Modified numerals revisited: The case of *fewer than 4* and *between 4 and 8*

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Take-home messages

- The goal is to account for this generalization in interpreting **non-increasing quantifiers** (e.g., *few*, *fewer than 4*):

	distributive predicate	non-distributive predicate
upper-bound reading	✓	#
existential reading	#	✓

- Following Brasoveanu 2013, I analyze the semantic contribution of these quantifiers as **two-fold**:
  - A maximization operator that introduces the maximal set of entities that satisfies their restrictor and nuclear scope;
  - A post-suppositional-style cardinality constraint.
- In sentences with a non-distributive predicate, the post-suppositional requirement is checked at the DP level.

Data: The interplay between predicate type and quantifier interpretation

(1) ***Fewer than 4 boys smiled.***

✓ **Upper-bound reading**: The maximal number of smiling boys is fewer than 4.

 # Existential reading: A group of boys smiled and its cardinality is fewer than 4.

(2) ***Fewer than 4 boys lifted the piano together.***

 # Upper-bound reading: The maximal number of boys that lifted the piano together is fewer than 4.

✓ **Existential reading**: A group of boys collectively lifted the piano and its cardinality is fewer than 4.

(3) ***Between 4 and 8 boys lifted the piano together.***

 # Upper-bound reading; ✓ Existential reading

(4) ***Few boys smiled.***

✓ Upper-bound reading; # Existential reading

(5) **** Few boys lifted the piano together.*** (Solt 2007)

(6) ***A few boys lifted the piano together.*** (Solt 2007)

 # Upper-bound reading; ✓ Existential reading

Problems of pragmatics-based accounts

- **Pragmatic blocking constraint** (Buccola 2015, 2016):
  - If an LF ***Φ*** contains a numeral ***n***, then for any numeral ***m*** distinct from ***n***, substituting ***m*** for ***n*** in ***Φ*** must yield different truth conditions.
- **Lexical Maximality + Pragmatic blocking constraint**:

  ⟦fewer than 4⟧ = $\lambda P_{\langle dt\rangle}.\max(P) < 4$
- **Separate Maximality + Pragmatic blocking constraint**:

  ⟦fewer than 4⟧ = $\lambda P_{\langle dt\rangle}.\exists n[n < 4 \wedge P(n)]$

  n may be substituted by $n_{\max} : n_{\max} = \lambda P_{\langle dt\rangle}.[\max(P) = n]$
- ⟦***fewer than 4 boys smiled***⟧

  a. $\max(\lambda n.\exists x[|x| = n \wedge \text{boys}(x) \wedge \text{smile}(x)]) < 4$

  b. $\exists x[|x| < 4 \wedge \text{boys}(x) \wedge \text{smile}(x)]$ **blocked**
- ⟦***fewer than 4 boys lifted the piano together***⟧

  a. $\max(\lambda n.\exists x[|x| = n \wedge \text{boys}(x) \wedge \text{lift-piano}(x)]) < 4$ **?**

  b. $\exists x[|x| < 4 \wedge \text{boys}(x) \wedge \text{lift-piano}(x)]$ **not blocked**
- **Problems of pragmatics-based accounts:**
- 1. They over-generate (see also Buccola 2016):
  - ***Fewer than 4 boys can lift that piano.***

    Generalizing reading: there is a number ***n*** (***n*** < 4) s.t. a group containing ***n*** boys can collectively lift the piano.

    $\exists n[n < 4 \wedge \forall_{\text{gen}} x[|x| = n \wedge \text{boys}(x) \rightarrow \Diamond\text{lift-piano}(x)]]$

    ✓Separate Max (SMax), #Lexical Max (LMax)

    Unattested readings generated by LMax and SMax:

    a. Any individual boy can lift that piano.

    $\forall_{\text{gen}} x[|x| < 4 \wedge \text{boys}(x) \rightarrow \Diamond\text{lift-piano}(x)]$

    b. The maximal number of boys that can lift the piano collectively is ***n*** (***n*** < 4):

    $\max(\lambda n.\forall_{\text{gen}} x[|x| = n \wedge \text{boys}(x) \rightarrow \Diamond\text{lift-piano}(x)]) < 4$
- 2. They cannot explain the use of *few*: there is no number involved, and the pragmatic constraint cannot be used.
- 3. New challenging data:
  - ***Fewer than 4 groups lifted that piano together.***

Proposal: maximization + post-supposition (see Brasoveanu 2013)

- A two-fold semantic contribution:
  - A maximization operator: ***σ***

    $[\![\sigma x(\phi)]\!]^{\langle g,h\rangle}$ = True

    a. $[\![[x] \wedge \phi]\!]^{\langle g,h\rangle}$ = True

    We store in ***x*** those items that satisfy ***Φ***.

    b. There is no ***h'*** such that

    $[\![[x] \wedge \phi]\!]^{\langle g,h'\rangle}$ = True and $h(x) < h'(x)$
  - A cardinality constraint on ***x***.
- ⟦***fewer than 4 boys smiled***⟧

  $\sigma x(\text{boys}(x) \wedge \text{smile}(x)) \wedge |x| < 4$
- ⟦***fewer than 4 boys lifted the piano together***⟧
  - The use of collective predicates requires the agent to be a sum.
  - The sum operator requires a non-empty set to be its input, and it returns the sum of the items in the set.
  - Thus the sum operator in turn enforces the use of a silent ∃(or ι) at the DP level. I.e., with the use of collective predicates, the post-suppositional-style constraint is checked at the DP level.
  - ⟦***fewer than 4 boys***⟧ — a group of boys

    $= \textbf{sum}(\lambda x.\exists x[\sigma x[\text{boys}(x)] \wedge |x| < 4])$
- ⟦***fewer than 4 boys can lifted that piano***⟧

  = Groups of 'fewer than 4' boys can lift-piano.

  $= \forall_{\text{gen}} x[x = \textbf{sum}(\lambda y.\exists y[\sigma y[\text{boys}(y)] \wedge |y| < 4]) \rightarrow \Diamond\text{lift-piano}(x)]$

Selected references:

Brasoveanu, A. 2013. Modified Numerals as Post-Suppositions. *JoS*. | **Buccola, B. 2015**. *Maximality in the Semantics of Modified Numerals.* Ph.D. thesis. McGill. | **Buccola, B. 2016**. Severing maximality from *fewer than*: evidence from genericity. *SuB 20*. | **Solt, Stephanie. 2007.** *Few* and *fewer*. Snippets.