A uniqueness puzzle: *How many*-questions and non-distributive predication[1]

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Abstract. We discuss a novel observation about the meaning of *how many*-questions, viz. a uniqueness implication that arises in cases that feature non-distributive predicates, such as *How many students solved this problem together?*. We attempt an analysis of this effect in terms of Dayal's (1996) *Maximal Informativity Presupposition* for questions. We observe that such an analysis must be reconciled with the unexpected absence of uniqueness implications in cases where the non-distributive predicate appears under a possibility modal. We explore two possible solutions: (i) the postulation of a scopally mobile maximality operator in degree questions of the sort proposed in Abrusán and Spector (2011); (ii) the proposal that the informativity to be maximized is based on pragmatic, contextual, entailment rather than semantic entailment. We explain why neither solution is satisfactory. We also observe that a Maximal Informativity Presupposition fails to capture uniqueness implications in *how many*-questions with predicates that are weakly distributive in the sense of Buccola and Spector (2016), such as *How many students in the seminar have the same first name?*. We conclude that uniqueness implications in *how many*-questions must have a source that is independent of Dayal's (1996) Maximal Informativity Presupposition.

Keywords: *how many*-questions, uniqueness presuppositions, non-distributive predication, Maximal Informativity Presupposition.

1. Introduction

Recent work posits that questions come with a *Maximal Informativity Presupposition* (MIP, Dayal 1996). The MIP is stated informally in (1), where an answer is to be understood as a member of the question extension under the Hamblin/Karttunen analysis of questions, the set of so-called Hamblin/Karttunen answers (Hamblin 1973, Karttunen 1977).

(1) Maximal Informativity Presupposition (MIP)
A question presupposes that it has a true answer that semantically entails any other true answer.

Dayal (1996) proposed the MIP in order to capture the uniqueness presupposition carried by singular *which* questions. It has since played a prominent role in the analysis of so-called weak islands (Fox and Hackl 2006, Schwarz and Shimoyama 2011, Abrusán and Spector 2011, Abrusán 2011, 2014).

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In this paper, we discuss a phenomenon which, at first sight, is naturally interpreted as another symptom of the MIP, viz. uniqueness presuppositions that arise from *how many*-questions with non-distributive predicates. The phenomenon is illustrated by (2).

(2) How many students solved this problem together?

A questioner who already assumes that two or more student groups of different sizes solved the problem will not consider (2) a fully appropriate vehicle for requesting information about the exact sizes of those groups; and an addressee who believes that two or more student groups of different sizes solved the problem will likewise judge (2) as not fully appropriate. In short, the question is judged to carry the presupposition that there is a unique size of student groups who solved this problem.[2]

Under familiar assumptions about *how many*-questions (Beck and Rullmann 1999), the set of Hamblin/Karttunen answers to (2) contains, for any cardinality n, the propositions that there is a group of n students who solved this problem together. We note that none of these Hamblin/Karttunen answers are related by semantic entailment. For different cardinalities n and m, the propositions that there is a group of n students who solved this problem together and the propositions that there is a group of m students who solved this problem together are semantically independent. As a consequence, the MIP is true if and only if exactly one of these Hamblin/Karttunen answers is true, hence if and only if there is exactly one cardinality n such that there is a group of n students who solved this problem together. This straightforwardly captures the uniqueness effect described above, which therefore appears to present a novel instantiation the MIP in (1).

A puzzle arises under this analysis from the study of examples that feature modal operators. In (3), example (1) is modified by embedding the wh-phrase's scope under the deontic possibility modal *allowed*. Clearly, this question does *not* presuppose that there is only one cardinality n such that it is allowed that a group of n students solves this problem.

(3) How many students are allowed to solve this problem together?

Yet this unattested uniqueness presupposition is precisely what the MIP derives. The reason is that in this case, too, the Hamblin/Karttunen answers are not related by entailment. For different cardinalities n and m, the propositions that it is allowed for n students to solved this problem together and the propositions that it is allowed for m students to solved this problem together are again semantically independent. What, then, accounts for the contrast between (2) and (3)?

In this paper, we explore two answers to this question. The first answer enriches the syntax-semantics of *how many* questions by positing a scopally mobile maximality operator of the

[2]We are not the first to study *how many*-questions with non-distributive predicates (Beck and Rullmann 1999, Abrusán 2014). However, to our knowledge previous work confined attention to examples with possibility modals, such as Beck and Rullmann's example *How many people can play this game?*. As we will discuss shortly, possibility modals obviate the uniqueness effect just described, which explains why no uniqueness effects were reported in those works.

sort posited in Abrusán and Spector (2011); the second answer proposes that for the purposes of the MIP, the proper notion of informativity is contextually supported entailment rather than semantic entailment. Unfortunately, however, we find that both solutions fall short of properly characterizing the relevant data, mischaracterizing the conditions under which the uniqueness effect arises in (2) or (3). In addition, we observe that a Maximal Informativity Presupposition fails to capture uniqueness implications in *how many*-questions with predicates that are weakly distributive in the sense of Buccola (2015) and Buccola and Spector (2016), such as *How many students in the seminar have the same first name?*. We conclude that, despite initial appearance, uniqueness implications in *how many*-questions must have a source that is independent of Dayal's (1996) Maximal Informativity Presupposition.

2. On the syntax-semantics of *how many*-questions

We adopt assumptions about the syntax and semantics of *how many*-questions familiar from, for example, Beck and Rullmann (1999). We assign to our running example (2) the logical form in (4), where *spt* abbreviates *solved this problem together*.

