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Free Choice, Simplification, and Innocent Inclusion*

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Abstract We propose a modification of the exhaustivity operator from Fox (2007), that on top of negating all the Innocently Excludable (IE) alternatives asserts all the "Innocently Includable" (II) ones. The main result of supplementing the notion of Innocent Exclusion with that of Innocent Inclusion is that it allows the exhaustivity operator to identify cells in the partition induced by the set of alternatives (assign a truth value to every alternative) whenever possible. We argue for this property of Cell Identification based on the Simplification of Disjunctive Antecedents (SDA) and the effects on Free Choice (FC) that arise as the result of the introduction of universal quantifiers (Chemla 2009b and Nouwen 2017). We further argue for our proposal based on the interaction of *only* with Free Choice disjunction (Alxatib 2014).

Keywords: Implicature, Exhaustification, Free Choice, Simplification of Disjunctive Antecedents, Innocent Inclusion, Innocent Exclusion

1 Introduction

As is well known, a sentence like (1) where an existential modal takes scope above *or* gives rise to the Free Choice (FC) inferences (1a)-(1b) (von Wright 1968; Kamp 1974). It is also well known that these inferences don't follow from standard assumptions about the semantics of *allowed* and *or*: $\Diamond(a \vee b)$ is equivalent to

$\Diamond a \vee \Diamond b$ rather than the conjunction $\Diamond a \wedge \Diamond b$.[1]

(1) **Free Choice disjunction**:
Mary is allowed to eat ice cream or cake. $\Diamond(a \vee b) \Leftrightarrow (\Diamond a \vee \Diamond b)$
a. $\rightsquigarrow$ Mary is allowed to eat ice cream. $\Diamond a$
b. $\rightsquigarrow$ Mary is allowed to eat cake. $\Diamond b$

Implicature-based accounts of FC (Kratzer & Shimoyama 2002; Fox 2007; Klinedinst 2007; Chemla 2009a; Franke 2011) maintain that the basic meaning of (1) can be stated as $\Diamond(a \vee b)$ using standard modal logic and that the FC inferences are explained by enriching the meaning using mechanisms that are independently motivated in the account of scalar implicatures.

Implicature accounts of FC are motivated both on conceptual grounds—they do not involve altering the meaning of logical words, and on empirical grounds—they correctly predict the behavior of FC disjunction under negation (Alonso-Ovalle 2005).[2] However, there are some lingering problems for such accounts, which will be the focus of this paper. First, when FC disjunction is embedded under *only* the FC inferences become part of *only*'s presupposition (Alxatib 2014). As (2) shows, the inferences project out of a question as though they were presuppositions. At the same time, deriving FC inferences in the scope of *only* (as an embedded implicature) leads to further problems, specifically it yields an assertive component which is weaker than attested.

(2) **Free Choice disjunction under *only***:
Are we only allowed to eat [ice cream or cake]$_F$?
a. $\rightsquigarrow$ We are allowed to eat ice cream.
b. $\rightsquigarrow$ We are allowed to eat cake.

Second, implicature approaches do not predict the distribution of "universal FC" inferences as in (3a)-(3b). Specifically, under current implicature-based accounts such inferences must be derived by embedded exhaustification, but there is evidence that a global derivation must be available (Chemla 2009b).

1 Here and throughout the paper we conflate logical forms and their truth conditions, e.g., $\Diamond(a \vee b)$ will at times stand for a logical form in which an existential modal takes scope over disjunction and at other times will stand for the corresponding truth conditions. We distinguish between the two only if we think things might be confusing otherwise.

2 We refer to such accounts of FC as Implicature accounts since they relate Free Choice to other scalar implicatures. However, our account, like many others, does not take FC to be an implicature in the traditional Gricean sense. Instead it is a logical inference of a particular parse of a potentially ambiguous sentence. This terminological choice could be at times problematic, as one still needs a term for traditional Gricean inferences—non-logical (abductive) inferences that are based on the fact that an utterance was made by a particular person in a particular context.

(3) **Universal Free Choice**:
Every boy is allowed to eat ice cream or cake. $\forall x \Diamond(Px \vee Qx)$

a. $\rightsquigarrow$ Every boy is allowed to eat ice cream. $\forall x \Diamond Px$
b. $\rightsquigarrow$ Every boy is allowed to eat cake. $\forall x \Diamond Qx$

Third, a similar case of FC inferences which is not predicted by implicature accounts has been discussed by Nouwen (2017);[3] this is a case where a universal quantifier intervenes between the existential modal and disjunction:

(4) The teacher is OK with every student either talking to Mary or to Sue. $\Diamond \forall x(Px \vee Qx)$

a. $\rightsquigarrow$ The teacher is OK with every student talking to Mary. $\Diamond \forall x Px$
b. $\rightsquigarrow$ The teacher is OK with every student talking to Sue. $\Diamond \forall x Qx$

Fourth, the phenomenon of Simplification of Disjunctive Antecedents (SDA), exemplified in (5), has been argued to pattern like FC and hence a unified explanation is called for. But such a unified explanation is not provided under all implicature accounts of FC (though see Klinedinst 2007; Franke 2011 which we will touch on in §7).

(5) **Simplification of Disjunctive Antecedents (SDA)**:
If you eat ice cream or cake, you will feel guilty. $(p \vee q) \,\Box\!\!\rightarrow r$

a. $\rightsquigarrow$ If you eat ice cream, you will feel guilty. $p \,\Box\!\!\rightarrow r$
b. $\rightsquigarrow$ If you eat cake, you will feel guilty. $q \,\Box\!\!\rightarrow r$

Our contribution in this paper is to propose a modification of Fox (2007), and to argue that it can provide the basis for a solution to these problems. Fox has defined an exhaustivity operator, $\mathcal{E}xh$, based on the notion of Innocent Exclusion. The results of applying that operator to a proposition and a set of alternatives are almost equivalent to the results of the algorithm proposed in Sauerland (2004) as a characterization of the outcome of neo-Gricean reasoning. One argument in favor of implementing Innocent Exclusion within a grammatical theory, namely letting $\mathcal{E}xh$ have a syntactic life, was that unlike the neo-Gricean implementation the grammatical one predicted recursive application of $\mathcal{E}xh$ over exhaustified alternatives to be available, which derived FC inferences.

The perspective put forth in the current paper involves a more radical departure from neo-Gricean reasoning. Instead of merely negating alternatives, we propose that $\mathcal{E}xh$ "attempts" to match alternatives with cells in the partition induced by the set of

3 The problem Nouwen (2017) focuses on is FC with ability modals, but (4) reveals the nature of the problem more transparently. See §6.

alternatives, whenever this is possible.[4] Our view of $\mathcal{E}xh$ as identifying cells leads to its definition as an operator which both assigns false to certain alternatives, what Fox calls Innocent Exclusion, and furthermore assigns true to some other alternatives, what we will call Innocent Inclusion. We will argue that the key to analyzing examples (3)-(5) lies in letting $\mathcal{E}xh$ identify cells whenever possible. As for (2), the distinction between Innocent Exclusion and Inclusion will allow us to claim that *only* too, just like $\mathcal{E}xh$, involves Inclusion; the only difference between the two operators would be that *only* presupposes the "Innocently Includable" alternatives, while $\mathcal{E}xh$ asserts them.

