really a problem).

terms) consistent with this presumption; and (3) the speaker thinks (and would expect the hearer to think that the speaker thinks) that it is within the competence of the hearer to work out, or grasp intuitively, that the supposition mentioned in (2) is required. [10]

Of course, this definition turns on what he means for a speaker to “say that  $p$ ”. It is this favored sense of “saying” that constitutes a conversational contribution, and what the maxims and cooperative principle will latch onto.

In the sense in which I am using the word *say*, I intend what someone has said to be closely related to the conventional meaning of the words (the sentence) he has uttered.

What someone has said, in other words, is closely related to the encoded content of the sentence uttered. This may include entailments and perhaps even presuppositions, but it clearly does *not* include conversational implicatures for Grice.

As mentioned above, the cooperative principle and the maxims are formulated to handle sentences (semantically well-formed, truth-evaluable expressions), as “conversational contribution” is, in Grice’s sense, the conventional content of an assertion. The calculation of embedded implicatures cannot even be considered under this model, as Grice excludes them from the domain of his implicature-generating machinery.

## 4 Localism versus Globalism

Embedded implicatures are too robust to ignore. Grice’s account, without revision or clarification, cannot explain them. This problem motivates two different courses of action: either we can amend Grice’s framework, or we can depart for other modes of explanation altogether.

Some researchers such as Chierchia (2004) and Fox (2006) want to explain scalar implicatures as either a semantic or a syntactic phenomenon, assigning them to the domain of the grammar. Their camp is labeled the “localists”, as they view embedded implicatures as being genuinely calculated at a “local” site (such as, at intermediate phrases).

Opposing them are the “globalists”, or those who wish to defend Grice’s original commitments (Sauerland 2004, Russell 2006). Globalists endorse techniques for calculating apparent embedded implicatures that use the whole sentence as the level of analysis.

Griceans and non-Griceans are, generally speaking, separable by whether or not they subscribe to globalism or localism respectively. This section will briefly explore the conceptual aspects of this contemporary work. Furthermore, it will discuss a recent, growing movement that fits into neither category: those who defend Grice while departing from globalism.

### 4.1 Tasks for Theories: The “What” and the “When”

In general, there are two tasks for theories of scalar implicature: (1) determine what the possible implicatures are, (2) determine when these possible implicatures are actual. We can reduce this to the distinction between “what” and “when”.

Theories of scalar implicature are charged with the task of predicting what is implicated given a context of utterance, a speaker, and a past utterance. This requires both computational techniques to calculate the inferences (what), and also an account of when we should or should not generate them (when). Furthermore, it must specify when we are to throw out implicatures which have been invalidated (cancellation, a sub-set of “when”). Each one of the fully-formed theories in the localist and globalist sections below commit themselves to solving both problems.

### 4.2 Localism/Defaultism

Some localist views are grammatical in nature. In other words, the strengthening effect characteristic of scalar implicatures is viewed to be a grammatical phenomenon (part of semantics or syntax), explainable without full recourse to pragmatic principles<sup>23</sup>.

Grammatical views can be classified as either semantic or syntactic; examples are Chierchia 2004 and Fox 2006, respectively. One loads the “strengthening”

---

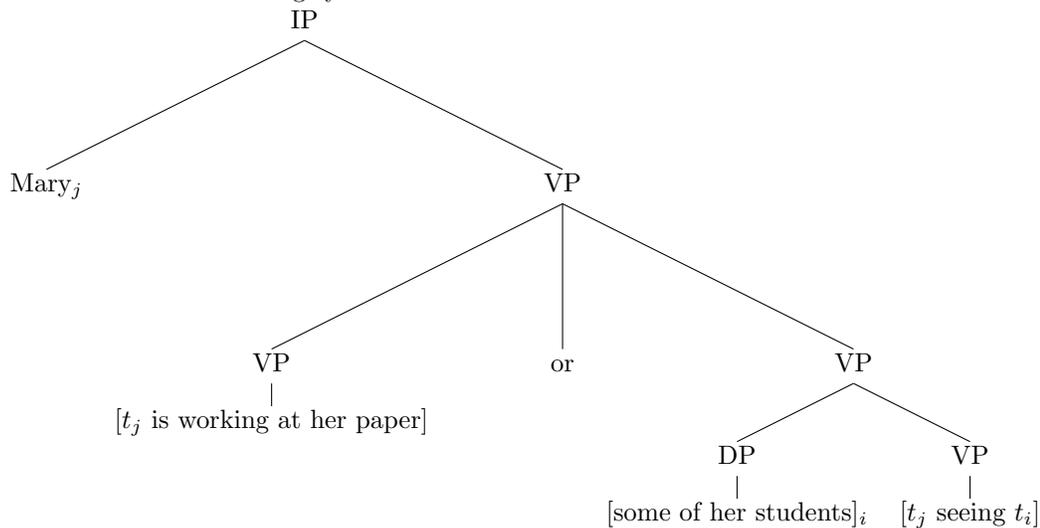
<sup>23</sup>Cohen is perhaps the first semantic-localist and presented in his 1971 work some of the first embedded counterexamples to Grice.

effect into the semantic composition, the other into a covert operator. The name of the game is to explain the strengthening effects of scalar implicatures without solely resorting to norm-driven reasoning.

Chierchia 2004 [2] views meaning as a “multi-dimensional phenomenon”. Both strengthened and plain meanings are calculated in parallel. Whenever plain values are preferred to strong ones (due to context, linguistic or otherwise), the interpreter throws out the strengthened value in favor of it.

Chierchia’s model of interpretation stands in stark contrast to Grice’s; instead of the semantic and pragmatic modules being “blind to the inner workings of the other”, they interleave and cooperate at intermediate stages<sup>24</sup>. More specifically, they interact at scope sites of type  $t$  (whenever a truth-evaluable intermediate phrase has been constructed). When we reach truth-evaluable phrases that contain a scalar item, in Chierchia’s model, we calculate an implicature and project it upwards. Scalar implicatures, though grammatically guided, consult the “pragmatic module”; however, the pragmatic module in Chierchia’s model serves a much different role than Grice’s.

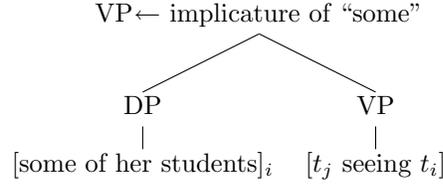
We will step through Chierchia’s example sequentially to give a better illustration of how his mechanism works. Consider the sentence  $\varphi$ : “Either Mary is working at her paper or she is seeing some of her students.” Chierchia represents this in the following syntactic tree:



Proceeding from the bottom up, we strengthen each constituent as we go according to the rules of Chierchia’s modified semantic recursion. Most terms end up receiving their “normal” semantic value when strengthened, so we’ll start with the first interesting case: where a truth-evaluable phrase contains a scalar

<sup>24</sup>Grice may have had a less black-and-white view than Chierchia attributes to him. If Grice were doing his work today, his view of the semantics/pragmatics interface might have been quite different. Even so, he might still view implicature as a pragmatic phenomenon driven by norm-governed reasoning; nothing about Chierchia’s view of semantics and pragmatics interleaving excludes this possibility.

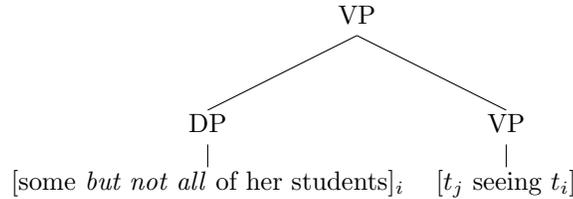
term. This occurs at the VP node above “some”.



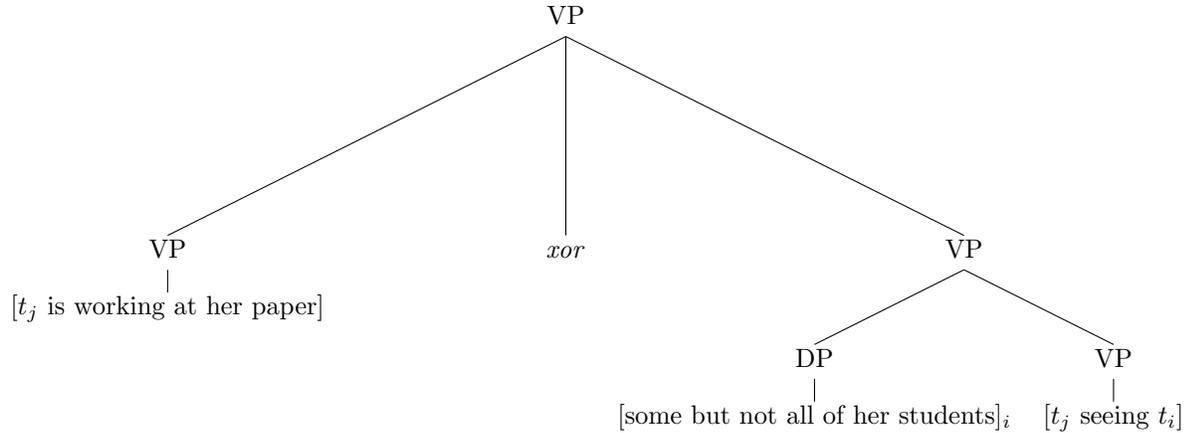
Let  $\phi$  denote this tree. We calculate  $\|\phi\|^S$ , or the “strengthened meaning” in Chierchia’s notation, with the following machinery:

**DEFINITION 10.** Let  $A$  be a set of expression alternatives and  $\beta$  an expression such that  $\beta \in A$ .  $S_\beta(A)$  (abbreviated  $S(A)$  when it’s unambiguous) denotes the weakest member of  $A$  such that it asymmetrically entails  $\beta$ .

**DEFINITION 11.** If  $\phi$  is a scope site (of type  $t$ ), then  $\|\phi\|^S = \|\phi\| \wedge \neg S(\phi^{ALT})$ , where  $\phi^{ALT}$  denotes the set of  $\phi$ ’s expression alternatives, and  $\|\phi\|$  denotes the “plain meaning” of  $\phi$ .



Thus, the “some but not all” implicature is projected upwards. Continuing the composition, another scalar implicature is calculated at the VP where “or” is composed.



“Or” is considered by Chierchia to be part of the scale  $\langle \text{and, or} \rangle$ , so at the top VP in the above tree, we see another application of the rules presented above. The result<sup>25</sup> is the strengthened proposition “Either Mary is working at

<sup>25</sup>“xor” denotes “exclusive or” above.

her paper or seeing some (but not all) of her students, but not both”.

This view of scalar implicatures is much different than Grice’s picture of norm-governed inferences. The pragmatic module, for Grice, is in the business of performing the necessary reasoning and backtracking to discover what the speaker must have meant in order to be following the cooperative principle and the maxims. Chierchia, while he doesn’t deny a Gricean view of pragmatics for other types of implicature, denies this view for scalar implicatures in favor of a grammatical one. He doesn’t exclude pragmatic effects from scalar implicatures altogether, but he provides them with a stripped down set of roles.