(4) how λn[[$\exists$ [n many] students] spt]

In (4), the wh-phrase *how* has extracted from the argument position of *many*; wh-movement leaves behind a trace *n* ranging over degrees or, more specifically, cardinalities; the phrase *n many students* is taken to form an existential generalized quantifier, headed by the silent existential determiner $\exists$; finally, wh-movement of *how* creates a derived predicate of cardinalities.

Again following Beck and Rullmann (1999), we take *many* to denote a relation between cardinalities and individual sums in the sense of Link (1983), which for ease of exposition we also refer to as *groups*. (Note that a group may consist of just one atomic individual.) As shown in (5), we take *many* to relate a cardinality n to a group x just in case the cardinality of x (i.e., the number of atomic individuals in x) equals n.

(5) $[\![\textit{many}]\!] = \lambda n_d.\ \lambda x_e.\ \lambda w_s.\ |x| = n$

The lambda abstract in (4) will accordingly denote the property of cardinalities in (6), which maps any cardinality n to the proposition that there is a group of size n that solved this problem.

(6) $\lambda n_d.\ \lambda w_s.\ \exists x[[\![\textit{students}]\!](x)(w) \wedge [\![\textit{spt}]\!](x)(w) \wedge |x| = n]$

For ease of exposition, we adopt the so-called functional approach to the semantics of questions (Krifka 2011), that is, we assume that the property in (6) is also the denotation of the question in (2) as a whole. Under this assumption, the set of Hamblin/Karttunen answers to a *how many*-question (or any wh-question) is the set of propositions that forms the range of the question's denotation.

3. The uniqueness effect

Before we turn to the analysis of the uniqueness effect described in section 1, we briefly elaborate on the nature of this effect. According to our description above, (2) presupposes that there is a *unique size* of student groups that solved the problem together. However, some speakers judge (2) to even carry the presupposition that there is a *unique group* of students who solved the problem together. We will refer to these two presuppositions as the *uniqueness of size* and the *uniqueness of group* presupposition, respectively. Note that the uniqueness of group presupposition is stronger than the uniqueness of size presupposition: if there is only one group who solved the problem, then there is a unique size of student groups who solved it, viz. the size of that unique group; however, the reverse entailment does not hold, as there could be several groups of the same size who solved the problem.

Whether or not (2) indeed carries an (obligatory) uniqueness of group presupposition, we will in this paper focus on the weaker uniqueness of size presupposition, whose existence we take not to be in doubt. However, we will briefly return to the potential uniqueness of group presupposition, and its significance for the puzzle we describe, in section 9 at the very end of this paper.

4. An account of the uniqueness effect

For any given question, the MIP stated in (1) requires that the question have a true Hamblin/Karttunen answer that entails any other true Hamblin/Karttunen answer. Under the functional question semantics that we have adopted, the MIP can be characterized as the proposition that there is a true proposition in the question denotation's range that entails any other true answer in the range. This is stated in (7), where the variable x ranges over members of the domain of the question denotation, which in the case of *how many*-questions is the set of cardinalities.

(7) For any (functional) question denotation Q:
$\text{MIP}(Q) = \lambda w_s.\ \exists x[Q(x)(w) \wedge \forall y[Q(y)(w) \rightarrow Q(x) \subseteq Q(y)]\ ]$

We observed in section 1 that the Hamblin/Karttunen answers to (2) are not related by entailment. For the denotation in (6), this means that it is *non-ordering* in the sense of (8): no proposition in the question denotation's range entails any other proposition in that range.

(8) Non-orderingness
f is *non-ordering* $:\Leftrightarrow\ \forall x,y[f(x) \neq f(y) \rightarrow f(x) \not\subseteq f(y)]$

The denotation in (6) is non-ordering because, as noted, for different cardinalities n and m, the proposition that there is a group of n students that jointly solved this problem and the proposition that there is a group of m students that jointly solved this problem are semantically independent.

The central observation, as stated in (9), is now that for a non-ordering question denotation, the MIP encodes a uniqueness presupposition: if all the propositions in the range of the question denotation are unrelated by entailment, then the MIP requires that there be at most (and at least) one of them that is true.

(9) For any non-ordering question denotation Q:
$\text{MIP}(Q) = \lambda w_s.\ \exists x[Q(x)(w) \wedge \forall y[Q(y)(w) \rightarrow Q(x) = Q(y)]\ ]$

For (2), the MIP therefore yields the proposition that exactly one proposition in the range of (6) is true, which we can describe as in (10), as the proposition that there is exactly one cardinality n such that a group of n students solved this problem together. In other words, the MIP delivers the uniqueness of size presupposition described above.

(10) $\lambda w_s.\ \exists!n[\exists x[[\![\textit{students}]\!](x)(w) \wedge [\![\textit{spt}]\!](x)(w) \wedge |x|=n]\ ]$

Note that the non-orderingness of the denotation of (2) is due to the semantic behaviour of the predicate *solved this problem together*. The property expressed by this predicate is itself non-ordering: for any two distinct groups x and y, the proposition that the members of x solved the problem together fails to entail the proposition that the members of y solved the problem together.

Of course, not all predicates are non-ordering. To illustrate, we contrast (2) with (11), where *solved this problem together* is replaced by *laughed*.

(11) How many students laughed?