The structure of the paper is as follows: in §2 we discuss motivations for the implicature account of FC. We then present our proposal in §3, which, as just mentioned, builds on Fox's notion of Innocent Exclusion and introduces in addition the notion of Innocent Inclusion; we show that together they consist of a mechanism that identifies cells whenever possible, thus providing a direct derivation of Free Choice disjunction which does not require the computation of exhaustified alternatives. The core argument of the paper is then discussed in §4-§8, where the four issues mentioned above, exemplified in (2)-(5), are argued to provide empirical support for Innocent Inclusion. Finally, §9 identifies some remaining issues that we have to leave for future research.

2 The implicature account of FC

2.1 Motivation for an implicature account

Alonso-Ovalle (2005), following Kratzer & Shimoyama (2002), argues that the Free Choice inference from (1) to (1a)-(1b) should be treated as a scalar implicature, due to the consequences of embedding the basic construction under negation as in (6).

(6) John isn't allowed to eat ice cream or cake. $\neg\Diamond(a \vee b)$

a. $\not\approx$ It's not the case that John is both allowed to eat ice cream *and* allowed to eat cake (but maybe he's allowed one of them). $\neg(\Diamond a \wedge \Diamond b)$

b. $\approx$ It's not the case that John is allowed to eat ice cream and it's not the case that he is allowed to eat cake. $\neg\Diamond a \wedge \neg\Diamond b$

A standard translation of the relevant sentences into modal logic will provide the

4 Implicature calculation as resulting in Cell Identification has been pursued in Franke's (2011) game theoretic approach, which is essentially a Cell Identification view of scalar implicatures. However, without various complications, Franke fails to derive several basic facts, such as FC disjunction with more than two disjuncts, universal FC inferences, and SDA with more than two disjuncts. Furthermore, it crucially relies on the assumption of equal priors for deriving FC (see Fox & Katzir 2018 for discussion).

correct interpretation when embedding under negation is involved: $\neg\Diamond(a \vee b) \Leftrightarrow \neg\Diamond a \wedge \neg\Diamond b$. If the conjunctive interpretation of the unembedded constructions could be derived by the theory that derives scalar implicatures, it would have the conceptual merit of keeping the underlying logic intact, and the empirical merit of predicting the correct result for embedding under negation (as scalar implicatures are rarely computed under negation).

2.2 A novel argument from VP-ellipsis

To strengthen the argument, we point out here that FC behaves similarly to scalar implicatures under VP-ellipsis. In VP-ellipsis constructions like (7) where the elided material is under negation but the antecedent isn't, the salient interpretation is one where the antecedent is interpreted conjunctively, having an FC meaning, while the elided material is interpreted disjunctively, lacking an FC meaning.

(7) Mary is allowed to eat ice cream or cake, and John isn't ~~allowed to eat ice cream or cake~~. ≈
   a. Mary is allowed to eat ice cream *and* allowed to eat cake, and
   b. John isn't allowed to eat ice cream and he isn't allowed to eat cake.

Such behavior under ellipsis is typical for scalar implicatures. For example, in (8) the antecedent VP contains *some* which is intuitively interpreted exhaustively as *some but not all*, while in the elided VP *some* is interpreted non-exhaustively under negation (see Fox 2004 for a similar data point, attributed to Tamina Stephenson, p.c.).

(8) Mary solved some of the problems, and John didn't ~~solve some of the problems~~. ≈
   a. Mary solved *some but not all* of the problems, and
   b. John didn't solve *any* of the problems.

Within the grammatical approach to scalar implicatures (Chierchia, Fox & Spector 2012), such facts are explained under strict theories of ellipsis which demand semantic identity between an elided constituent and a pronounced antecedent (Sag 1976; Williams 1977), since the exhaustivity operator $\mathcal{E}xh$ that generates the *some but not all* or the FC inferences can be base generated above the antecedent.[5]

5 And, of course, they are also explained when weaker demands are made as in Rooth (1992); Heim (1996)—the "parallelism domain" for ellipsis (the domain of ~ in Rooth 1992; Heim 1996) need not to contain $\mathcal{E}xh$. If this operator was forced to be inside the parallelism domain, we would expect the implicature to be derived for the elided material as well; see Crnič (2015) for arguments that the presence of $\mathcal{E}xh$ inside the parallelism domain indeed has the predicted consequences. We will return

(7) is especially problematic for an ambiguity approach to FC such as Aloni (2007); within such an approach the semantics of *allowed to eat ice cream or cake* is different when under negation than when unembedded. Hence it is impossible to derive the meaning in (7a)-(7b) while at the same time respecting the semantic identity required in VP-ellipsis between the antecedent and the elided material.[6]

2.3 Comments on other approaches to FC

As William Starr has pointed out to us, the argument from ellipsis doesn't carry over (at least not straightforwardly) to other accounts with a non-standard semantics which don't rely on ambiguity like Aloni (2007) does (see recently Starr 2016; Aloni 2016; Willer 2017). In order to deliver the right truth conditions for sentences like (6) without assuming ambiguity, those approaches rely on defining truth and falsity conditions separately (bilaterally) for *allowed* and for *or*.[7] Once this is done, negation can be defined so that it interacts in the desired way with the bilateral meaning of *allowed to eat ice cream or cake*. As Aloni (2016) points out, this considerably weakens the explanatory power of those approaches, since the "behaviour under negation is postulated rather than predicted: Allowing to pre-encode what should happen under negation, bilateral systems are more descriptive than explanatory".[8,9]

Our goal in this paper is then to seek an explanatory account of FC. As mentioned in the introduction, we believe that deriving FC via an exhaustivity operator can in principle meet this goal. However, there are remaining empirical challenges that motivate modification in the theory of $\mathcal{E}xh$.

Some challenges to a theory of FC as an implicature are not fully addressed in this paper: First, it has been claimed that FC inferences behave differently than other

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to the issue of Free Choice with ellipsis and the relevance of parallelism to exhaustivity in §5.3.

6 A similar argument against ambiguity comes from non-monotonic contexts (see Bar-Lev 2018: §1):

(i) Exactly two girls are allowed to eat ice cream or cake.
  a. $\rightsquigarrow$ Two girls are both allowed to eat ice cream and allowed to eat cake.
  b. $\rightsquigarrow$ No more than two girls are allowed ice cream or cake.

One might consider the following fix for salvaging the ambiguity approach in light of (7) and (i): *allowed A or B* gives rise to a special kind of ambiguity, which requires truth on all of its resolutions (along the lines of the ambiguity approach to Homogeneity phenomena in Spector 2013; Križ & Spector 2017; Spector 2018). Since it does not seem to extend to every kind of ambiguity resolution we do not pursue this route.

7 This is not strictly speaking true of Starr (2016), whose proposal nonetheless involves non-standard modifications of logical operators, a move we are trying to avoid.

8 In addition, some of those approaches (Starr 2016; Aloni 2016) predict FC inferences when disjunction takes wide scope above the modal, a result argued against in Bar-Lev (2018: §2).

9 This criticism is reminiscent of Schlenker's (2009) criticism of dynamic semantics approaches.

scalar implicatures (e.g., Chemla & Bott 2014); Second, it has been claimed that FC inferences are attested even when disjunction takes scope above the modal (i.e., with sentences of the form $\Diamond a \vee \Diamond b$), which isn't predicted by implicature accounts. We briefly discuss the first issue in §9.1; both are discussed more fully in Bar-Lev (2018: §2).

3 Proposal: Innocent Inclusion

3.1 Disjunction and its alternatives

A key point in analyzing FC disjunction is understanding the distinction between (9) and (10). While from simple disjunction we infer the exclusive inference in (9a), for Free Choice disjunction we infer what we might call the opposite inference in (10a), namely FC.