First, pragmatics is responsible for fixing the “granularity” of the quantitative scale at play. “For example, in talking of age, we do not typically consider months (unless it’s babies); and sometimes we talk in terms of decades”. In some cases, ⟨all, most, some⟩ is relevant, and in others only ⟨all, some⟩. It’s the role of the pragmatic module, in considering how a truth-evaluable phrase should be strengthened, to determine the granularity of the scale contextually.

Furthermore, it takes some pragmatic reasoning to figure out if an implicature should be contextually canceled. For instance, if John utters “I drank some of the beer” while clumsily holding up an empty case of beer, the “I didn’t drink all of the beer” implicature is contextually canceled. Such a determination cannot be predicated on purely linguistic data; observations of John’s behavior along with some reasonable assumptions are required.

Chierchia’s framework allows him to do what Gazdar could not. He can calculate the strengthened version of embedded phrases and “drop in” the intended implicature directly into the embedded position. Since he views scalar implicature as a grammatically-guided phenomenon, his account is nothing more than an amendment of the standard semantic recursion that calculates scalar implicature at each truth-evaluable phrase. This is Chierchia’s answer to “what”.

Fox’s answer to “what” is slightly different. Rather than viewing strengthening as semantic in nature, Fox views it as syntactic. In other words, he believes there is a “covert exhaustivity operator” present in our syntax that accounts for the strengthening effects. For each phrase, it’s possible for an “*exh*” operator to be present; if it is present, then a sentence  $S$  becomes its strengthened counterpart,  $S^+$  [4].

If such a theory is correct, we might think of *exh* as a syntactic device designed (“by a super-engineer”) to facilitate communication in a pragmatic universe governed by B-MQ [maxim of quantity].

In Fox’s model, every sentence has at least two readings: one where there is an *exh* present, and one where it is absent. As sentences get more complex, the possibilities expand quickly. The role of pragmatics, in such a case, is to select amongst these syntactic representations to arrive at the intended one. Just as in Chierchia’s model, pragmatics is given a “negative” role: pragmatic considerations cull potential scalar implicatures rather than generate them.

Due to Chierchia’s commitment to the grammatical nature of scalar implicatures, his answer to the “when” question is determined. His theories is

“defaultist”; it predicts scalar implicatures are “default inferences”, calculated by default. This stands in stark contrast to Grice’s framework, where a speaker must intend to implicate. Fox, on the other hand, does not provide an answer to “when”.

### 4.3 Globalism/Griceanism

In this section, Sauerland 2004 and Russell 2006 will be examined as examples of those who defend globalism. In defense of globalism, they show that from the perspective of the whole sentence one can calculate embedded implicatures.

Sauerland’s proposal for “what” can be split into a few steps. First, he deals with generating scalar alternatives for embedded cases by treating the possibilities as a cross product between the various scales at play. Second, he alters the traditional Horn scale for disjunction, making it into a partially ordered set (instead of the well-ordered set  $\langle \text{and, or} \rangle$ ). This is due to the fact that one of the specific cases he considers involves embedding in a disjunct.

Sauerland’s trick is simple yet elegant. In order to get at the individual disjuncts from the sentence as a whole, he has to isolate them. He does this by adding more operators to the Horn scale for disjunction, making it a partially-ordered set instead of a well-ordered one. In essence, he adds the two disjuncts themselves to the set of alternatives.

The new Horn scale that Sauerland proposes doesn’t have lexical items on it, but rather sentences. Thus, the scale that he proposes is a scale schema of sorts. He proposes  $\langle A \wedge B, \{A, B\}, A \vee B \rangle$  as the scale schema. Conjunction is the strongest element, disjunction the weakest. In the middle, with no entailment relation between them, are the two disjuncts appearing on their own.

Of course, this means that any sentence is part of an infinity of Horn scales. This means that implicatures could get calculated along an infinity of scales unless we have a way to distinguish between using a sentence  $B$  with respect to one scale rather than another. To isolate particular scales even when one of the possible disjuncts is uttered in isolation, Sauerland replaces  $A$  and  $B$  with formally different elements  $A \text{ L } B$  and  $A \text{ R } B$ . The semantic definitions for these are as follows:

**DEFINITION 12.**  $\|A \text{ L } B\| = \|A\|$

**DEFINITION 13.**  $\|A \text{ R } B\| = \|B\|$

L and R effectively manage to tag sentences as being part of particular Horn scales. This blocks irrelevant implicatures (otherwise, choosing any sentence at all could entail the negation of its corresponding “ $B$ ” sentence in an infinity of Sauerlandian Horn scales). Thus the Sauerlandian Horn scale for disjunction, for an arbitrary disjunction  $p$  or  $q$ , is  $\langle p \text{ and } q, \{p \text{ L } q, p \text{ R } q\}, p \text{ or } q \rangle$ .

Scalar alternatives, for Sauerland, are calculated using a cross product.

**DEFINITION 14.** Suppose  $Q_X$  and  $Q_Y$  are quantitative scales. Let  $X$  and  $Y$  be scalar terms such that  $X \in Q_X$  and  $Y \in Q_Y$ . Suppose  $\phi$  is a sentence such that it contains  $X$  and  $Y$ , and no other scalar terms.

$$ScalAlt_{Sauer}(\phi(X, Y)) = \{\phi(X', Y') \mid X' \in Q_X, Y' \in Q_Y\}$$

Informally, Sauerland’s alternatives contain every possible permutation of scalar terms in the sentence given the possibilities granted by the terms’ quantitative scales<sup>26</sup>.

Let our scale of generalized quantifiers be ⟨all, some⟩. In an example such as “Kai had the broccoli or some of the peas last night”, the set of Sauerlandian scalar alternatives consists of:

{Kai had the broccoli or some of the peas last night,  
 Kai had the broccoli or all of the peas last night,  
 Kai had the broccoli L Kai had some of the peas last night,  
 Kai had the broccoli R Kai had some of the peas last night,  
 Kai had the broccoli L Kai had all of the peas last night,  
 Kai had the broccoli R Kai had all of the peas last night,  
 Kai had the broccoli and some of the peas last night,  
 Kai had the broccoli and all of the peas last night}

As  $p \text{ L } q$  evaluates to  $p$  and  $p \text{ R } q$  evaluates to  $q$ , Sauerland is able to “get at” the embedded phrases of the original sentence. Springing from this method of defining alternatives, Sauerland defines a set of “primary” and “secondary” scalar implicatures:

**DEFINITION 15.** If  $\psi \in ScalAlt(\phi)$  and  $\psi \models^A \phi$ , then  $\neg K\psi$  is a *primary implicature* of  $\phi$ .

**DEFINITION 16.** If  $\neg K\psi$  is a primary implicature of  $\phi$  and  $K\neg\psi$  is consistent with the conjunction of  $\phi$  and all primary implicatures of  $\phi$ , then  $K\neg\psi$  is a *secondary implicature* of  $\phi$ .

These definitions also answer the “when” question for Sauerland. A speaker implicates her ignorance straight-away. Whenever a knowledge implicature ( $K\neg\psi$ ) is consistent with the original utterance and its primary “ignorance” implicatures, it is a secondary implicature of the utterance. This “epistemic step” is meant to explain why in some cases we have merely ignorance implicatures, but in others we have stronger knowledge implicatures of Gazdar’s form.

First, primary implicatures of the form “The speaker is not certain whether  $\phi$  holds” are computed for certain  $\phi$ . In the second step, primary implicatures are strengthened to implicatures of the form “The speaker is certain that  $\phi$  doesn’t hold,” which I have called secondary implicatures. . . . My contribution here has been to show one case in which the step from primary to secondary implicatures is blocked because the additional assumption normally made to justify this step contradicts the assertion and its primary implicatures. [13]

---

<sup>26</sup>My definition of scalar alternatives in chapter 2 captures this for arbitrarily large constructions.

To answer “when”, then, primary implicatures are always calculated. Secondary implicatures are always calculated and are kept whenever consistent.

Russell 2006 provides a defense against Chierchia’s puzzles in a different way. Rather than providing new technical machinery with which to solve Chierchia’s challenges, Russell simply argues that the Gricean framework already has the resources to account for embedded implicatures.

One of Chierchia’s main challenges is that the Gricean framework does not account for certain strong implicatures. For instance, in the case of the example  $\varphi$ : “John believes that some of his students are coming”, the implicature  $\psi$ : “John believes that some (but not all) of his students are coming” would not be predicted by a Gricean (according to Chierchia). Chierchia assumes that the Gricean is limited to negating the whole of the stronger expression alternative, in Gazdar’s sense. The result of this would be  $\psi'$ : “It’s not the case that John believes all of his students are coming” which is too weak.

Russell’s reply demonstrates that the Gricean framework *can* deal with these examples<sup>27</sup>. He starts by pointing out that in every context where John has an opinion about whether or not all of his students are coming,  $\psi'$  leads to  $\psi$ . In other words, if we have reason to believe that John is opinionated about whether or not all students will come, then the fact that he doesn’t believe one of the disjuncts implies that he believes the other. In the case that we have no reason to believe he’s opinionated (or in cases where we believe he’s not), the weaker implicature predicted by Griceans is the ignorance inference one would expect.

#### 4.4 The Excluded Middle

Unconventional Gricean replies exist: Simons 2007 and Geurts 2006. This section will address their arguments, as not only do they step beyond the localist/globalist separation between non-Griceans and Griceans, but also ultimately serve as the inspiration for my account.

Simons’ argument is fairly straight-forward:

There is no reason why, as theorists, we have to take Grice’s views as an unseparable whole. The fundamental ideas underlying the notion of implicature are clearly separable from Grice’s expressed views about the nature of “what is said” and about the semantic content of particular items. For example, Grice argued that natural language conditionals have the semantics of material implication, and used the notion of conversational implicature to try to make this view plausible. Formal linguists have by now (I think) universally rejected this account of conditionals; but this does not prevent us from maintaining a Gricean account of conversational inference. Similarly, the Cooperative Principle and the associated maxims are a first stab at formulating the appropriate principles, not the last word on the matter. One can maintain a broadly Gricean outlook

---

<sup>27</sup>In his paper he deals with all of Chierchia’s cases, not just cases of unary operators.

while modifying the details of the account of how implicatures are generated. [14]

The notion of conversational implicature as being norm-governed is perhaps more “Gricean” than tenaciously holding to his views on what constitutes a conversational contribution. Simons’ move, then, is to reformulate the maxims and the cooperative principle to deal with non-asserted sentence parts as conversational contributions.

But suppose we read Quantity 1 this way: Provide as much information as is required about the situation you are describing. Or, utilizing the notion of strength: Provide the strongest description of the situation you aim to describe compatible with the requirements of Relevance. The idea is this: a speaker’s choice of words is always an indication of some belief she has about the situation she is describing. In the case where the utterance describes this situation as actual, the beliefs in question will be beliefs about what is the case. In the case where the utterance describes the situation as hypothetical, or as merely possible or probable, or as the content of someone’s propositional attitude, the beliefs will be beliefs about what is possible or probable, or about another agents beliefs.