The properties expressed by *laugh* and *students* are distributive in the sense of (12), where $\sqsubseteq$ is the mereological part-of relation between groups of individuals. For any two groups of individuals x and y such that x is included in y, the proposition that y are students (or is a student) who laughed entails the proposition that x are students (or is a student) who laughed. This keeps the denotation of the logical form (13a), shown in (13b), from being non-ordering. In fact, distributivity ensures that (13b) is *downward scalar* in the sense of (14) (Beck and Rullmann 1999).[3]

(12) Distributivity
f is *distributive* $:\Leftrightarrow\ \forall x,y[x \sqsubseteq y \rightarrow f(y) \subseteq f(x)]$

(13) a. how λn[[$\exists$ [n many] students] laughed]
b. $\lambda n_d.\ \lambda w_s.\ \exists x[[\![\textit{students}]\!](x)(w) \wedge [\![\textit{laughed}]\!](x)(w) \wedge |x| = n]$

(14) Downward scalarity
f is *downward scalar* $:\Leftrightarrow\ \forall n,m[m \leq n \rightarrow f(n) \subseteq f(m)]$

[3] Since it presupposes reference to the intrinsic ordering of cardinalities or other degrees, the notion of downward scalarity selectively applies to the denotations *how many*-questions and other degree questions, not to question in general.

The denotation in (13b) is downward scalar because, given distributivity of *students* and *laughed*, the existence of a group of n students who laughed guarantees the existence of a group of m students who laughed, for any m≤n; hence for any such n and m, the proposition that there is a group of n students who laughed semantically entails the proposition that there is a group of m students who laughed. Consider now the set of cardinalities that a given downward scalar question denotation maps to a true proposition. Suppose that this set has a (unique) maximal element. The downward scalar function will map that maximal element to a proposition that entails all the other true propositions in its range, and hence the MIP will be met; so, as stated in (15), the MIP will merely require that the set of cardinalities that the question denotation maps to a true proposition have a maximal member in terms of the intrinsic ordering of cardinalities.

(15) For any downward scalar question denotation Q:
$\text{MIP}(Q) = \lambda w_s.\ \exists n[Q(n)(w) \wedge \forall m[Q(m)(w) \rightarrow m \leq n]\ ]$

But in order for a set of cardinalities to have a (unique) maximal member, all it takes is for that set to be non-empty and finite. Applied to the downward scalar denotation of (11) in (13b), the MIP therefore yields the proposition stated in (16), the presupposition that there is a group of students of some (finite) size who solved this problem together.

(16) $\lambda w_s.\ \exists n[\exists x[[\![\textit{students}]\!](x)(w) \wedge [\![\textit{laugh}]\!](x)(w) \wedge |x|=n]\ ]$

So in this case, the MIP amounts to a mere existence presupposition. The content of the MIP is predicted to be more obvious, however, in cases that feature non-ordering properties, as illustrated by the correctly predicted uniqueness of size presupposition for (2).

5. The puzzle: modal obviation of the uniqueness effect

Under the MIP-based account for the uniqueness of size presupposition in *how many*-questions, an interesting puzzle emerges from examples that feature modal expressions such as deontic *required* and *allowed*. Consider the pair of questions in (17), where (17b) repeats (3). Extrapolating from our analysis of (2), we arrive at assigning to these questions the logical forms in (18a) and (19a) and the denotations in (18b) and (19b).[4]

(17) a. How many students are required to solve this problem together?
b. How many students are allowed to solve this problem together?

(18) a. how λn[required [[$\exists$ [n many] students] spt]]
b. $\lambda n_d.\ \lambda w_s.\ \forall v[v \in \text{ACC}(w) \rightarrow \exists x[[\![\textit{students}]\!](x)(v) \wedge [\![\textit{spt}]\!](x)(v) \wedge |x|=n]\ ]$

(19) a. how λn[allowed [[$\exists$ [n many] students] spt]]
b. $\lambda n_d.\ \lambda w_s.\ \exists v[v \in \text{ACC}(w) \wedge \exists x[[\![\textit{students}]\!](x)(v) \wedge [\![\textit{spt}]\!](x)(v) \wedge |x|=n]\ ]$

[4]We only consider the "de dicto" readings of the relevant questions (Cinque 1990), where the existential generalized quantifier containing *many* plus modified noun is interpreted within the scope of the modal.

So, the denotation assigned to (17a), shown in (18b), maps any cardinality n to the proposition that in every permissible world, there is a group of students of size n who solved the problem together; likewise the denotation of (17b), shown in (19b), maps any cardinality n to the proposition that there is a permissible world in which there is a group of students of size n who solved the problem together. Note that the non-orderingness of the denotation of *solve this problem together* ensures that the question denotations in (18b) and (19b), too, are once again non-ordering. For any distinct cardinalities n and m, the proposition that in every permissible world there is a group of n students who solved the problem together does not entail the proposition that in every permissible world there is a group of m students who solved the problem together; likewise the proposition that there is a permissible world where a group of n students solved the problem together does not entail the proposition that there is a permissible world where a group of m students solved the problem together. This means that for both of those cases the MIP once again delivers a uniqueness presupposition. Given (18b), the MIP assigned to (17a), shown in (20a), is the proposition that that there is a unique cardinality such that a student group of that cardinality is required to solve the problem; and given (19b), the MIP assigned to (17b), shown in (20b), is the proposition that there is a unique cardinality such that a student group of that cardinality is allowed to solve the problem.

