The number sensitivity of modal indefinites
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Abstract. In this study, I take up the interaction between modal flavor and the number morphology of existential wh-indefinites in Mandarin, a phenomenon first discussed in Lin (1998). I argue that there are three essential theoretical pieces to this interaction: (i) types of nominal reference (quantized vs. cumulative), (ii) degrees of modal variation (total vs. partial), and (iii) achieving modal variation by exhaustification. I show that while exhaustifying alternatives contributed by quantized wh-indefinites may give rise to either total or partial modal variation, exhaustifying alternatives contributed by cumulative wh-indefinites may only give rise to partial modal variation.

Keywords: alternatives, exhaustification, modal indefinites, plurality

1. Introduction

Indefinites that give rise to modal inferences are called modal indefinites. Well-known examples of modal indefinites include those introduced by determiners like English any (Kadmon and Landman 1993, Dayal 1998, Chierchia 2013, a.o.) German irgendein (Kratzer and Shimoyama 2002, a.o.,) Spanish algún (Alonso-Ovalle and Menéndez-Benito 2010) and Romanian vreun (Fäläus 2014). Take German irgendein indefinites as an example. They give rise to different modal inferences under the scope of epistemic modals and deontic modals (Aloni and Port 2010, Aloni and Franke 2013), as exemplified by (1) and (2) (taken from Aloni and Franke 2013). Specifically, in (1), the irgendein indefinite is in the scope of a deontic modal, and it gives rise to an inference that any man can be a marriage option for Mary; by contrast, in (2), the irgendein indefinite is in the scope of an epistemic modal, and it leads to an inference that signals speaker ignorance or indifference.

(1) Mary musste irgendeinen Mann heiraten.
‘Mary had to marry a man.’

~~ Any man was a permitted marriage option for Mary.

(2) Juan muss in irgendeinem Zimmer im Haus sein.
‘Juan must be in some room in the house’

~~ The speaker does not know or does not care about which room Juan is in.

Aloni and Franke (2013) called the inference in (1) ‘total variation inference,’ and the inference in (2) ‘partial variation inference.’ These terms are based on the Modal Variability Hypothesis, which attributes distinct degrees of modal variation to epistemic and deontic modals (see also Aloni and Port 2010). According to Aloni and Franke (ibid), deontic modals require total

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modal variation, but epistemic modals only require partial modal variation, where total and partial modal variations are defined below (Aloni and Franke 2013:110): 2

(3) a. Total (modal) variation: \( \forall x \phi(x) \)
All alternatives in the relevant domain qualify as a possible option.

b. Partial (modal) variation: \( \exists x \exists y (\phi(x) \land \phi(y) \land x \neq y) \)
More than one (but not necessarily all) alternatives in the relevant domain qualify as a possible option.

The inference in (1) clearly follows the definition of total variation. The inference in (2) is identified as partial variation, because it is felicitous in the ‘hide and seek’ scenario described by Alonso-Ovalle and Menéndez-Benito (2010).

(4) María, Juan, and Pedro are playing hide-and-seek in their country house. Juan is hiding. María and Pedro haven’t started looking for Juan yet. Pedro believes that Juan is not hiding in the garden or in the barn: he is sure that Juan is inside the house. Furthermore, Pedro is sure that Juan is not in the bathroom or in the kitchen. As far as he knows, Juan could be in any of the other rooms in the house.

In the above scenario, Pedro can felicitously utter (2), even though not all the rooms are epistemic possibilities for him—he knows that Juan is not in the bathroom or in the kitchen.

In this paper, I extend this line of research to include modal indefinites in Mandarin. I demonstrate that the distinction in total vs. partial modal variation has a reflex on the number morphology of modal indefinites. Adopting the Alternatives and Exhaustification approach outlined in Chierchia (2013), I show that total modal variation is not compatible with domain alternatives triggered by a modal indefinite that exhibit cumulative reference in the sense of Krifka (1989, 1998). This analysis is able to account for a long standing puzzle in Mandarin, i.e., a modal indefinite must occur with a numeral classifier under the scope of deontic modals (Lin 1998).

This paper is organized as follows. Section 2 introduces the core data and clarifies the correlation between modal flavor and degrees of modal variation. Section 3 provides an account for the interaction, which incorporates plurality into the framework of Alternatives and Exhaustification (Chierchia 2013, Fox 2007, a.o.). Section 4 concludes.