(9) **Simple disjunction**:
Mary ate ice cream or cake $a \vee b$
a. ⇝ Mary didn't eat both ice cream and cake. $\neg(a \wedge b)$

(10) **Free Choice disjunction**:
Mary is allowed to eat ice cream or cake. [=(1)] $\Diamond(a \vee b)$
a. ⇝ Mary is allowed to eat ice cream and allowed to eat cake. $\Diamond a \wedge \Diamond b$

To see if we can view both the exclusive inference in (9a) and the FC inference in (10a) as scalar implicatures, we should start by asking what alternatives are generated in each case (since scalar implicatures can only be determined when alternatives are specified).

Various arguments have been provided for the following sets of alternatives: disjunction gives rise to disjunctive alternatives, that is alternatives where the disjunction is replaced by the individual disjuncts (see, e.g., Kratzer & Shimoyama 2002; Sauerland 2004; Katzir 2007);[10] and to a conjunctive alternative—an alterna-

10 There are several reasons for assuming that disjunctive alternatives are generated: First, their existence provides an explanation for the fact that from a sentence like (i) we infer (ia)-(ib), i.e., the negation of the disjunctive alternatives (*You are required to solve problem A*, *You are required to solve problem B*). See Sauerland (2004); Spector (2007); Katzir (2007); Fox (2007).

(i) You are required to solve problem A or problem B.
a. ⇝ You are not required to solve problem A.
b. ⇝ You are not required to solve problem B.

Second, if Free Choice inferences are to be treated as scalar implicatures, as we assume here, disjunctive alternatives are required. Third, the independently motivated structural approach to the generation of alternatives (Katzir 2007) predicts such alternatives to be generated, being the result of tive where disjunction is replaced with conjunction. When applied to the sentences in (9) and (10), we generate the alternatives in (11a) and (11b), respectively.

(11) a. **Set of alternatives for simple disjunction**:

$$Alt(a \lor b) = \{ \underbrace{a \lor b}_{\text{Prejacent}}, \underbrace{a, b}_{\text{Disjunctive alts.}}, \underbrace{a \land b}_{\text{Conjunctive alt.}} \}$$

b. **Set of alternatives for Free Choice disjunction**:

$$Alt(\Diamond(a \lor b)) = \{ \underbrace{\Diamond(a \lor b)}_{\text{Prejacent}}, \underbrace{\Diamond a, \Diamond b}_{\text{Disjunctive alts.}}, \underbrace{\Diamond(a \land b)}_{\text{Conjunctive alt.}} \}$$

Looking at these sets of alternatives in light of the inferences we get in (9) and (10), we can see a striking difference between simple disjunction and FC disjunction with regard to the conjunction of their disjunctive alternatives:

(12) **Observation**:

a. From simple disjunction we infer that the conjunction of the disjunctive alternatives $(a \land b)$ is **false**.

b. From Free Choice disjunction we infer that the conjunction of the disjunctive alternatives $(\Diamond a \land \Diamond b)$ is **true**.

What distinguishes the two cases and yields the opposite results in (12)? We will adopt the answer in (13) provided by Fox (2007); Chemla (2009a); Franke (2011), according to which *closure under conjunction* is the distinguishing property: whereas $Alt(a \lor b)$ is closed under conjunction, $Alt(\Diamond(a \lor b))$ is not.

(13) a. The conjunction of the disjunctive alternatives a and b, i.e., $a \land b$, **is** a member of $Alt(a \lor b)$.

b. The conjunction of the disjunctive alternatives $\Diamond a$ and $\Diamond b$, i.e., $\Diamond a \land \Diamond b$, **is not** a member of $Alt(\Diamond(a \lor b))$.

To see how this distinction might yield opposite results for the two cases, let us focus on the account of FC in Fox (2007).

3.2 Innocent Exclusion

Within the version of the grammatical theory assumed by Fox (2007), scalar implicatures are generated by applying a covert exhaustivity operator, $\mathcal{E}xh$, akin to overt *only*. This operator takes a prejacent and a set of alternatives. What should it return as output? If we let $\mathcal{E}xh$ assign false to every alternative, we would sometimes get a contradictory result, even if we are "smart enough" and restrict the procedure to

replacing a constituent (*P or Q*) with a sub-constituent (*P*, *Q*).

those alternatives whose falsity is consistent with the prejacent. To see this, consider the case of exhaustifying $a \vee b$ with respect to $Alt(a \vee b)$. Since both a and b are non-weaker than $a \vee b$, $\mathcal{E}xh$ would assign them false and yield the contradiction $(a \vee b) \wedge \neg a \wedge \neg b$.

$\mathcal{E}xh$ must be a bit "smarter" than this: it must find a way to exclude as many alternatives as possible, while carrying about overall consistency. To achieve this, Fox (2007) argues in favor of using the notion of Innocent Exclusion (a generalization of the algorithm provided in Sauerland 2004 incorporated into a grammatical theory).

(14) **Innocent Exclusion procedure**:

a. Take all maximal sets of alternatives that can be assigned false consistently with the prejacent.

b. Only exclude (i.e., assign false to) those alternatives that are members in all such sets—the **Innocently Excludable** (=IE) alternatives.

Let us show now how Innocent Exclusion avoids contradiction when applied to $a \vee b$ and to $\Diamond(a \vee b)$. To apply Innocent Exclusion to $a \vee b$ we first have to identify the maximal sets of alternatives in $Alt(a \vee b)$ that can be assigned false consistently with the prejacent. There are two such sets: $\{a, a \wedge b\}$ and $\{b, a \wedge b\}$. The second step, by the Innocent Exclusion procedure, is to exclude the alternatives which are in all of those sets; there is only one such alternative, $a \wedge b$, which is thus the only IE alternative.

A parallel result is derived by applying Innocent Exclusion to $\Diamond(a \vee b)$. The maximal sets of alternatives in $Alt(\Diamond(a \vee b))$ that can be assigned false consistently with the prejacent are $\{\Diamond a, \Diamond(a \wedge b)\}$ and $\{\Diamond b, \Diamond(a \wedge b)\}$, and consequently the only IE alternative is $\Diamond(a \wedge b)$. The result of Innocent Exclusion for the two cases is represented schematiclally in figure 1. At this level, for simple disjunction we derive that the prejacent $a \vee b$ is true and that the conjunctive alternative $a \wedge b$ is false, and similarly for Free Choice disjunction: the prejacent $\Diamond(a \vee b)$ is true and the conjunctive alternative $\Diamond(a \wedge b)$ is false.[11] This result gives us the first hint as to

11 As pointed out by Simons (2005), FC inferences are not always accompanied by the inference that excludes the conjunctive alternative $\Diamond(a \wedge b)$. Our derivation of FC in what follows does not depend on deriving the falsity of the conjunctive alternative (in fact it follows even if it's omitted altogether from the set of alternatives). See §9.1 and Bar-Lev (2018: §2) for reasons why this alternative will be assigned false only if it's taken to be relevant. Of course, one would then need to say why the same reasoning cannot apply to simple disjunction: why can't the conjunctive alternative in this case be irrelevant, and thus lead to a conjunctive reading for simple disjunction? The answer that has been given in the literature (e.g., Fox & Katzir 2011) is that it results from relevance being closed under Boolean operations (conjunction and negation). It is thus possible for $\Diamond a$ and $\Diamond b$ to be relevant without $\Diamond(a \wedge b)$ being relevant; in contrast, once a and b are relevant, $a \wedge b$ must be relevant as well. A simplifying assumption in the discussion here and in what follows is that all alternatives are relevant, but this should not be seen as an empirical claim that we always infer the falsity of the IE