(20) a. $\lambda w_s.\ \exists!n[\forall v[v\in ACC(w) \rightarrow \exists x[[\![\mathit{students}]\!](x)(v) \wedge [\![\mathit{spt}]\!](x)(v) \wedge |x|=n]\ ]\ ]$
b. $\lambda w_s.\ \exists!n[\exists v[v\in ACC(w) \wedge \exists x[[\![\mathit{students}]\!](x)(v) \wedge [\![\mathit{spt}]\!](x)(v) \wedge |x|=n]\ ]\ ]$

For the question in (17a), this prediction appears adequate, supported by intuitions similar to those about the non-modal example in (2). A speaker using (17a) indeed seems to exclude the possibility of there being more than one cardinality such that it is required for a student group of that cardinality to solve the problem. However, for the question in (17b), the predicted uniqueness of size presupposition is clearly incorrect. The question can be used very naturally in contexts where there is assumed to be a range of permissible group sizes. For example, the interlocutors' common knowledge might entail that there is an upper bound on the permissible sizes, that this upper bound is between five and ten, and that there are no further constraints on permissible group sizes. In this case, common knowledge is consistent with there being between five and ten permissible sizes, and the speaker may use (17b) to obtain further information.

So the uniqueness effect is expectedly preserved under the addition of the necessity modal, but it is unexpectedly obviated by the addition of the possibility modal. How can this obviation effect of possibility modals be accounted for? In sections 6 and 7, we examine two different conceivable strategies to answer this question that preserve the idea that the uniqueness effect is due to to a maximal informativity presupposition.

6. First try: a maximality operator

We are not the first to encounter a problem of unwanted uniqueness presuppositions derived by the MIP. Abrusán and Spector (2011) describe much the same problem as they develop an analysis of degree questions that lets gradable predicates denote relations between individuals and intervals of degrees. Abrusán and Spector report that such an interval semantics comes into conflict with the assumption that questions carry a maximal informativity presupposition. For example, they report that under their interval-based analysis, a degree question like (21) is incorrectly assigned the MIP that there is only one length that the paper is allowed to have.

(21) How long is the paper allowed to be?

This presupposition is inconsistent with intuitions about (21), which clearly indicate that the question can be used in contexts where the paper is permitted to have a range of different lengths. For example, (21) can be used in a context where the interlocutors's common knowledge entails that there is an upper bound on the paper's permitted lengths, that this upper bound is somewhere between 10 and 20 pages, that there is a lower bound of 6 pages, and that there are no further constraints on the on permissible lengths. In such a scenario, common knowledge is consistent with there being between 5 and 15 different permitted paper lengths (measured in terms of numbers of pages). The speaker might then use (21) to obtain further information.

The problem that Abrusán and Spector (2011) describe is transparently analogous to our puzzle of modal obviation of the uniqueness of size effect in *how many*-questions. Therefore, since Abrusán and Spector offer a solution to the problem that arises in their interval semantics, one might hope that this solution is transferrable to our puzzle. That is the line of attack that we explore below.

Abrusán and Spector propose that gradable predicates (such as *long* or *many*) introduce a scopally mobile operator Π (first defined in Heim 2006) that creates derived properties of degrees. Here we minimally adapt Abrusán and Spector's semantics for Π to ensure consistency with our assumptions. We take Π to denote a Curryed version of a function from properties of cardinalities to properties of cardinalities, which relates any set of cardinalities that has a unique maximal member to the singleton set containing that maximal member. This denotation is spelled out in (22) (where *max* maps a set of cardinalities to its unique maximal member). We note that a nearly identical maximality operator has been employed for somewhat different purposes in Buccola and Spector (2016).

(22) $[\![\Pi]\!] = \lambda m_d.\ \lambda P_{d(st)}.\ \lambda w_s.\ max\{n\text{: } P(n)(w)\} = m$

To illustrate the syntax and the semantic effect of Π, we return to the question in (11), repeated below in (23). As shown in (24a), we assume that Π enters the syntactic derivation as part of a phrase, the Π-*phrase*, that also includes the wh-phrase *how* and that sits in the argument position of *many*; in (24b), the Π-phrase has extracted from its base position to the edge of the sentence, forming a derived predicate of cardinalities in its scope; and in (24c), *how* has subextracted from the Π-phrase, once again forming a predicate of cardinalities in its scope.

(23) How many students laughed?

(24) a. [∃ [[Π how] many] students] laughed
b. [Π how] λn[[∃ [n many] students] laughed]
c. how λm[[Π m] λn[[∃ [n many] students] laughed]]

Given the denotation of Π shown in (22), the logical form in (24c) receives the denotation in (25). This question denotation maps any cardinality m to the proposition that m is the largest cardinality n such that there is a group of n students who laughed. More transparently, (25) maps any cardinality to the proposition that the set of all students who laughed has that cardinality.

(25) $\lambda m_d.\ \lambda w_s.\ \max\{n\colon \exists x[[\![\textit{students}]\!](x)(w) \wedge [\![\textit{laughed}]\!](x)(w) \wedge |x| = n]\} = m$

The propositions in the range of this function are mutually incompatible. For any distinct cardinalities n and m, the proposition that n is the number of all students who laughed contradicts the proposition that m is the number of all students who laughed. Therefore, the MIP derived from (25) is satisfied as long as there are (finitely many) students who laughed. Therefore, just like under the analysis in section 4 (where we appealed to the distributivity of the meaning of *laugh*) the MIP for (23) amounts to a mere existence presupposition.

For completeness, we note that (23) may well be considered ambiguous between the meaning in (25) and the meaning we derived in in section 3, shown in (13b). Repurposing related hypotheses entertained in Buccola and Spector (2016), we can see two possible ways of recovering (13b). One way is to assume that the presence of Π is optional and that (13a), too, is a well-formed logical form for (23). Another option is to assume that the Π-phrase can scope within the containing noun phrase, below the existential determiner ∃. As readers are invited to verify, with such narrow scope, the presence of Π winds up having no effect on the question denotation as a whole, replicating the meaning assigned to the logical form (13a), where Π is not present in the first place.