2. Modal indefinites in Mandarin

Mandarin makes use of \(wh\)-phrases, rather than a distinct determiner, to construct modal indefinites (Lin 1998, see also Li 1992). Consider the example in (5). This sentence makes an existential claim, but additionally conveys a modal inference that the speaker does not know

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2 As a terminological note, the term total (modal) variation is often taken to be synonymous with free choice, while the term partial (modal) variation is often simply referred to as modal variation, after Alonso-Ovalle and Menéndez-Benito (2010). I take modal variation to be a cover term for both total and partial modal variation in this paper.
which book(s) Zilu read. Due to the modal inference, the speaker of (5) may not identify
the book, whether by its name or its physical ostension, as suggested by the infelicity of the
‘namely’ continuation (see also Liao 2011: 8–9). The same infelicity is not shared by an ordi-
nary indefinite, which lacks the modal inference, as shown in (6).

(5) Zilu kenengE kan-le shenme shu. # Jiushi Halibote.
Zilu possibly read-ASP what book namely Harry.Potter
‘Zilu possibly read some book(s), namely *Harry Potter*.’

Zilu possibly read-ASP one-CL book namely Harry.Potter
‘Zilu possibly read a book, namely *Harry Potter*’

In addition, Mandarin wh-indefinites can also occur within the scope of deontic modals (sub-
scripted with D), as exemplified in (7). Intuitively, this sentence conveys that any book in a
contextual relevant set can be an option for Zilu.

(7) Zilu keyiD du yi-ben shenme shu.
Zilu can read one-CL what book
‘Zilu can read any book.’

Based on this set of data, Liao (2011) argues that wh-indefinites in Mandarin should be taken
to be modal indefinites. She proposes an analysis based on Alternatives and Exhaustification
(Chierchia 2006, 2013, Fox 2007) to capture the modal variation effect and the fact that modals
are required for these wh-indefinites to be well-formed.

Mandarin wh-indefinites are similar to German irgendein indefinites in their interactions with
modals with different flavors. Specifically, a wh-indefinite under an epistemic modal expresses
partial modal variation, but the same indefinite under a deontic modal expresses total modal
variation. This contrast is verified by the following contexts.

**Context 1: Total variation**

*John and Mary were planning a trip to Europe. John suggested:*

(8) Women keyiD qu yi-ge Ouzhoude shenme chengshi.
we can go one-CL European what city
‘We can go to an European city (whichever will work).’

*Mary knew that they could only visit an European city where they had a friend to stay with. Since they only had a friend in London and a friend in Berlin, she added.*

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3Mandarin lacks plural morphology, so depending on the context Zilu could have read one or more books. This property of bare noun phrases is central to the discussion of this paper.

4I chose not to narrow down the domain to a single city so that it can be clear that the objection is not to the lack of modal variation, but to how freely one can choose among the cities in Europe.
(9) Bu dui. Women zhi keyi qu Lundun huo Bolin.  
no right we only can visit London or Berlin.  
‘No, we can only go to London or Berlin.’

Mary’s response is felt to be felicitous in this context. Under the deontic modal, which requires total modal variation, the sentence in (8) expresses that all the cities in Europe that are in the domain are open options for their visit. This total modal variation inference can be challenged by a denial, as shown in (9).

**Context 2: Partial variation**

*John and Mary knew that Peter went on a trip last week, but they did not know where he went. They were talking about where Peter could have gone. John suggested:*

(10) Ta keneng qu-le yi-ge Ouzhoude shenme chengshi.  
he possibly go-ASP one-CL European what city  
‘He could have gone to an European city.’

*Mary knew that Peter stayed with a friend during his trip, and Peter only had two overseas friends, one in London and one in Berlin. So, she added:*

(11) #Bu dui. Ta zhi keneng qu-le Lundun huo Bolin.  
not right he only possible go-ASP London or Berlin  
‘No, he could only have gone to London or Berlin.’

This time, the response is felt to be infelicitous. Although modal variation is still perceived with (10), partial modal variation is enough for the epistemic environment. This means that not necessarily all European cities in the domain are taken into consideration. In this case, challenging total modal variation is not felicitous, because no total modal variation inference is generated. I argue that this is the reason why the response is odd.5

Although Mandarin *wh*-indefinites have modal variation inferences similar to German *irgend-\*dein* indefinites, they show a lesser known and intriguing interaction between the types of licensing modals and numeral classifiers in *wh*-phrases. In particular, as observed by Lin (1998), modal *wh*-indefinites under deontic modals must be accompanied by a numeral classifier (underlined), as evidenced by (12), whereas epistemic modals are compatible with *wh*-phrases

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5A limitation of this diagnostic is that the contrast fails to hold between deontic and epistemic necessity modals. Using English as an example, we can see that (ib) is way more natural as a objection to (ia) than (ii b) is to (iia):

(i)  a. A: John may read a book from the shelf.  
   b. B: No. He may not read *Harry Potter*.  