Such a reformulation allows a Gricean analysis of embedding such as I provided in the previous chapter. If Sally utters “I believe that John drank some of the beer”, then we can treat her description of her belief (the dependent clause “John drank some of the beer”) as a conversational contribution subject to Simons’ maxim of quantity. If this is the case, then we can apply otherwise typical Gricean reasoning to this unasserted sentence part; if Sally believed that John drank *all* of the beer, she should have said so (so on and so forth).

Geurts 2006 provides a similarly unconventional Gricean outlook, except he effectively wishes to relax the maxims in two different directions. Working from the perspective of discourse representation theory (Kamp & Reyle 1981), Geurts makes the point that conversational contributions can also be viewed as *sets of DRS-conditions*<sup>28</sup> [6]. In other words, subsets of discourses could be considered subject to the maxims. This view captures the fact that implicature-like effects are seen both above and below the clausal level.

The arguments Geurts adduces for “discourse” implicatures of the multi-sentential form will not be discussed here, as they take us too far afield. However, his recommendation will be taken seriously insofar that I will use discourse representation theory to explore how fruitful it is for scalar implicatures. Since scalar implicatures are triggered by the use of particular terms, we will only be dealing with singleton sets of DRS-conditions. As we will see, my theory takes Simons’ and Geurts’ suggestions as inspiration for developing a computational framework of this general style.

---

<sup>28</sup>Just what these are is covered in the next section.

## 5 A Very Short Primer on DRT

This section will provide a very short introduction to discourse representation theory (Kamp & Reyle 1981). Many introductions to this subject already exist, and many do a better job than this section. In particular, Bart Geurts and David I. Beaver’s entry in the Stanford Encyclopedia of Philosophy [8] is recommended for those who wish for a more adequate crash course. In fact, much of this presentation borrows heavily from Geurts and Beaver’s perspicuous explication. The purpose of this section is merely to give readers unfamiliar with DRT an adequate enough understanding for the upcoming proposal, in the interest of keeping the content of this thesis as self-contained as possible.

### 5.1 What is DRT?

Discourse representation theory (DRT) is a representationalist theory of semantics. In other words, it provides a language for semantic representation that is meant to mirror actual mental objects. The level of representation, for DRT, is essential to a theory of interpretation.

DRT was constructed with particular problems in its sights. In particular, it was the problem of discourse anaphora that helped motivate its development. DRT’s motivations and the particular problems it helps to solve will be excluded from our treatment, however. Understanding how the construction of objects in DRT solves problems in discourse anaphora resolution is important for the critical reader to understand why it’s more than just a fancy first-order language. For a more detailed examination of these issues, the reader is referred to Geurts’ article. The presentation provided here is meant simply to familiarize the reader with the syntax and semantics of DRT’s language.

### 5.2 Syntax

This section will introduce the language of DRT.

Discourse representation structures (referred to as “DRS’s”) are putatively mental representations constructed by an interpreter as a discourse unfolds<sup>29</sup>. DRT is a theory of interpretation from the standpoint of a passive interpreter.

DRS’s are set theoretic objects consisting of two parts: a set of “discourse referents”, and a set of “DRS-conditions”. Discourse referents are the objects under discussion. Loosely speaking, DRS-conditions express content asserted about the discourse referents.

Defining DRS’s and DRS-conditions comprises the entire specification of DRT’s language. Both of these objects are defined by simultaneous recursion, as DRS’s contain DRS-conditions, and DRS-conditions can be comprised of DRS’s:

---

<sup>29</sup>DRT’s notion of “discourse” is perhaps impoverished compared to common usage. Strictly speaking, a discourse is an ordered set of sentences uttered by a single speaker.

**DEFINITION 17.** A DRS  $K$  is a pair  $\langle U_K, Con_K \rangle$ , where  $U_K$  is a set of discourse referents, and  $Con_K$  is a set of DRS-conditions.

All of the members of  $Con_K$  are said to be “immediately dominated” by  $K$ .

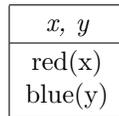
**DEFINITION 18.** If  $P$  is an  $n$ -place predicate, and  $x_1, \dots, x_n$  are discourse referents, then  $P(x_1, \dots, x_n)$  is a DRS-condition.

If  $x$  and  $y$  are discourse referents, then  $x = y$  is a DRS-condition.

If  $K$  and  $K'$  are DRS's, then  $\neg K$ ,  $K \Rightarrow K'$ , and  $K \vee K'$  are DRS-conditions.

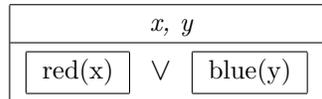
If  $K$  and  $K'$  are DRS's,  $x$  is a discourse referent, and  $Q$  a generalized quantifier, then  $K \langle Qx \rangle K'$  is a DRS-condition.

What is presented in the above definitions is the “linear notation” of DRT. In the rest of this thesis, I will use the “box notation”, which I feel is more transparent for the presentation of my algorithms. The box notation is fairly intuitive.

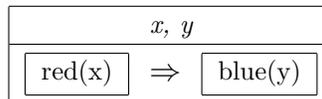


What is depicted above is a DRS (call it  $\varphi$ ). Above the line, the set of discourse referents is enumerated. Beneath the line, the DRS-conditions are enumerated. We will review semantics in a moment more formally, but they are fairly intuitive. A model  $M$  verifies  $\varphi$  if and only if a function  $f$  exists mapping the discourse referents of  $\varphi$  to elements of  $M$  such that  $f(x)$  is red and  $f(y)$  is blue.<sup>30</sup>

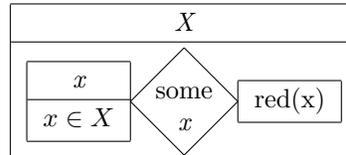
If, instead of “ $x$  is red and  $y$  is blue” we wished to express something such as “either  $x$  is red or  $y$  is blue”, we would have written:



Or if we wished to write “If  $x$  is red then  $y$  is blue”:



We can also, as alluded to in the above definition, express quantification using any generalized quantifier. “Some  $X$ 's are red” would come out as follows:



<sup>30</sup>Unsurprisingly, “snow is white” is true if and only if snow is white.

### 5.3 Semantics

As alluded to in the previous section, DRT’s semantics are what we would expect from any straight-forward, model-theoretic semantics. A DRS  $\varphi$  is satisfied by a model  $M$  if and only if there exists an “embedding function”  $f$  that maps the discourse referents of  $\varphi$  to individuals in  $M$  such that whatever is asserted of them in  $\varphi$ ’s DRS-conditions is the case in  $M$ .

An embedding function, as we’ve mentioned, is a partial function that maps discourse referents in a DRS to individuals in a model. In order to fully specify our semantics, we’ll have to first define what it is for one embedding function to “extend” another with respect to a DRS.

**DEFINITION 19.** An embedding function  $f$  *extends* another embedding function  $g$  with respect to DRS  $K$  (written  $f[K]g$ ) if and only if  $f \subseteq g$  and the domain of  $g$  ( $Dom(g)$ )  $\subseteq Dom(f) \cup U_K$ .

If we define a model as an ordered pair  $\langle D, I \rangle$  where  $D$  is a set of individuals, and  $I$  is an interpretation function assigning  $n$ -tuples of  $D$  to  $n$ -place predicates, we can define when an embedding function verifies a DRS in a model:

**DEFINITION 20.** Let  $f$  be an embedding function.  
 $f$  verifies a DRS  $K$  iff  $f$  verifies all conditions in  $Con_K$ .  
 $f$  verifies  $P(x_1, \dots, x_n)$  iff  $\langle f(x_1), \dots, f(x_n) \rangle \in I(P)$ .  
 $f$  verifies  $x = y$  iff  $f(x) = f(y)$ .  
 $f$  verifies  $\neg K$  iff there is no  $g$  such that  $f[K]g$  and  $g$  verifies  $K$ .  
 $f$  verifies  $K \vee K'$  iff there is a  $g$  such that  $f[K]g$  and  $g$  verifies  $K$  or  $K'$ .  
 $f$  verifies  $K \Rightarrow K'$  iff, for all  $f[K]g$  such that  $g$  verifies  $K$ , there is an  $h$  such that  $g[K']h$  and  $h$  verifies  $K'$ .  
 $f$  verifies  $K \langle Qx \rangle K'$  iff, for  $Q$  individuals  $d \in D_M$  and for all  $f[K]g$  such that  $g(x) = d$  and  $g$  verifies  $K$ , there is an  $h$  such that  $g[K']h$  and  $h$  verifies  $K'$ .

**DEFINITION 21.** A DRS  $K$  is *true* in a model  $M$  iff there exists an embedding function  $f$  such that  $Dom(f) = U_K$  and  $f$  verifies  $K$  in  $M$ .



## 6 The Account

The purpose of this chapter is to establish an algorithm for the calculation of scalar implicatures in layered discourse representation theory (Geurts, Maier 2003). First we will outline the computational challenges that the theory must overcome. Then we will step through each of the solutions in turn. Each solution builds upon the material in the previous one, gradually building up a complete computational model.

### 6.1 Goals

The main goal of the theory presented in this chapter is to answer the question of how scalar implicatures should be calculated, and under what linguistic circumstances they should be culled from the discourse. The theory is fully constructive and this is taken to be an important methodological point; for a more in-depth discussion of constructiveness and its importance for theories of interpretation, consult Appendix C.

The question of “what” must be split apart into a few sections. First the question of entailment must be discussed; following other Gricean accounts, I cash out “informativeness” in terms of entailment. In order for my theory to be constructive, then, an account of entailment must be given. Instead of taking on such a formidable task, I restrict the domain to that of scalar alternatives, making the project tractable.

This definition of strength, formulated in DRT, will then be used in a layered DRT (LDRT) driven account. This primitive account will calculate all potential implicatures present in the discourse, culling only those that are linguistically inappropriate. The goal of this theory is to produce predictions similar to a defaultist account such as Chierchia 2004, the difference being a refinement and precisification of cancellation.

The last goal is to solve the “when” problem; in other words, to determine when we should calculate the scalar implicatures. A Gricean extension of the defaultist account will be offered to this end; beliefs about the context, the speaker, and the speaker’s intention will need to be modeled in order to achieve a total solution. This is, however, a project in itself; the theory presented here will only sketch the interface for these extra-linguistic factors. Thus we conclude at the end of this chapter that purely linguistic theory cannot provide us with a complete (“what” and “when”) theory of scalar implicature.

### 6.2 Scalar Entailment ( $\models_S$ )

For Gazdar, the calculation of a “stronger set of alternatives” was computationally simple. Given that Gazdar only examines logically simple cases, sentences that use a scalar term “to the left” of other alternatives asymmetrically entail those alternatives. In other words, entailment between scalar alternatives in this model is determined by the relative positions of expressions’ scalar terms

in their shared quantitative scale. This is the case because of how the scales are ordered: by entailment relations between simple sentences.

Sauerland’s account likewise uses a notion of scalar alternatives, and requires a “stronger” set of these alternatives in order to calculate primary and secondary implicatures. In Sauerland’s cases, however, which include a multitude of scalar terms in logically complex sentences, we can no longer do a simple lookup in a quantitative scale to calculate the “stronger alternative” set.