We are now ready to return to our problematic example (3), repeated again below in (26). We are focusing on a logical form for this question where the Π-phrase has moved past *allowed* to again take widest scope, turning (27a) into (27b), followed by short subextraction of *how*, yielding (27c). The denotation assigned to (27c) is shown in (28).

(26) How many students are allowed to solve this problem together?

(27) a. allowed [[∃ [[Π how] many] students] spt]
b. [Π how] λn[allowed [[∃ [n many] students] spt]]
c. how λm[[Π m] λn[allowed [[∃ [n many] students] spt]]]

(28) $\lambda m_d.\ \lambda w_s.\ \max\{n\colon \exists v[v{\in}\mathrm{ACC}(w) \wedge \exists x[[\![\textit{st}]\!](x)(v) \wedge [\![\textit{spt}]\!](x)(v) \wedge |x|{=}n]\ ]\ \} = m$

This denotation maps any cardinality m to the proposition that m is the largest cardinality n such that in some permissible world, there is a group of n students who solved this problem together. More transparently, (28) maps any cardinality m to the proposition that m is the largest permitted size of groups of students that solve this problem. Since there can be at most one such largest permitted group size, the propositions in the range of (28) are again pairwise incompatible. Therefore, the MIP once again yields a mere existence presupposition, though in this case the presupposition that there is a *largest* permitted size of student groups solving the problem.

So the Π-phrase scoping over the possibility modal in (26) has the welcome effect of preempting the derivation of the unattested uniqueness of size presupposition, that is, the presupposition that there is a *unique* permitted size of student groups solving the problem. Moreover, the weaker MIP now derived for (26) appears weak enough to be consistent with intuitions about the meaning of the question.

This might be an encouraging finding, especially since the Π operator employed here, or some version of it, may have independent support from domains other than *how many*-questions (Heim 2006, Buccola and Spector 2016, Abrusán and Spector 2011, Kennedy 2015). However, there remains an obvious open question that the proposal as stated fails to answer. Why is the obviation of the uniqueness effect tied to the presence of a possibility modal? After all, one might expect that the questions in (2) and (17a), repeated below in (29a) and (29b), also allow for logical forms where the Π-phrase takes wide scope, that is, the logical forms in (30).

(29) a. How many students solved this problem together?
b. How many students are required to solve this problem together?

(30) a. how λm[[Π m] λn[[∃ [n many] students] spt]]
b. how λm[[Π m] λn[required [[∃ [n many] students] spt]]]

These logical forms, too, would preempt the uniqueness of size presuppositions described in sections 1, 3, and 5. The MIP that the logical form in (30a) would give rise to is the proposition that there is a *largest* size of student groups who solved the problem. But once again, this proposition amounts to a mere existence presupposition. Whenever there is at least one student group (of finite size) who solved the problem, there is guaranteed to be exactly one that is as large as any of the others. Hence the Π-phrase in (30a) incorrectly obviates the very uniqueness of size presupposition that served as the starting point of our investigation. Similarly, the MIP based on (30b) is the proposition that there is a *largest* cardinality such that it is required for there to be a student group of that cardinality that solves the problem. Once again, this is weaker than the uniqueness presupposition described above, namely the proposition that there is a unique required cardinality of student groups solving the problem.

Therefore, if a MIP-based account of the uniqueness effects in (29a) and (29b) is to be preserved, logical forms like those in (30) must be excluded as well-formed inputs to semantic interpretation. Unfortunately, however, there appears to be no independent rationale for such exclusion. In particular, it is not promising to explore the hypothesis that there is something wrong with the syntactic distance that the moving Π-phrase has travelled in the logical forms in (30), since an equal or greater distance is covered by the Π-phrase movement posited in the logical form in (27c). We conclude that, as long as the uniqueness of size effect is sought to be accounted for in terms of the MIP, the obviation of this effect by a scopally mobile Π-phrase must be excluded, which presumably requires excluding the logical form in (27c) along with those in (30). We then require a different account of the puzzling obviation of the uniqueness of size effect by possibility modals.

7. Second try: contextual entailment

Having reverted to the position that the relevant question denotations are as initially presented in sections 2 and 5, we now target the MIP itself for revision. Our formulation of the MIP in (7) above follows Dayal (1996) and subsequent literature in that it construes the relevant notion of informativity as semantic entailment. We will now explore the consequences of instead construing informativity for the purposes of the MIP as pragmatic, contextual, entailment. Let c be the context set in the sense of Stalnaker (1978). As stated in symbols in (31), we say that a proposition p contextually entails a proposition q in context set c just in case q is semantically entailed by p in conjunction with c.

(31) Contextual Entailment
$p \subseteq_c q \;\; :\Leftrightarrow \;\; p \cap c \subseteq q$

We use this definition to minimally revise the statement of the MIP in (7) as shown in (32), relativizing it to the context set and substituting contextual entailment for semantic entailment.

(32) For any (functional) question denotation Q and context set c:
$\text{MIP}_c(Q) = \lambda w_s.\ \exists x[Q(x)(w) \wedge \forall y[Q(y)(w) \rightarrow Q(x) \subseteq_c Q(y)]\ ]$

The immediate benefit of this revision is that the problematic modal obviation example in (3), repeated once more in (33), is no longer predicted to carry a uniqueness of size presupposition in all contexts.

(33) How many students are allowed to solve this problem together?