(ii) a. A: John must read a book from the shelf.  
    b. B: # No. He may not read *Harry Potter*.  

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with or without numeral classifiers, as illustrated by the well-formedness of (13).\(^6\) This generalization is true regardless of the modal force.

(12) Zilu yao\(_D\)/keyi\(_D\) kan *(yi-ben) shenme shu.
Zilu must/can read one-CL what book
‘Zilu must/can read some book(s) (I don’t care which).’

(13) Zilu kending\(_E\)/keneng\(_E\) kan-le (yi-ben) shenme shu.
Zilu certainly/possibly read-ASP one-CL what book
‘Zilu possibly read some book(s) (I don’t know which).’

Represented configurationally, the classifier puzzle has the following form:

(14) a. Epistemic modal . . . (NUM-CL) wh-phrase
b. Deontic modal . . . *(NUM-CL) wh-phrase

Combining the terminology used in the literature of bare noun phrases and modal indefinites, I refer to existential wh-indefinites without numeral classifiers as ‘bare wh-indefinites’. In contrast, I refer to existential wh-indefinites with numeral classifiers as ‘non-bare wh-indefinites’.

In this paper, I develop an account for the classifier puzzle, couched primarily in the Alternatives and Exhaustification approach (Chierchia 2006, 2013, Liao 2011, a.o.), but incorporates insights from research on nominal reference (Link 1983, Krifka 1989, a.o.). Assuming that wh-indefinites are translated as existential quantifiers (Karttunen 1977), I argue that it is possible to tease apart bare and non-bare wh-indefinites in terms of their domain restrictions. The former are restricted by nominal predicates which denote sets containing atomic and plural individuals closed under sum formation. Following the classical terminology, we say that this type of predicates exhibit cumulative reference. The latter are restricted by nominal predicates whose denotations are not closed under sum formation. These predicates are known to exhibit quantized reference. The cumulativity-quantization distinction goes back at least to Link (1983) (who credits Quine 1960 for cumulativity) and is discussed in detail by Krifka (1989).

In addition, I adopt an emerging view from the literature of modal variation that epistemic modals and deontic modals have different free choice potentials (see Aloni and Port to appear b, Fäläus 2014). Once this contrast is in place, we can see that wh-indefinites under epistemic modals express partial modal variation, while those under deontic modals express total modal variation.

After incorporating the distinctions between cumulativity vs. quantization and between total vs. partial modal variation, an analysis based on Alternatives and Exhaustification reveals the source of the classifier puzzle. Specifically, deriving total modal variation with a modal indefinite restricted by a cumulative predicate results in a contradictory inference. This contradiction can be avoided in two ways, either by replacing a cumulative predicate (i.e., a bare

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\(^6\)Some aspect markers on verbs that are available under epistemic modals are not so under deontic modals. Le is such an example. At this stage, it’s not clear to me whether this difference in aspect marking is related to the classifier puzzle.
wh-indefinite) with a quantized predicate (i.e., a non-bare wh-indefinite), or by lowering the degree of modal variation. The former explains why numeral classifiers can help deontic modals ‘license’ wh-indefinites. The latter accounts for the optionality of numeral classifiers with wh-indefinites in epistemic modal environments.

3. An Alternatives and Exhaustification approach to modal variation

3.1. Key ingredients

My proposal is based on the Alternatives and Exhaustification approach initiated by Chierchia (2006, 2013) and Fox (2007), and especially follows the extension of these studies to Mandarin wh-indefinites by Liao (2011). Following these studies, a modal indefinite is taken to be an implicature trigger with a bi-dimensional denotation. An exhaustification operator makes use of this bi-dimensional denotation, giving rise to a modal inference and conditioning the distribution of the modal indefinite.

Concretely, a wh-indefinite in the form of *shenme shu* ‘what book’ is a generalized quantifier with a covert quantificational determiner ∃ (cf. Karttunen 1977), as shown in (15a). Following Hamblin (1973) and recent works on Alternative Semantics (e.g., Kratzer and Shimoyama 2002), I take this wh-NP to denote a set of alternative individual books. For instance, in a model with two books a and b, this wh-NP denotes the set {a, b}, as illustrated in (15b).

(15) a. [DP ∃ [NP what book]]
   b. [[what book] = {x | bookw(x)}] = {a, b}

Combining the existential operator with the wh-NP gives rise to the ordinary denotation of the wh-indefinite, which is a generalized existential quantifier, as shown in (16a). In addition, it has a special, focus-like denotation, which is a set of alternatives to the ordinary denotation (16b).