As the theory presented in this thesis is computational, we cannot rely on judgments of entailment in selecting stronger alternatives. A computational theory should be fully reducible without relying *directly* on intuition. The strategy we adopt will be simple: using the information we have in standard Horn scales (that reflect entailment relations in logically simple sentences), we will develop machinery to decide entailment relations between arbitrarily complex scalar alternatives.

Consider the following logically simple case:

$H = \langle \text{all, some} \rangle$ .  
 $\phi$  : “John drank some of the beer.”  
 $\psi$  : “John drank all of the beer.”

Whether or not  $\psi$  is stronger than  $\phi$  is easy to calculate. We simply find  $\psi$ ’s term, “all”, in  $H$  and compare it versus the position of  $\phi$ ’s term, “some”. Since it occurs to the left of “some”, we can conclude that  $\psi \models^A \phi$ .

Consider the following logically complex case, on the other hand:

$H = \langle \text{all, most, some} \rangle$ .  
 $\phi$ : “Some of the students didn’t fail all of their tests.”  
 $\psi$ : “Some of the students didn’t fail most of their tests.”

This case is harder to work out than the logically simple case above. Intuitively,  $\psi \not\models^A \phi$ . If some of the students didn’t fail most of their tests, then some of the students didn’t fail all of them (namely, the ones that didn’t fail most). Sauerland 2004’s theory, for instance, requires these intuitive judgments to be made in order to calculate a stronger set of alternatives.

Again, for a fully computational theory, we need a general method for working out whether or not one scalar alternative is stronger than the other. Thus, in this section, I will develop an inductive definition for “scalar entailment”; the scalar entailment operator will, in finite time, determine whether or not one scalar alternative entails another. This definition relies on Horn scales which are fixed initially by intuition. The important notion, however, is that the theory can carry out its own predictions once these intuitions are fixed once and for all. This ability to automatically generate predictions given data is essential for a theory to be appropriately termed “computational”.

The definition is by induction over the construction of objects in discourse representation theory. Each case is stipulative in the sense that they don’t derive from any general characteristics or “first principles”; rather, they are empirical generalizations. Some logical operators are  $\text{mon}\uparrow$  and some are  $\text{mon}\downarrow$ ; depending

on which a particular operator is, it affects the contribution of the content under its scope. This feature is captured by the definition. For example, in order for  $\neg\phi$  to asymmetrically entail  $\neg\psi$ , it must be that they are scalar alternatives and  $\psi$  entails  $\phi$ <sup>31</sup>.

**DEFINITION 22.** If  $\varphi$  and  $\psi$  are both scalar terms from the same quantitative scale, then  $\varphi \geq \psi$  if and only if  $\varphi$  occurs to the left of  $\psi$  or  $\varphi = \psi$ .  $\varphi < \psi$  if and only if  $\varphi \not\geq \psi$ .

**DEFINITION 23.** Let  $\phi$  and  $\psi$  be any two DRS-constructions such that  $\text{ScalAlt}(\phi, \psi)$ . If  $\phi \models \psi$ , we say that  $\phi$  *scalarly entails*  $\psi$  ( $\phi \models_S \psi$ ) and vice versa. We will define  $\models_S$  by induction.

**CASE 1:**  $\phi$  contains no scalar terms<sup>32</sup>.

$\phi \models_S \psi$  vacuously.

**CASE 2:**  $\phi$  is a  $k$ -ary predicate of the form  $P(\theta_1, \dots, \theta_k)$  where  $P$  is a scalar term and  $\theta_1, \dots, \theta_k$  are discourse referents,  $\psi$  is of the form  $P'(\theta_1, \dots, \theta_k)$  where  $P'$  is a scalar sibling of  $P$ .

$\phi \models_S \psi$  if and only if  $P' \leq P$ .

**CASE 3:**  $\varphi$  and  $\psi$  are DRS's<sup>33</sup>. Let  $\{x_1, \dots, x_n\}$  be the set of  $\varphi$ 's DRS-conditions and  $\{y_1, \dots, y_n\}$  be the set of  $\psi$ 's DRS-conditions.

$\varphi \models_S \psi$  if and only if  $\bigwedge_{i=1}^{i=n} \bigvee_{j=1}^{j=n} x_i \models_S y_j$

**CASE 4:**  $\varphi$  is of the form  $\theta \vee \sigma$  and  $\psi$  is of the form  $\alpha \vee \beta$ .

$\varphi \models_S \psi$  if and only if  $(\theta \models_S \alpha \text{ and } \sigma \models_S \beta)$  or  $(\theta \models_S \beta \text{ and } \sigma \models_S \alpha)$ .

**CASE 5:**  $\phi$  is of the form  $\theta(Q_i x)\sigma$ <sup>34</sup>,  $\psi$  is of the form  $\alpha(Q_j y)\beta$  where  $\theta, \sigma, \alpha, \beta$  are DRS's,  $x, y$  are discourse referents such that  $x \in \theta$ ,  $y \in \alpha$ , and  $Q_i, Q_j$  are scalar siblings.

$\phi \models_S \psi$  if and only if  $\theta \models_S \alpha, \sigma \models_S \beta$ , and  $Q_j \leq Q_i$ .

**CASE 6:**  $\phi$  is of the form  $\neg\sigma$ ,  $\psi$  is of the form  $\neg\alpha$  where  $\alpha, \sigma$  are DRS-constructions.

<sup>31</sup>“John didn’t drink some of the beer” entails “John didn’t drink most of the beer” ; “John drank most of the beer” entails “John drank some of the beer”. This is, in essence, the “scale reversal” we observed in chapter 3.

<sup>32</sup>Hitherto I have characterized scalar terms as expressions of English. “Scalar terms” in this definition refers to the predicates, quantifiers, and logical operators (such as disjunction, in some cases) in DRT-language that represent these English expressions.

<sup>33</sup>This distinguishes them from the other cases, as DRS-conditions are not DRS’s (though they may contain DRS’s as sub-parts).

<sup>34</sup> $\varphi(Qx)\psi$  denotes a duplex condition where  $\varphi$  is the left-hand DRS,  $\psi$  is the right-hand DRS,  $Q$  is a quantifier, and  $x$  is a discourse referent.

$\phi \models_S \psi$  if and only if  $\alpha \models_S \theta$ .

**CASE 7:**  $\phi$  is of the form  $\theta \Rightarrow \sigma$ ,  $\psi$  is of the form  $\alpha \Rightarrow \beta$ , where  $\theta, \sigma, \alpha, \beta$  are DRS's.

$\phi \models_S \psi$  if and only if  $\alpha \models_S \theta$  and  $\sigma \models_S \beta$ .

The above definition is used in the following theory to pick out the set of stronger alternatives in a computable fashion.

### 6.3 A Defaultist Theory

This section will outline and motivate the defaultist version of my theory. The defaultist account only establishes “what”. It generates all of the possible implicatures present, excluding only those that produce inconsistencies. Thus, only a partial solution to “when” is provided.

#### 6.3.1 Layered DRT

“Layered” DRT is a class of extensions proposed by Geurts, Maier 2003<sup>35</sup>. The idea and its motivation are simple:

Utterances carry content about the world as it is according to the speaker, but also about speakers’ attitudes, the way they speak, what has been said before, and so on. There are many kinds of information that are conveyed by way of language, and differences in kind correlate with differences in status. Presupposed information exhibits a distinctive projection behaviour; conversational implicatures are cancellable in a way that asserted information is not; a pronoun’s gender may help to determine a referent, but is otherwise truth-conditionally inert; and so on. [9]

Different kinds of linguistic information behave differently. The idea behind LDRT is to separate out these different kinds with “tags” into “layer sets” determined by the tags; these tags, which are simply markings meant to keep different “layers” of content separated, provide different algorithms with information when treating content.

The semantics of LDRT is also straight-forward: “instead of specifying what is the truth-conditional content of an LDRS  $\varphi$ , we have to define what is the truth-conditional content of a selection L of layers in  $\varphi$ ”. Different layers for an object in LDRT can come out as true; the layered DRS, however, only comes out as true in a model if all of its layers are satisfied.

We won’t need to get too far into the details of LDRT here. The main notion is that of layer sets and the ability to treat them differently; for our purposes, we will treat implicated content as separate from encoded (literal) content. The semantics for the implicature layer will be the same as the encoded layer. The

---

<sup>35</sup>This paper can be found on Bart Geurts’ website but otherwise is unpublished as far as I know.

only reason to keep these two sorts of information separate is for the benefit of our cancellation algorithm (not to refine semantics for implicated content).

### 6.3.2 The Machinery

This section will establish the computational machinery of the default theory. The strategy is simple. First, we find the location of each potential scalar implicature. Then, we identify the relevant sub-structure for its calculation. Upon doing this, we calculate the strengthened reading, and substitute it directly back into that sub-structure.

For this purpose, I have to make a working assumption about the nature of this sub-structure, and how to pick it out.

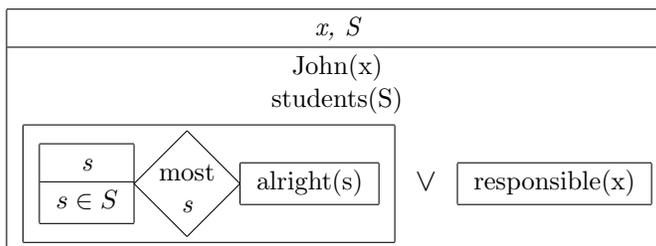
**Sub-structural Assumption.** The scope of a scope-taking scalar operator is the relevant sub-structure for the calculation of embedded scalar implicatures. These sub-structures can be treated as if they were unembedded and strengthened accordingly.

**DEFINITION 24.** We will call disjunctive conditions, duplex conditions, and scalar predicates (predicates that appear in quantitative scales) *scalar conditions*<sup>36</sup>. DRS-conditions immediately dominated by the DRS's of scalar conditions are called *implicative roots*.

**DEFINITION 25.** Let  $t$  be a scalar condition and  $H$  be a Horn scale. We say  $t \in H$  if  $t$  contains a scalar term  $s$  such that  $s \in H$ .

Implicative roots are the “relevant sub-structures” for scalar implicature calculation<sup>37</sup>. The name of the game, then, is to find each scalar term in our DRS, pick out its implicative root, and then to calculate its set of scalar alternatives with respect to this one scalar term.

We will examine an example of implicative roots before continuing. Consider the sentence “Either most of the students were alright or John was responsible.” In DRT:



Suppose we’re dealing with the following Horn scales:  $H = \langle \text{all, some, most} \rangle$ ,  $H' = \langle \text{great, alright, bad} \rangle$ . Given the above considerations, there are two implicative roots to this DRS. The first is the simple DRS-condition “alright(s)”,

<sup>36</sup>If DRT is extended to model beliefs and other intentional concepts, this definition can be extended.

<sup>37</sup>This is justified by automated tests I ran after implementing this account in Java. Randomly generated tests were run with DRS-schema and tested versus intuition.

as it is immediately dominated by the DRS of a scalar condition. Likewise,

the DRS condition  $\frac{s}{s \in S}$   $\left\{ \begin{array}{c} \text{most} \\ s \end{array} \right\}$   $\text{alright}(s)$  is an implicative root, as it is

immediately dominated by the DRS of a scalar condition itself. Our working assumption says that we can deal with these roots as if they were unembedded when calculating the embedded implicatures they carry.

