Variable-free gradability and comparison

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Abstract. Distributive lattices and standard assumptions about the cognitive structure of prototypical concepts explain (1) the positive degree of declension denotations of gradable-adjective antonym pairs of opposite polarity like *tall/short* predicated of quantities; (2) the prototypes (together with their psychological and statistical implications) that separate the denotations of those adjective pairs and generate the positive-degree lattices; (3) the individuals of whom the gradable adjectives are *apparently* predicated; and (4) any measures applied to quantitative properties of those individuals, of which the gradable adjectives are actually predicated thanks to the polysemy of terms referring to individuals; (5) the distinct adjective denotations that gradable-adjective antonym pairs require in comparative constructions with degree morphemes, where those constructions simply denote lattice operations, including complementation when possible.

Keywords: gradability; comparison; adjectives

1. The phenomena to be explained and how to explain them[1]

Basic singular Quantity is a mode of existence, the physical mode in fact. Particular quantities are not basic individuals but physically realized existence-properties of individuals ranging from photons to galaxies. These properties can be abstracted, so that two individuals can have the "same" quantity of height just as they can have the "same" hat. However, being individual-level physical properties, quantities themselves can also be treated by language as non-sharable inalienably possessed secondary individuals, i.e. as relational nouns. Just as parts of individuals such as arms or heads can be treated as individuals, so too heights, weights and so on serve as such. Thus, it is impossible to put quantities into single semantic categories.

[1] The term *degree* suggests that gradability phenomena necessarily involve measurement, and indeed Kennedy (1999 et pass.) defends that idea. This paper uses the term *quantity* because, if one takes quantities to be measurements, the result is a misunderstanding of the notion of an *ordered set of quantities* which is the basis of gradable adjective concepts and their semantics. Details of the misunderstanding are given at the end of this section.

Furthermore, though basic quantities are physical, language is made more interesting by the fact that it extends the notion of quantities to more abstract and subjective properties like cleanness or beauty. This is possible because the semantics of quantities is based on a signature abstract structural framework corresponding to the notion of a lattice (Link 1983, 1998), motivated in this section and defined later.

Gradable adjectives are syntactically used to talk about individuals (*Bob is tall*) but semantically, they seem pretty clearly to be used to talk about quantities of those individuals (Bob's height is in a certain range). Going back to just-mentioned quantity-denoting relational nouns for a moment, the examples *Mary asked Bob to measure her height, ??Mary asked Bob to measure (the) height* illustrate the semantic need to indicate the possessor of the quantity. A typical attempt to analyze *Bob is tall* using a bound extent variable to satisfy this need is 'There is an e such that Bob extends vertically to e and e is above the norm of human height'. This variable use has the following problems: (1) it actually neglects the possession relation, i.e. the relational nature of the quantity, or better, the fact that it is a property of the subject; (2) it unnecessarily, as will be seen, refers to the prototypical height quantity (as well as being phrased in the language of measurement); (3) above all, it takes *tall* to be predicated of *Bob* rather than Bob's height. The neglect of the crucial possession of e or the property nature of e can be solved by not notationally separating the quantity from its possessor. Instead, let the two be represented by a modification of the symbol for the possessor: if x stands for a discourse referent (DR), let |x| denote the relevant quantity for x.[2] It turns out that this solves two of the three problems and

[2] The grammatical framework that is presupposed but not actually employed in this paper because it does not deal with syntax is the Discourse Representation Theory version of Combinatory Categorial Grammar presented by Steedman (2012), which does away not only with phrase structure and ∃ but also with using type-raising as a scoping device. Along with Steedman's important additions to the set-theoretic truth-functional approach to semantics, the

opens the way for solving the third. Take referring to the prototypical height: if 'tall' is directly predicated of |x|, that accomplishes several things. First, together with the nominal referent, it identifies the quantity dimension/type of quantity (and this is not because *tall* happens to double as a dimension label, because *short* does the same thing in *Bob is short*).[3] Second, it simply means that |x| falls in a certain set, one which is defined as being above the prototype, where that set definition is part of the more basic human-height-dimension concept, to be illustrated directly, so that it is not necessary to mention the prototype in the adjective meaning. Third, the discrepancy between syntactic *Bob is tall* and the semantic fact that a height is being discussed is removed by positing an independently justified relation between apparently referring to an individual and actually referring to that individual's quantitative properties, to |x| in other words. Part of the justification for |x| is that there is a variety of actually very frequent types of nominal polysemy, for example, count-mass alternations (*Becky raised a 4H lamb, The lamb is well-done*), figure-ground-reversals (*Close the window! He looked out of the window*), place-inhabitant alternations (*New York voted for Clinton, New York is sinking*) and others. The present phenomenon is therefore assumed to be a polysemy of the same general logical type, with the surface use of individuals being a matter of relevance and the semantics being based on their

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paper also adopts additions from Pustejovsky (1995). So, instead of syntactic variables/assignment functions, there are just *direct discourse referents*: place-holders or blanks (variables, if you want, but more generally some kind of representation of syntactic requirements) occur only in the lexicon and using a lexical item requires that its argument place-holders/blanks be situationally or semantically filled in with DRs *as soon as possible*. The paper focuses on semantics, but it is kept extensional, which excludes the meanings of expressions like 'adjective enough', 'too adjective'.

[3] Both the adjective and the noun it modifies are necessary for identification of the dimension. As pointed out by Hellan (1981: 46f.), *old/young* and *old/new* mean that *old* is polysemous and a noun is required for choosing between the two dimensions. This also shows that the dimension/antonym pair is the basic semantic unit for gradable adjectives, a fact recognized since Sapir (1944), Osgood et al. (1957).

quantities. See Pustejovsky (1995) for a general semantic analysis of that logical type. In summary, extent variables like e, d or whatever, will be abandoned in this paper, thereby enabling a more direct relation between syntax and semantics (direct compositionality) and also importantly, avoiding any temptation to mimic measurement.[4] The metaphysical implications of the |, | notation are ignored and left to philosophers.

The formal structure of quantity concepts is determined by how quantities occur in the world. If there were only one kind of entity in the universe and it were always composed of the same amount of matter or energy and uniformly distributed in spacetime, there would be no need for quantities in language. It is the variation of quantities that makes their recognition important for survival. The more frequent a phenomenon is, the more likely that it will be remembered, meaning that its concept will form. Variation of an existence-property like size leads to a probablistic kind of concept of an entity based on relative frequencies and again due to the need for adaptation to the world. If members of the property's extension (set of exemplars) are inherently variable in the conceptual property, as in shades of red or in height, the conceptual probability distribution will take that into account: the most frequent shade of red or amount of height will correspond to a prototype. Linguistically, some concepts also display a further efficiency adaptation: Cartesian concepts like height are among those for which there are no words for the least newsworthy, most expected parts of the distributions, the prototypes, but only for the two tails. So, there are English etymologically unrelated pairs of words (so they can be short and efficient for these frequently used concepts) for extremes of height (*tall, short*),

[4] Such variables are also objectionable because thanks to measurement technology, they suggest the *general* availability of symbolic manipulations of independent quantities, for example, arithmetic operations on them as in carpentry. To repeat, that measurement view is cognitively indefensible. There are, however, kinds of linguistic quantities that have some *logical* independence, namely, Boolean ones, as we shall see.

comfortable temperature (*hot, cold*), weight (*heavy, light*), and so on, but not for the most ordinary and expected quantities of these (color concepts are not among those displaying this linguistic efficiency adaptation). In short, quantity concepts are *necessarily going to be prototypical ones*, and the prototypical exemplars are their most predictable, least linguistically relevant ones.

Do quantity concepts have any other structure? Indeed, they do, and it is abstractly captured by the mathematical notion of an ordered set, a set with a transitive and antisymmetric relation defined on it (all its members participate). If the conceptual representation of a set of varying quantities is unordered, the represented variation is random and incoherent and *no prototype is possible*. Since quantity prototypes clearly exist, conceptual sets of quantities are both ordered and probability distributions. It will turn out to be helpful for modeling to also let the order relation be reflexive (or non-strict), i.e., allow members to be equal to each other, which will enable an ordered set to be of a particular kind, called a lattice (lattices will be formally defined later in this paper).

How much lattice structure should be recognized in a language? Consider Figure 1, which represents the lattice structures of English *tall/short* and *taller/less tall, shorter/less short*.

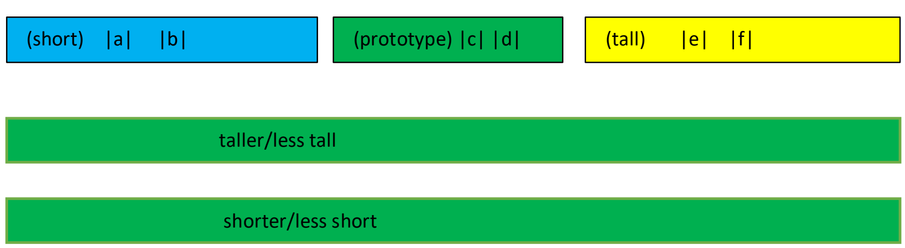

Figure 1. The height dimension

The upper part of Figure 1 represents a segmentation of an ordered set <P; ≤ > of abstract quantities (vertical extensions) |a|, |b|, |c|, etc. for a given human population. One can anticipate, by the way, that the cognition of adjective dimensions and their quantities is as imagistic as it can possibly be (Barsolou 1999).[5] Each population, be it everyone, or just ten-year old American children, or just professional basketball players, may be so represented, and each will be a different segmentation of a different set of height quantities (distances from sole to crown for each human member). Most notably, each will have a different prototype, which can be taken as generating it. This requires that the extensions for 'tall' and 'short' be determined by selectiv4 binding (Pustejovsky 1995) of the adjective and a kind concept which is either linguistically or situationally given. Furthermore, each such segmented set is also a likelihood distribution that approximates more and more closely a normal probability distribution as the population size (set of exemplars) increases. Maximum populations will be kinds. In other words, the prototype -- neither short nor tall -- is the mean area of greatest likelihood, and the two more or less symmetrical tails have decreasing likelihood as one moves to the left for *short* and to the right for *tall*. The gaps between the colored rectangles are the so-called truth-value gaps, areas of rapidly changing probability where discrimination of the heights they contain is difficult. Note also something that is not usually recognized: discrimination of heights is also difficult in the crowded prototype, the relative size of which is no doubt exaggerated in the figure. However, the prototype is outside the domain of any characteristic function not because of that but because the

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[5] Take the basic questions *What is similarity to an image? How does one state a characteristic function for that?* In the absence of answers, one must explicitly recognize what was implicit all along but can be learned from textbooks like Dowty, Wall and Peters (1979): formal semantics is a simple mathematical metaphor (model) for what is really psychological and complicated. Representing an adjective as a function f just means that, psychologically, the combination of the adjective meaning with a modified noun or a subject noun meaning somehow results in a third (more or less) unique meaning different from that of the noun alone. One doesn't need to know the *how* to talk about the *what*. One can go farther still by developing a non-mathematical imagistic but still metaphorical language for meaning composition; that is the project of Cognitive Linguistics.

information it contains would be most predictable. Therefore, semantics includes the information that most people are neither tall nor short, that most elephants are neither big nor small, and so on because the most probable and least newsworthy ranges of these distributions are not covered by the two antonyms. Language learning results in large numbers of these newsworthy extensions contained in prototypical concepts.

In summary, the extensions of *tall* and *short* combined with the nouns they modify or predicate of are completely ordinary, just prototype-free sets. To put an important innovation of this paper in a nutshell, a conceptual property like vertical extension (height) has an internal structure that is *independent of language* but whose parts provide the denotations of adjectives like *tall*, *short* and their equivalents in other languages. *Those adjectives (or their degrees of declension) do not have meanings that themselves structure the concepts of height, etc. by incorporating standards or norms (prototypes)*. [6] This point alone is enough to differentiate this paper from previous formal approaches to gradability and comparison -- see Section 2 for details of those. That adjectives depend on some independent conceptual structure rather than entirely creating it is true even for highly culturally determined and context-dependent adjectives like *expensive/cheap* -- that pair depends on a pre-existing technology of money and an economic system based on it that determines the prices of goods and services.

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[6] Note too that for an antonym-pair concept like *tall/short*, the relation between the pair's prototype and membership in the adjective denotations is quite different from that for concepts like 'cup' or 'red'. Closeness to the prototype cannot possibly correlate with *membership* in 'tall' or 'short' (it has been recognized that *tall*, *wide*, etc. denotations do not have central values/prototypes themselves, e.g. by Kamp and Partee 1995). Correspondingly, one cannot use Voronoi tesselations to model gradable antonym pairs, contrary to Verheyen and Égré (2018), because the conceptual boundary *is in effect the prototype* rather than being as far away from the prototype as the system of related concepts allows.

The lower part of Figure 1 represents the denotation of the comparative forms, the antonyms *taller/less tall* and *shorter/less short* representing the *tall* class of adjectives. Unique to this *tall* class, prototypical structure completely vanishes in the comparative ordering (the green color can be taken as representing the idea suggested in this paper that the positive to comparative semantic mapping for this adjective class simply expands the prototype bounds to cover the denotation of the whole dimension). At any rate, other adjective classes have different kinds of semantic mappings from positive to comparative denotations and because the type of mapping depends on the adjective, it must be learned for each adjective. There is no simple one-size-fits-all mapping.[7] However, the mappings are all distributive lattice mappings, making Category Theory an applicable and insightful mathematical framework for their analysis.

The syntactic nature of the mappings between positive and comparative lattices depends on the language. A Particle language like English (Stassen 2013) uses what have been called degree morphemes (adverbs, determiners) *more/-er than, enough*, etc. while a Locational language (ibid.) like Haida uses adpositions, adverbs, and auxiliary verbs.[8] The existence of the mapping is most evident in the fact represented by the color difference between the two parts of Figure 1: the denotations of the antonyms can and do change from one degree of declension to the other, in a way depending on the (class of) the adjective in English. This change in denotations has long been noticed (see Cruse 1986) but it has not been properly explained due to a failure to recognize distinct positive and comparative denotations and a mapping between them

[7] This rules out, for example, an analysis of the mapping as simply due to -*er/more* and *less* (in the English case). Furthermore, it indicates that the products of the mappings are learned, not calculated as needed. In short, the meanings of English comparative adjectives are unlikely to be "inflectional," contrary to some accounts such as that of Klein (1980, 1982).