(16) a. [DP] = λQ.λw.∃x[x ∈ [[what book] ∧ Qw(x)]]
   b. [DP]A = {λQ.λw.∃x[x ∈ D′ ∧ Qw(x)] : D′ ⊂ [[what book] ∧ D′ ≠ Ø} ∪
                   {λQ.λw.∀x[x ∈ [[what book] → Qw(x)]}

The alternative set consists of two different kinds of alternatives. The first one is domain alternatives. They are alternatives constructed from non-empty proper subsets of a quantificational domain, as shown in (17a). If the domain contributed by [[what book] has two atomic books, as suggested by (15b), then the set of domain alternatives has two members, each a proper subset of this domain, as exemplified in (17b).

(17) a. [DP]DA = {λQ.λw.∃x[x ∈ D′ ∧ Qw(x)] : D′ ⊂ [[what book] ∧ D′ ≠ Ø}
   b. {D′ : D′ ⊂ {a, b}} = {{a}, {b}}

The second variety is the scalar alternative. Following Chierchia (2013), it is determined by replacing existential quantification in the ordinary denotation with universal quantification.\(^7\)

\(^7\)Alonso-Ovalle (2008) has offered a different mechanism for deriving scalar alternatives, which crucially relies...
These two types of alternatives have to be operated on by an alternatives-sensitive operator, often abbreviated as $\mathbb{O}$ (after only, or $\mathbf{Exh}$ (after exhaustification)). Based on Chierchia (2013), I offer the following lexical entry for $\mathbb{O}$:

$$(18) \begin{align*}
\text{a. } & \llbracket \mathbb{O} S \rrbracket = \lambda w[\llbracket S \rrbracket(w) \land \forall q \in \llbracket S \rrbracket^{\text{Exh-Alt}}(q \rightarrow \lambda w'[\llbracket S \rrbracket(w') \subseteq q]) \\
\text{b. } & \llbracket S \rrbracket^{\text{Exh-Alt}} = \{\mathbb{O}_{\text{IE}}(p) : p \in \llbracket S \rrbracket^{\Lambda}\} 
\end{align*}$$

This is often known as a ‘recursive’ exhaustification operator. When applying to a sentence $S$, it first applies pointwise to the set of alternatives of $S$, generating a set of pre-exhaustified alternatives, as done in (18b). Note that pre-exhaustification is subject to a condition called ‘Innocent Exclusion’ (Fox 2007, see also Sauerland 2004), which is defined below:

$$(19) \begin{align*}
\text{a. } & \text{IE-ALT}_p = \cap\{X \subseteq ALT : \text{CONS}(p \land \neg \cap X) \land \forall q \in ALT[(\text{CONS}(p \land \neg \cap X \land \neg q) \rightarrow q \in X]\}, \text{where CONS}(p) = p \text{ is consistent} \\
\text{b. } & \mathbb{O}_{\text{IE}}(p) = p \land \forall q \in ALT[(q \in \text{IE-ALT}_p \land p \not\subseteq q) \rightarrow \neg q] 
\end{align*}$$

The role of Innocent Exclusion in pre-exhaustification is to avoid excluding alternatives that may lead to contradictory inferences. Each pre-exhaustified alternative that is not entailed by the assertion is negated, helping to generate a strengthened assertion.  

With the mechanism to derive alternatives and the operator that operates on alternatives, this approach derives the ill-formedness of $wh$-indefinites when they are not in the scope of a modal as a logical contradiction. Since explaining the modal requirement is not the primary concern of this paper, I refer the reader to Liao (2011) and Chierchia (2013) for detailed discussions on this point. How modal variation is achieved can be found in Section 3.3.

3.2. Pluralities

Since the seminal work of Link (1983), it has been well accepted into the semantic literature that (i) the domain of individuals ($D_e$) consists of both atomic and plural individuals, (ii) plural individuals are sums of atomic individuals or sums of other plural individuals, and (iii) $D_e$ is closed under sum formation. For example, a domain with three atomic individuals $a$, $b$, and $c$ have the following mereological structure ($\oplus$ is the sum formation operator; each line represents a partial order):

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8Two notes on the exhaustification operator. First, it is based on Chierchia (2013) but differs from it in having the exhaustification of scalar alternatives and the exhaustification of domain alternatives merged, for the purpose of simplification. Second, it differs from Fox’s (2007) exhaustification operator in being able to lead to contradictions. It is worth noting that whether an exhaustification operator with an innocent exclusion component should be contradiction-free is subject to debate. For instance, while Fox (2007) argues for the desirability of a contradiction-free exhaustification operator, Gajewski (2012) and Chierchia (2013) point out a few merits for allowing an exhaustification operator to derive contradictions. For Chierchia (2013), one of these merits is that it explains why modal indefinites have a restricted distribution in many languages: they are illegitimate precisely when contradictory inferences are induced.