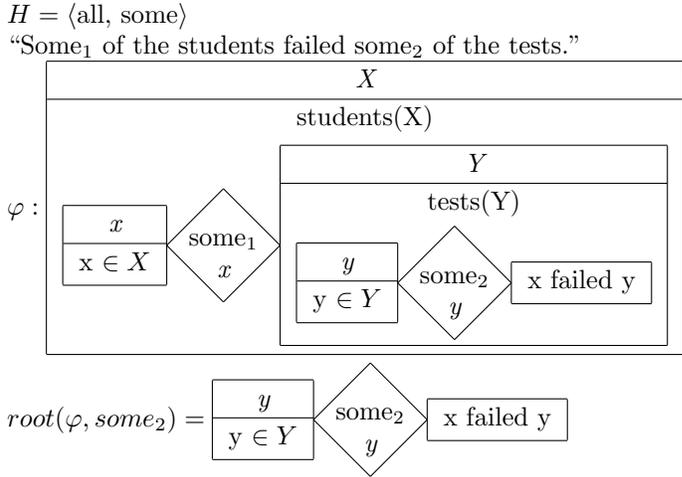
An elegant way to do this is to pick out the scalar terms depth-first. This ensures that as we augment each implicative root with its appropriate implicature, embedded implicatures under the scope of scalar terms get carried along when their implicatures are calculated as well.

First, we'll define the implicative root for any given scalar term more precisely.

**DEFINITION 26.** The implicative root of a scalar condition  $t$  occurring in an expression  $\varphi$ , written  $root(\varphi, t)$ , is one of the following two DRS-conditions:

- (1)  $t$  itself, if  $t$  is immediately dominated by the main DRS.
- (2) The DRS-condition immediately dominated by the closest scalar condition that contains  $t$ , otherwise.

A general procedure to finding the implicative root is to trace up each “parent” DRS until finding a DRS that fulfills either condition. Consider the following example, which we will trace throughout the development of this theory:



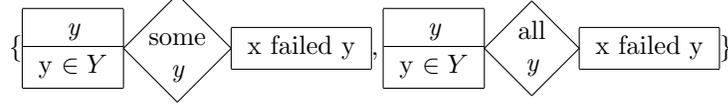
Scalar alternatives can then be relativized to a single scalar condition in an expression. In other words, we can take the scalar alternatives of an expression with respect to just one of its scalar conditions. In doing so, we can address the deepest untouched scalar condition at a time.

**DEFINITION 27.** Let  $\varphi$  be any expression and  $t \in H$  be some scalar condition occurring in  $\varphi$ .  $ScalAlt(\varphi, t)$  (read: “the set of scalar alternatives of  $\varphi$  with respect to  $t$ ”) is the set:

$$\{\text{root}(\varphi, t)[t'/t] \mid t' \in H\}$$

Continuing the example from above:

$\text{ScalAlt}(\varphi, \text{some}_2)$  :



This is our familiar notion of scalar alternative, except we now fix the scalar condition at which it occurs. Furthermore, we are calculating the alternatives across implicative roots instead of the entire expression.

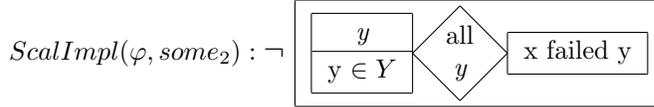
Now the strategy is the traditional one used by all accounts: negate the stronger alternatives and count these amongst the scalar implicatures. We will merely negate the next strongest alternative, as this will entail the other intended implicatures. Now that we have the  $\models_S$  operator at our disposal, we can handle arbitrarily complicated implicative roots automatically.

**DEFINITION 28.** Let  $\varphi$  be any expression and  $t \in H$  be some scalar condition occurring in  $\varphi$ .  $\text{ScalImpl}(\varphi, t)$  (read: “the embedded scalar implicature generated by  $\varphi$  with respect to  $t$ ”) is the DRS-condition:

$\neg\psi$  where the following three conditions are all met:

- (1)  $\psi \in \text{ScalAlt}(\varphi, t)$
- (2)  $\psi \models_S^A \varphi$ <sup>38</sup>
- (3)  $\forall \eta \in \text{ScalAlt}(\varphi, t), \eta \models_S^A \varphi \rightarrow (\eta = \psi \vee \eta \models_S^A \psi)$

Continuing our example:

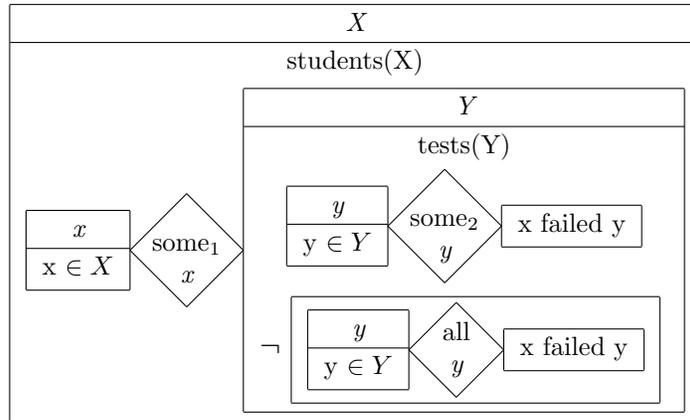


This is the implicature we want locally. In order to make sure this gets placed in at the embedded level, we can perform the following substitution:  $\varphi[\text{ScalImpl}(\varphi, \text{some}_2), \text{root}(\varphi, t)/\varphi]$ . In other words, we can substitute the conjunction of the scalar implicature with the implicative root that generated it in for that root itself<sup>39</sup>.

In terms of our example:

<sup>38</sup> $\psi \models_S^A \varphi$  if and only if  $\psi \models_S \varphi$  and  $\varphi \not\models_S \psi$ .

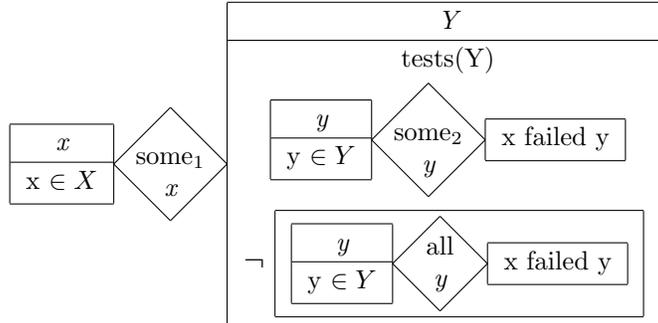
<sup>39</sup>When encoded content gets scrapped, so should its implicatures.



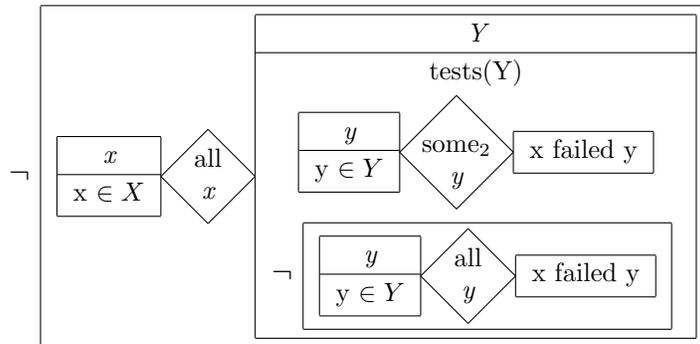
“Some of the students failed some (but not all) of the tests.”

Naturally, we want to keep going. Once we’ve completed the calculation of the embedded implicature at  $some_2$ , we should step back and do the same thing for  $some_1$ . The difference now is that when we calculate the implicature of  $some_1$ , the embedded implicature under its scope gets carried along for the ride.

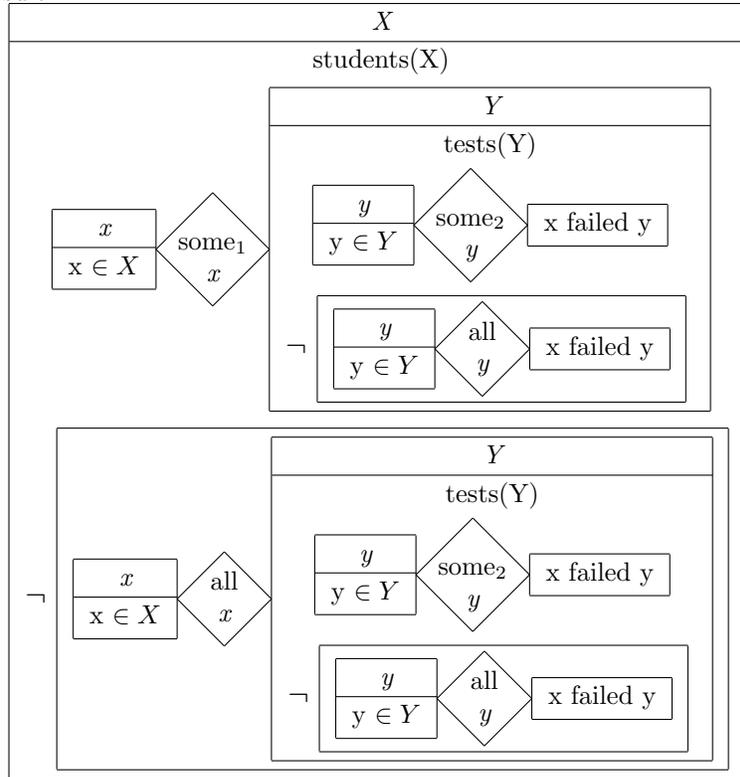
$root(\varphi, some_1)$  :



$ScalImpl(\varphi, some_1)$  :



End result:



“Some of the students failed some (but not all) of the tests, but not all of the students failed some (but not all) of the tests.”

Which gets all of the scalar implicatures we’d expect if they are indeed default.

Now the only step is to take what we’ve outlined and generalize it to an algorithm. This algorithm applies to layered DRS objects and augments them in the ways shown above. The only difference is that it tags newly implicated content with an  $n$ , and retags pre-existing  $n$ -tagged content with  $o$ . This is to keep newly and previously implicated content separated. (Why this is important will become apparent later.)

Our algorithm will take two inputs,  $\phi$  and  $\psi$ . The first call to the algorithm should feed the same DRS into both positions; since the algorithm is recursive, it requires two arguments. After the first recursive call, the identity between the two parameters no longer holds.

**ALGORITHM 1.** (*Default Implicature Calculation*)

*Input:* DRS-construction  $\phi, \psi$ .

*First, take each DRS-condition with the  $n$  label and relabel it  $o$ .*

*Let  $I_\phi = \emptyset$ .*

For each DRS-construction that isn't the antecedent of a conditional<sup>40</sup> in  $\psi$  (call it  $\eta$ ):

Call algorithm on  $(\phi, \eta)$ .

If  $\psi$  is a scalar condition:

Let  $\phi := \phi[(\text{root}(\phi, \psi) \wedge \text{ScalImpl}(\phi, \psi))/\text{root}(\phi, \psi)]$ .