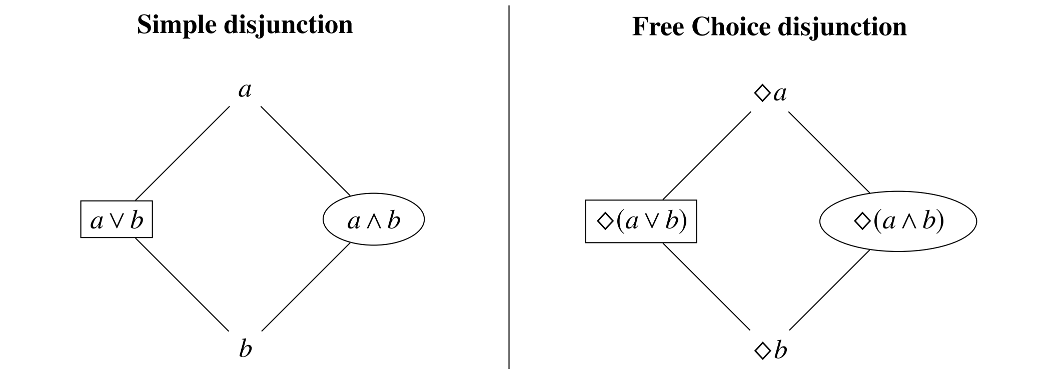

Figure 1 Results of Innocent Exclusion for simple and Free Choice disjunction. The lines represent entailment relations from right to left, the prejacent is marked with ▭ and the IE alternatives with ⬭.

why we might derive opposite results for the two cases:

Since $Alt(a \vee b)$ is closed under conjunction, the output of applying Innocent Exclusion to $a \vee b$—which ensures the falsity of $a \wedge b$—*is not comaptible* with the truth of both disjunctive alternatives a and b. In contrast, since $Alt(\Diamond(a \vee b))$ is not closed under conjunction, the output of applying Innocent Exclusion to $\Diamond(a \vee b)$—which ensures the falsity of $\Diamond(a \wedge b)$ (but crucially not the falsity of $\Diamond a \wedge \Diamond b$)—*is comaptible* with both disjunctive alternatives $\Diamond a$ and $\Diamond b$ being true.

But FC is not yet derived. Given what we have said so far we can only explain why it would be in principle possible to derive a conjunctive meaning for disjunction when embedded under an existential modal but not for simple disjunction: a conjunctive meaning is consistent with the result of applying Innocent Exclusion in the case of FC disjunction but not in the case of simple disjunction. We still have to find a way to actually derive the conjunctive meaning.

The Innocent Exclusion procedure in (14) leads to the lexical entry for the exhaustivity operator $\mathcal{E}xh^{\text{IE}}$ in (15a): given a set of alternatives C and a prejacent p, it would assign true to the prejacent and false to all the IE alternatives (defined in (15b)).

(15) **Innocent-Exclusion-based exhaustivity operator**: (Fox 2007)

a. $[\![\mathcal{E}xh^{\text{IE}}]\!](C)(p)(w) \Leftrightarrow p(w) \wedge \forall q \in IE(p,C)[\neg q(w)]$

b. $IE(p,C) = \bigcap\{C' \subseteq C : C'$ is a maximal subset of C, s.t. $\{\neg q : q \in C'\} \cup \{p\}$ is consistent$\}$

alternatives. See Fox & Katzir (2011); Singh, Wexler, Astle-Rahim, Kamawar & Fox (2016) for a relevant discussion.

In the next section we introduce the notion of Innocent Inclusion, and suggest a different lexical entry for $\mathcal{E}xh$ than (15a), one that implements both Innocent Exclusion and Innocent Inclusion and can directly derive FC inferences.

3.3 Introducing Innocent Inclusion

How do we derive the Free Choice inferences for Free Choice disjunction? This is where we depart from Fox (2007). The FC inferences $\Diamond a$ and $\Diamond b$ are derived by Fox *indirectly*, by applying $\mathcal{E}xh^{\text{IE}}$ recursively. Our proposal is to define $\mathcal{E}xh$ differently from (15a), with the consequence that those inferences are derived *directly*. Before presenting our proposal, we first illustrate in §3.3.1 how Fox (2007) derives FC with a recursive application of $\mathcal{E}xh^{\text{IE}}$. As we have mentioned in the introduction, Fox's approach faces some empirical challenges, such as universal FC and SDA. We will argue in §3.3.2 that these challenges can be seen to reveal different instantiations of a single generalization that Fox's account fails to capture. We then move on in §3.3.3 to demonstrate that the generalization is predicted when the notion of Innocent Inclusion is incorporated into the definition of $\mathcal{E}xh$.

3.3.1 Deriving FC with recursive $\mathcal{E}xh^{\text{IE}}$

Applying $\mathcal{E}xh^{\text{IE}}$ once to $\Diamond(a \vee b)$ we get (see figure 1):

(16) $\mathcal{E}xh^{\text{IE}}_C \Diamond(a \vee b) \Leftrightarrow \Diamond(a \vee b) \wedge \neg\Diamond(a \wedge b).$ (where $C = Alt(\Diamond(a \vee b))$)

We can now apply $\mathcal{E}xh^{\text{IE}}$ once more, over $\mathcal{E}xh^{\text{IE}}_C \Diamond(a \vee b)$. The alternatives for the second application of $\mathcal{E}xh^{\text{IE}}$ are the exhaustified alternatives of $\Diamond(a \vee b)$:

(17) $$Alt(\mathcal{E}xh^{\text{IE}}_C \Diamond(a \vee b)) = \{\underbrace{\mathcal{E}xh^{\text{IE}}_C \Diamond(a \vee b)}_{\text{Prejacent}}, \underbrace{\mathcal{E}xh^{\text{IE}}_C \Diamond a\,,\ \mathcal{E}xh^{\text{IE}}_C \Diamond b}_{\text{Disjunctive alts.}}, \underbrace{\mathcal{E}xh^{\text{IE}}_C \Diamond(a \wedge b)}_{\text{Conjunctive alt.}}\}$$

Computing the exhaustified alternatives, we get the following:

(18) $$Alt(\mathcal{E}xh^{\text{IE}}_C \Diamond(a \vee b)) = \{\underbrace{\Diamond(a \vee b) \wedge \neg\Diamond(a \wedge b)}_{\text{Prejacent}}, \underbrace{\Diamond a \wedge \neg\Diamond b\,,\ \Diamond b \wedge \neg\Diamond a}_{\text{Disjunctive alts.}}, \underbrace{\Diamond(a \wedge b)}_{\text{Conjunctive alt.}}\}$$

Now, applying $\mathcal{E}xh^{\text{IE}}$ with respect to this set we get (19) (since all the alternatives are IE):