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Following Link (1983), Schwarzschild (1996) and many others, I assume that noun phrases with distinct number morphologies have different denotations. Take noun phrases in English as an example. A singular count noun like book denotes a subset of $D_e$, which contains only atomic books. For instance, if a and b in $D_e$ are books, then the denotation of book is as shown in (20a). By contrast, a bare plural noun like books denotes as its extension a set involving the atomic books as well as their sums, as in (20b).

$$(20) \begin{align*}
\text{a. } \lbrack \text{book}\rbrack &= \{a, b\} \\
\text{b. } \lbrack \text{books}\rbrack &= \{a, b, a \oplus b\}
\end{align*}$$

According to Krifka (1989) (see also Quine 1960, Link 1983), these two types of predicates differ in terms of nominal reference. A predicate has cumulative reference iff, when it holds of two distinct individuals, it also holds of their sums (Krifka 1989:78, D12). Obviously, the bare plural books has cumulative reference, since the sum of any two books still belongs to the denotation of books. On the contrary, a predicate has quantized reference iff, when it holds of an individual, it doesn’t hold of any proper subparts thereof (Krifka 1989:78, D14). The singular count noun book has quantized reference, since the sum of a and b is not in the denotation of book. Likewise, in English predicates with numerals like two books are also quantized (despite being plural), because a collection of two units of two books results in four books, rather than two books.

Chierchia (1998) argues that Mandarin bare nouns, which lack plural morphology, share the same type of denotation with English bare plurals. In other words, the denotation of shu ‘book’ is comparable to that of books (see also Yang 2001; Jiang 2012), ignoring the implicature associated with the plural morphology (Krifka 1989, Sauerland 2003). This view, which is rather standard in the current literature, meets the empirical requirement that shu may refer to one or more books in Mandarin. Hence, we can conclude that Mandarin bare nouns exhibit cumulative reference. To create quantized predicates, the use of a numeral classifier is needed. In this regard, the function of numeral classifiers is very similar to the function of numerals in English—they both create quantized predicates. It is easy to demonstrate that nouns with numeral classifiers are quantized: the sum of two units of yi-ben shu ‘one book’ is not yi-ben shu, but liang-ben shu ‘two books’.

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9In the neo-Carlsonian approach, bare nouns are kind terms and numeral classifiers are lexical predicativizers that help bring a kind term to its predicate use (Krifka 1995, Chierchia 1998, Yang 2001). This view is entirely compatible with the cumulativity-quantization distinction defended here, as long as we bring in the type-shifting that are part and parcel in the neo-Carlsonian approach. However, to avoid complicating the semantic composition, I omit the derivation of bare nouns from kind terms to predicates and assume that bare nouns start out as predicates.
I argue that the cumulativity-quantization distinction can be extended to *wh*-indefinites. In particular, bare *wh*-indefinites are cumulative predicates while non-bare *wh*-indefinites are quantized predicates. Recall that a *wh*-phrase provides the quantificational domain for a covert existential quantifier (see Section 3.1). As shown in (21), the existential determiner quantifies over individuals restricted by *books*, which is cumulative. Assume that there are two atomic books, *a* and *b*. The denotation of *shenme shu* ‘*what books’* is a set of alternative books closed under sum formation, as illustrated in (21a). The bi-dimensional denotation of the *wh*-indefinite is derived as (21b) and (21c).

(21)  
a.  \[ \text{\{what books\}} = \{x \mid \text{books}_w(x)\} = \{a, b, a \oplus b\} \]

b.  \[ \exists \text{what books} = \lambda \lambda. \exists x[x \in \text{\{what books\}} \land Q_w(x)] \]

b. \[ \exists \text{what books} \lambda. \exists x[x \in \text{\{what books\}} \land Q_w(x)]: D' \subset \text{\{what books\}} \}

\[
\{ \lambda \lambda. \exists x[x \in \text{\{what books\}} \land Q_w(x)] \}
\]

After putting the bare *wh*-indefinite in a sentential context, the bi-dimensional meaning in (22) is obtained (the propositional correlates of the individuals are set in boldface). The underlined parts represent the extra contribution of the cumulative nominal predicate after plurality is brought into the picture.