Let  $I_\phi := I_\phi \cup \langle \text{root}(\phi, \psi), \text{ScalImpl}(\phi, \psi) \rangle$ .

The  $I_\phi$  set in the above algorithm is simply to track ordered pairs of generating structures and their scalar implicatures. The members of  $I_\phi$  are ordered pairs  $\langle \alpha, \beta \rangle$  such that  $\alpha$  is encoded content that implicates  $\beta$ . We'll exploit this information during cancellation.

## 6.4 Cutting into “When”: Cancellation

As I will argue later on, the question of when implicatures should be calculated is a question that requires extra-linguistic theory to solve. However, parts of the “when” problem can be cut into with purely linguistic analysis. These parts consist of cancellation, as it is normally called.

Cancellation, in its purely linguistic cases, is when implicated content is contradicted by newly asserted content. Consider the following example:

Fred: “John ate some of the fries. In fact, he ate all of them.”

The encoded content of Fred's second sentence cancels the “some but not all” implicature normally generated by the first. This implicature is old news; Fred's second assertion carries much more weight and overrides his weaker inference.

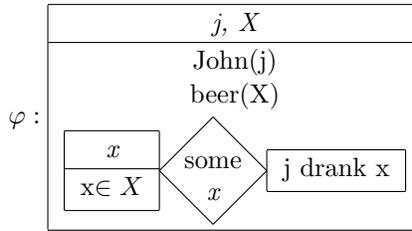
We can capture this notion formally and add it into our model. Using a proof calculus for discourse representation theory (either Sauerer 1993 or van Eijck 1999 would do), we can add consistency checking as a post-process to our default calculation algorithm.

The idea is simple: we perform a consistency check at the level of each DRS, examining the conditions that are immediately dominated by it. We expand the scalar statements out to capture all of their entailments (for the purposes of the deductive system) and check point-wise to see if any absurdities arise. In the case of a contradiction between any two items, we remove the first  $o$  tagged item we come across. This means that old implicated content makes room for asserted content.

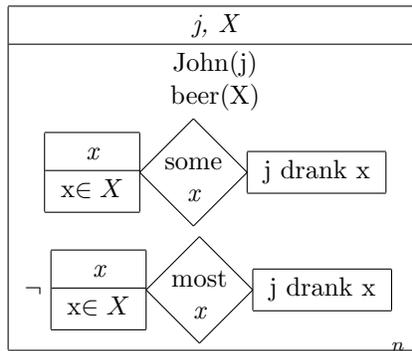
Time for an example. Take the following sentence: “John drank some of the beer” with a scale  $\langle \text{all}, \text{most}, \text{some} \rangle$ .

---

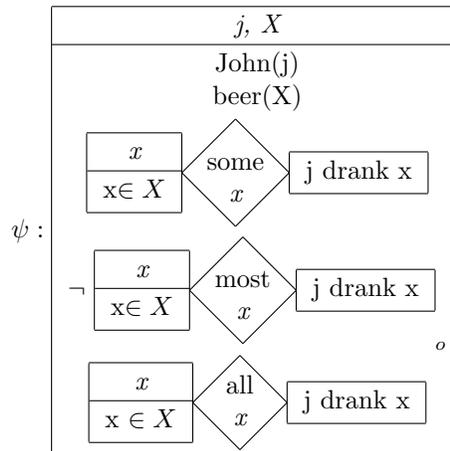
<sup>40</sup>This is stipulative; more work needs to be done to understand how embedded implicatures work in the antecedents of conditionals. This topic will not be covered in this thesis, as it requires a project of its own.



Augmented with scalar implicatures:

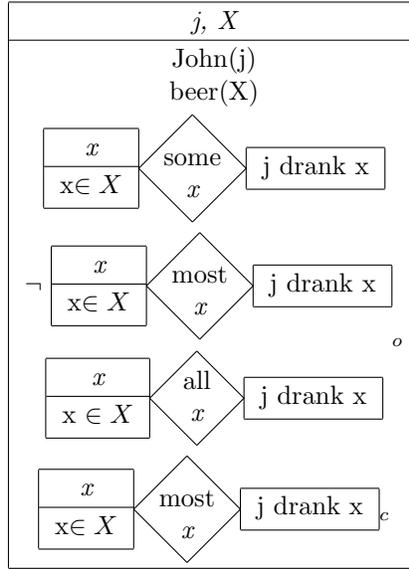


Suppose we add the sentence “John drank all of the beer” to this. We’d result in a discourse representation structure which we’ll label  $\psi$ :

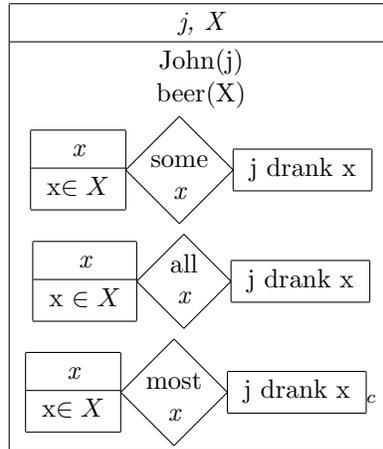


Clearly an inconsistency arises. If John drank all of the beer, then he had to have had *most* of it as well. Using a proof calculus as our mechanism for detecting inconsistencies, however, we would never detect this. Thus, we must temporarily augment our DRS with the scalar alternatives of our encoded DRS-conditions, such that these alternatives are entailed. We will call this the “consistency” layer, tagged *c*, and it will only live temporarily (during consistency checking). After consistency checking, all *c* content is deleted.

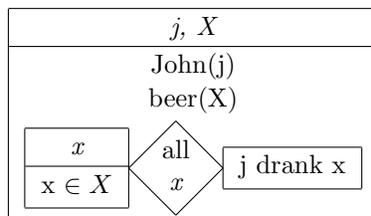
We will call the process of adding our consistency layer “scalar augmentation” (written  $Aug(.)$ ). We are, in essence, adding the following set for each condition  $\varphi : \{\psi \mid \psi \in ScalAlt(\varphi, t) \wedge \varphi \models_S \psi\}$ , where  $t$  is the highest-scope scalar term in  $\varphi$ .



Now, in checking point-wise across the elements in our DRS, we can see that  $c$  content conflicts with  $o$  content. In this case, we want to discard the  $o$  content. Another way of saying this is that not only does asserted content trump old implicatures, but so does the asserted content’s entailments.



After this, we want to drop  $c$  along with any content that is entailed by other content. This results in the following DRS:



Which is what we expect. Uttering that John drank all of the beer not only cancels the previous “not all” implicature, it also absorbs the weaker “John drank some of the beer” statement, as it entails it.

The general procedure is pretty clear. We want to augment our DRS first so that scalar statements at all levels of embedding add their  $c$  level content. Once this happens, we check each level for any consistencies that may arise. If any do, we drop the  $o$  content that led to the inconsistency. So far, so good.

**ALGORITHM 2.** (*Cancellation*)

*Input:* DRS-construction  $\phi$ .

Let  $\psi = Aug(\phi)$ .

Check pointwise for  $\perp$ .

If two conditions,  $\alpha, \beta \vdash \perp$ , then remove the element tagged  $o$  unless one of the elements is tagged  $n$ , in which case remove this instead (call it  $r$ ).

Remove any tuple from  $I_\phi$  such that its first element is  $r$ , and also remove its second element from the DRS.

For each  $\eta$  immediately dominated by  $\psi$ :

Call algorithm on  $\eta$ .

When scalarly implicated content comes into conflict with encoded content, the story doesn’t always unfold as above. There are times when a speaker uses focal stress to force through an implicature over the top of encoded content. In other words, a speaker can signal with focal stress that her implicature should revise previously asserted content.

Suppose we’re dealing with the following example:

Fred: “John drank all of the beer. Actually, he drank *most* it.”

If Fred were not trying to revise his first sentence by way of implicating “John didn’t drink all of the beer”, then uttering his second sentence would be absurd. Furthermore, if implicated content always gave way before asserted content, his second sentence would be pointless; it would immediately be discarded from the discourse. Intuitively, however, his implicature should stand and it’s the first sentence’s content that should move out of the way.

This motivates the following generalization.

**Focal Stress Generalization.** New implicatures generated by scalar terms with focal stress survive cancellation; it is the encoded content that gets cancelled instead. Call this process *revision*.

We can now add revision to our above algorithm.

**ALGORITHM 3.** (*Cancellation and Revision*)

*Input: DRS-construction  $\phi$ .*

*Let  $\psi = \text{Aug}(\phi)$ .*

*Check pointwise for  $\perp$ .*

*If two conditions,  $\alpha, \beta \vdash \perp$  and one is tagged  $o$  or  $n$  without focal stress, then remove the element tagged  $o$  unless one of the elements is tagged  $n$  without focal stress, in which case remove this instead (call it  $r$ ).*

*Else if two conditions,  $\alpha, \beta \vdash \perp$  and one is tagged  $n$  with focal stress, then remove the other element (call it  $r$ ).*

*Remove from  $I_\phi$  any tuple where  $r$  is the first element, and also remove its second element from the DRS.*

*For each  $\eta$  immediately dominated by  $\psi$ :*

*Call algorithm on  $\eta$ .*

## 6.5 A Gricean Extension

The above treatment, as with most theories of scalar implicature, leaves the separation between the speaker and the interpreter murky at best. An extension of the above model to a more realistic model would involve two moves: (1) an LDRT structure to represent each interlocutor, (2) an extra step of reasoning determining whether the implicature is an ignorance inference or a “strong” implicature, in Gazdar’s sense.

To understand whether or not a strong implicature is warranted in any given case, an interpreter needs to have certain beliefs about the speaker and the context of utterance. As in the example above, if John says “I drank some of the beer” while holding up an empty case proudly, the implicature normally associated with this utterance is not warranted. Any interpreter witness to this utterance must have certain beliefs about John and what’s going on; that John is the type to do this sort of thing, and that drunks are usually proud of the damage they do.

The type of scalar implicature, either primary or secondary in Sauerland’s terminology, depends fundamentally upon our beliefs about the speaker. Sauerland claims that the secondary ( $K\neg\psi$ ) style implicatures arise when consistent. However, they should never arise in cases where we believe the speaker to be non-committal or otherwise incompetent. In cases where the speaker does not project authority, the  $K\neg\psi$  form implicatures should never arise. The inconsistency that arises is not between linguistic data and the secondary implicature, but between beliefs and the secondary implicature.

Psychological extensions are required to solve this aspect of the “when” problem. The basis of what is calculated is the same; the difference resides in whether the strengthened value is denied or explicitly unknown. Clearly, no matter what generalizations we make to determine when an implicature should be generated,

we can always generate counterexamples by fixing a context. Without the ability to reason with a richer set of beliefs than just a discourse representation, any formalism is unable to cope with the flexibility with which language is used and accommodated. Until then, theory is limited to “what” and only a partial solution to “when”.



## 7 Discussion of the Account

This section will discuss the advantages of my theory and the places where it needs to be tightened up. Its limitations will be discussed with the hope that they are roped off honestly for further research.