(19) $\mathcal{E}xh^{\text{IE}}_{Alt(\mathcal{E}xh^{\text{IE}}_C \Diamond(a \vee b))} \mathcal{E}xh^{\text{IE}}_C \Diamond(a \vee b)$

$$\Leftrightarrow \underbrace{\Diamond(a \vee b) \wedge \neg\Diamond(a \wedge b)}_{\text{Prejacent}} \wedge \underbrace{\neg(\Diamond a \wedge \neg\Diamond b) \wedge \neg(\Diamond b \wedge \neg\Diamond a)}_{\text{negation of disjunctive alternatives}}$$
$$\Leftrightarrow \underbrace{\Diamond(a \vee b) \wedge \neg\Diamond(a \wedge b)}_{\text{Prejacent}} \wedge \underbrace{\Diamond a \wedge \Diamond b}_{\text{FC inferences}}$$

3.3.2 Cell Identification

Note that the result of the recursive application of $\mathcal{E}xh^{\text{IE}}$ in (19) entails that the prejacent $\Diamond(a \vee b)$ and the disjunctive alternatives $\Diamond a$ and $\Diamond b$ are true, and that the conjunctive IE alternative $\Diamond(a \wedge b)$ is false. That is, recursive exhaustification, in this particular case, determines the truth value of every alternative in C. But recursive exhaustification of $\mathcal{E}xh^{\text{IE}}$ does not always have this property. Our argument for a modification of $\mathcal{E}xh$ will be based on the observation that it doesn't in cases where it should. To understand the argument it is useful to define a function, $Cell(p,C)$, that takes a prejacent p and a set of alternatives C and returns a proposition that is true if all members of $IE(p,C)$ are false and all other members of C are true:

(20) $Cell(p,C) = p \wedge \bigwedge\{\neg q : q \in IE(p,C)\} \wedge \bigwedge(C \setminus IE(p,C))$

This proposition, of course, determines a truth value for every member of C. Our empirical claim is that whenever this determination is consistent (whenever the output of the function is distinct from $\bot$) it should be the output of exhaustification.

Consider the partition induced by the set of alternatives, that is the partition of logical space into sets of worlds that assign the same truth value to every member of C—$Partition(C)$. $Partition(C)$ can be written in a format reminiscent of our definition of $Cell(p,C)$ as the set of non-contradictory propositions, each of which is the conjunction of a subset C' of C and the conjunction of the negations of all remaining alternatives (i.e., alternatives in $C \setminus C'$):

(21) $Partition(C) = \{p : p \neq \bot \wedge \exists C' \subseteq C[p = \bigwedge\{\neg q : q \in C'\} \wedge \bigwedge(C \setminus C')]\}$

It is easy to see that $Cell(p,C)$ is either a contradiction or a cell in $Partition(C)$.[12] Given a prejacent p and a set of alternatives C, the proposition returned by $Cell$ is true if and only if the prejacent is true, all the IE alternatives are false, and all the non-IE alternatives are true. In the case of $\Diamond(a \vee b)$ the following holds, namely recursive application of $\mathcal{E}xh^{IE}$ turns out equivalent to the application of $Cell$:

(22) For $p = \Diamond(a \vee b)$ and $C = Alt(\Diamond(a \vee b))$:
$\mathcal{E}xh_C^{\text{IE}^2}(p) \Leftrightarrow Cell(p,C)$

12 As long as $p \in C$ and C is finite, the first conjunct p on the right hand side of (20) is redundant. See fn. 16.

(where $\mathcal{E}xh^{\mathrm{IE}^2}$ stands for two applications of $\mathcal{E}xh^{\mathrm{IE}}$)

The case of $\Diamond(a \vee b)$, we will argue, is but one manifestation of the following generalization:

(23) **Cell Identification (when possible)**:
Let S be a sentence with denotation p and C be the set of denotations of its alternatives. If $Cell(p,C)$ is not a contradiction, then S can have $Cell(p,C)$ as a strengthened meaning.

We will discuss in §5-§8 several cases that demonstrate that (23) is not predicted to hold if our strengthening tool $\mathcal{E}xh$ is $\mathcal{E}xh^{\mathrm{IE}}$; we will however provide evidence that it holds empirically. We thus propose in what follows a redefinition of $\mathcal{E}xh$ that validates (23).

3.3.3 Innocent Inclusion and Cell Identification

Assuming that (23) is indeed an empirical generalization and one that does not follow from recursive exhaustification, a shift in perspective is called for. We will suggest that exhaustification should not only lead to the exclusion of as many alternatives as possible (as in $\mathcal{E}xh^{\mathrm{IE}}$); it should also lead to the "inclusion" of as many alternatives as possible given what has been excluded.

More specifically, we suggest a direct implementation of this idea, one in which $\mathcal{E}xh$ has a dual role: it doesn't only *exclude* certain alternatives, assigning them the truth value false, it also *includes* some other alternatives, assigning them the truth value true.

At this point we would like to mention the underlying conception that has guided our thinking, namely (24).

(24) **Possible underlying conception**:
Exhaustifying p with respect to a set of alternatives C should get us as close as possible to a cell in the partition induced by C, $Partition(C)$.

Namely the goal of $\mathcal{E}xh$ is to come as close as possible to an assignment of a truth value to every alternative, i.e., to a cell in the partition that the set of alternatives induces. In other words, $\mathcal{E}xh$ is designed such that when possible it would yield a complete answer to the question formed by the set of alternatives.[13] If this conception is correct, one would think that $\mathcal{E}xh$ shouldn't only exclude, i.e., assign false to as many alternatives as possible, but should also include, i.e., assign true to as many

13 See Fox (2017) for the relevance of this notion of exhaustivity as Cell Identification to issues having to do with the semantics of questions.

alternatives as possible once the exclusion is complete.

What are the alternatives we want $\mathcal{E}xh$ to include? One possibility we might entertain is that $\mathcal{E}xh$ blindly includes all non-IE alternatives. But this would sometimes lead to contradictions: exhaustifying $a \vee b$ with respect to $Alt(a \vee b)$ would yield a contradiction, because including a and b, which are both non-IE, would contradict the derived falsity of $a \wedge b$. In other words, Inclusion just like Exclusion must apply *innocently*, so as to avoid contradictions. We thus suggest the procedure of Innocent Inclusion in (25).

(25) **Innocent Inclusion procedure**:

a. Take all maximal sets of alternatives that can be assigned true consistently with the prejacent and the falsity of all IE alternatives.

b. Only include (i.e., assign true to) those alternatives that are members in all such sets—the **Innocently Includable** (=II) alternatives.

Note the similarity between Innocent Exclusion and Innocent Inclusion. Innocent Inclusion is only different from Innocent Exclusion in two respects: (i) that we include instead of exclude, and (ii) that we check for consistency not only with respect to the prejacent but also with respect to the falsity of all the IE alternatives.[14]

Let us see now how Innocent Inclusion applies to simple and Free Choice disjunction to derive the desired results. Having Innocent Inclusion changes nothing for simple disjunction: The only II alternative is the prejacent $a \vee b$. This is since the maximal sets of alternatives that are consistent with the truth of $a \vee b$ (the prejacent) and the falsity of $a \wedge b$ (the IE alternative) are $\{a \vee b, a\}$ and $\{a \vee b, b\}$, and the only member in their intersection is the prejacent $a \vee b$.