(22)  
a.  \[ \text{Zilu read what books} = a \lor b \lor a \oplus b \]

b.  \[ \text{Zilu read what books} = \lambda \lambda. \exists x(x \in D' \land Q_w(x)) \}

\[
\{ \lambda \lambda. \exists x(x \in \text{\{what books\}} \land Q_w(x)) \}
\]

Note that the assertion in (22a) has an additional disjunct, contributed by the plural individual in the domain (21a). Since the domain alternatives are derived from the proper subsets of the domain, the addition of the plural individual to the domain has non-trivial effects on the set of domain alternatives. Specifically, there is a weakest alternative, i.e., \( a \lor b \), which is truth-conditionally equivalent to the assertion \( a \lor b \lor a \oplus b \). In addition, there is a strongest alternative, in this case \( a \oplus b \), which is truth-conditionally equivalent to the scalar alternative \( a \land b \), when lexical distributivity is assumed. Throughout this paper, I assume lexical distributivity and hence the equivalence of the maximal alternative and the scalar alternative. Whenever the maximal alternative is present, I suppress the scalar alternative.

Now, I turn to non-bare *wh*-indefinites, i.e., those with numeral classifiers. Like bare existential *wh*-phrases, a covert determiner \( \exists \) is still posited for this type of expression. The structure of *yi-ben shenme shu* ‘*one-CL what books’* is (23).

(23)  \[ \text{\{DP \exists [NP one-CL what books]}} \]

Just like (21a), *shenme shu* ‘*what books’* denotes a set of alternative individual books including both atomic and plural individuals. This is repeated in (24a). A numeral classifier serves as a function that can select a subset from a set. The denotation of the numeral classifier under consideration is formulated as in (24b). It takes the set denoted by *what books* as an argument and returns a set whose only members are those in \([\text{what books}]\) and have one atomic part.\(^{10}\) As shown in (24c), the denotation of *NP* essentially excludes the maximal individual from

\(^{10}\)Following the notation in Landman (2004), the function \( \text{ATOM} \) maps an individual to its atomic parts.
what books], giving rise to a quantized predicate. Finally, the denotation of DP is represented in (24d) and the set of its alternatives is shown in (24e).\footnote{I have ignored the potential scalar alternatives associated with the numeral.}

\begin{equation}
\begin{aligned}
\text{what books} &= \{x \mid \text{books}_w(x)\} = \{a, b, a \oplus b\} \\
\text{one-CL} &= \lambda D \{x \mid x \in D \land |\text{ATOM}(x)| = 1\} \\
\text{NP} &= \{x \mid x \in \text{what books} \land |\text{ATOM}(x)| = 1\} \\
\text{DP} &= \lambda Q.\lambda w.\exists x [x \in \text{what books} \land |\text{ATOM}(x)| = 1 \land Q_w(x)] \\
\text{DP} &A = \{\lambda Q.\lambda w.\forall x [x \in \text{what books} \land |\text{ATOM}(x)| = 1 \land Q_w(x)] : D' \subset \text{what books}\} \cup \\
&\{\lambda Q.\lambda w.\forall x [x \in \text{what books} \land |\text{ATOM}(x)| = 1 \rightarrow Q_w(x)]\} \\
\end{aligned}
\end{equation}

The difference between this example and the case with a bare \textit{wh}-indefinite is most clearly seen in a sentential context. Consider a sentence with a non-bare \textit{wh}-indefinite:

\begin{equation}
\begin{aligned}
\text{Zilu read one-CL what books} &= a \lor b \\
\text{Zilu read one-CL what books} & A = \{a, b, a \land b\} \\
\end{aligned}
\end{equation}

A noticeable difference is that the maximal individual \(a \oplus b\) is no longer part of the ordinary denotation. Relatedly, it is also absent in the set of alternatives.

3.3. Deontic modals: contradiction and repairing

This section answers the first part of the classifier puzzle, namely, why deontic modals militate against bare \textit{wh}-indefinites. Consider the following ill-formed sentence:

\begin{equation}
\begin{aligned}
\text{*Zilu keyi}_D \text{ kan shenme shu.} \\
\text{‘Zilu may read what book} \\
\text{‘Zilu may read some book(s) (I don’t care which).’} \\
\end{aligned}
\end{equation}

In the Alternatives and Exhaustification approach, the interaction of epistemic modals and modal indefinites is captured by recursively applying the exhaustification operator \(\ominus\) (Fox 2007, Liao 2011, Chierchia 2013). Therefore, the LF structure of (27) can be represented as follows:

\begin{equation}
\begin{aligned}
\text{*[IP4 } \ominus [\text{IP3 may}_D [\text{IP2 } [\text{DP } \exists [\text{what books}]]_1 \text{[IP1 Zilu [VP read } t_1]]]]]] \\
\text{The bi-dimensional denotation of IP2 is equivalent to example (22), i.e., (28). Then, the denotation of IP3 incorporates the contribution of the modal ([} is a shorthand for a deontic possibility modal), as illustrated in (29).}
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
\text{IP2} &= a \lor b \lor a \oplus b \\
\text{IP2} &A = \{a \lor b, a, b, a \oplus b, a \land b \land a \oplus b\} = \{a, b, a \lor b, a \oplus b\} \\
\end{aligned}
\end{equation}
(29) a. \[ \text{IP3} = \{ \phi (a \lor b \lor a \oplus b) \} \]
b. \[ \text{IP3}^\land = \{ \phi (a \lor b), \phi a, \phi b, \phi (a \oplus b) \} \]