Under (32), the felicity of (33) is expected to be consistent with the existence of multiple true Hamblin/Karttunen answers, as long as there is one among them that contextually entails all the others. The most natural contexts of this sort are contexts that ensure that the denotation of (33) in (19b), repeated in (34), is *contextually scalar*.

(34) $\lambda n_d.\ \lambda w_s.\ \exists v[v \in \text{ACC}(w) \wedge \exists x[[\![\textit{students}]\!](x)(v) \wedge [\![\textit{spt}]\!](x)(v) \wedge |x|=n]\ ]$

The denotation in (34) is contextually scalar in the intended sense in contexts where the relative contextual strength of any two Hamblin/Karttunen answers is predictable from the intrinsic ordering of the two cardinalities that (34) maps to these propositions. In particular, (34) might be *contextually downward scalar* in the sense defined in (35).

(35) Contextual downward scalarity
f is *contextually downward scalar* in c $:\Leftrightarrow \;\; \forall n,m[m \leq n \rightarrow f(n) \subseteq_c f(m)]$

According to (35), the denotation (34) is contextually downward scalar in contexts where for any cardinalities n and m such that m≤n, the proposition that there are permissible worlds where a group of n students solves the problem contextually entails the proposition that there are permissible worlds where a group of m students solves the problem. In such a context, the truth of the contextual MIP for (33) merely requires that the set of permissible student group sizes have a unique maximal member. Assuming contextual downward scalarity, the proposition that (34) maps that maximal member to will be true and will contextually entail all the other true propositions in the range of (34).

We submit that contexts that make (34) contextually downward scalar are rather natural. Such a context could arise in a scenario where, for some cardinality n, the relevant authority, say, a university teacher, is known to have stated that groups of no more than n students may be formed to collaborate on solving this problem, and where it is moreover understood that there are no further restrictions on permitted group sizes. In such a scenario, the contextual downward scalarity of (34) is common knowledge: if it were to emerge, for example, that groups of five students are permitted, an interlocutor could justifiably infer that groups of four or less are permitted as well. A speaker who lacks complete information about the identity of n might appropriately use (33) in the hope of acquiring such information.

Once again, the obvious question is whether the approach under consideration captures our finding that the obviation of the uniqueness of size presupposition is only observed with possibility modals. The non-modal example (2) and the case with a necessity modal in (17a) are repeated again in (36), together with their denotations in (37), repeated from (6) and (18b).

(36) a. How many students solved this problem together?
b. How many students are required to solve this problem together?

(37) a. $\lambda n_d.\ \lambda w_s.\ \exists x[[\![\textit{students}]\!](x)(w) \wedge [\![\textit{spt}]\!](x)(w) \wedge |x| = n]$
b. $\lambda n_d.\ \lambda w_s.\ \forall v[v \in ACC(w) \rightarrow \exists x[[\![\textit{students}]\!](x)(v) \wedge [\![\textit{spt}]\!](x)(v) \wedge |x|=n]\ ]$

The expectation is, of course, that in the examples in (36), the uniqueness effect might disappear in suitable scenarios. The most plausible conceivable cases of this sort would again be scenarios that render the denotations in (37) contextually scalar. But now, again focusing on contextual downward scalarity, let us consider what such scenarios would be like.

For (37a), contextual downward scalarity would require a scenario where for any cardinalities n and m such that m≤n, the existence of a student group of size n who solved the problem contextually guarantees the existence of a student group of size m who solved the problem. In such a scenario, upon learning that there was a group of, say, six students who solved the problem together, one would be able to infer that there was also a group of five that did so, as well as a group of four, and so on. So in scenarios of this type, the existence of a group of n students who solved the problem would allow one to infer that there are at least n−1 other groups who did so as well, whose cardinalities moreover cover the entire range from 1 to n−1. We suggest that such a scenario is outlandish enough to not readily come to mind to speakers that interpret sentence (36a). The robustness of the uniqueness of size presupposition carried by

(36a) would then not be a matter of linguistic structure, but of the complexity and implausibility of the type of scenario under which that uniqueness presupposition would be weakened in the way it can be weakened in cases with possibility modals.

This line of analysis arguably accommodates the case of (36b), as well. For the denotation in (37b), contextual downward scalarity would require a scenario where for any cardinalities n and m such that m≤n, the proposition that a group of n students solved the problem in every permissible world contextually entails the proposition that a group of m students solved the problem in every permissible world. In such a scenario, upon learning that it is required for a group of n students to solve the problem, one would be justified in drawing the inference that it is also required that n−1 other groups of students solve the problem, whose cardinalities moreover must cover the entire range from 1 to n−1. Once again, we submit that such a scenario is sufficiently implausible to not readily come to mind to speakers judging the question in (36b), which serves to account for the robustness of the uniqueness effect in that example.

Thus, unlike our attempted account in terms of the Π-operator, the move to a revised MIP based on contextual entailment promises to capture the fact that the obviation of the uniqueness of size effect is limited to cases with a possibility modal.

Even so, we must report that, unfortunately, this account does not seem to fully capture the conditions on the use of the case with the possibility modal. While it affords a welcome weakening of the conditions on the use of (33), we can observe that this weakening does not go far enough. Consider a scenario where a teacher is known to have stipulated, for two different cardinalities n and m, that students must form groups to solve the problem together and that each group must have one of the sizes n and m. In this scenario, without having any further beliefs about the permitted group sizes, a speaker may felicitously use (33) to learn about the identity of n and m. Yet in this scenario, no contextual entailment holds between the proposition that a group of n students solves the problem in some permissible world and the proposition that a group of m students solves the problem in some permissible world. This scenario not only conflicts with the presupposition given by the original MIP in (7) but likewise with the weaker presupposition given by the revised MIP, based on contextual entailment.