For FC disjunction, on the other hand, we derive the desired FC inferences with our procedure, since $\Diamond a$ and $\Diamond b$ are II. In this case, all the alternatives which are not IE are together consistent with the truth of $\Diamond(a \vee b)$ (the prejacent) and the falsity of $\Diamond(a \wedge b)$ (the IE alternative). That is, we only have one maximal set of alternatives to consider, $\{\Diamond(a \vee b), \Diamond a, \Diamond b\}$, and all alternatives within this set are II. Therefore applying Innocent Exclusion and Innocent Inclusion yields a cell in the partition (a

14 Why do we have to consider the set of IE alternatives for determining the set of II alternatives, and not vice versa? Let us consider what would happen if we first considered what's II: take for example the sentence *some boy came* and its alternative *every boy came*. If we were to include first, we would derive that the alternative *every boy came* is true. Namely exhaustifying over *some boy came* would yield a meaning equivalent to *every boy came*. This would make for a very inefficient tool to use in conversation: by choosing an utterance from the set of alternatives {*some boy came*, *every boy came*} an opinionated speaker would only be able to convey one epistemic state she might be in (one cell in the partition); she would not be able to convey an epistemic state that entails *some but not all boys came*. In other words, it would be a bad tool for answering questions (see Fox 2017). By prioritizing exclusion over inclusion we allow the speaker to convey more epistemic states.

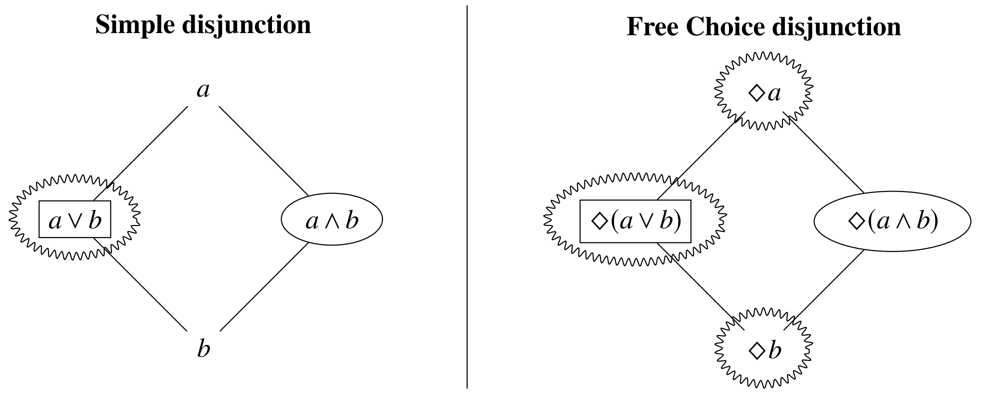

Figure 2 Results of Innocent Exclusion and Innocent Inclusion for simple and Free Choice disjunction. The lines represent entailment relations from right to left, the prejacent is marked with ▭, the IE alternatives with ⬭, and the II alternatives with ✺.

complete answer) in this case; the output tells us of every alternative whether it is true or false. The results of Innocent Exclusion and Innocent Inclusion for the two cases are represented schematiclally in figure 2.

We have seen that applying Innocent Exclusion and Innocent Inclusion yields the desired results for simple and Free Choice disjunction: an exclusive *or* meaning for simple disjunction and an FC meaning for Free Choice disjunction. For these two cases, the results we derive are identical to those Fox (2007) derives with the recursive application of $\mathcal{E}xh^{\text{IE}}$.

The lexical entry of the exhaustivity operator we are assuming here, $\mathcal{E}xh^{\text{IE+II}}$, implements both Innocent Exclusion and Innocent Inclusion. We first define the sets of IE and II alternatives: the set of IE alternatives remains as in Fox (2007), and the set of II alternatives is defined in parallel in (26b), with the two key differences between Innocent Exclusion and Innocent Inclusion discussed above.

(26) Given a sentence p and a set of alternatives C:

a. $IE(p,C) = \bigcap\{C' \subseteq C : C' \text{ is a maximal subset of } C\text{, s.t. } \{\neg q : q \in C'\} \cup \{p\} \text{ is consistent}\}$ [=(15b)]

b. $II(p,C) = \bigcap\{C'' \subseteq C : C'' \text{ is a maximal subset of } C\text{, s.t. } \{r : r \in C''\} \cup \{p\} \cup \{\neg q : q \in IE(p,C)\} \text{ is consistent}\}$

With these definitions at hand we can write the lexical entry of $\mathcal{E}xh^{\text{IE+II}}$ in (27): Given a set of alternatives C and a prejacent p, it would assign false to all the IE alternatives and true to all the II alternatives.[15,16]

(27) **Innocent-Exclusion+Innocent-Inclusion-based exhaustivity operator**:
$$[\![\mathcal{E}xh^{\text{IE+II}}]\!](C)(p)(w) \Leftrightarrow \forall q \in IE(p,C)[\neg q(w)] \wedge \forall r \in II(p,C)[r(w)]$$

Note that $\mathcal{E}xh^{\text{IE+II}}$ has the property of Cell Identification in (23): it follows from its definition that the output of a single application of $\mathcal{E}xh^{\text{IE+II}}$ would be the *Cell* interpretation whenever that is not a contradiction.

(28) **$\mathcal{E}xh^{\text{IE+II}}$ validates Cell Identification (when possible)**:[17]
For any proposition p and any set of alternatives C:
If $Cell(p,C) \neq \perp$, then:
a. $C \setminus IE(p,C) = II(p,C)$, and (therefore)
b. $\mathcal{E}xh^{\text{IE+II}}_C(p) \Leftrightarrow Cell(p,C)$

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15 An alternative one could attempt to pursue is to stick to the definition of $\mathcal{E}xh^{\text{IE}}$ and derive results parallel to those we derive using $\mathcal{E}xh^{\text{IE+II}}$ by having different assumptions than Fox (2007) about the alternatives of exhaustified constituents. For example, assume that the only alternatives $\mathcal{E}xh^{\text{IE}}$ projects are its sub-domain alternatives, i.e., alternatives generated by replacing the set of alternatives $\mathcal{E}xh$ operates on with its subsets. Indeed, (ia) is semantically equivalent to (ib) for any choice of p and C we checked.

(i) a. $\mathcal{E}xh^{\text{IE}}_{C'}\ [\mathcal{E}xh^{\text{IE}}_{C}\ p]$ (where $C' = \{\mathcal{E}xh^{\text{IE}}_{C''}(p) : C'' \subseteq C\}$)
b. $\mathcal{E}xh^{\text{IE+II}}_{C}\ p$

Even if these two formulas are semantically equivalent in general, as we suspect, a theory of Cell Identification based on $\mathcal{E}xh^{\text{IE+II}}$ will allow us to capture facts that we cannot capture with $\mathcal{E}xh^{\text{IE}}$ when coupled with the specific assumption about projection in (ia) (the identity of C'). Specifically, our proposal in §4 for the semantics of *only* and the different treatment considered for IE and II alternatives in §9.1 and Bar-Lev (2018: §2) crucially rely on the distinction between IE and II alternatives, which is only available with $\mathcal{E}xh^{\text{IE+II}}$. Furthermore, distributive inferences for sentences of the form $\forall x(Px \vee Qx)$ can be derived with recursive application of $\mathcal{E}xh^{\text{IE+II}}$ along the lines of Bar-Lev & Fox (2016) (see discussion in Gotzner & Romoli 2017) using Fox's assumptions about how alternatives project; recursive application of $\mathcal{E}xh^{\text{IE}}$ with the assumption about projection in (ia) won't do in this case.

16 Note that p (the prejacent) can never be in $IE(p,C)$ and it will always be in $II(p,C)$, assuming that the prejacent p must be in C and that C is finite. Namely, $p(w)$ in (i) would be redundant under these assumptions since it would be entailed by $\forall r \in II(p,C)[r(w)]$. So (i) would be equivalent to (27), where $p(w)$ is taken out.