Recall that combining \( \bigcirc \) and IP3 requires the affirmation of the assertion of IP3 and the negation of all the pre-exhaustified alternatives of IP3. The assertion of IP3 is given in (30a) and the set of pre-exhaustified alternatives is given in (30b) and elaborated in (30bi)–(30biv). Note in (30biv) that pre-exhaustifying the maximal alternative returns itself, as this alternative is the strongest member in the set.

(30) a. \[ \text{IP3} = \{ \phi (a \lor b \lor a \oplus b) \} \]
b. \[ \text{IP3}^\text{Exh-Alt} = \{ \bigcirc \phi (a \lor b), \bigcirc \phi a, \bigcirc \phi b, \bigcirc \phi (a \oplus b) \} \]
   (i) \[ \bigcirc \phi (a \lor b) = \phi (a \lor b) \land \neg \phi (a \oplus b) \]
   (ii) \[ \bigcirc \phi a = \phi a \land \neg \phi b \land \neg \phi (a \oplus b) \]
   (iii) \[ \bigcirc \phi b = \phi b \land \neg \phi a \land \neg \phi (a \oplus b) \]
   (iv) \[ \bigcirc \phi (a \oplus b) = \phi (a \oplus b) \]

When \( \bigcirc \) combines with IP3 to give rise to the meaning of IP4, an undesirable inference is generated:

(31) \[ \text{IP4} = \phi (a \lor b \lor a \oplus b) \land \phi a \rightarrow (\phi b \lor \phi a \oplus b) \land \phi b \rightarrow (\phi a \lor \phi (a \oplus b)) \]

\[ \land \phi (a \lor b) \rightarrow \phi (a \oplus b) \land \neg \phi (a \oplus b) \]

It is undesirable as the combination of the underlined conjuncts, derived from the weakest and the strongest alternative, leads to the following inference, which contradicts the assertion:

(32) \[ \neg \phi (a \oplus b) \land (\phi (a \lor b) \rightarrow \phi (a \oplus b)) = \neg \phi (a \lor b) \land \neg \phi (a \lor b) \]

The contradiction can be repaired by adding a numeral classifier to the \( wh \)-phrase. In other words, replacing the bare \( wh \)-indefinite in (26) with a non-bare one, as shown in (33), can avoid the undesirable result in (32).

(33) Zilu \( keyi_D \) kan \( yi-ben \) \( shenme \) shu.
    Zilu may read one-\( CL \) what book
    ‘Zilu may read some book (I don’t care which).’

As exemplified in (24), repeated in (34), the denotation of the \( wh \)-phrase remains unchanged, denoting a set of books closed under sum formation, and the numeral classifier is a subset selection function, applying to the \( wh \)-denotation and collecting those members with only one atomic part. The resulting denotation (34c) is the set \{\( a, b \} \), which is not cumulative. After applying the existential closure operator, the bi-dimensional denotation of the DP containing one-\( CL \) \( what \) \( books \) is given in (34d) and (34e).
3.4. Epistemic modals: partial variation

This section is devoted to the account of why epistemic modals are compatible with bare *wh*-indefinites as well as non-bare ones. Let us begin with bare *wh*-indefinites. Consider the following sentence, whose LF is given in (39).

(38) Zilu **keneng** kan-le **shenme** shu.
Zilu possibly read-ASP what book
‘Zilu possibly read some book(s). (I don’t know which).’

(39)  
**[IP4 ⊗ [IP3 possibly [IP2 [DP ∃ [what books]]]1 [IP1 Zilu [VP read t1]]]]**
If we derived the denotation of IP2 in the same way as IP2 in (27), we would get the same contradiction. However, this undesirable result can be avoided if the modal variation property of epistemic modals is taken into consideration. Specifically, as shown in Section 1, epistemic modals require partial modal variation, whose definition is repeated in (40).

\[
\text{(40) Partial (modal) variation: } \exists x \exists y (\Box \phi(x) \land \Box \phi(y) \land x \neq y)
\]

More than one (but not necessarily all) alternatives in the relevant domain qualify as a possible option.