So we are still left without a solution to the puzzle of modal obviation of the uniqueness of size effect. If we were to insist that this effect is a symptom of the MIP, we would still need to find a different way of understanding why the relevant *how many*-questions with possibility modals do not carry the expected presuppositions. In the next section, we present a type of *how many*-question that raises the reverse problem, by virtue of carrying a uniqueness of size presupposition that the MIP fails to derive.

8. Another problem: weak distributivity and weak downward scalarity

The *how many*-question in (38) gives rise to a similar observation as the question in (2) that we presented as our first illustration of the uniqueness of size presupposition. That is, (38) suggests that there is only one size of groups of students in the seminar that have the same first name.

(38) How many students in the seminar have the same first name?

In support of this assessment, we present two concrete scenarios in the form of the class rosters in (39). Imagine that (39a) is the roster for the seminar and that an addressee who has access to this information is presented with the question in (38). It seems clear that the addressee's cooperative response would be *three*, given that the class roster in (39a) shows a group of students with the same first name (viz. Ann), which has cardinality 3, and given that it does not show any other group of students that share their first name.

(39) a.

Adams, Ann	Durant, Bill
Baker, Ann	Ellis, Chris
Collins, Ann	Foster, Dan

b.

Adams, Ann	Durant, Bill
Baker, Ann	Ellis, Bill
Collins, Ann	Foster, Dan

In contrast, the class roster in (39b) shows a second group of students with the same first name (viz. Bill), which has cardinality 2. We submit that an addressee who has this information would be hard pressed to answer the question in (38). While such an addressee could truthfully assert that there are groups of two and three students with the same first name, the question in (38) does not appear to be a suitable vehicle for eliciting this information. Accordingly, a speaker who takes a class roster like (39b) to be a good possibility would presumably refrain from using (38) as a request for information. These observations confirm that (38) indeed carries a uniqueness of size presupposition.

Interestingly, however, this uniqueness of size presupposition cannot be understood as an instantiation of the MIP. This is a consequence of the semantic behaviour of the denotation of *have the same first name* and the denotation of the question in (38) as a whole. In contrast to the property expressed by *solve this problem together*, the property expressed by *have the same first name* is not non-ordering. For example, it is clear that the proposition that Adams, Baker, and Collins have the same first name semantically entails the proposition that Adams and Baker have the same first name. In fact, this example illustrates that the property expressed by *have the same first name* is weakly distributive in the sense of definition (40). That is, the property is distributive down to groups of at least two individuals: for any groups x and y such that x is included in y and x has cardinality 2 or more, the proposition that the individuals in y have the same first name semantically entails the proposition that the individuals in x have the same first name.[5]

(40) Weak distributivity
f is *weakly distributive* $:\Leftrightarrow \; \forall x,y[x \sqsubseteq y \wedge 2 \leq |x| \rightarrow f(y) \subseteq f(x)]$

[5]What keeps the property from being distributive simpliciter is that it does not apply to any atomic individuals. Setting aside the so-called discourse anaphoric reading of *same* (Beck 2000), the statement that, say, Adams has the same first name is not meaningful. That some collective predicates are weakly distributive in this way was observed in, e.g., Champollion (2010), Buccola (2015), and Buccola and Spector (2016), who illustrate the phenomenon with the verb *gather*. We borrow the term *weak distributivity* from Buccola and Spector (2016) (although in a slightly different meaning).

Under our current assumptions about the syntax and semantic of *how many*-questions, (38) has the logical form in (41a) and the denotation in (41b), where *hsfn* abbreviates *have the same first name*.

(41) a. how λn[[$\exists$ [n many] students] hsfn]
b. $\lambda n_d. \lambda w_s. \exists x[[\![\textit{students}]\!](x)(w) \wedge [\![\textit{hsfn}]\!](x)(w) \wedge |x| = n]$

The weak distributivity of the denotation of *have the same first name* (together with the distributivity of the denotation of *students*) guarantees that the question denotation in (41b) is *weakly downward scalar* in the sense of definition (42).

(42) Weak downward scalarity
f is *weakly downward scalar* $:\Leftrightarrow \forall n,m[m\leq n \wedge 2\leq m \rightarrow f(n) \subseteq f(m)]$

That is, for any cardinalities n and m such that $m\leq n$ and $2\leq m$, the proposition that a group of n students have the same first name semantically entails the proposition that a group of m students have the same first name.

For an argument that will be familiar from section 4, consider now the set of cardinalities (greater than 1) that a given weakly downward scalar question denotation maps to a true proposition. Suppose that this set has a (unique) maximal element. The weakly downward scalar function will map that maximal element to a proposition that entails all the other true propositions in its range, and hence the MIP will be met; so, as stated in (43), the MIP will merely require that the set of cardinalities that the question denotation maps to a true proposition have a (unique) maximal member in terms of the intrinsic ordering of cardinalities.

(43) For any weakly downward scalar question denotation Q:
$\mathrm{MIP}(Q) = \lambda w_s. \exists n[Q(n)(w) \wedge \forall m[Q(m)(w) \rightarrow m\leq n]\ ]$

But, again, in order for any set of cardinalities to have a maximal member, all it takes is for that set to be non-empty and finite. Applied to the weakly downward scalar denotation of (38) in (41b), the MIP therefore yields the proposition stated in (44), the presupposition that there is a group of students in the seminar of some size (greater than 1 and finite) who have the same first name. So, just like questions with (strongly) downward scalar denotations, (38) winds up with a mere existential presupposition.