### 7.1 Computational Advantages of DRT

Discourse representation theory is an incredibly strong formalism for this sort of work. Admittedly, the results of the default theory’s predictions all but match Chierchia 2004’s results; it looks similar, and even proceeds in a “depth-first” fashion (even though it remains sensitive to context in a way that Chierchia 2004 does not).

One advantage DRT has over Chierchia’s formalism (and other formalisms in general) is how it models the discourse, or what can be regarded as the “linguistic context”. By keeping track of what has been said and judging new content against this background, we have a naturally extendable system in which to do implicature generation and consistency checking. As the content of the discourse thus far is fully accessible to us in a convenient fashion and a proof calculus is readily available, we can perform consistency checking with ease.

A critic might argue that Chierchia’s system, along with many others, could be easily extended to handle integration with the context. A formalism such as the one that appears in Gazdar 1979 [5] could work, perhaps. However, such systems are cumbersome compared to DRT, and even worse, are tacked-on to the theory as an afterthought. While this could be viewed as a mere formal or technical difficulty, the difficulty is really much deeper than that.

A good analogy can be found in mathematics with numeral systems. We could practice modern mathematics using Roman numerals. It’s possible that all of man’s mathematical achievements could have been made in this numeral system. However, Arabic numerals ended up becoming the dominant numerical notation; why? The reason why is because this form of representation is more amenable to elegant, simple algorithms.

Algorithms using Roman numerals were clunky and cumbersome. When men from the Italian peninsula traded with Arabs from North Africa, they were shocked at the ease and speed with which the ancient Arabs calculated sums. The numerals that the Arabs were using “got out of the way”; the notation they used was so well-designed that it sped up their computation. As Whitehead puts it (quoted by Hersh and Davis):

By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and, in effect, increases the mental power of the race. [3]

While one can argue that mathematics itself doesn’t depend on the numerals we use, mathematical research *does* depend on mathematical researchers. Similarly for science; a good theory is not just accurate, it is computationally

tractable and psychologically friendly. A well-organized, systematic notation not only speeds up the computational runtime of a theory, it also confers insights to researchers through its clarity.

## 7.2 Psychological Approaches to “When”

Another plus to DRT is its ability to, in time, address the question of “when”. As mentioned in the previous chapter, psychological theory will be needed to perform such a task. At the very least, belief representation and choice theory will be needed.

The nice thing about DRT is the layered component pioneered by Geurts 2003. Layers could be used to model beliefs of various kinds; common knowledge, beliefs about the speaker, background knowledge, etc. Even if Geurts’ extension did not exist, other attempts at modeling belief have been developed (Asher 1985).

DRT is amenable to proof calculi and the development of modal logic and belief representation. The notion of context here is key; we want to represent extra-linguistic elements of context that is of a purely mental nature. In this sense, we want to hold linguistic data of various kinds (the “discourse representation”) away from beliefs altogether. Again, LDRT is a natural choice for such a task.

## 7.3 Interactions with Deductive Closures

Consider the following case:

Only John drank beer.  
John drank some of the beer.  
There’s no beer left<sup>41</sup>.

In such a case, the “some but not all” implicature in “John drank some of the beer” should be cancelled by the deductively valid inference that John drank all of the beer. This is just another example of how content of various types interact differently with implicatures.

Of course, being able to detect cases like this require calculating some finite portion of a deductive closure. The typical size and content of this finite portion is an open question, along with the precise way in which it interacts with better-known discourse features. Solving this problem is a project in and of itself, and is something I am forced to shelve here.

---

<sup>41</sup>Credit goes to Clark Glymour for giving me a hard time with this one.

## 8 Conclusion

In this thesis I have provided a computational theory of embedded scalar implicature that accounts for the canonical cases. Furthermore, I have provided a recursive explication of the entailment operator restricted to scalar alternatives. This definition could be adapted to any entailment-based theory of scalar implicature that wishes to produce predictions in an automated fashion.

The theory presented in this thesis is not a complete one. One glaring inadequacy is its failure to deal with conditionals. In fact, the unpredictable behavior of implicature-like effects in the antecedents of conditionals raises doubts, in my mind, that a purely formal account of scalar implicature can do a complete job.

The question of how to solve conditional cases along with the question of how to completely solve the “when” problem can, in my opinion, best be addressed through models of choice and reasoning. In other words, I believe that we need a better understanding of how belief representation interacts with discourse representation in order to solve these issues. Given this belief, I think one of my theory’s virtues is its amenability to extension in these directions; DRT is a good framework for representing belief alongside discourse.

Though I have Gricean leanings, the theory in this thesis could be interpreted a variety of ways as it stands. I interpret it in a Gricean fashion, but those with views similar to Chierchia or Fox’s could interpret it in their favored way. In its current form, the theory is simply a computational model. It tells us *how* these implicatures are calculated when they are licensed; the theory is not currently equipped to distinguish whether or not they are the result of norm-governed reasoning or grammar.

The issue will be settled as research continues to progress. In my opinion, models of belief representation, choice, and reasoning will show a lot of promise in rounding out the shortcomings of theories such as mine. Once these are better developed, it could be more clearly seen whether or not viewing scalar implicature as a grammatical phenomenon produces better or worse predictions than viewing it as a pragmatic phenomenon.

Either way, the theory in this thesis brings us a step closer to a computational model capable of understanding scalar implicatures. Each computable solution to a problem in natural-language pragmatics brings us ever closer to an automated system capable of sophisticated linguistic performance. Furthermore, it gives us a picture of how we may operate as natural-language processors ourselves.



## A Scale Reversal

Here’s an alternate way to explain the “reversal” of entailment under negative polarity items that more clearly captures the notion. Consider an arbitrary sentence  $\varphi$  such that it contains a single occurrence of a term  $t$ , which is a member of a quantitative scale  $H$ .  $t$  is the only occurrence of a scalar term in  $\varphi$  and does not occur under the scope of a logical operator. Let  $\psi = \varphi[t'/t]$  where  $t' \in H$ .

We will now define a set,  $S$ .

- (1)  $\langle \varphi, \psi \rangle \in S$
- (2)  $\langle f(\varphi), f(\psi) \rangle \in S$  where  $f$  is any scope-taking operator.

We can divide  $S$  into two sets,  $S \uparrow$  and  $S \downarrow$ . Let  $S \downarrow$  be the set including all pairs  $\langle f(\varphi), f(\psi) \rangle \in S$  such that  $f$  is a monotone decreasing operator ( $\text{mon}\downarrow$ )<sup>42</sup>, and  $S \uparrow$  be its complement in  $S$ .

Now suppose that  $\varphi \models^A \psi$ . Given that this is the case, for any  $\langle \alpha, \beta \rangle \in S \uparrow$ ,  $\alpha \models^A \beta$ . On the other hand, for any  $\langle \alpha, \beta \rangle \in S \downarrow$ ,  $\beta \models^A \alpha$ . If we take any arbitrary pair in  $S$ , its position to the left or the right of the entailment operator depends upon whether it’s in  $S \downarrow$  or  $S \uparrow$ . This is what is meant by reversal; it refers to the behavior of different arbitrary alternative pairs.

## B Definitions and Notation

Let  $\varphi$  and  $\psi$  be any two expressions.

**DEFINITION 29.**  $\varphi \models^A \psi$  if and only if  $\varphi \models \psi$  and  $\psi \not\models \varphi$ . Read this “ $\varphi$  asymmetrically entails  $\psi$ ”.

**DEFINITION 30.**  $\varphi \equiv \psi$  if and only if  $\varphi \models \psi$  and  $\psi \models \varphi$ .

**DEFINITION 31.**  $\varphi \rightsquigarrow \psi$  if and only if  $\varphi$  conversationally implicates  $\psi$ .

**DEFINITION 32.**  $\varphi[t'_1/t_1, \dots, t'_n/t_n]$  is the expression with  $t'_i$  substituted in for  $t_i$ , where  $1 \leq i \leq n$ .

*For the next three definitions, let  $E$  be our domain of discourse.*

**DEFINITION 33.** A quantifier  $Q$  is *monotone increasing* ( $\text{mon}\uparrow$ ) if  $X \in Q$  and  $X \subseteq Y \subseteq E$  implies  $Y \in Q$  (i.e. for any set  $X \in Q$ ,  $Q$  also contains all the supersets of  $X$ .) [1]

**DEFINITION 34.**  $Q$  is *monotone decreasing* ( $\text{mon}\downarrow$ ) if  $X \in Q$  and  $Y \subseteq X \subseteq E$  implies  $Y \in Q$  (i.e. for any set  $X \in Q$ ,  $Q$  also contains all the subsets of  $X$ .) [1]

**DEFINITION 35.** A determiner  $D$  is *monotone increasing* (or *decreasing*) if it always gives rise to monotone increasing (or decreasing) quantifiers  $\|D\|(A)$ . [1]

---

<sup>42</sup>Defintion in Appendix B.



## C Autonomy, Determinacy, and Indeterminacy

This section will propose a distinction between theories that make use of non-constructive methods, and those that do not. The ones that avoid use of non-constructive methods we will call *autonomous*<sup>43</sup>. This section will argue for the virtues of autonomous theories and that, where possible, theorists should make the necessary assumptions to secure autonomy. The arguments for the virtues of autonomy will be used to motivate the original theory presented later in this thesis.

### C.1 Autonomy

As we alluded to above, autonomous theories are theories that do not use non-constructive methods. The term “non-constructive” is used here in a similar way to how it is used in mathematics. In math and logic, a non-constructive proof is the proof of an object’s existence without a method for its construction. We will call a set or function definition *non-constructive* if and only if it picks out a set, but without specifying a method for doing so. The use of such set and function definitions in theory is what we refer to when we say “non-constructive methods”.

The term “autonomy” has been selected due to an important property of theories that use purely constructive methods - they can make predictions without what we will call “intelligent intervention”. In other words, calculating a prediction is a simple matter of computation; there are no gaps which a human being must fill with her own linguistic competence.

Consider the following example, drawn from Sauerland’s theory of scalar implicature.

If  $\psi \in \text{ScalAlt}(\phi)$  and  $\psi \models \phi$  and not  $\phi \models \psi$ , then  $\neg K\psi$  is a *primary implicature* of  $\phi$ . [13]

Sauerland is, in essence, defining the following set:  $\{\neg K\psi \mid \psi \in \text{ScalAlt}(\phi) \wedge \psi \models^A \phi\}$ . While this specification is mathematically precise, it does not give us a method by which to decide whether or not  $\psi \models^A \phi$ . Barring such an analysis, a human being must “plug in” her intuitions to the predictive framework in order to get a result. As such, Sauerland’s theory is *non-autonomous*; it cannot make predictions on its own, but requires a human being to perform a non-trivial cognitive task. This is what is meant by “intelligent intervention”.

This stands in stark contrast to Gazdar’s method for the calculation of unembedded scalar implicatures. Gazdar’s method, beyond being purely formal, is also constructive. That is, the calculation of scalar implicatures, in his model, follows a complete, explicit method that requires no linguistic intuition or ability to execute.