(i) $[\![\mathcal{E}xh^{\text{IE+II}}]\!](C)(p)(w) \Leftrightarrow p(w) \wedge \forall q \in IE(p,C)[\neg q(w)] \wedge \forall r \in II(p,C)[r(w)]$

17 If $Cell(p,C)$ is not a contradiction, then $\{p\} \cup \{\neg q : q \in IE(p,C)\} \cup \{r : r \in C \setminus IE(p,C)\}$ is consistent. And if this is the case, there is only one maximal subset of C consistent with the prejacent and the

Throughout the paper we will occasionally use the property of Cell Identification to justify a shortcut: whenever the *Cell* interpretation is non-contradictory, that would be what we get by applying $\mathcal{E}xh^{\text{IE+II}}$. In such cases the set of II alternatives would be identical to the set of non-IE ones.

To summarize, building on Fox's notion of Innocent Exclusion we have introduced the notion of Innocent Inclusion. We have suggested a revision of the exhaustivity operator such that it would not only assign false to all the IE alternatives but also assign true to all the II alternatives. As a conceptual motivation we have suggested that $\mathcal{E}xh$ should be able to assign a truth value to every alternative as long as this doesn't lead to a contradiction (or involves an arbitrary choice between alternatives).

3.3.4 Empirical evidence for Cell Identification and where to find it

At this point, though, we have not yet presented any empirical motivation to prefer $\mathcal{E}xh^{\text{IE+II}}$ over a recursive application of $\mathcal{E}xh^{\text{IE}}$; as mentioned above, for both simple disjunction and Free Choice disjunction the view promoted here yields the same result Fox (2007) derives by applying $\mathcal{E}xh^{\text{IE}}$ recursively. The remainder of this paper will be mainly dedicated to providing empirical evidence that Cell Identification in (23), repeated here, is a correct generalization, thereby arguing in favor of $\mathcal{E}xh^{\text{IE+II}}$ which predicts this generalization.

(29) **Cell Identification (when possible)**:
Let S be a sentence with denotation p and C be the set of denotations of its alternatives. If $Cell(p,C)$ is not a contradiction, then S can have $Cell(p,C)$ as a strengthened meaning.

Where should we look for evidence for (29)? Note that if we take a 'standard' set of alternatives derived by replacements of only one scalar item with others, we normally derive a totally ordered set of alternatives with respect to entailment (e.g., {*some students came*, *most students came*, *all students came*}). For any prejacent with such a set of alternatives, a *Cell* interpretation would be derived by virtually any theory of scalar implicatures. (Putting aside cases of the sort discussed in Fox & Hackl 2006, all the alternatives that asymmetrically entail the prejacent would be assigned false and all alternatives that are entailed by the prejacent would be assigned true). To be more precise, as long as the set of alternatives has no subset which is symmetric relative to the prejacent, it will be difficult to distinguish theories

falsity of the IE alternatives, one that contains all the non-IE alternatives. The set of II alternatives is hence the set of all non-IE alternatives: $II(p,C) = C \setminus IE(p,C)$. Since $\mathcal{E}xh_C^{\text{IE+II}}(p) \Leftrightarrow p \wedge \bigwedge\{\neg q : q \in IE(p,C)\} \wedge \bigwedge II(p,C)$, it is also true that $\mathcal{E}xh_C^{\text{IE+II}}(p) \Leftrightarrow Cell(p,C)$.

in which (29) holds from those in which it doesn't.[18]

Moreover, not every prejacent with symmetric alternatives will help us figure out whether (29) holds. What we need are cases with symmetric alternatives for which the *Cell* interpretation is non-contradictory. As we have seen in §3.2, both simple and Free Choice disjunctions give rise to sets of alternatives that have a symmetric subset: the set $\{a, b\}$ is symmetric relative to $a \vee b$, and the same relation holds between $\{\Diamond a, \Diamond b\}$ and $\Diamond(a \vee b)$. However, only for FC disjunction is the *Cell* interpretation non-contradictory; for simple disjunction the *Cell* interpretation is contradictory and Cell Identification is subsequently impossible. We are thus looking for prejacents having sets of alternatives with a symmetric subset which, like FC disjunction, are not closed under conjunction and thus the *Cell* interpretation may be non-contradictory. In §5-§8 we will then consider more data involving disjunctions which readily give rise to symmetric alternatives and for which the *Cell* interpretation is non-contradictory. We argue that in all of those cases, the *Cell* interpretation is empirically attested, even where previous theories fail to predict it. Those data points, which have been briefly introduced in §1, will be utilized to argue for Innocent Inclusion.

Before we turn to these cases, we will deal first in §4 with an issue which immediately arises from our definition of $\mathcal{E}xh$ in (27), namely its relation with the overt exhaustivity operator *only*. We argue for implementing Innocent Inclusion in the semantics of *only*, providing an explanation for the data in (2) and maintaining a close connection between $\mathcal{E}xh$ and *only*. In §5 we motivate Cell Identification on the basis of the problem of universal FC brought up by Chemla (2009b), showing that Innocent Inclusion provides a global derivation of FC for structures of the schematic form $\forall x \Diamond(Px \vee Qx)$, illustrated in (3). §6 is concerned with a similar problem due to Nouwen (2017), namely deriving FC for structures of the schematic form $\Diamond \forall x(Px \vee Qx)$ exemplified in (4). §7 applies Innocent Inclusion to derive Simplification of Disjunctive Antecedents (SDA) exemplified in (5). §8 discusses further predictions of our account of simplification, mainly that not every conditional with a disjunctive antecedent will simplify (McKay & van Inwagen 1977) and that simplification inferences will be found outside the realm of conditionals.

18 We say that a set C' is symmetric relative to a prejacent p *iff* $\forall r \in C' : p \wedge \neg r \neq \bot$ and $p \Rightarrow \bigvee C'$. We also say that an alternative q is symmetric to alternatives $q_1, \ldots, q_n$ given a prejacent p whenever the set $C' = \{q_1, \ldots, q_n\}$ isn't symmetric relative to p but $C' \cup \{q\}$ is.

4 The presupposition of *only*

4.1 The connection between $\mathcal{E}xh$ and *only*

$\mathcal{E}xh$ was stated originally as a covert analog of *only*, with the minimal difference that while *only* presupposes its prejacent, $\mathcal{E}xh$ asserts it (see Fox 2007):

(30) a. $\mathcal{E}xh^{\text{IE}}$ **asserts** that its prejacent is true and asserts that all IE alternatives are false.
  b. *only* **presupposes** that its prejacent is true and asserts that all IE alternatives are false.

When we add Innocent Inclusion into the definition of $\mathcal{E}xh$ in (27), is the analogy disrupted? We claim it is not. The minimal difference can still be maintained if we assume that Innocent Inclusion is at play in the case of *only* too: What *only* presupposes is the positive part of the meaning $\mathcal{E}xh$ asserts, namely Inclusion. The analogy can then be stated as in (31): while *only* presupposes all the II alternatives, $\mathcal{E}xh$ asserts them.

(31) a. $\mathcal{E}xh^{\text{IE+II}}$ **asserts** that *all II alternatives* are true and asserts that all IE alternatives are false.
  b. *only* **presupposes** that *all II alternatives* are true and asserts that all IE alternatives are false.