[8] Particle and Locational languages tend to have only what can be called strict comparatives, that is, comparative constructions based on morphemes denoting lattice meet and join operations (see Section 3). The languages of Stassen's other types tend not to have them.

(for example, see Rett (2007) for a feature-based analysis). Furthermore, there is a distinction between the English words *very, rather, quite, how, this/that*, *so (that s),*, etc. which comprise mostly what have been called degree adverbials and *enough (that/to s), too (for np/to s), more/less than np/s, six feet* etc. which have been called degree morphemes, amounting to which lattice they operate in, and therefore they differ in whether they are able to induce or mark the mapping to the comparative lattice.[9] Consider the data in (1).

(1a.i)	He is short enough.	(He can be tall)
(ii)	He is shorter than Rex.	(He can be tall)
(iii)	He is too short.	(He can be tall)
(iv)	He is very short.	(He must be short)
(v)	How short is he?	(He must be short)
(vi)	He is that short.	(He must be short)
(vii)	He is so short that . . .	(He must be short)
(viii)	*He is six feet short.	

(1b.i)	He is tall enough.	(He can be short)
(ii)	He is taller than Rex.	(He can be short)
(iii)	He is too tall.	(He can be short)
(iv)	He is very tall.	(He must be tall)
(v)	How tall is he?	(He can be short)
(vi)	He is that tall.	(He can be short)
(vii)	He is so tall that . . .	(He must be tall)
(viii)	He is three feet tall.	(So, he can be short)

There are *three mutually independent factors* at work here. First, the difference between (1a.viii) and (1b.viii) is a matter of mathematics (what determines where measures are possible) and will be dealt with in Section 3; it is not relevant to the point here. Second, the difference between (1b.v,vi) and (1a.v,vi) is a matter of *tall* but not *short* being the label for the whole dimension -- the dimension comprises the whole tripartite upper lattice of Figure 1, not just its set ordered

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[9] I have rearranged the commonly proposed membership in the degree-adverbial and the degree-morpheme categories according to whether a particular word operates in the positive-degree lattice (comprising mostly former degree adverbials) or the comparative-degree lattice (comprising a reduced set of degree morphemes).

above the prototype. If *tall* were not a dimension label, then (1b.v,vi) would require that the subject be tall, *just as (1a.v,vi) require that he is short*. Not all dimensions have labels. The criterion for dimension-label use is precisely the contrasting behavior of the two antonyms just illustrated (Cruse 1986, Lehrer 1985). Thus, the two known members of the *tall* class without a dimension label, *abstract/concrete* and *pungent/bland*, do not show that contrasting behavior: one can say *How abstract is it? It is that abstract* both requiring some quantity of abstractness and *How concrete is it? It is that concrete* both requiring some quantity of concreteness. That these quantities are required (i.e. there is no dimension-label use here) is in turn shown by the fact that one can answer meta-linguistically, *It has no abstractness/ concreteness at all*; compare the dimension label *tall*: *How tall is he? *He has no tallness (height) at all*, at best implying that he does not exist in the real world. However, leaving dimension-label use, I want to emphasize the third factor here, the positive-comparative lattice distinction and mapping. The remaining difference between the first three sentences of (1a) and (1a.iv,vii), similarly between the first three sentences of (1b), and (1b.iv,vii), is due to the adverbs *enough, -er than, too* in the first three sentences of (1a,b) being restricted to the comparative degree of declension, while the adverbs *so, very* in (1a.iv,vii) and (1b.iv,vii) are restricted to the positive degree.

It is intuitively obvious that the ordered sets of Figure 1 do not themselves have the structure of a linear measure though of course they can be mapped to one.[10] If members of a set

[10] Nevertheless, the available alternative to Figure 1 takes the denotation of a gradable antonym pair to be a scale of the positive real numbers, as in Seuren (1978), Bierwisch (1989), and Kennedy (1999, 2001, et pass.), among others. This idea naively derives from experience of measurement, one speaks of *extents* starting above zero, rather than *quantities*, and there is no natural/automatic account of the order direction of negative-polarity adjectives (the only order direction is increasing and decreasing order must be specified ad hoc). The order direction problem alone is enough to disconfirm this extent model, but there are also other problems. For the natural/automatic account of order direction, see Section 3.

of comparable quantities (say heights of people) are measured, the same scale composed of an ordering of measurement units is applied to each, with the result that each quantity is associated with an ordered subset of measurement units, its measurement, drawn from the same set. So then, if linear additive measurement is applicable, because it starts at zero one can assume that *measurements* a and b (but not the quantities corresponding to them) are in a subset relationship, a $\subseteq$ b corresponding to a $\leq$ b or a $\supseteq$ b corresponding to a $\geq$ b or both. However, there is no experimental or other evidence that heights or weights themselves are perceived and thought of as composed of scalar units, let alone scalar units reused across the individuals perceived, and both ideas are intuitively weird and to be avoided. Therefore, not even members of ordered sets of additive quantities are required to be in any subset relationship with each other. On the contrary, quantities are completely distinct unless *specifically defined* as overlapping sets: a $\geq$ b normally does *not* correspond to a $\supseteq$ b. The fact that not all linguistically recognized quantities can be measured additively makes a degree approach even less attractive. *The general notion of a quantity should not be defined as a subset of a (measurement) scale.* It is important to begin this paper with the correct notion of an ordered set of quantities because it presents a model of how the notion is linguistically realized.

The phenomena of importance for formal-semantic modeling of a language using quantities as in Figure 1 are the following (Section 3 is especially important here). First, what causes the tripartite segmentation of an ordered set of quantities? To put it another way, why do gradable adjectives come in pairs, or why are there gradable-adjective dimensions? (My answer: the variable nature of the world and the nature of concept formation; most directly, the formation of a central prototype forcing the partition.). Second, why is the left segment treated linguistically as decreasing and the right segment as increasing? (My answer: this is mathematically required and therefore predictable.) Third, what, then, is the denotation of the positive degree of declension of a gradable adjective (to make this easier, let's focus on a Cartesian adjective pair like *tall/short*)? (My answer: a tripartite linear lattice of exemplar quantities supporting a probability distribution. Note that the whole dimension is one complex concept and that it is not enlightening to treat an antonym in isolation.) Fourth, what is the denotation of the comparative degree of declension of a gradable adjective? (My answer: the denotation of each comparative antonym is a separate linear lattice, also known as a chain, no probabilities and thus no prototypes being involved.).

The next section is a selective comparison of the literature with the approach of this paper. Section 3, the central section, then covers the logic of gradability and comparison. Section 4 presents English positive-to-comparative lattice mappings by drawing on the sample of 152 English gradable antonym pairs given in Lehrer (1985) to show that they fall lattice-wise into six distinct behavioral classes, the four already discerned by Cruse (1986) plus two more.[11]. Section 5 looks briefly at the resulting overall picture of English gradable adjectives.

2. Criticism of previous formal attempts to model gradability and comparison

The guiding critical principles are as follows. This selective review of the literature shows that all have been violated for many years.

(1) Because argumenthood is syntactic, there cannot be an argument that is always phonetically unrealized.

[11] The Lehrer (1985) list comprises 152 pairs, but *sane/mad* was dropped because I did not consider myself competent to judge British dialects, and the presumably American *fast/quick*, *serious/humorous* were dropped as not being antonyms in my dialect. I added *aggressive/unaggressive* (*clever* class), ending up with 150 pairs.

(2) Regarding gradable adjectives in particular, the syntactic and semantic frameworks should be able to handle the *full range of types of gradable adjectives, and the full range should be modeled* from the beginning.

(3) Importantly, a *clear and detailed psychological model of quantity concepts* is needed from the beginning.

(4) Importantly, the cognitive organization of quantity variation under single (often prototypical) concepts is *not limited to humans or to language users in particular*.

(5) Certain basic assumptions of cognitive psychology, such as what can be evidence for a particular cognitive process, should be adhered to when trying to model cognition.

Points (3) and (4) are the key to the difference between this paper and preceding work. An important point documented here is that everyone else has assumed that the denotations of positive-degree of declension gradable adjectives must be linguistically defined with respect to prototypical values.

The general format will be four paragraphs for each publication: (1) modeling conceptual quantities perceived in the world (principles (3) and (4) are relevant); (2) modeling positive degree of declension *tall* with and without a measure (all principles are relevant); (3) modeling the comparative degree of declension *taller than* (all principles are relevant); (4) optional additional comments.

2.1. Cresswell (1976, 1990)

(1) C. recognized primitive ordered sets of comparable conceptual quantities including their order direction <U; < > , <U; > >, said to be somehow associated with individuals; for

instance, a set of distances from sole to crown each associated with a human.[12] These sets were not specified as pre- or post-linguistic concepts. There was said to be a norm of height for U_{tall} for sets of humans but it was mentioned much later than U_{tall} itself in connection with the morpheme *pos* (below), and the implications of norms for statistical structure of the ordered sets were not explored.

(2) C. dealt only with one class of gradable adjectives, the *tall* class, and only with increasing *tall*, and furthermore with attributive rather than predicate adjectives. Based on the intuition that *tall* is about height and on sentences like *Bob is a six-foot tall man* in which the degree expression *six-foot* is clearly not a canonical argument, *tall* was said to have a degree *argument*, a syntactic error not recognized as such in the paper and later requiring a work-around in the form of *pos*. The analysis is montegovian though C. did not curry categories: a syntactic function from common nouns <e, t> to common nouns <d, <e, t>> where d is the quantity argument: <<e, t>, <d, <e, t>>>, where individual e is implicitly assumed to be the possessor of quantity d, such that a common noun is in the domain of *tall* iff it thereby becomes a function from individuals to propositions such that a world is a member of that proposition iff an individual is of the noun's description augmented with a height d. However as noted, *Bob is tall* does not include anything that can be called an overt degree argument and also, the semantic analysis needs to specify that Bob's height is above the norm. C. solved both these problems at once with the phonologically unrealized morpheme *pos*, short for 'positive degree of declension'.

[12] First, Cresswell actually began with the comment that the semantics of degree is intuitively based on scales composed of points called degrees (p. 266) but on the next page he switched to talking about *tall* as denoting distances, i.e. *quantities,* and he maintained that point of view for the rest of the paper. Second, on page 681 of his 1991 review paper, Klein helpfully but misleadingly pinned down the vague Cresswellian assumption of a correspondence between quantities and individuals as a one-to-one partial function from degrees onto order-equivalence classes of individuals.

Pos combines with adjectives like *tall* to satisfy the quantity argument and results in an ordinary non-prototypical adjective, so its category is <<e, t>, <d, <e, t>>>, <<e, t>, <e, t>>>. The semantic contribution of *pos* was given syncategorematically: a quantity satisfying its combination with *tall* must be above (>) some average quantity u "toward the top end" of the set of ordered quantities of height for the relevant population, thus above and excluding the average. This means that *pos* performs a comparison. In sum, if we fudge the problem of syntactic-semantic subject discrepancy by using the noun *height*, *tall* is semantically λx.have.height.above.norm'(x). In criticism, a phonological form, the strongest form of evidence for *pos*, does not exist, so it violates any reasonable theory of argument structure and only compounds the earlier error. Despite the fact that C. included d as an argument of positive *tall* because *tall can* occur with an overt quantity as in *Bill is a six-foot tall man*, he caused problems with *pos* for the analysis of such sentences with an overt quantity but without comparative *-er than.* If nothing else is said, they will wrongly be subject to *pos*, that is, *pos* somehow applies only if there is no overt quantity. Furthermore, the norm is not relevant for such a sentence with an overt quantity, since one can say for an adult Bob, *He is three feet tall*. Thus, some restrictions need to be put on *pos,* but they were omitted. Furthermore, for other classes of English gradable adjectives, the denotation of the positive degree of declension is sometimes identical with that of the comparative degree of declension, so the *pos* solution would require multiple kinds of *pos*.

(3) Comparative *Bill is taller than Tom* was taken as a reduced version of *Bill is a tall man more than Tom is a tall man* containing two clauses using the positive degree of declension (note that this incorrectly means that both individuals are tall); (2) each clause was taken as a nominalized predicate of quantities formed by abstraction λd.[tall(man)](d)(Bill/Tom); (3) the nominalizations were syncategorematically due to the complex comparative marker *-er than*

taking two nominalizations of this type as arguments, yielding a sentence meaning 'the quantity of Bill being tall is more than the quantity of Tom being tall' (definite *the quantity* because C. assumed that each person has only one quantity of height, an assumption made explicit in Cresswell (1990: 246). C. noted as an advantage of his account that it handled comparisons of two different dimensions with their requirement of two full clauses, provided the two dimensions were of the "same type", as with *Bill is taller than Ophidia (a snake) is long* with two linear dimensions of the same type, but not **Bill is taller than Tom is clever* with a first linear dimension and a second intelligence dimension or reduced **The book is longer than the rope* with a first text dimension and a second linear dimension. However, there is no reason in C.'s account of comparison for why dimensions have to belong to the same type -- his claim that it is a selectional restriction of *er than* merely restated the facts.

(4) Cresswell (1976) is unique and to be commended for ending by recognizing and wondering about the extension of quantities from physical dimensions to abstract and subjective ones like happiness or reluctance. C.'s key point here was that we have the cognitive ability to learn a general semantics for physical quantities because we are presented with sensory evidence of different kinds of quantities all the time, but we also have the ability to at least imagine that two things can be equally or non-equally beautiful, etc.[13] Even if this is imagination, it serves to induce an ordering of quantities of beauty, etc. Thus, we extend the formal semantics developed for physical quantities to other kinds. This was the point of his suggestion in both the 1976 article and the 1990 book that quantities be formally defined to be equivalence classes of exemplars, real or not, similar to the Frege/Russell/ Whitehead definition of positive integers

[13] See Thurstone (1927, 1959), who was first to experimentally address this very issue of scales of abstract, subjective quantities.

based on metamathematical set cardinality, even though this may seem to be superfluous when one deals with perceptible physical quantities, those being, as far as formal semantics is concerned, evident properties/sets of entities.