(44) $\lambda w_s. \exists n[\exists x[[\![\textit{students}]\!](x)(w) \wedge [\![\textit{hsfn}]\!](x)(w) \wedge |x|=n]\ ]$

This result entails that the MIP does not capture the uniqueness of size presupposition of (38), identified above. In the scenario (39b), for example, the MIP of (38) would be satisfied. In this scenario, the question has two true Hamblin/Karttunen answers, viz. the proposition that there is a group of two students who have the same first name and the proposition that there is a group of three students who have the same first name. MIP is satisfied in this scenario because, due to the weak distributivity of the property of having the same first name, the latter proposition semantically entails the former.

We conclude that for *how many*-questions with weakly distributive predicates, a MIP-based account delivers a presupposition that is too weak to capture the attested uniqueness effect.

9. Conclusion

To recap, we described a uniqueness presupposition carried by certain *how many*-questions with non-distributive predicates, such as *solve this problem together*. We explored an approach to this uniqueness effect that credits it to (a version of) Dayal's (1996) Maximal Informativity Presupposition (MIP). However, we found two symmetric problems for this approach. (i) The MIP derives an overly strong presupposition for *how many*-questions where a non-distributive predicate is embedded under a possibility modal. For such cases, the MIP is very obviously inadequate when informativity is construed as semantic entailment, and substituting contextual entailment for semantic entailment does not go far enough in weakening the MIP. We saw that a Π-operator which can outscope the possibility modal would weaken the MIP sufficiently to be consistent with intuitions; but such a Π-operator would be expected to incorrectly remove the uniqueness effect across the board, and leave one without an explanation for why the effect is ever observed to begin with. (ii) The MIP derives a presupposition that is to weak for *how many*-questions with a weakly distributive predicate.

Our finding about *how many*-questions with a weakly distributive predicate brings us back to our discussion of the uniqueness effect in section 3. There we noted that some speakers report that the question in (2), repeated one more time in (45a), does not merely carry a uniqueness of size presupposition, but a stronger uniqueness of group presupposition. That is, the relevant speakers take (45a) to presuppose that there is a unique group of students who solved the problem together. We suspect that this observation extends to (38), repeated in (45b), that is, that (45b) is naturally read as presupposing that there is only one group of students in the seminar that have the same first name.

(45) a. How many students solved this problem together?
b. How many students in this seminar have the same first name?

So, at least for some speakers, the MIP-based approach delivers presuppositions that are overly weak in two different ways. It fails to derive attested uniqueness of size presuppositions in *how many*-questions with certain collective predicates, and it fails to derive uniqueness of group presuppositions across the board.

The problem of predicted presuppositions being too weak becomes even more pronounced under a recent proposal in Fox (2013). In the context of analyzing so-called mention-some readings of questions, Fox proposes a revision of Dayal's (1996) definition of the MIP. Translated into our format, Fox proposes to replace Dayal's MIP in (7), repeated in (46), with the weaker version in (47).

(46) For any (functional) question denotation Q:
$\mathrm{MIP(Q)} = \lambda w_s.\ \exists x[Q(x)(w) \wedge \forall y[Q(y)(w) \rightarrow Q(x) \subseteq Q(y)]\ ]$

(47) For any (functional) question denotation Q:
$\text{MIP}(Q) = \lambda w_s.\ \exists x[Q(x)(w) \wedge \neg\exists y[Q(y)(w) \wedge Q(x) \supset Q(y)]\ ]$

The MIP as defined in (46) requires that the set of true Hamblin/Karttunen answers have a member that is maximal in terms of the ordering given by semantic entailment, that is, a member that entails any other member. This maximal member is necessarily unique. Hence according to (46) the MIP excludes the existence of two true Hamblin/Karttunen answers that are not related by entailment. In contrast, the MIP defined in (47) merely requires that the set of true Hamblin/Karttunen answers have *at least one* maximal element in terms of the ordering given by semantic entailment, that is, at least one member that is not entailed by any other member. This allows for the existence of two true Hamblin/Karttunen answers that are not related by entailment.

Recall now that in all of the *how many*-questions with a non-distributive predicate like *solve this problem together*, the Hamblin/Karttunen answers are semantically unrelated. But that means that for those questions, the MIP as defined in (47) will amount to a mere existence presupposition, requiring that the question have at least one true Hamblin/Karttunen answer. Hence the weak MIP does not derive a uniqueness of size presupposition, let alone a uniqueness of group presupposition, for any of the questions we have studied in this paper.

In all, the negative results that we have reported suggest that the uniqueness of size effect we set out to understand is after all not a symptom of the MIP, and that an alternative account of this effect is to be sought, one that also captures uniqueness of group presuppositions and that extends to *how many*-questions with weakly distributive predicates. We must leave the development of such an alternative account to future work. But we will conclude by pointing to one possible approach that may have some promise: We have analyzed *many* as a relational adjective, which combines with a cardinality denoting expression to form an intersective modifier. In contrast, Hackl (2000) proposes that *many* is a so-called parametrized determiner. Hackl takes this determiner to have existential quantificational force. Adapting Hackl's proposal, one could think of *many* as being more similar to a singular definite article in that it triggers a presupposition of uniqueness. Uniqueness presuppositions would then be contributed by conventional lexical meaning, and would not be tied to the interpretation of questions in the way they are under the approach explored in this paper. We hope to spell out and evaluate this idea in future work.

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