Therefore, even though we only consider singleton subdomains generated by \(wh\)-indefinites, as defined in (41), partial modal variation is still achieved.

\[
\text{(41) a. } \exists \lambda Q. \lambda w. \exists x (x \in D' \land Q_w(x)) : D' \subset \exists \lambda Q. \lambda w. \exists x (x \in D' \land Q_w(x))
\]

So the set \(\{a, b\}\) does not count as a singleton subdomain, it plays no role in the derivation of singleton domain alternatives. As a consequence, no weakest alternative is created, despite the presence of domain cumulativity. The denotations of IP2 and IP3 in (39) can be derived as below (\(\Box\) stands for epistemic possibility modals):

\[
\text{(42) a. } [IP2] = a \lor b \lor a \oplus b
\]

\[
\text{b. } [IP2]^A = \{a, b, a \oplus b\}
\]

\[
\text{c. } [IP3] = \Box (a \lor b \lor a \oplus b)
\]

\[
\text{d. } [IP3]^A = \{a, b, a \oplus b\}
\]

Recursive exhaustification makes use of the pre-exhaustified alternatives of IP3, as given in (43a). After affirming the assertion of IP3 and negating all the pre-exhaustified alternatives, the modal variation implicature is derived, i.e., MV implicature in (43b).

\[
\text{(43) a. } [IP3]^{\text{Exh-Alt}} = \{\Box \Box a, \Box \Box b\}
\]

\[
\text{b. } [IP4] = \Box (a \lor b \lor a \oplus b) \land \Box (a \oplus b) \land \Box a \rightarrow (\Box b \lor \Box a \lor b) \land \Box a \rightarrow (\Box a \lor \Box a \lor b)
\]

As a result, we derive a modal variation effect that holds true of \(a\) and \(b\), namely, a subset of the the individuals in the domain. This is essentially how partial modal variation is defined, according to studies like Alonso-Ovalle and Menéndez-Benito (2010), Chierchia (2013) and Fäläs (2014).
Under the scope of epistemic modals, non-bare \textit{wh}-indefinites do not lead to a contradiction, either. Take as an example the following sentence, whose LF structure is represented as (45).

(44) Zilu \textit{keneng}\textsubscript{E} kan-le yi-ben \textit{shenme} shu.
    ‘Zilu possibly read-ASP one-CL what book (I don’t know which)’

(45) \[\text{IP}_4 \Box \text{IP}_3 \text{possibly} [\text{IP}_2 \text{one-CL what book}] [\text{IP}_1 \text{Zilu[VP read } t_1\text{]]]]\]

Similar to what we have discussed in the last section, the numeral classifier excludes the maximal individual from the domain given by the \textit{wh}-phrase, as shown by (24) and repeated in (46a). Hence, the denotations of IP2 are derived as (46b) and (46c) (see also (25)).

(46) a. \[\text{one-CL what books} = \{a, b\}\]
    b. \[\text{IP}_2 = a \lor b\]
    c. \[\text{IP}_2 \land = \{a, b, a \land b\}\]

Adding the epistemic modal results in (47), which is the denotation of IP3. Exhaustification makes use of the denotation of IP3 and the set of pre-exhaustified alternatives in (47b).

(47) a. \[\text{IP}_3 = \Diamond (a \lor b)\]
    b. \[\text{IP}_3^{\text{Exh-Alt}} = \{\Diamond \Diamond a, \Diamond \Diamond b, \Diamond \Diamond (a \land b)\}\]

Finally, exhaustification affirms the assertion of IP3 and negates all the pre-exhaustified alternatives not entailed by the assertion, generating the following inference, which is free of contradiction:

(48) \[\text{IP}_4 = \Diamond (a \lor b) \land \neg \Diamond (a \land b) \land \Diamond a \rightarrow \Diamond b \land \Diamond b \rightarrow \Diamond a \land \neg \Diamond a \land \neg \Diamond b\]

4. Conclusion

In this study, I have argued that the interaction between modal flavors and the number morphology of existential \textit{wh}-indefinites in Mandarin should be understood along the lines of Alternatives and Exhaustification, as long as nominal references are allowed to play a role in structuring the domain of alternatives. In particular, total modal variation, facilitated by deontic modals, is shown to be only compatible with existential \textit{wh}-indefinites with quantized reference. This is why numeral classifiers are required in existential \textit{wh}-indefinites in a deontic environment. On the other hand, partial modal variation, allowed by epistemic modals, is shown to be compatible with both existential \textit{wh}-indefinites with quantized reference and cumulative reference, hence explaining why numeral classifiers are optional with existential \textit{wh}-indefinites in an epistemic environment.
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