2.2. Klein (1980, 1982)

(1) There is an ordered set of comparable individuals C (for 'comparison class') assumed to have varying quantity-properties like height, weight, etc. C is said to be contextually given, e.g., in a discussion about professional basketball, the heights of centers could be included in the discussion and comprise a C. The relation of C to a norm was not immediately discussed.

(2) K. semantically analyzed predicative rather than attributive adjectives, and focused on the *tall* class, briefly mentioning the distinct behavior of *clever*, a member of a different class. A gradable adjective like *tall* predicates a quantity range of an individual based on the C it belongs to; a quantity argument is deliberately excluded. For example, in the basketball discussion just mentioned, Klein would suggest that the tallest center would naturally be picked out as tall and the other centers would be not-tall. At any rate, the adjective *tall* means $\lambda C \lambda x_{\in C}$.tall'Cx which therefore has a characteristic function with value 1 if the individual is judged tall relative to other members of C, 0 if not-tall, and undefined if in a so-called extension gap *which was taken to include the norm.* This inclusion of the norm (usually understood in the statistical sense of prototype, but not by Klein) in the extension gap makes the latter not equivalent to the truth-value gap of Fine (1975), Kamp (1975), et al. because the latter's truth-value gap excludes people in the prototype (whose heights *are* distinguished as non-tall given the current grain of

discrimination), only people whose heights fall on the tall-prototype boundary so that it is *impossible* to distinguish them as tall or non-tall given the current grain of discrimination (for which compare, for instance, rounding off to whole centimeters vs rounding off to whole millimeters). In other words, Klein went against the ordinary meanings of *non-tall* and *short* (equivalently, *non-short* and *tall*). For him, *non-tall* had the special meaning 'short' and the extension gap comprised the norm -- those who everyone else calls 'neither tall not short' -- plus the truth value gap between the norm and *tall*.[14] Interestingly, Klein admitted that *tall* predicates a quantity of an individual *based* on a norm, but he idiosyncratically took that norm to be defined by just C rather than by some wider population. In criticism, first, if the so-called norm has no statistical basis, it is not a norm but a highly suspect *purely arbitrary* criterion. Second, it is therefore unsurprising that Klein's so-called norm arbitrarily distinguishes between the *tallest* member of C and all the rest. Third, it simply does not work for the so-called norm to be defined by C because one *cannot* say of the tallest member of a C composed of two very short people, *She is tall*. The real height norm still governs the use of *tall* and to pretend otherwise is again contrary to English.[15] Leaving the linguistic criticism of class C aside, the treatment of the norm as an arbitrary quantity was needed for the analysis of comparatives in (3) based on ideas of Lewis (1972), who said he got them from Kaplan. In summary, Klein avoided basing the analysis of comparatives on an ordered set of quantities together with its prototype in the world because to do so, he claimed, would be circular and non-compositional, and consequently he avoided

[14] von Stechow (1984) notes that this criticism was also made by Hoepelman (1982).
[15] Klein's analysis was facilitated by his use in the 1980 paper of illustrative sets of objects for which there is *no population-wide prototype*, namely rectangular figures of different lengths. Of course, I do not intend by this criticism to suggest that it is impossible to do without d, e variables.

using d or e variables, but he used a C variable instead and his analysis based on it does not work.[16]

(3) K.'s analysis of comparison is based on (2), where f is a function yielding a norm. The idea is that *any such function* should be constrained so that the ordering of individuals in the world is respected -- the influence of supervaluation theory is obvious here. To make the idea of functions yielding arbitrary norms linguistically respectable, Klein suggested that what he calls degree modifiers including *six feet, very, as, more, less, that, how* (the list lumps together syntactically/semantically distinct items) are such functions -- their semantic effect is to induce a two-set partition of the comparison class, say those entities which satisfy *very tall* and everything else. Function f, then, is any degree modifier added to the positive degree of declension form, here *tall*, which makes the following expression true:

(2) f(tall(x)) & ¬f(tall(y)), e.g. let f = degree-modifier that':
that'(tall'(bill)) & ¬that'(tall'(joe)) must reflect the world ordering, i.e. be true

(Compare the similar Boolean-operator analysis of comparison in Seuren (1973); however, Seuren used quantity variables and quantification, not quantity adverbs.) Although the degree modifier in (2) is just a device to give the desired partition of C, it should respect the linguistic facts. As shown in Section 1, the use of the degree modifier *that* which I have used in (2) requires the dimension-label use of *tall*, meaning that (1) Bill can actually be short, and (2) *tall* is not actually the positive degree of declension form. Only if the adjective does not have a dimension label does *that* operate in the positive degree of declension semilattice. K. did not recognize the complex behavior of so-called degree modifiers covered in Section 1, and if he had

[16] Just as he did not reject ordering in the world, Klein did not totally reject d or e variables -- see his discussion of those in Klein (1982). He only rejected them for the analysis of the comparative, and only because he claimed that using them would be circular/non-compositional.

unknowingly chosen one that operates in the comparative lattice, then his analysis of the comparative would be circular. Thus, *enough(tall)* would already be comparative without the structure in (2). Therefore, (2) is linguistically compromised, failing to guarantee an analysis of the comparative based on the positive degree of declension.

To get the desired effect of the order of individuals in the world, norm-shifting f in (2) is assumed to be subject to meaning postulate (3) for g = gradable adjectives:

(3) $\forall xyg[\exists f(f(g(x)) \mathbin{\&} \neg f(g(y)) \rightarrow \forall f\, [f(g(y) \rightarrow f(g(x))]]$.

This postulate rules out the possibility left open by (2) that the latter is true because f(g(y)) is actually bigger in extent than f(g(x)).

Ironically, a norm is irrelevant for comparatives of the *tall* class, so using a *norm* to distinguish the two comparative partition classes for that class of adjectives was misguided.

(4) von Stechow (1984) remarked in connection with Klein (1980) that quantity variables are absolutely indispensable for the analysis of is *six inches taller than, is twice as fat as*, and *is more tall than broad*. Klein (1991) suggests in response that this is not so. Section 3 of this paper shows that von Stechow's claim is wrong. The difference between the semantic representations of adjectives of the positive degree of declension and the comparative degree of declension is more complicated and less "inflectional" than Klein realized; in fact, it requires lattices and specific lattice mappings to relate them. Nevertheless, it is true, as Kamp (1975) recommended and Klein hoped to show, that it is the positive degree of declension forms that are the morphosyntactically and semantically basic ones

2.3. von Stechow (1984)

(1) Being a survey paper, von Stechow (1984) is a useful gauge of the amount of advance since Cresswell's paper, which was not overwhelming despite various intervening publications

and despite relevant advances in the psychology of concepts. von Stechow closely followed Cresswell (1976) for much of his own analysis, beginning with assumptions about the world, and thereby acknowledged Cresswell as the central figure in the semantic analysis of gradability and comparison.

(2) The analysis of the positive degree of declension attributive *tall* was essentially that of Cresswell (1976), with a change in *pos* to remedy some of the problems with it. *Pos* was given an individual argument in order to explicitly specify that the quantity that an individual has is above the norm: if w is a world, A is a gradable adjective, C a common noun, and x an individual, then $w \in ||pos||(A)(C)(x)$ iff $\exists d[d$ is an A-degree & $d > average(A,C)$ and & x is d in w and $w \in C(x)]$. v. S. explicitly says that the function *average* is an "empirical question" outside the domain of truth-functional semantics; however, with this analysis he imports it into semantics. At any rate, the preceding serves to make it impossible for *pos* to be used if *six-foot* is also present as an argument of A.

(3) v. S.'s analysis of the comparative, however, abandoned Cresswell's "generalized quantifier" analysis in favor of one closer to Government and Binding Logical Form and making heavy use of existential quantification of degrees. Abstraction/nominalization of both compared quantities was tied to scoping them in Logical Form along with quantifying-in.[17] To handle quantity adverbs in comparison like *is six inches taller than* and equatives like *is twice as heavy as*, v. S. adopted Hellan's (1981) Measure-Theory based analysis of quantity representation. v. S correctly concluded that comparative standards are definite, but incorrectly attributed this to an

[17] The invocation of the role of LF here may prompt some readers to wonder how I can ignore syntax and especially syntactic variables in this paper as the title promises. As already stated in footnote 2, much of the machinery used to construct earlier accounts of gradability and comparison has been discarded for other reasons as unworkable.

ad hoc max operator in the standard that was also motivated by plural standards: *Ede drinks more than any of us* means that Ede drinks more than *the maximum* that any of us drinks. A parallel min operator would be needed for negative-polarity adjectives but was omitted. These operators are made obsolete by the logical analysis of comparison to be given in the next section. It must be noted here that the definiteness of standards is correlated with a universal feature of strict comparative constructions that has been ignored in the literature, namely, their *presupposition* of the comparative standard against which a greatest lower bound or a least upper bound is asserted.[18] That presupposition is most evident in English when verbs or nouns rather than gradable adjectives are compared with *more* and *less* because, as already noted, many of the adjectives relax their denotations in the comparative degree as compared with the positive degree. However, just as an utterance of *Bill did too* requires that Bill did the same contextually specified thing as someone else, so does comparative *More girls came in* require that some other people, asserted to be less numerous than the girls, also came in, and *Bill is taller* require that he is taller than the height of someone, perhaps just himself at a younger age, though it does not require that that person be or have been tall. Being a presupposition rather than an assertion, a standard in its full propositional form is defeasible. Thus, in *If Bill was there, I was there longer*

[18] See footnote 8 on strict comparison. The presuppositions of comparative operators *more/-er* and *fewer* were noted only by Sapir (1944), Keenan and Faltz (1985), and Keenan and Stavi (1986), the latter remarking that that positive *many/much* and *few/little* are "implicitly comparative", making value judgements against a standard. The fact that von Stechow consistently gave raised standards not only definiteness but also wide scope is a clue to their presupposition. Section 3 shows that definiteness of standards is forced by the logic but whether one can attribute the presupposition to that fact is moot -- it may be an independent phenomenon. At any rate, a presupposed standard is important and welcome because it reduces the semantic and syntactic complexity of comparative constructions, transferring much of interpretation to the resolution of anaphora and enabling so-called comparative ellipsis. This paper does not go into the compositional semantics of comparative constructions and so does not further discuss their presuppositions and how they enable comparative ellipsis.

than him the standard of *longer*, namely *than him* = 'Bill was there for period x' is no longer in force -- maybe he was not there at all. The preceding examples with common anaphoric omission of comparative standards serve to bring out the fact that those are presupposed, because otherwise anaphoric omission of arguments in English is limited compared to many other languages (see van der Sandt (1992), Geurts (1999) on presupposition and anaphora, Enrico (2021) on anaphoric omission as evidence for argument structure).

2.4. Kennedy (1999 et pass.)

(1) Kennedy is explicit about adopting the extent model (see footnote 10). There are sets of individuals with comparable and varying extents unique to their kinds. At the beginning of Kennedy (1999), he suggested that recognition of this size variation is regimented by language rather than being independent (pre- or non-linguistic):

> In the vague predicate analysis, the ordering on the domain is presumed; in the scalar analysis, however, the adjective imposes an ordering on its domain by relating objects to a scale. (page 42)
>
> . . . the ordering that can be imposed on its domain is derived from a semantic property of the adjective itself . . . (pages 42-43).

However, this point of view makes it impossible to account for prototypes/norms/standards, so it was never developed further. Instead, prototypes were recognized and modeled by an averaging function STND from sets of extents to their mean values (1999, Section 2.3.2).

(2), (3) A predicative gradable adjective like *tall* maps an individual onto an extent from zero to some degree (real number) on an increasing scale, here of height: λx.tall'x = d. Again, the *tall* class was focused on. Each gradable adjective is a measure in the mathematical sense of

Measure Theory: a homomorphism from the lattice of quantities in the world to the degree lattice by virtue of a zero in the world being mapped to a zero degree and fusions of quantities in the world being mapped to sums of degrees, with the latter of course being equal steps.[19] This solved the problem first encountered with Cresswell -- that quantities are not canonical arguments of gradable adjectives. A first problem with Kennedy's approach is that a comparison class "in the real world" cannot in general be ordered in equal steps, which are required as practical approximations of real numbers. A second problem is that such a mapping makes a gradable adjective more cognitively complicated than is empirically justified, because first, measurement is a technological achievement that is distinct from natural eyeballing, hefting, etc. types of comparison and second, appropriate scales have to be assumed for a very large number of *never-before-measured* gradable dimensions (nine dimensions in Lehrer's (1985) sample of 152 have measures). However, the complication is glossed over by never examining how the supposed measurement occurs: no homomorphisms between real-world orders and adjective scales are ever suggested. A third problem is that the positive degree of declension without a measure as in *Bob is tall* continues to be treated like a comparative; in fact, it is collapsed with the comparative in a degree-morpheme construction, just the opposite of the treatment of the two degrees of declension in this paper. That construction gives equal treatment to all of a revised phonetically null *pos* morpheme ABS (for absolute) plus -*er/more*, *less, as*. ABS is now semantically λGλPλx.ABS(G(x))(STND(G)(P)) with G = gradable adjective, P = the nominal kind of which x is a member, ABS = the phonetically null degree morpheme, and STND a function supplying the

[19] Logarithmic and non-additive (interval) scales must be allowed, however (e.g. the most familiar non-additive scales are temperature scales).

mean value of the degrees of the G scale applied to P.[20] In other words, ABS *compares* the degree supplied by the gradable adjective applied to the individual with the degree supplied by the standard function, which also indirectly depends on the individual: the individual's degree must be equal to or greater than the standard degree for a proposition based on ABS to be true. A fourth problem was mentioned in footnote 10: the extent model's lack of a non-ad hoc account of polarity. A final fifth problem also was mentioned in that connection, the distortion of the order relation by the requirements of measurement. In an extent model, extents *themselves* (as opposed to *lattices* to which they belong) have internal (real-number) structure and are increasing or, in an ad hoc fashion, decreasing.[21]

3. Introducing the logic of gradable adjectives with the *tall* class

Let P be a partially ordered set <P; ≤ >, for example, the extension of a comparative degree of declension gradable adjective like *taller*. Because a ≤ b iff b ≥ a, the existence of <P; ≤ > requires the existence of <P^{∂}; ≥ >, called the dual of P, with matching properties, including the required equivalence $P = P^{\partial}$. P^{∂}, then, is the extension of the antonym *shorter*.[22] Order direction (increasing or decreasing) in the *linguistic case*, not the mathematical case, is called polarity. The term polarity is used because linguistic application of the negative order ≥ will reverse the

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[20] The primary goal of general phrase-structure templates covering as much territory as possible is the syntactic version of the structuralist-linguistic goal that was set with the discovery of the phoneme, -*emic structures* that each cover as much contextually determined variation as possible.