One merely calculates the expression alternatives, which is a matter of syntactic substitution. Once this is done, we find the expression alternatives with

---

<sup>43</sup>This term has no relation to the concept of “autonomy” in the philosophy of psychology.

scalar terms that appear *to the left* of the original term on their respective scale. A “ $\neg K$ ” is appended to the front of these, and thus we have our scalar implicatures. Nowhere in this process do we require a human being to perform a sophisticated judgment. Once we have our Horn scales in front of us, the theory can make predictions on its own.

It’s worth pointing out, however, that Horn scales themselves are “given by intuition”. This is the case since every attempt at defining Horn scales, as of yet, admits of non-standard models. This does not blur the line between Gazdar’s autonomy and Sauerland’s lack thereof; the relevant notion for autonomy is intelligent intervention *mid-prediction*. Horn scales, as they are compiled before any predictions are made, are not an example of mid-prediction intervention.

The difference between pre and mid-prediction intervention is an important one. Language is ultimately arbitrary; there is no way to “construct” the basic facts of human language, they are merely contingent matters of fact. Thus, empirical generalizations (such as Horn scales or axioms for entailment) must conform to intuitions and behaviors which are, at the bottom, arbitrary. This does not mean that these intuitions and behaviors are not systematic, however; if they were not, then linguistic study would be impossible.

The important point is that a foundation that relies on non-constructive elements *pre-prediction* has no bearing on a theory’s autonomy. The important aspect is that given its set of relevant tools, the theory could calculate its predictions algorithmically with no outside help.

## C.2 Determinacy and Intuition

Later we will discuss the positive aspects of autonomous theories, aspects that should make autonomy attractive to theorists. This section, however, will discuss the negative aspects of *non-autonomous* theories. Avoidance of the potential complications of non-autonomy will be cast as an incentive for autonomy.

Sauerland’s account, as we saw above, is non-autonomous as it does not specify a method for sorting out entailment relations. Gazdar’s account does, but admittedly, Gazdar has a much easier task than Sauerland. Given the restriction he makes to consider solely unembedded cases, Gazdar can simply appeal to the ordering of the relevant Horn scale to determine entailment.

Since Sauerland deals with embedded cases, his account is on the hook to deal with arbitrarily complicated examples. For instance, it’s possible that the content of a disjunct is embedded under negation. Without a general account of how to sort out the entailment relations, his account is not autonomous. One way to think of it is we could not implement his account on a computer, as a human being has to sort out the entailment relations mid-prediction.

However, Sauerland’s account is precise. Intuition is determinate most examples. Given enough time, even with arbitrarily complex cases, humans should be able to determine the entailment relation (if there is any) between two scalar alternatives. As such, Sauerland’s theory, while it fails to provide us with a full method to make predictions, still does make a determinate prediction. We will call such a theory a *determinate* theory. All autonomous theories are de-

terminate in that they specify not only a result, but also a method for constructing that result. Some non-autonomous theories, such as Sauerland’s, are determinate as well. As we will see soon, there is a class of non-autonomous “indeterminate” that lack determinacy of prediction.

The reason why Sauerland’s theory is determinate, though non-constructive, is that his non-constructive set is sharply defined by intuition. In other words, for most cases, intuition is clearly settled on the matter. At any rate, for entailment, there seems to be a “correct” way of sorting it all out. Therefore, even in the absence of a precise method, human intuition settles the matter enough for Sauerland’s theory to make a determinate prediction.

This mid-predictive role of intuition easily goes unnoticed in sciences such as linguistics, where the descriptions are reflexive in nature. In other words, linguistic theory describes human abilities that we only have an implicit command over. What linguistic theory attempts to predict and describe is a set of behaviors humans are perfectly capable of predicting on their own, based on their implicit knowledge of language. The endeavor, then, is to make it explicit.

Making it explicit all at once is a monstrous task. Thus, there is a role for non-autonomous, determinate theory. It allows us to erect scaffolding, as theorists, for predictive frameworks. A theory of scalar implicature like Sauerland’s, then, is not committed to an analysis of entailment; to be on the hook for every interacting linguistic phenomenon would demand a complete linguistic theory for any explanation whatsoever. And insofar as Sauerland’s theory makes determinate, precise predictions, it is a good theory; he has managed to rope off the issues irrelevant to the matter at hand appropriately.

### C.3 Non-autonomous, Indeterminate Theories

In contrast to non-autonomous, determinate theories such as Sauerland’s are non-autonomous theories that do not make a clear prediction. We will call these theories *indeterminate* theories. Depending on how the theory’s mechanisms are made specific, it could result in quite different predictions. Because of this, indeterminate theories can be said to specify a class of possible theories rather than an actual theory itself.

A good example of a non-autonomous, indeterminate theory is Fox’s syntactic theory of scalar implicature. As Geurts (2009) points out, Fox’s theory proposes syntactic ambiguity but gives no indication of how to select amongst the potential options. As we’ve seen above, Fox’s theory posits a covert exhaustivity operator, which Geurts writes as “MONLY” (standing for “mute only”, as the operator works as an unarticulated “only”).

Consider the following sentence.

- (2) You may have an apple or a pear. [7]

Depending on whether or not MONLY takes wide scope or is embedded under the modal, we get the following two possibilities:

- (3) You MONLY [may have an apple or a pear].  
(4) You may [MONLY have an apple or a pear].

Thus so far for (2) we have three possible readings; either the MONLY is absent, it appears outside of the modal, or it appears inside the modal. But there are more possibilities present; Geurts points out that Fox’s “natural” reading of (2), which is that you may have an apple or a pear (but not both) is derived from the following representation:

(5) MONLY MONLY [you may have MONLY an apple or MONLY a pear].

It doesn’t matter that the order of ambiguities is so high; even a simple two-member ambiguity would suffice in making Fox’s theory indeterminate. As scalar implicatures can be cancelled or present, this means that every sentence, under Fox’s view, has at least two possible syntactic representations.

As Geurts observes, Fox’s theory requires a “module for selecting among the various, and often numerous, readings predicted to be available for sentences containing scalar expressions”. In other words, Fox has provided us with a non-constructive method, but we have no intuitions about covert MONLY operators to settle the matter. Thus, it is impossible to tell what his account would predict on any particular case; not only would a human be required to make the prediction, but a charitable one with an eye for the right result.

The selection module that Geurts calls for is not separable from Fox’s theory, as how it is specified will determine the content of his predictions. In many ways, the predictive success of Fox’s framework depends on how this is filled out, as we have no intuitions about covert operators to guide prediction. In lieu of this, the best we can do is specify a syntactic form that matches different readings of scalar expressions; such an account fails to predict.

## C.4 Virtues of Autonomy

We’ve already seen one reason to care about autonomy; when theories are non-autonomous, they run the risk of being indeterminate. Every autonomous theory, on the other hand, is determinate. What I’ve provided thus far is an argument to care about determinateness; indeterminate theories don’t make testable predictions. This section will provide reason to believe that autonomy is a step above sheer determinateness.

As opposed to indeterminate theories, both autonomous and non-autonomous determinate theories are testable. The autonomous theory has an extra perk, however; it can generate predictions in an automated fashion. An autonomous theory can run on a body of data and produce predictions across that data automatically. These predictions can then be tested versus our intuition and, in the case that it conforms to intuition across its inputs, provides us with a higher degree of confirmation for the theory.

If a theory is capable of the flexibility required to stand on its own and make predictions without the benefit of human oversight, it’s a more robust theory. The autonomous theory, while it may be stipulative, provides us with a *full, possible story*. In other words, the autonomous theory has a chance at being the correct, full description, whereas the non-autonomous theory leaves gaps.

Autonomous theories hold automatic prediction as more important than parsimony. In other words, assumptions may be required that make the theory overly stipulative (such as Gazdar's stipulation barring the handling of scalar terms under the scope of logical functors). However, ensuring autonomy ensures a theory that can be implemented. Even though Gazdar's theory does not account for all cases involving scalar implicature, it could be implemented for unembedded cases. Such an account has fruitfulness that Sauerland's account does not; it could serve to enrich our artificial intelligence with a more sophisticated understanding of human conversation.

Of course, autonomous theories do not need to be limited by their stipulations. They can also, with the help of extra assumptions, widen the scope of cases they can handle. The theory presented in this thesis serves as an example; axioms for entailment between scalar alternatives are given that would render Sauerland and Chierchia's accounts autonomous. Over time, the assumptions can either be relaxed or justified from first principles. The important notion, however, is that the theory is immediately deployable and practically capable of more robust testing.

Any determinate theory will do, but taking the view of autonomy sets the sights of the theorist in a different direction. As opposed to searching for the simplest explanation, regardless of whether or not it relies on human intuition to bridge the gaps, it advocates for the construction of a full solution regardless of the ontological commitments. Once the theory enjoys sufficient predictive success, we can take it in two quite different directions; either we can deploy it for practical good, or we can ratchet the assumptions down in an attempt to find a finer grained description of human linguistic ability.



## References

- [1] Jon Barwise and Robin Cooper. Generalized quantifiers in natural language. *Linguistics and Philosophy*, 4:159–219, 1981.
- [2] Genaro Chierchia. Scalar implicatures, polarity phenomena, and the syntax/pragmatics interface. In Adriana Belletti, editor, *Structures and Beyond*, volume 3. Oxford University Press, 2004.
- [3] Philip J. Davis and Reuben Hersh. *The Mathematical Experience*. Mariner Books, 1999.
- [4] Danny Fox. Free Choice and the Theory of Scalar Implicatures. In U. Sauerland and P. Stateva, editors, *Presupposition and Implicature in Compositional Semantics*, page 71120. Palgrave Macmillan, 2007.
- [5] Gerald Gazdar. *Pragmatics: Implicature, Presupposition, and Logical Form*. Academic Press, 1979.
- [6] Bart Geurts. Implicature as a discourse phenomenon. In Estela Puig-Waldmueller, editor, *Proceedings of Sinn und Bedeutung*, volume 11, pages 261–275. University Pompeu Fabra, Barcelona, 2006.
- [7] Bart Geurts. Scalar implicature and local pragmatics. *Mind and Language*, 24:51–79, 2009.
- [8] Bart Geurts and David I. Beaver. Discourse representation theory. *Stanford Encyclopedia of Philosophy* (<http://plato.stanford.edu/entries/discourse-representation-theory/>).
- [9] Bart Geurts and Emar Maier. Layered drt. *Ms, University of Nijmegen*, 2003.
- [10] Paul Grice. Logic and conversation. In Peter Cole and Jerry L. Morgan, editors, *Syntax and Semantics*, volume 3. Academic Press, 1975.
- [11] Julia Hirschberg. *A Theory of Scalar Implicature*. PhD thesis, University of Pennsylvania, 1985.
- [12] Laurence R. Horn. *A Natural History of Negation*. The University of Chicago Press, 1989.
- [13] Uli Sauerland. Scalar implicatures in complex sentences. *Linguistics and Philosophy*, 27:367391, 2004.
- [14] Mandy Simons. A gricean view on intrusive implicatures. In *Meaning and Analysis: New Essays on Grice*. Palgrave Studies in Pragmatics, Language and Cognition Series, (Forthcoming).