[21] An obvious objection is that the order direction from which a quantity is observed is due to the linguistic context in which it is referred to, not to the quantity itself. There is no other way to determine if a glass is half-full or half-empty. Also, though trying to use any version of Measure Theory to model linguistic quantities would be a mistake, a proper integration of logical order direction and Measure Theory would have been to use signed measurement (using the whole real number line, positive and negative).

[22] We start with the comparative lattices because they are simpler.

direction of both negative and positive orders in its scope, just like multiplication by a negative number will reverse the polarity of both negative and positive numbers or, similarly, sentence negation will reverse the polarity of both positive and negative sentences. The reversing property of negative polarity is obvious in comparative constructions, in which positive *more* is increasing but negative *less* is decreasing and reversing, as in *less small* = *bigger*, *less big* = *smaller*. Positive-polarity *more* has no such reversing effect and therefore the negative-polarity adjective is, intuitively, more cognitively complex than the positive-polarity one.[23] One more point: as just seen, both comparative antonyms come in both positive (*taller, shorter*) and negative (*less tall, less short*) polarities. This means, as will become clear when we get to the definition of a lattice, that there are *two* comparative lattices for each antonym pair, one for *taller/less tall* and one for *shorter/less short,* and in the case of the *tall* class, those two comparative lattices share the set P = P^{∂}. By the same argument, all gradable adjectives have two comparative lattices, though they are not always duals as they are here. Their shared set in the case of the *tall* class, however, is why *Bill is six inches taller than Mary* means the same as *Mary is six inches shorter than Bill* (recall that two sets are equal iff they have the same members).[24]

The *tall* class is the English version of the set of most frequent and cognitively simplest gradable adjectives/stative verbs in any language. The sample used here comprises 29 members: *abstract/concrete, abundant/scarce, big/little, compact/diffuse, complex/simple, conformist/non-conformist, deep/shallow, expensive/cheap, far/near, fast/slow, fine/coarse, hard/soft,*

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[23] Clark (1969) experimentally demonstrated this greater cognitive complexity. However, his more or less off-the-cuff assumption that this takes the form of the meanings of negative-polarity adjectives being contradictory negations of positive-polarity ones must be rejected because, e.g., 'short' is not equivalent to 'not tall'. One would have to suggest a covert contrariety operator equivalent to *in-*/*un-*, but ideally there are better accounts of the complexity of negative polarity.

[24] Kennedy (1999: 186-187) observing this fact for the *tall* class, incorrectly asserted that it holds for all gradable adjectives.

heavy/light, high/low, large/small, long/short, loud/soft, many/few, old/new, old/young, outgoing/shy, plentiful/scarce, pungent/bland, shiny/dull, tall/short, thick/thin, tight/loose, warm/cool, wide/narrow. The first canonical feature of the class is that members are mostly based on very salient physical and usually additive dimensions, a few being based on very salient cultural properties such as cost and wealth. The high cultural relevance and salience of the dimensions covered by this class is evident in the sample number which allow numerical measures: 8 of the 29 *tall*-class members in the sample occur with measures, and the sample total of the five other classes combined yield only one pair occurring with numerical measures, namely *early/late* in the *hot* class. The second canonical feature is the cultural/behavioral fact that each antonym is sufficiently important/relevant/salient that each justifies its own efficient monolexemic word, therefore suppletive (*rich* vs. *poor*/**non-rich*/**unrich*). The second canonical feature is revealed by the first of Cruse's (1986) tests defining the class, namely that the positive-degree of declension pairs display an extension gap that allows well-formed *He is neither tall nor short, It is neither cheap nor expensive,* etc., examples which are equivalent to *He is of average height, It is of average cost*, respectively. This gap and its meaning are explained by the prototypical statistical structure of the positive-degree lattice. As noted in Section 1, these adjectives justify their existence by denoting the newsworthy extremes of their dimensions -- height, weight or whatever -- rather than the ho-hum predictable prototypes in a population. Importantly, because a statistical distribution is by definition a single set (here an ordered set of quantities), the positive degree of declension of an antonym pair of this class is a *single lattice* simultaneously serving as a statistical distribution. Indeed, by systematically going through Lehrer's (1985) sample, we will find that all positive-degree adjectives, not just in the *tall* class, denote single lattices (though not all are prototypical).

Cruse's second semantic test isolating the *tall* class is that in the comparative degree, both antonyms each cover the whole population-wide set of exemplar quantities that the positive-degree lattice covers with two separate sets plus a mean set. That is, it is possible to say both *Podiatrists and chiropractors are scarce here but the latter are* ***more abundant*** *than the former* and *Podiatrists and chiropractors are abundant here but the latter are* ***scarcer*** *than the former.* Compare *smart/dumb* in another class: *Bob is dumb but he's* ***smarter*** *than Bill, *Bob is smart but he's* ***dumber*** *than Bill*, showing that the denotata of *dumber* are not included in those of positive-degree *smart* while the denotata of *smarter* do include those of positive-degree *dumb*, i.e. the two comparative denotations are not duals. Finally, Cruse's third test isolating the *tall* class reveals that the extensions of these adjectives do not have upper or lower bounds: **He is absolutely/perfectly tall, *He is completely short*.

Let us now look at something else that the mathematics of partial orders in general explains or predicts about these adjectives. An important property is the possibility of down-sets and, dually, up-sets in a partially ordered set P. Let $P \subseteq Q$ for $<Q; \leq>$. P is a down-set or order ideal in Q if whenever $x \in P$, $y \in Q$ and $y \leq x$, then $y \in P$. Dually, P is an up-set or order filter in Q if whenever $x \in P$, $y \in Q$, and $y \geq x$, $y \in P$. Up-sets and down-sets in the same ordered set Q have opposite orders. Now, suppose that there is an ordered set Q. The following lemma is important.

Lemma 1. P is a down-set in Q iff Q\P (the complement of P in Q) is an up-set in Q.

Proof. By duality, if P is a down-set in Q then there exists an up-set R (it is irrelevant that if Q = P that up-set will be $\emptyset$). The elements of that up-set also comprise, by definition, Q\P, so R = Q\P ■

The consequence is that a down-set and an upset can co-exist in the same ordered set. Lemma 1 predicts and explains the existence of opposite-polarity gradable-adjective antonym extensions making up a single larger *dimension* extension as is required by the latter's prototypicality (statistical nature). The two opposite-polarity extensions are commonly but not necessarily separated by the prototype region (there may be no prototype, depending on the context -- see footnote 15). Furthermore, recognition of a mean height extension-gap region in a probability distribution for height, etc. automatically leads to two antonyms of opposite polarity -- the prototype can be said to generate the tripartite positive degree of declension representation.

Another property important in this paper because it leads to lattices, which in turn enable comparison, is the possibility of upper and lower bounds. Let $<P; \leq>$, $<P^{\partial}; \geq>$ be dual partially ordered sets. One optional matching property of dual ordered sets is the existence of lower bounds of P, upper bounds of P^{∂}. Then a lower bound of P is an $x \in Q$ where $P \subseteq <Q; \leq>$such that $y \leq x$ for all $y \in P$, dually for an upper bound of P^{∂}. Note that upper bounds and lower bounds of an ordered set may or may not be members of that set. The least upper bound (lub) of P, if it exists, is the member x of the set of all upper bounds y of P such that $x \leq y$ for all y, dually for the greatest lower bound (glb) of P^{∂}. The lub of P is also called the supremum of P (sup P), and the glb the infimum of P^{∂} (inf P^{∂}). If a supremum/lub exists, it is unique (suppose a, b are both lub's of an ordered set P; then $a \leq b$ and $b \leq a$, and by antisymmetry, $a = b$). Since $P = P^{\partial}$, any partially ordered set can have both a supremum and an infimum.

A poset *lattice* $<L; \leq>$ is a partially ordered set L (or dually $<L^{\partial}; \geq>$) such that sup{x, y}, inf{x, y} exist for all $x, y \in L$. This poset view of lattices is usually replaced by an algebraic view in which inf and sup become dual binary operations, $\wedge$ (inf or meet) and $\vee$ (sup or join) defined as $a \vee b = a$ iff $b \leq a$ and $a \wedge b = a$ iff $a \geq b$, or as obeying the following identities.

(4)	$a \vee b = b \vee a$, $a \wedge b = b \wedge a$	(commutative laws)
	$a \vee (b \vee c) = (a \vee b) \vee c$, $a \wedge (b \wedge c) = (a \wedge b) \wedge c$	(associative laws)
	$a \vee a = a$, $a \wedge a = a$	(idempotent laws)

$a \wedge (a \vee b) = a, a \vee (a \wedge b) = a$ (absorption laws)

Poset $<L; \leq >$ is equivalent to algebraic $<L; \vee, \wedge >$ and poset $<L^{\partial}; \geq >$ is equivalent to algebraic $<L^{\partial}; \wedge, \vee >$, that is, the order of the operations meet and join in the angled brackets indicates whether the order direction of an algebraic lattice is $\leq$ or $\geq$. In this linguistic paper, for convenience $<L; \leq >$, $<L; \vee, \wedge >$ will be referred to as positive-polarity lattices and $<L; \geq >$, $<L; \wedge, \vee >$ as negative-polarity ones. The algebraic view makes it easy to define denotations as up-sets and down-sets that are closed with respect to only one operation, which is important for modeling positive degree of declension adjectives. As will be shown next, it also makes it easy to represent the comparative operators 'more', 'less' with the lattice operations join and meet, respectively, with some assistance from implicature to eliminate equality. Note that each comparative antonym makes use of both join and meet, i.e. is a full lattice.

Finally, the lattices evoked by gradable adjectives and their comparison are in general *chains*, which means that they satisfy the additional identity of linearity: for all a, b in L, $a \leq b$ or $b \leq a$, ruling out branching, which corresponds to pairs of members outside the domain of $\leq$. It also means that they fall into the large class of distributive lattices, that is, that they obey two equivalent distributive identities which are omitted here, and they can be taken to make up categories in the sense of Category Theory.

Thus, important for cognitive modeling, cognitive equivalents of lattices are evoked (made cognitively accessible) by gradable adjectives in two linguistically distinct ways, i.e. there are two distinct types of lattice structures, one contributed by the base (positive-degree) forms of the adjectives and the other by their occurrence in comparative constructions -- the two degrees of declension are logically distinct and must be examined separately for each antonym pair. Furthermore, each antonym pair demonstrably corresponds to a single positive-degree lattice and to two comparative-degree lattices. Therefore, it is necessary to determine whether the single positive-degree lattice is positive-polarity or negative-polarity. Positive degree of declension lattices will be assumed to be default positive-polarity for the following reasons. First, as already noted, positive polarity is cognitively simpler. Second, there is evidence that positive polarity is acquired earlier than negative polarity (Donaldson and Wales 1970, Palermo 1973, Holland and Palermo 1975, Gärdenfors 2014: 143, among others). Third, negative-polarity antonyms are often formed by prefixing positive-polarity ones, as with *possible/impossible, happy/unhappy* (see Horn 1989, Chapter 5 for discussion). Fourth, if one antonym is used as a dimension label, it is normally the positive-polarity one (see Section 5). Fifth, if the culture allows for additive measurement, as is most common for the *tall* class and its correspondents in other languages, one must measure upward from zero (positive-polarity).

Summarizing to this point, it is a lattice up-set or down-set (more usually referred to as a lattice filter and ideal, respectively) that occurs as the denotation determined by selective binding of a positive- or negative-polarity adjective plus noun. In the case of the *tall*-class, these sublattices, the ideal for *short* and the filter for *tall*, are each an open interval $(a, m_1) = \{x \mid a < x < m_1\}$ and $(m_3, b) = \{x \mid m_4 < x < b\}$ respectively, separated by mean exemplars $m_2, \ldots, m_3$. The whole structured lattice $(a, m_1)(m_2, m_3)(m_4, b)$ is the domain of a full two-tailed likelihood distribution.

Argument structures for *tall/short* are given in (5a). In fact, the forms in (5a) cover the syntax of all English gradable adjectives. Degree adverbials like *quite, rather, very* select subsets X of the unmodified extensions at either their low or their high ends.

(5a) One positive-degree lattice

(5a.i)	$tall_1$+n/$short_1$+n	*subj* $\|x\| \in$ 'tall'/'short'
(5a.ii)	degree adverbial+$tall_2$+n/$short_2$+n	*subj* $\|x\| \in X \subset$ 'tall'/'short'

(5b) Two comparative-degree lattices, of which only that for 'tall' is used here
(5b.i) degree morpheme+$tall_3$+n *subj* $|x| \rightarrow |y|$
(5b.ii) er/less+$tall_4$+n *subj, comparative* $|x| \wedge |y| = |y|$ or $|x| \vee |y| = |x|$
(5b.iii) degree morpheme+-er/less+$tall_5$+n *subj, comparative* $|x| \in |y| \cup |z|$ for 'x is z more' or *subj, comparative* $|y| \in |x| \cup |z|$ for 'x is z less' where $|z| \rightarrow |w|$

The argument structure in (5b.i) using the two comparative lattices requires the presence of a degree expression derived from measurement, e.g. *three inches*, *six feet*. Such an expression evokes a homomorphism from the quantity lattice to a measure lattice, and the latter must be positive-polarity. The arrow $\rightarrow$ in (5b) represents that homomorphism. Consequently, the domain of the homomorphism, the quantity lattice, must also be positive-polarity, e.g. 'tall' but not 'short'. So even though comparative-degree *$short_3$* covers the whole comparative lattice just like *$tall_3$*, it has the wrong polarity to map to a measure lattice.

The term *comparative* in (5b.ii, iii) refers to the comparative standard, the English *than* expression. Recall that because of the lattice idempotency and absorption identities in (4), repeated here, the two lattice operations are defined as $a \vee b = a$ iff $b \leq a$ and $a \wedge b = a$ iff $a \geq b$, so they appear at first glance unsuitable for natural-language comparison with its strict orders.

(4) $a \vee a = a$, $a \wedge a = a$ (idempotent laws)
$a \wedge (a \vee b) = a$, $a \vee (a \wedge b) = a$ (absorption laws)

However, Gricean principles as revised by Atlas and Levinson (1981) plus the comparative context account for the fact that $a \vee b = a$ is taken to mean $a \wedge b \neq a$, and therefore $a \neq b$; instead, it must be that $a \wedge b = b$, i.e., $b < a$. The Q principle as stated by Levinson (2000: 76) is "Select the informationally strongest paradigmatic alternate that is consistent with the facts". Selection of $a \vee b = a$ thereby excludes the paradigmatic alternate $a \wedge b = a$ and logical $\leq, \geq$ are automatically interpreted in a comparative context as $<, >$, respectively. The meaning of *Bob is taller than Bill* is derived as follows. *$Taller_4$* evokes (makes cognitively accessible) a full lattice $<L; \vee, \wedge>$

minus any distinguished mean segment, simultaneously placing in it the quantities of the relevant discourse referents.[25] The perspectival argument of the adjective -- the subject -- is in the relation |bob| $\vee$ |bill| = |bob| in that lattice, where |α| denotes the relevant type of quantity for the discourse referent α, here vertical extension. This implicates |bill| < |bob|. Mutatis mutandis for *Bill is shorter than Bob*. The meaning of more complicated *Bob is less short than Bill* is derived as follows. *Less short$_4$*, like *taller$_4$*, evokes a full lattice minus a mean segment, $<L; \vee, \wedge>$, simultaneously placing in it the quantities of the relevant discourse referents. The perspectival argument of the adverb plus adjective -- the subject -- is in the relation |bob| $\vee$ |bill| = |bob| in that lattice, and that implicates |bill| < |bob|. In the rest of this paper, the implicatures involved in understanding comparison will be taken for granted.

(5b.iii) is the comparative of (5b.ii) plus a degree expression giving the amount by which the standard is exceeded or by which the standard is not met. $\cup$ is Link's (1983) quantity fusion. The degree expression can either be a measure as in *Bob is five inches taller/shorter than Bill, Bob is five inches less tall/less short than Bill* or else *much, very much, somewhat, a bit*, *a lot*, etc. The preceding examples show with differential quantities (here measured as five inches worth of height) that quantities themselves are polarity-neutral, contrary to Kennedy (1999, Chapter 3). Other argument structures properly belonging under (5b) include intensional

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[25] This eliminates Kennedy's (1999) "cross-polar anomaly", as with **Mona is happier than Jude is sad* and "incommensurability" as with **Larry is more tired than Mike is clever*. The possibility of his "comparison of deviation" as with *Robert is more happy than Jude is sad* is explained below in this section and in Section 4.4. The apparent violations of incommensurability as with *The plant is taller than the ceiling is high* are explained by (1) the fact that eyeballing or minimal manipulations are enough to overcome incommensurability in such cases, and, if one wants to bring in measures, then (2) the fact that certain measures are mapped to by more than one dimension because the technology of measures has been developed as economically as possible. Comparative incommensurability is a fact about the world, not about language.

tall+n+enough(+infinitive), too+tall+n(+infinitive), which are not covered here due to the paper's restriction to extensions.

The *tall*-class mapping between (1) the single *positive-degree* lattice $(a, m_1)(m_2, m_3)(m_4, b)$ composed of two semilattices $<(a, m_1); \wedge >$ and $<(m_4, b); \vee >$ plus the mean interval and (2) the dual *comparative-degree* lattices $<L; \vee, \wedge >$, $<L; \wedge, \vee >$ can be viewed as a shifting of m_1 and m_4: by identifying the former with b and the latter with a, one obtains a single L = (a, b).[26] Concurrently and automatically with the shifting, the domains of the two operations become identical to L. Two dual lattices sharing L instead of one lattice with dual operations, however, is a fact of language, not mathematics, being required to accommodate antonyms and having the consequence of making language more expressive.

So far, only two-place $\wedge$, $\vee$ have been used. Comparative standards, however, can be plural or quantified: *Joe is taller than Pete, Tom and Lou* (conjunction), *Joe is taller than everyone in his class* (extensional quantifier), *Joe is taller than I will ever be* (intensional quantifier). Such sentences raise three issues. First, how do lattice operators and Boolean operators *and, or, not* interact in general (scope-wise)? Second, there is a need to generalize the lattice operations to arbitrary sets, calling for another lemma. Third, call {Pete, Tom, Lou}, etc. above comparison classes; what are their properties? These are taken in order.

The interaction of modeling with a Boolean lattice vs. with a Boolean algebra is an interesting topic because often either is possible, and therefore they can compete.[27] Take, for example, a set of 100 people divided into five sets of 20 each, making three organizational (set-theoretic, Boolean-algebra) levels of 1, 5, and 100 sets, respectively. Boolean-lattice-wise, apply

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[26] A more conventional mapping to L would be to form the linear sum $(a, m_1)\oplus(m_2, m_3)\oplus(m_4, b)$ but this does not so nicely motivate the extensions of the domains of the two operations.
[27] Boolean lattices are discussed in Section 4.4.

three alternative arrangements of decision-making power (lattice structure, Boolean lattices): the five sets can arrive at decisions by consensus of the 20 members of each or by having just one of those sets make the decisions or by having 100 individuals arrive at decisions by consensus among themselves. In the real world, one would probably find the second lattice alternative to be the most practical, and that fact would force an organizational (Boolean-algebra) change, the development of an executive or officer corps whose members are embedded in the major units. The forces modeled as lattice structure, therefore, tend to determine organizational or set structure. The same is true in language, which on the one hand is thoroughly Boolean in organization (Keenan and Faltz 1985) but on the other hand has to accommodate gradability and comparison. This is manifested by the fact that the linguistic Boolean-algebra operators *and, or* (leaving *not* for separate discussion) must be within the scope of the linguistic lattice operators *more, less* as shown by the possibility of distribution. The comparative operators distribute over the Boolean operators as in (6a) but the Boolean operators do not distribute over the lattice operators as in (6b), which is uninterpretable (meaningless).

(6a) $|x| \wedge_L (|y| \text{ and/or } |z|) = (|x| \wedge_L |y|) \text{ and/or } (|x| \wedge_L |z|)$
Bob's height is more than Mary's height and Joe's height = Bob's height is more than Mary's height and more than Joe's height.
Bob's height is more than Mary's height or Joe's height =
Bob's height is more than Mary's height or more than Joe's height.

(6b) $|x| \text{ and } (|y| \wedge_L |z|) = (|x| \text{ and } |y|) \wedge_L (|x| \text{ and } |z|)$

Turning to the generalization of the lattice operators needed to handle sentences with arbitrary set standards, the lemma necessary for one operator has conveniently been provided by Birkhoff and Mac Lane (1965), as follows, and the other follows dually.

Lemma 2 (Birkhoff and Mac Lane 1965: 318). Let f and g be two expressions formed from all of the letters $a_1, \ldots, a_n$ using only $\wedge$'s. Then f = g.

Proof. Follows from the idempotent, commutative and associative laws of (4) ■

This lemma makes it possible to abbreviate all the multiple expressions it refers to as simply $\bigwedge_i^n a_i$. Consequently, *Joe is taller than Pete, Tom and Lou* is semantically |joe| ∨ $\bigvee\{|pete|, |tom|, |lou|\}$.

Regarding the third issue mentioned above, the properties of the comparison class, just as the perspectival term (usually taken as the first term) of a comparative operator is unique *in its comparison class*, being the least upper bound or the greatest lower bound of that class, so too is the other term of the operator in the reduced comparison class that excludes the perspectival term. Although this is obvious, being justified by definition and Lemma 2, just to be certain it is understood, it is given here as a lemma for finite sets.

Lemma 3. Let $a_i \in$ <P; ≤ > for $1 \leq i \leq n$. If n =2, let $\bigvee_i a_i = a_1 \vee a_2 = a_1$; if n > 2, let $\bigvee_i a_i = a_1 \vee (\bigvee_2^n a_i) = a_1$. Then $\bigvee_2^n a_i$ is itself a unique member of $\{a_2, \ldots, a_n\}$ (dually for ∧).

Proof. By definition of ∨ and Lemma 2. If P has only two members, then Lemma 3 is especially trivial since there is only one a_i remaining after a_1 is separated off by ∨ ■

This formalizes the intuition that the comparative operators replace in a logically grounded way von Stechow's ad hoc max and min operators in the comparative standard. Since by Lemma 3 they pick out a unique (least) upper bound or a (greatest) lower bound of the subset that excludes the perspectival member, they are definite expressions. That is to say, $\bigvee\{|pete|, |tom|, |lou|\}$, the meaning of the comparative standard, is definite. There may or may not be a causal relation between the comparative standard being (pragmatically) presupposed and its being logically (semantically) definite.

It is clear from examples like **Bob's height is more than not Mary's height*, etc. that the Boolean unary negation operator is not freely possible in the scope of a comparative operator.

That is, as first noted by Lees (1961), there are problems with $|x| \vee |y|'$, the apostrophe symbolizing the complement. The first thing to note here is that the reason is *not* the comparative construction but rather a basic constraint on *what kinds of nominal expressions can denote members of ordered sets*. The denotations of some kinds of expressions are simply not orderable, most negative expressions being prime examples: 'not Mary's height', etc. Naturally, then, not only are these bad as comparative standards, they are also bad as comparative subjects: **Not Mary's height is less than Joe's.*[28] The second point is that negation is not uniformly bad in a standard, being able to occur when different adjectives are themselves compared, as in *Bill is more overworked than not willing.*[29] Distinct adjectives require distinct lattices, so such a sentence is comparing quantities in two distinct lattices. Cross-lattice comparison is only possible if the lattices are special Boolean comparative degree of declension lattices, that is bounded at both top and bottom, because that licenses ratio quantities and those can be imported into other Boolean lattices (the general Boolean class of gradable adjectives is discussed in Section 4.4 and the special ones used in cross-lattice comparison are examined further there). The only lattices with their own negative operators are Boolean ones, and therefore sentences like those in (7) are possible. [30]

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[28] Therefore, the ill-formedness of negation in comparative standards is not due to von Stechow's (1984) max operator, contrary to what is often claimed.

[29] This is what Kennedy (1999) called "comparison of deviation," a term that will not be used here. Comparison of different predicates has two possible interpretations, apparently first noted by Bolinger (1950): comparison of the "ordinary" quantities which can be described with each (quantities of time, quantities of objects, etc.), or complete denial of the appropriateness of use of the second predicate. For instance, (8a) in the first sense generically compares the amounts of Bob's Mexican-type and non-Mexican type behaviors but in the second sense it denies the appropriateness of the application to Bob of *non-Mexican*. The second sense is an extreme and meta-linguistic development based on the first sense and will not be covered here.

[30] This type of cross-lattice predicate-comparison construction was earlier studied by among others, Napoli (1983), Napoli and Nespor (1986). It involves more predicates than just adjectives -- see (11) below. In English and Italian, as shown by those authors (cf. also Pinkham (1972) for

(7a) He is more lazy than not qualified. (giving reasons for (bounded) poor performance)
(7b) The gunfights in that film were more ridiculous than not justified. (two film critics splitting hairs over gunfights in bounded film)
(7c) Driving to Tucson was more annoying than not exciting. (splitting hairs over a non-enjoyable trip)

In case the second of two compared adjectives is the negation of the first, then both will be compared in a single Boolean lattice, Boolean of course because of the negation:

(8a) Bob is more Mexican than not Mexican.
(8b) Ben is more hungry than not hungry/satisfied.
(8c) It's more full than empty/not full.
(8d) It's more clean than dirty/not clean.

Note that the sentences in (8) involve comparison of antonyms, cross-polar comparison in other words, and they are fine. The so-called cross-polar comparisons that have been noted to be impossible prior to this paper (e.g. by Kennedy 1999) are of the type **Ed is heavier than Sally is light*, which differ from (8) in two ways: first, rather than *more heavy*, *heavier* is used and second, instead of the same subject for the two adjectives, they have different subjects. If I say, *Bob is more heavy than light*, I am comparing adjective denotations (or, more generally, predicate denotations) for applicability to Bob, not weights, and what I mean is that, *as far as I can tell*, Bob's single weight seems to be on or toward the heavy side (maybe Bob, who is of average height, is not fat but rather solidly built). If I say *Bob is heavier than Sally* I am comparing two distinct weights. The comparative degree of declension of an adjective is only possible if two distinct quantities belonging to the same adjectival comparative lattice are used, hence two distinct quantities of heaviness or two distinct quantities of lightness, but not a mixture, explaining Kennedy-type cross-polar impossibility. Therefore also, the comparative

French), it even affects syntax, with *more* able to assume the Boolean polymorphic syntactic category schema X\(X/X) like the Boolean coordinators *and*, *or* (for which see Shieber 1992).

degree of declension is not used in acceptable cross-polar comparisons. This difference between comparison of predicates and of individuals is discussed further in Section 4.4.

The *not* in (7) and (8) is simultaneously a Boolean-algebra unary operator and a Boolean-lattice unary operator belonging to a Boolean lattice, and in both interpretations, it is truth-functional rather than pragmatic.[31]. This is easiest to see if the adjective is kept the same: *more lazy than not lazy*. The *eventualities* realizing adjectives and their negations can be represented together in a single Boolean lattice because the total set of eventualities required as evidence (the top bound) can be well-defined contextually, e.g. for (8a), the set of observations of Bob's behavior over some time period. Negating non-subject nouns, however, does not result in a well-defined set of eventualities -- even with a potentially bounded constraining verb, the set of possibilities opened by negation of the noun is too broad:

(9a) ??Iggy buys comics more than not comics.
(9b) ??Iggy takes Advil more than not Advil.
(9c) ??Abe works with Millicent more than not Millicent.

However, the use of an adjective depends on the adjective's lattice having the required bounds. The following are either bad or impossible for that reason:

(10a) ??It's more quiet than not quiet/than noisy.
(10b) *He's more tall than not tall/than short.

Besides certain adjectives, the polymorphic predicate-comparison construction also occurs with gerunds, infinitives, predicate nouns, manner adverbs, and locational adverbials, to name only some of the possible categories (which include categories that cannot be captured in phrase-structure accounts, for which see the data in Napoli and Nespor 1986):

(11a) Lily likes breathing more than not breathing.
(11b) To keep a horse is more expensive than to not keep a horse

[31] It does not seem correct to call this an ambiguity because the two interpretations are simultaneously required rather mutually exclusive.

(11c) Bill is more (of) a professor than not a professor.
(11d) Mary dressed fashionably more than not/unfashionably yesterday.
(11e) He sleeps on the floor more than not on the floor.
(11f) She flew the kite in the park more than not in the park

Other cases in which no Boolean lattice can be constructed are given in a footnote for the reader to figure out why. [32] In conclusion, this construction illustrates that if a sentence can be construed as presupposing a fully bounded (Boolean) domain of comparison, it will be. Boolean construal is preferred because of its nice logical implications for practical reasoning and is accommodated wherever possible. In summary, negation is restricted in comparative standards first because it often results in unorderable extensions and second, because the construction of a Boolean lattice is necessary but sometimes impossible.

The last noteworthy logical property of comparatives to be mentioned is that they are downward entailing in their right argument. Downward entailment is first defined in terms of $\subseteq$ and then the fact that the comparative constructions are downward entailing is stated as Lemma 4. This accounts for their ability to host negative polarity items as in *Fred broke more track records than anyone anywhere ever*, *Tom is taller than anyone else in any Detroit school*. The example just given breaks down into the binary relation R(A,B) = 'taller'(tom, 'anyone in the school').

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[32] *Bob is more/less Mexican than not Croatian, *Ben is more/less curious than not worried, *Lily likes eating more than not sleeping, *Igor buys comics more than not novels, *Igor takes Advil more than not Tylenol, *Rube more than not Rob loves nachos. *Mary helpfully more than unwisely loaned her key *Bob ate more than few/not many other guests. (cf. many), *Bob drew more attention than less than five other people. (cf. five), *Bob ate more than he seldom/hardly ever does (cf. usually), *This one is more full than that one isn't (full), *This car runs better than that one doesn't (run well), *Sunset is earlier than it wasn't (early) last week.

Def. A relation R between two sets A, B is downward entailing in its right argument (right monotone decreasing) if, when $C \subseteq B$, then $R(A,B) \rightarrow R(A,C)$.

Lemma 4. Comparative constructions are right monotone decreasing.

Proof. Assume $\bigvee_i a_i = a_1 \vee (\bigvee_2^n a_i) = a_1$. Let $A = \{a_1\}$, $B = \{a_2, \ldots, a_n\}$, $C = \{a_2, \ldots, a_j\}$ for $j<n$. Then $a_1 \vee (\bigvee_2^j a_i) = a_1$, or R(A,C), is obviously true. A dual result follows if one assumes $\bigwedge_i a_i = a_1 \wedge (\bigwedge_2^n a_i) = a_1$ ■

Note the nice fact that we get for free an explanation of negative-polarity *Tom is less inhibited/more difficult than anyone else in the school*.

It is worth pointing out that contradictory negatives and other downward entailing expressions do not occur in downward entailing environments, as illustrated by **No man doesn't walk, *No/??Few girls/??Less than three girls don't like Ringo.* One can force oneself to understand such sentences with effort, but that effort makes one avoid them in favor of *Every man walks, Many girls like Ringo, All but two or three girls like Ringo.* This may contribute to the difficulties with negation in comparative standards, but it is hard to see how one could test that.

Summarizing the framework presented in Sections 1 and 3, a familiar logical (mathematical) set-based structure, the distributive lattice, and standard assumptions about the cognitive structure of prototypical concepts bring together tightly and in an explanatory way all of (1) the positive degree of declension denotations of gradable-adjective antonym pairs of opposite polarity like *tall/short* predicated of quantities; (2) the prototypes (together with their psychological and statistical implications) that separate the denotations of those adjective pairs and generate the positive-degree lattices; (3) the individuals of whom the gradable adjectives are *apparently* predicated; and (4) any measures applied to quantitative properties of those individuals, of which the gradable adjectives are actually predicated thanks to the polysemy of terms referring to individuals; (5) the distinct adjective denotations gradable-adjective antonym pairs require in comparative constructions with degree morphemes, where those constructions simply denote lattice operations, including complementation when possible.

4. The general picture: The other positive to comparative lattice mappings

4.1. The *clever* class [33]

The *clever* class sample comprises the 30 pairs *aggressive/unaggressive, aggressive/timid, brave/timid, clever/dull, clever/stupid, controlled/impulsive, dominant/submissive, extroverted/introverted, friendly/unfriendly, generous/selfish, generous/stingy, good/bad, harsh/mild, honest/dishonest, impulsive/restrained, industrious/lazy, nice/nasty, nice/awful, optimistic/pessimistic, pleasant/annoying, positive/negative, sharp/dull, sharp/blunt, shrewd/naive, smart/dumb, smart/stupid, strong/weak, strict/lenient, sturdy/delicate, successful/unsuccessful.* The first membership criterion is the same extension gap found with the tall class, the assumption of which explains the fact that $(\neg clever \nrightarrow dull)$ & $(dull \rightarrow \neg clever)$. Therefore, the positive degree of declension pair *clever/dull* is like *tall/short* in denoting two opposed extreme ordered sets of quantities, here on a dimension of intelligence,

[33] The label of this class is carried over from Cruse's (1986) canonical *clever/dull* pair; some readers may find it easier to access intuitions about American *smart/dumb*.

justifying the assumption of a *tall/short*-type lattice with prototypical/statistical structure (and intelligence is usually recognized to be statistically distributed). Each positive degree of declension antonym pair in this class, then, denotes a paired lattice ideal for the negative polarity antonym and a lattice filter for the positive polarity one, the two separated by the mean area in their hosting lattice.

The second criterion is again like one for the *tall* class, the absence of bounds: one cannot say **She is absolutely clever/dull*.

The third criterion for this class has to do with the lattice mapping of the positive degree lattice onto the two comparative ones. Recall that the lattice for *shorter*$_4$ shared its set L with *taller*$_4$, so that one could say *Our center is quite a bit shorter than yours* even though basketball centers are always tall. That is, my describing an arbitrary person as *shorter* even though everyone knows him to be tall and who therefore is in the domain of *taller* requires that those two adjectives have the same domains (this is a duality test). In contrast, *cleverer/duller* allow one to say *Those boys are both dull but Bob is cleverer than Bill*, but one cannot say **Those boys are both clever but Bill is duller/less dull than Bob*. Nor is it the case that *Bob is cleverer than Bill* iff *Bill is duller than Bob*.[34] Thus, comparative *cleverer, less clever* behave like *taller*, *shorter*, covering persons who fall into the whole positive statistical lattice comprised of the *clever* set (filter F_{clever}), the *dull* set (ideal I_{dull}), and neither (the mean set M) -- this union $F \cup I \cup M$ comprises $L_{intelligence}$. The lattice for *cleverer, less clever* is therefore $<L_{intelligence}; \vee, \wedge>$. Comparative *duller, less dull*, however, cover only people who fall into the positive lattice's *dull*

[34] This fact contradicts Kennedy's conclusion that antonym pairs are always "inverses" (Kennedy 1999, Section 3.3.2). Members of the *hot* class will be seen to also be counterexamples. The lattice mapping approach explains which pairs are in fact "inverses".

set, the ideal I_{dull}, and the lattice for them is $<L_{dull}; \wedge, \vee>$.[35] The mapping from the lattice (a, m_1)(m_2, m_3)(m_4, b) for the positive pair *clever/dull* to the paired lattices for the comparative pair, therefore, shifts (m_4, b) to (a, b) but leaves (a, m_1) unchanged. The two comparative lattices have distinct sets so that they are not duals. Though not the same set, one is a subset of the other.

The interesting question is why the positive to comparative mapping leaves the set of *duller, less dull* unchanged, in effect forcing these two forms to apply only to people who are actually on the low end of the intelligence curve, as in *Bob is less dull than Bill* (even though uttering this could be viewed as praise of Bob, both Bob and Bill must be dull). This is undeniably a semantic phenomenon but it appears to occur only for a pragmatic reason, namely, it strengthens the negative-affect force of *duller, less dull* because applying those words to compare two people absolutely excludes the possibility that either of them is bright. This pragmatically motivated semantic strengthening applies throughout the *clever* class and it uniformly applies to that affect-laden antonym that can be used to depreciate, the negative-affect antonym. As for the choice of one antonym as a dimension label, it is obvious that the negative-affect antonym does not have the proper denotation for that, leaving the field to the positive-affect one.

4.2. The *clean* class

The *clean* class sample has 29 members: *active/passive, ambitious/unambitious, brave/cowardly, calm/agitated, clean/dirty, clear/hazy, difficult/easy, dry/wet, dry/sweet, dynamic/static, even/uneven, fancy/plain, flexible/rigid, fresh/stale, helpful/unhelpful, impartial/partial, important/unimportant, intellectual/unintellectual, light/dark, noisy/quiet,*

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[35] Therefore, application of the implicational duality test, *Bob is cleverer than Bill* $\rightarrow$ *Bill is duller than Bob*, by requiring that both Bob and Bill be dull also shows that duality does not exist.

practical/impractical, relaxed/tense, smooth/rough, sober/drunk, steady/capricious, straight/crooked, transparent/opaque, unbiased/biased, interesting/uninteresting.

The first membership test shows the lack of a prototype; one cannot say **It is neither dry nor wet*, etc. The antonyms thus behave like complementaries, prompting Cruse (1980, 1986) to call this the class of gradable complementaries. This behavior follows from the most striking feature of the *clean* class -- that, as recognized by Cruse, one antonym denotes the total absence of any physical manifestation of a certain property while the other denotes some physical manifestation of it (see also Yoon 1996). Therefore, both **It is neither clean nor dirty, *It is clean and dirty* are bad. Looking at the positive-degree forms, say *clean* and *dirty*, only the latter is physically real, and the former simply denotes its negation, as in Figure 2.

|
|
|
$|x| \langle L; \vee, \wedge \rangle$ dirtiness
|
(0 = ~dirty = clean)

Figure 2. *Dirty* lattice

The second membership test is that the non-statistical lattice of Figure 2 has a lower bound of cleanness that is excluded from the set of quantities of dirtiness, i.e. that set is $L = (0, q)$ (cf. Paradis 2001, Rotstein and Winter 2004, Kennedy and McNally 2005 on the lower bounds for these pairs).

It is clear that antonym pairs in this class are also in affect-laden territory, and the third membership test is that the negative-affect members must have restricted comparative domains while the positive-affect members have unrestricted ones. **These shirts are clean but this one is dirtier* is therefore contradictory while *These shirts are dirty but this one is cleaner than the rest* is fine. As with the *clever* class, the consequence is that the negative-affect antonyms cannot become dimension labels. Comparative *dirtier*, *less dirty* therefore correspond to the quantity lattice of Figure 2 -- positive-degree of declension to comparative-degree mapping here involves no change.

The fourth membership criterion for the *clean* class deals with its most remarkable property: that the mapping of the lattice of Figure 2 to the two comparative lattices produces a full negative-polarity (decreasing) lower-bounded lattice for *cleaner, less clean*, but this lattice must be "fictitious", which is to say, its quantities are unmeasurable as for any decreasing lattice. Its set is L = [0, q) of fictitious quantities of cleanness. The inclusion of the lower bound is shown by the *absolutely* test: *It is absolutely clean.* The two lattices, real and fictitious, are illustrated in Figure 3, the ordering on the left being upward and on the right downward. Note that the two involve different sets, one without 0 and one with, and are therefore technically not duals. However, practically speaking, a non-zero quantity in one lattice is identical with a quantity in the other, so that it is true that *This is dirtier than that* iff *That is cleaner than this*. Exactly the same picture emerges for the pairs *active/passive, easy/difficult, even/uneven, quiet/noisy* and 24 others: there are real comparative lattices for *wetter, less wet, noisier, less noisy,* and so on, and corresponding fictitious ones for *drier, less dry, quieter, less quiet,* etc.

```
|                               |
|                               |
|                               |
|x| <L; ∨, ∧ > dirtiness        |x| <L; ∧, ∨ > cleanness
|                               |
(0 = clean)                     0
```

Figure 3. Increasing dirtiness quantity = decreasing cleanness quantity

The zero bound in the single positive degree of declension *clean*-class lattices can correspond to either a positive-affect or a negative-affect antonym. The sample of 29 pairs includes 17 positive-affect zero bounds, e.g. *calm* of *calm/agitated*, *easy* of *easy/difficult*, and 12 negative-affect ones, e.g. *passive* of *active/passive, plain* of *fancy/plain*. The positive-affect antonym that serves as a dimension label, if one exists (four *clean*-class pairs in the sample lack dimension labels), can therefore correspond to either a zero bound or to a positive-polarity lattice.

Given the contrast of real set (0, q) and fictional set [0, q) in Figure 3, it may be no surprise that the introduction of bounds into the sets of this class of lattices commonly goes even further. In fact, two things often simultaneously happen to the positive degree of declension lattice in Figure 2. First, it receives a lower bound 0 and an upper bound 1 denoting total dirtiness and cleanness or whatever, encompassing [0, 1]. Second, dirtiness, etc. is reduced to the single operation ∨. The result is a positive-polarity Boolean lattice encompassing both cleanness and dirtiness, quantities of these now being treated as complementary sets corresponding to dual operations. See Figure 4. This alternative treatment is generally possible for gradable complementary antonym pairs, and it may be referred to as the accommodation of a Boolean lattice, since the result is just that, <L; ∨, ∧, ', 0, 1>. It is now possible to non-figuratively use the adverb *completely* as well as other ratio adverbs, saying, *This shirt is completely/half dirty*, and to indirectly measure quantities of the decreasing antonym.

```
        1
        |  ∧ clean
        |
        |x|
∨ dirty |
        |
        0
```

Figure 4. Accommodated Boolean lattice <L; ∧, ∨, ', 0, 1 >

The possibility of Boolean accommodation is a fifth membership criterion. Some additional reasons for Boolean accommodation are given in the next section.

Finally, consider the following pairs (of which only *coordinated/clumsy* and *fancy/plain* occur in Lehrer's (1985) sample) which have the interesting property that one member seems to denote the mean range of a statistical distribution: *normal/abnormal, fancy/plain, typical/ untypical, usual/unusual, coordinated/clumsy.* However, these fail the test for a statistical distribution, e.g., 'not normal' implies 'abnormal', 'not fancy' implies 'plain', 'not coordinated' implies 'clumsy'. In fact, they are all members of the *clean* class.

4.3. The *just* class

The *clean* class is characterized by positive-degree of declension lattices with zero bounds and therefore increasing quantities of activity, ambitiousness, agitation, etc. modeled with positive polarity lattices <L; ∨, ∧ >. It has not been noticed before that there is a very similar and equally complicated class without prototypes and with contrasting 1 (or upper) bounds in positive-degree lattices representing complete or full states and quantities that are likewise increasing and therefore measurable. The first criterion for membership is the lack of a prototype. The second criterion is the 1 upper bound representing the full presence of a generally complex state of affairs (see the list of members below). The third criterion is that the comparative-degree mapping keeps this positive-degree lattice unchanged and produces an additional decreasing/negative lattice <L; ∧, ∨ > with a (1) upper bound for the antonym. The canonical member of this class is *just/unjust* -- see Figure 5. Just as the zero bound of the *clean* class is picked out by the quantity adverb *absolutely*, so the 1 bound is picked out by *fully* or *completely*: *The ruling was completely/fully just* means that every aspect of a probably complex state of affairs was properly taken into account in the judgment and found to require the ruling. If any aspect was not properly considered, there would be a decrease in completeness and the ruling would be more unjust. The other pairs in the 22-member sample for this class are *accurate/inaccurate, comfortable/uncomfortable, coordinated/clumsy, distinct/vague, dominant/subordinate, fair/unfair, faithful/unfaithful, healthy/sick, mature/immature, neat/messy, obvious/subtle, organized/unorganized, pure/impure, ripe/unripe, safe/dangerous, sane/insane, sane/crazy, similar/different, stable/unstable, true/false, usual/unusual.* Unlike for the *clean* class, the 1 bound in the *just* class uniformly corresponds to the positive-affect antonym.

This class includes English past particles with their productive antonyms formed with the prefix *un-*: *read/unread, lived in/unlived in, washed/unwashed,* etc.

```
1
|
|
|        <L; ∨, ∧ >
|
|
```

Figure 5. *Just* lattice

The two comparative lattices for real *more/less just* and the fictitious *more/less unjust* are given in Figure 6.

```
1                              (1 = just)
|                              |
|                              |
|                              |
|x| <L; ∨, ∧ > justice         |x| <L; ∧, ∨ > injustice
|                              |
```

Figure 6. Increasing justice quantity = decreasing injustice quantity

Here, too, there is a fourth criterion, the possibility of Boolean accommodation with its added zero bound, particularly in the case of past participles. Thus, one can non-figuratively say, *This* *book is absolutely unread, This house is absolutely unlived in,* etc., and one can measure the amount of a book that is unread, the amount of an apple that is uneaten, etc.

4.4. The Boolean class

Several non-statistical gradable complementary antonym pairs are always interpreted with Boolean lattices, including *complete/incomplete, full/empty, certain/uncertain, total/partial, perfect/imperfect,* and therefore there is a small but distinct Boolean class. Only one pair in Lehrer's 152-pair sample, *full/empty*, is Boolean. The first criterion, lacking a prototype, cannot be shown with the contrariety test suggested by Lehrer and Lehrer (1982) -- that both antonyms can be false but both cannot be true -- because Boolean antonyms are bounds and therefore there is a whole lattice between them in which neither can be true and both are false. An alternative and pragmatic test for being statistical is that the notion of a normal, average quantity (prototype) is meaningful. That suffices to separate off the Boolean class from the statistical classes that have so far been recognized only on the basis of the contrariety test.

Like the *hot* class to be covered in the next section, Boolean antonym denotations are mutually exclusive: just as something described as hot cannot be described as cold, something described as perfect cannot be described as imperfect (cf. the different behavior of *tall/short, clean/dirty*, etc.). The second criterion is therefore complete failure to pass the duality test: **The box and bag are both full but the bag is emptier than the box, *The box and bag are both empty but the box is fuller than the bag.*

The third criterion is that mapping from a positive-degree Boolean lattice to a comparative degree one involves no changes in the case of the positive-polarity antonym, so that *full/empty* and *fuller/less full* are both <L;∨, ∧, ', 0, 1>. However, the negative-polarity antonym receives its own comparative lattice <L; ∧, ∨, ', 0, 1> for *emptier/less empty* in that mapping.

Regarding bounds in Boolean lattices, as we have already seen in Figure 4, a quantity $|x|$ of fullness or completeness corresponds to a complement quantity $|x|'$ of emptiness or incompleteness such that $|x| \wedge |x|' = 0$, $|x| \vee |x|' =1$, and $\wedge$ yields the upper-bounded semilattice of quantities $|x|$ of incompleteness while $\vee$ yields the lower-bounded semilattice of quantities $1– |x|$ of completeness. Here too there are fictional quantities: only quantities $1– |x|$ of completeness are physically realized (think of a house being built), but because of the relation $|x| \vee |x|' =1$ where the bound 1 represents the building plan, virtual quantities $|x|$ of incompleteness are deducible from the corresponding fusional equation $|x| \cup |x|' = 1$. The practical consequences of Boolean lattices, however, go beyond this.

First, comparison of predicates as in *Bob is more rich than wise* was earlier introduced with the remark that it requires two special Boolean comparative-degree lattices, as in Figure 7.

```
                 1                     1
        ^        |                     |        ^
not-rich         |                     |        not-wise
                 |x|                   |
                 |                     |
        v        |                     |y|
rich             |                     |        v
                 0                     0        wise
```

Figure 7. Two comparative Boolean lattices for predicate comparison

In the above *rich/not-rich* lattice, despite the fact that *rich* is *not* a Boolean adjective (it is a member of the statistical *hot* class), it is possible to compare $|x|$, the amount of wealth and $1– |x|$, the amount not-wealth, yielding *Bob is more rich than not rich*, but it also possible to compare $|x|$ and $|y|$, the amount of wisdom, which must be imported from the rightmost lattice in Figure 7. The upper 1 bounds in Figure 7 are rather interesting. They seem to be idealizations -- necessarily definitizations -- of the relevant prototypes, say *the* average income level for the left lattice, *the* average wisdom level for the right. The fact that amounts $|x|$ and $|y|$ of predicates *themselves* are being compared with *more* corresponds to the fact that one says *Bob is more rich than wise* rather than **Bob is richer than wise,* **Bob is richer than wiser.* If I were to say, *Bob is richer than Bill*, I would be constructing the comparative lattice of the positive-polarity member of the *hot* class, not the left lattice in Figure 7.

Yet another nice consequence of Boolean lattices is the pair of De Morgan laws for distributed complemented lattices given in (13): the *greater* of two quantities of completeness is identical with the *lesser* of the two complement quantities of incompleteness, and vice versa.

(13) $(a \wedge b)' = a' \vee b'$
$(a \vee b)' = a' \wedge b'$

Suppose Mary puts a quantity of mashed potatoes and a platter with more or less that same quantity of pork chops on the table for her boys Bill and Bob, with the understanding that they will share the food and eat all of it (thereby invoking a Boolean lattice). Let Bill's share of pork chops be a and his share of potatoes be b, so that Bob's shares are a' and b' respectively. If Bob eats less potatoes than pork chops, then Bill gets more potatoes than pork chops. Clearly, everyday reasoning about actions is heavily dependent on Boolean lattices.[36]

4.5. The *hot* class

The positive (= non-comparative) pair *hot/cold* is like *tall/short* in denoting two opposed extreme ranges of values on a statistically distributed simple physical (though non-additive) dimension of heat energy, and therefore, one may assume, it corresponds to a *tall/short*-type lattice with statistical structure, thereby explaining the fact that $(\neg hot \nrightarrow cold)$ & $(hot \rightarrow \neg cold)$.[37] This is the first criterion for the class. But whereas comparative *taller/shorter*

---

[36] Including, of course, linguistic phenomena besides gradable adjectives, such as completive and incompletive verbal aspect (see Krifka (1989).
[37] To say that (earthly) heat energy is statistically distributed just means that it is hardly ever constant at any one spot on Earth.

respectively mean 'greater amount of height', 'lesser amount of height', so that it is possible to say of two very tall men Bob and Bill, *Bill is shorter than Bob*, and similarly for two very short men, Tim and Tom, *Tom is taller than Tim*, it is not possible to say for two very hot stoves, *This one is colder than that one*, nor for two very cold rooms, *That one is hotter than this one* (in fact one must use a different pair of words, *warmer/cooler*, to make the preceding temperature comparisons). Consequently, neither of comparative *hotter/colder* covers the whole range of temperatures -- the former is restricted to the hot range and the latter to the cold range, and the two adjectives fail the duality test: *This is hotter than that* is not extensionally equivalent to *That is colder than this*. While the *clever* and *clean* classes display similar restrictions for only their negative-affect comparative antonyms, here one finds restriction of both comparative antonyms. This lattice mapping is the second criterion for the *hot* class. Note that neither of *hotter, colder* is affect-laden, nor is there a dimension label -- *How hot is the pot? How cold is the pot?* require that the pot be hot and cold, respectively. These properties are commonly but not always found for members of this class, so are not criterial.

More formally, positive-degree *hot* and *cold* correspond to a filter F_h and an ideal I_c, respectively, in a single statistical temperature lattice, separated by a mean region M. Comparative *hotter* and *colder* correspond to the full lattices $<L_h; \vee, \wedge>$ and $<L_c; \wedge, \vee>$, respectively, with $F_h = L_h$ and $I_c = L_c$. The two distinct sets L_h and L_c mean that the two lattices are not duals. The mapping from the positive lattice to the full comparative lattices is similar to that for the *clever* class, but here the whole positive lattice $(a, m_1)(m_2, m_3)(m_4, b)$ remains unchanged.

The maintenance of the positive lattice for the comparative can only be due to the mutual semantic incompatibility of *hot* and *cold*: no pragmatic motivation is involved. This is even clearer with the pair *fat/thin* belonging to the same class. While affect-loading may exist in this case, it seems to occur for both members, and the basic reason for *fat/thin* being in the *hot* class is that no person or animal considered fat can be considered thin, and conversely. The other members of the 39-pair sample of this class (the largest in the sample) are *agile/clumsy, aggressive/defensive, altruistic/egoistic, austere/lush, beautiful/ugly, bright/dark, calm/violent, colorful/drab, delicate/rugged, feminine/masculine, ferocious/meek, friendly/hostile, generous/skimpy, graceful/awkward, graceful/clumsy, happy/sad, important/trivial, interesting/boring, kind/cruel, late/early, lush/barren, moral/immoral, peaceful/violent, pleasant/unpleasant, pleasant/displeasing, powerful/powerless, pretty/homely, reassuring/frightening, reassuring/threatening, rich/poor, ripe/green, sociable/unsociable, sturdy/fragile, useful/useless, valuable/cheap, valuable/worthless, wise/foolish.* Of these, the first members of *interesting/boring, kind/cruel, late/early, moral/immoral, pleasant/unpleasant, pleasant/displeasing, powerful/powerless, ripe/green, valuable/worthless, valuable/cheap, useful/useless, sturdy/delicate,* and *sociable/unsociable* serve as dimension labels, leaving 26 pairs without labels. The restrictions on denotations of both antonyms in the comparative lattices would lead one to predict that in fact there would be *no* dimensional labels at all in this class. Whatever it is that drives the development of a label can override the lack of an unrestricted comparative antonym, forcing one antonym to lose its restriction for label purposes.

4.6. What lattices and their mappings explain

The order-theoretic approach proved its explanatory value in Section 3. The importance of Boolean *lattices* alone for cross-lattice comparison and of Boolean accommodation for practical reasoning about quantities, is enough to justify that approach. In less theoretical Section 4, lattice structure has been shown to provide all the distinctions semantically relevant for the six classes of positive-degree and comparative-degree English gradable adjectives, including mappings. The hypothesis that lattices for the positive degree of declension and the comparative degree are distinct in *several different ways* so that a *variety of mappings* between them are required is strengthened by several facts. First, this paper has shown that there actually exist all and only the theoretically possible numbers of restrictions on the statistical sets in these mappings -- see Table 1.

	POSITIVE	COMPARATIVE
tall	2	0
clever	2	1
hot	2	2

Table 1. Numbers of restrictions on statistical sets by class

Second, the comparative lattices *explain* the standard Cruse *comparative* criteria based on -*er/more* and *less* that have been used here, including whether members of an antonym pair are "inverses" in Kennedy's (1999) sense (see footnote 34). Third, the comparative lattices similarly *explain* the behavior of the other degree morphemes that were claimed in Section 3 to operate in comparative lattices. That behavior has not so far been examined in this paper. Starting with the *tall* class, note the lack of restrictions in both comparative *Bob and Bill are tall but Bob is shorter* and non-comparative *Bob and Bill are tall but Bill is short enough to get through doors without ducking.* For the *clever* class, note *Bob and Bill are timid but Bob is brave enough* versus **Bob and Bill are brave but Bob is timid enough*, just as one finds for comparatives *braver* and *more timid* in place of *brave enough* and *timid enough*, respectively. Skipping to the *hot* class to save time, neither of the following is acceptable: **These coffees are hot but this one is cold enough, *These coffees are cold but this one is hot enough.*

5. Brief descriptive comments on dimensions, polarity, affect and quantities

A dimension arises from some culturally driven need to communicate about both the (degree of) presence and the (degree of) absence of what is considered to be a single quantity property. Somewhat circularly and as recognized early on by Sapir (1944), Osgood (1957, 1975) and his colleagues, it is then *the dimensional property that defines* top-down what is a degree of presence (positive polarity) and what is a degree of absence (negative polarity), most clearly for the *tall* class where actual matter either increases or decreases in the obvious dimensions involved.[38] Dimensions and accordingly increase and decrease are less obvious for pairs like *fancy/plain, transparent/opaque, subordinate/dominant, sturdy/delicate*. Therefore, one needs a test for order direction. The most plausible numerical measure of quantities, however imaginary, picks out the increasing adjective. For instance, *fancy/plain* is most plausibly quantized in terms of number of decorative (non-functional) frills, making *fancy* positive-polarity, *transparent/opaque* is most plausibly quantized in terms of amount of blockage of light transmission, making *opaque* positive-polarity, etc. The analysis in this paper of the *clean* and *just* classes is based on discerning the most plausibly real quantities and associating them with the positive-polarity (increasing) adjectives.

Turning from polarity values to affective values, those too are not always obvious, so a test is needed. For example, which of *fancy/plain, transparent/opaque, sturdy/delicate* has positive affect out of context? In many cases -- these three, for example -- one must conclude

[38] The primacy of dimensions is revealed by the participation of homonymous or polysemous adjectives in several dimensions, e.g. *light/dark, heavy/light, dense/light* and by the existence of synesthetic metaphors like *light/dark* for types of fiction, humor, etc.

that a pair is non-affective, no negative context being presupposed. It is often helpful to accentuate any negative meaning by adding the adverb *too*: *too dirty* is natural but *too clean* requires a special context such as evidence for having performed a job that will make any clean clothing dirty. *Too fancy, too plain* both require special contexts. Pairs in the sample without affective meaning include all of the 19 *tall* class pairs, eight of the *clever* class pairs, five of the *clean* class pairs, two of the *just* class pairs, and six of the *hot* class pairs, a total of 40, leaving 110 pairs (73%) with affective meaning.

There is a strong correlation between positive polarity and positive affect. The only deviating pairs are fifteen members of the *clean* class for which the positive-polarity adjective carries negative affect: *agitated, dirty, hazy, difficult, uneven, stale, partial, noisy, tense, rough, drunk, capricious, crooked, opaque* and *biased*. The reason for the correlation remains unexplained.

Turning to the choice of one member of a pair as a dimension label, one must ask whether that choice is correlated with the variables polarity or affect. Only 2 of the 29 *tall* class pairs do not have one member used as a dimension label, those two being *abstract/concrete* and *pungent/bland*. The only variable that could possibly affect the occurrence of a label for this class is polarity, and 26 of the 27 labels are the positive-polarity members.

Only 3 of the 30 *clever*-class pairs do not have dimension labels. All 27 with labels use the positive polarity antonyms in their pairs. Here the variable of affect also enters the picture, and all 27 of the positive polarity members occurring as dimension labels are also positive-affect. The strong correlation between positive polarity and positive affect is therefore also evident.

The picture is more complex for the *clean* class because of the possibility of positive polarity occurring with negative affect. However, out of the 29 pairs, only three do not have dimension labels, namely, *light/dark, noisy/quiet,* and *relaxed/tense*. Of the 26 labels, four are for dimensions without presupposed affect: *dry/wet, dry/sweet, fancy/plain, flexible/rigid*. Of the remaining 22, however, all are the positive-affect antonym. Of those 26 labels, 11 are the positive-polarity antonym and 15 are the negative-polarity antonym. Thus, affect has a stronger correlation than does polarity with label status in this class.

In the *just* class, only one of the 22 pairs, *usual/unusual*, does not have a dimension label. All the 21 labels are the positive-affect and positive-polarity members of their pairs.

The *hot* class has the least use of dimension labels, with only 13 of its 39 pairs having one. All 13 of those labels are both the positive-polarity and positive-affect members of their pairs.

In summary, the existence of a dimension label is due to historical/usage factors relating to the cultural importance and saliency of the dimension, with the choice strongly in favor of the cognitively simpler positive polarity antonym but above all in favor of positive affect.

The most striking logical conclusion of this paper is that quantities are only partly physically real (measurable), even though real (physical) quantities are indisputably the foundation for the development of cognition about quantity in general. The related conclusion in close second place for unexpectedness is that the concept of quantity is a formal rather than solely a perceptual one, highly abstract and therefore easily extended from simple perceptions of physical properties to complex aesthetic judgments that can be both subjectively and culturally determined (see footnote 13). Thus, both physical *clean/dirty* and subjective/cultural *important/unimportant* belong to the formal *clean* class, both physical *delicate/rugged* and subjective/cultural *smart/dumb* belong to the *clever* class, and both physical *hot/cold* and subjective/cultural *beautiful/ugly* belong to the formal *hot* class. Therefore, even without any way to additively measure extremely context-dependent importance, intelligence, or beauty, one can still talk about their quantities.

References

Atlas, Jay D. and Stephen Levinson. 1981. *It*-clefts, informativeness, and logical form. In Peter Cole (ed.), *Radical pragmatics*, pp. 1-61. New York: Academic Press.

Barsalou, Lawrence W. 1999. Perceptual symbol systems. *Behavioral and Brain Sciences* 22: 577-609.

Bierwisch, Manfred. 1989. The semantics of gradation. In Manfred Bierwisch and Ewald Lang (eds.), *Dimensional adjectives: Grammatical structure and conceptual interpretation*, pp. 71-261. Berlin: Springer.

Birkhoff, Garrett and Saunders Mac Lane 1965. *A survey of modern algebra*. New York: Macmillan.

Bolinger, Dwight. 1950. The comparison of inequality in Spanish. *Language* 26: 28-62.

Clark, Herbert H. 1969. Linguistic processes in deductive reasoning. *Psychological Review* 76: 387-404.

Cresswell, Maxwell J. 1976. The semantics of degree. In Barbara H. Partee (ed.), *Montague grammar*, pp. 261-292. New York: Academic Press.

Cresswell, Maxwell J. 1990. *Entities and indices*. Dordrecht, Holland: Kluwer.

Cruse, David Alan. 1980. Antonyms and gradable complementaries. In Dieter Kastovsky (ed.), *Perspektiven der lexicalischen Semantik: Beiträge zum Wuppertaler Semantik-kolloquium vom 2-3 Dezember, 1977, Bonn*, pp. 14-25.

Cruse, David Alan. 1986. *Lexical semantics*. Cambridge: University of Cambridge Press.

Donaldson, Margaret and Roger J. Wales. 1970. On the acquisition of some relational terms. In John R. Hayes (ed.), *Cognition and the development of language*, pp. 235-268. New York: Wiley.

Dowty, David R., Robert E. Wall and Stanley Peters. 1981. *Introduction to Montague semantics*. Dordrecht, Holland: D. Reidel.

Enrico, John. 2021. Argumenthood. Published online with ResearchGate.

Fine, Kit. 1975.Vagueness, truth and logic. *Synthese* 30: 265-300.

Gärdenfors, Peter. 2014. *The geometry of meaning*. Cambridge, MA: MIT Press.

Geurts, Bart. 1999. *Presuppositions and pronouns*. Leiden: Brill.

Hellan, Lars. 1981. *Towards an integrated analysis of comparatives*. Tübingen: Gunter Narr.

Hoepelman, Jaap. 1982 Adjectives and nouns: a new calculus. In Rainer Bäuerle, Cristoph Schwarze and Arnim von Stechow (eds.), *Meaning, use and interpretation of language*, pp. 190-220. Berlin: de Gruyter.

Holland, V. Melissa and David S. Palermo. 1975. On learning "less": Language and cognitive development. *Child Development* 46: 437-443.

Horn, Lawrence 1989. *A natural history of negation*. Chicago: University of Chicago Press.

Kamp, Hans. 1975. Two theories about adjectives. In Edward L. Keenan (ed.), *Formal semantics for natural language*, pp. 123-135. Cambridge: Cambridge University Press.

Kamp, Hans and Barbara Partee. 1995. Prototype theory and compositionality. *Cognition* 57: 129-191.

Keenan, Edward L. and Leonard M. Faltz. 1985. *Boolean semantics for natural language*. Dordrecht, Holland: Kluwer.

Keenan, Edward L. and Jonathan Stavi. 1986. A semantic characterization of natural language determiners. *Linguistics and Philosophy* 9: 253-326.

Kennedy, Christopher. 1999. *Projecting the adjective*. New York: Garland.

Kennedy, Christopher. 2001. Polar opposition and the ontology of 'degrees'. *Linguistics and Philosophy* 24: 33-70.

Kennedy, Christopher and Louise McNally. 2005. Scale structure, degree modification, and the semantics of gradable predicates. *Language* 81: 345-381.

Klein, Ewan. 1980. A semantics for positive and comparative adjectives. *Linguistics and Philosophy* 4: 1-45.

Klein, Ewan. 1982. The interpretation of adjectival comparatives. *Journal of Linguistics* 18: 113-136.

Klein, Ewan. 1991. Comparatives. In Arnim von Stechow and Dieter Wunderlich (eds.), *Semantics: An international handbook of contemporary research*, pp.673-691. Berlin: de Gruyter.

Krifka, Manfred. 1989. Nominal reference, temporal constitution and quantification in event semantics. In Renate Bartsch, Johan van Benthem and Peter van Emde Boas (eds.), *Semantics and contextual expression*, pp. 75-115. Berlin: De Gruyter.

Lees, Robert B. 1961. Grammatical analysis of the English comparative construction. *Word* 17: 171-185.

Lehrer, Adrienne and Keith Lehrer. 1982. Antonymy. *Linguistics and Philosophy* 5: 483-501.

Lehrer, Adrienne. 1985. Markedness and antonymy. *Journal of Linguistics* 21: 397-429.

Levinson, Stephen. 2000. *Presumptive meanings*. Cambridge, MA: MIT Press.

Lewis, David. 1972. General semantics. In Donald Davidson and Gilbert Harman (eds.), *Semantics of natural language*, pp. 169-218. Dordrecht, Holland: D. Reidel.

Link, Godehard. 1983. The logical analysis of plurals and mass terms: A lattice-theoretical approach. In Rainer Bäuerle, Christoph Schwarze and Arnim von Stechow (eds.), *Meaning, use, and interpretation of language*, pp. 302-323. Berlin: de Gruyter.

Link, Godehard. 1998. *Algebraic semantics for language and philosophy*. Stanford, CA: Center for the Study of Language and Information.

Napoli, Donna Jo. 1983. Comparative ellipsis: A phrase structure analysis. *Linguistic Inquiry* 14: 675-694.

Napoli, Donna Jo and Marina Nespor. 1984. Comparative structures in Italian. *Language* 62: 622-653.

Osgood, Charles E., George J. Suci and Percy H. Tannenbaum. 1957. *The measurement of meaning.* Urbana, IL: University of Illinois Press.

Osgood, Charles E., William H. May and Murray S. Miron. 1975. *Cross-cultural universals of affective meaning*. Urbana, IL: University of Illinois Press.

Palermo, David S. 1973. More about less: A study of language comprehension. *Journal of Verbal Learning and Verbal Behavior* 12: 211-221.

Paradis, Carita. 2001. Adjectives and boundedness. *Cognitive Linguistics* 12: 47-65.

Pinkham, Jessie E. 1982. The formation of comparative clauses in French and English. Doctoral dissertation, Harvard University.

Pustejovsky, James. 1995. *The generative lexicon*. Cambridge, MA: MIT Press.

Ross, John R. 1968. A proposed rule of tree pruning. In David A. Reibel and Sanford A. Schane (eds.), *Modern studies in English*, pp. 288-299. Englewood Cliffs, NJ: Prentice-Hall.

Rotstein, Carmen and Yoad Winter. 2004. Total adjectives vs. partial adjectives: Scale structure and higher-order modifiers. *Natural Language Semantics* 12: 259-288.

Sapir, Edward. 1944. Grading: A study in semantics. *Philosophy of Science* 11: 93-116. Reprinted in David G. Mandelbaum (ed.), *Selected writings of Edward Sapir*, pp.122-149, Berkeley: University of California Press.

Seuren, Pieter A. M. 1973. The comparative. In Ferenc Kiefer and Nicolas Ruwet (eds.), *Generative grammar in Europe*, pp. 528-564. Dordrecht, Holland: D. Reidel.

Seuren, Pieter A.M. 1978. The structure and selection of positive and negative gradable adjectives. In Donka Farkas, Wesley M. Jacobson and Karol W. Todrys (eds.), *Papers from the parasession on the lexicon*, pp. 336-346. Chicago: Chicago Linguistic Society.

Shieber, Stuart. 1992. *Constraint-based grammar formalisms*. Cambridge, MA: MIT Press.

Stassen, Leon. 2013. Comparative constructions. In Matthew S. Dryer and Martin Haspelmath (eds.), *The World Atlas of Language Structures* online. <wals.info/chapter/121>

Steedman, Mark. 2012. *Taking scope*. Cambridge, MA: MIT Press.

Thurstone, Louis L. 1927. The method of paired comparisons for social values. *Journal of Abnormal and Social Psychology* 21: 384-400.

Thurstone, Louis L. 1959. *The measurement of values*. Chicago: University of Chicago Press.

van der Sandt, Rob. 1992. Presupposition projection as anaphora resolution. *Journal of Linguistics* 9: 333-377.

Verheyen, Steven and Paul Égré. 2018. Typicality and graded membership in dimensional adjectives. *Cognitive Science* 42: 2250-2286.

von Stechow, Arnim. 1984. Comparing semantic theories of comparison. *Journal of Semantics* 3: 1-77.

Yoon, Youngeun. 1996. Total and partial predicates and the weak and strong interpretations. *Natural Language Semantics* 4: 217-236.