We propose the lexical entry for *only* in (33) which, following (31), is only different from the entry for $\mathcal{E}xh^{\text{IE+II}}$ in presupposing rather than asserting that all II alternatives are true.[19]

(32) $$[\![\mathcal{E}xh^{\text{IE+II}}]\!](C)(p) = \lambda w.\forall r \in II(p,C)[r(w)] \wedge \forall q \in IE(p,C)[\neg q(w)]$$ [=(27)]

(33) $$[\![\text{only}]\!](C)(p) = \lambda w : \forall r \in II(p,C)[r(w)].\ \forall q \in IE(p,C)[\neg q(w)]$$

4.2 Motivation for Innocent Inclusion with *only* (Alxatib 2014)

An empirical motivation for the entry we suggested in (33) comes from work by Alxatib (2014) on the interaction between FC disjunction and *only*. Embedding FC

19 Since this is a strong presuppositional analysis, we will have to deal with arguments that *only*'s presupposition is weaker than its prejacent (Ippolito 2008). We hope that there is a way to deal with these arguments that will not destroy the picture we are trying to draw here.

disjunction in the scope of *only*, as in (34), yields the FC inferences in (34a)-(34b).[20]

(34) We are only allowed to eat [ice cream or cake]$_F$.

a. $\rightsquigarrow$ We are allowed to eat ice cream.

b. $\rightsquigarrow$ We are allowed to eat cake.

Furthermore, Alxatib (2014) claims that FC inferences become presuppositions when FC disjunction is embedded in the scope of *only*. As (35) (repeated from (2)) shows, they project out of questions, as we would expect from presuppositions. The contrast between (35) and (36) shows that *only* is the culprit: in the absence of *only*, as in (36), we do not infer FC.[21]

(35) Are we only allowed to eat [ice cream or cake]$_F$?

a. $\rightsquigarrow$ We are allowed to eat ice cream.

b. $\rightsquigarrow$ We are allowed to eat cake.

(36) Are we allowed to eat ice cream or cake?

a. $\not\rightsquigarrow$ We are allowed to eat ice cream.

b. $\not\rightsquigarrow$ We are allowed to eat cake.

Given the entry for *only* in (33), the FC inferences in (34) and (35) are straightforwardly predicted to be part of the presupposition triggered by *only*. Since *only* presupposes all the II alternatives, applying *only* to FC disjunction $\Diamond(a \vee b)$ and its set of alternatives $Alt(\Diamond(a \vee b))$ would presuppose $\Diamond a$ and $\Diamond b$, which are II as has been established in §3. Without the entry in (33), it is not trivial to explain why the FC inferences of FC disjunction under *only* should become presuppositions.[22,23]

20 (34) bears a family resemblance to scalar implicatures under operators which are Strawson-DE but not DE, discussed in Gajewski & Sharvit (2012); Spector & Sudo (2017); Marty (2017); Anvari (2018). It is not clear to us however that FC disjunction gives rise to similar facts more generally, e.g., when embedded under *sorry* or *surprised*.

21 As Chris Barker pointed out to us, a *yes* answer to (36) could lead (in certain contexts) to the inference that we are free to choose between ice cream and cake, and a *no* answer would naturally convey that we are allowed neither ice cream nor cake. Note that a similar situation arises in other cases in which a scalar implicature generating sentence is used to form a yes/no question:

(i) Did John do some of the homework?

In this case too, a *yes* answer can lead to the inference that John did not do all of the homework, whereas a *no* answer would mean that John did not do any of the homework.

22 Alxatib suggests two possible accounts, both relying on the assumption that there is an embedded exhaustivity operator other than *only*. However, it is not clear why this exhaustivity operator should be *obligatorily* embedded.

23 Our entry for *only* in (33) together with our analysis of SDA in §7 predicts (i) to presuppose the disjunctive alternatives of *only*'s prejacent, in parallel to (34).

Alxatib's data then provide an argument for Innocent Inclusion with *only*. We move on now to argue in favor of having Innocent Inclusion in the definition of $\mathcal{E}xh$, by providing empirical evidence for Cell Identification; our first argument comes from Free Choice disjunction embedded in the scope of a universal quantifier, which we discuss in the next section.

5 The problem of Universal Free Choice

5.1 A local derivation for universal Free Choice

Chemla (2009b) discussed sentences like (37) (repeated from (3)) where FC disjunction is embedded under universal quantification, and pointed out that they give rise to the embedded FC inferences in (37a)-(37b) (in fact, Chemla presents experimental evidence that such embedded FC inferences are as robust as in the unembedded case of (1)).

(37) Every boy is allowed to eat ice cream or cake. $\forall x \Diamond (Px \vee Qx)$

a. $\rightsquigarrow$ Every boy is allowed to eat ice cream. $\forall x \Diamond Px$

b. $\rightsquigarrow$ Every boy is allowed to eat cake. $\forall x \Diamond Qx$

A prima facie plausible analysis of the inferences in (37) may be based on a local derivation of FC: *every boy* takes scope over an enriched FC meaning, with whatever mechanism we might have for enriching (1) applying in the scope of *every boy*. (See Singh et al. 2016 for a possible explanation for the relative robustness of this putative local implicature.) As Chemla (2009b) points out, a local derivation is in fact the *only* way standard implicature-based accounts can derive universal FC (see fn. 27 for the results of applying Fox's mechanism globally). In this light, (37) may just

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(i) Only if you work hard or inherit a fortune do you succeed.

The simplification inferences (i) gives rise to are however somewhat weaker than expected: (i) doesn't seem to presuppose that if you work hard you succeed, but rather the weaker presupposition that if you work hard you *might* succeed. We believe the problem is more general, since the presupposition of *only if* sentences is weaker than expected on most accounts regardless, that is even for simple sentences that do not involve disjunction (see von Fintel 1997). We hope this can be captured with a modification of *only*'s presupposition (see in this connection fn. 19).

In parallel to our discussion of FC with *only*, we further expect simplification inferences to survive embedding in a question only in the presence of *only*. This seems to be borne out: while from (iia) we infer that if you work hard you might succeed, this is not an inference of (iib).

(ii) a. Is it true that you succeed only if you work hard or inherit a fortune?

b. Is it true that you succeed if you work hard or inherit a fortune?

seem like yet another argument in favor of deriving implicatures at an embedded level (Cohen 1971; Landman 1998; Levinson 2000; Chierchia 2004; Chierchia et al. 2012).

5.2 Negative universal Free Choice as an argument for a global derivation

However, as Chemla (2009b) notes, a local derivation cannot explain a very similar universal inference that arises in the negative case in (38):

(38) No student is required to solve both problem A and problem B.
$\neg\exists x\Box(Px \wedge Qx) \Leftrightarrow \forall x\Diamond(\neg Px \vee \neg Qx)$

a. $\rightsquigarrow$ No student is required to solve problem A. $\neg\exists x\Box Px \Leftrightarrow \forall x\Diamond\neg Px$

b. $\rightsquigarrow$ No student is required to solve problem B. $\neg\exists x\Box Qx \Leftrightarrow \forall x\Diamond\neg Qx$

The inferences from (38) to (38a)-(38b) are logically parallel to the universal FC inferences in (37). As the formulas on the right indicate, the inference can be restated as a universal FC inference: from *every student is allowed not to solve problem A or not to solve problem B* (=(38)) to *every student is allowed not to solve problem A* (=(38a)) and *every student is allowed not to solve problem B* (=(38b)).

An account of (38) parallel to an account of (37) is probably needed. However, a local derivation is not applicable in this case: such a derivation would require an embedded syntactic position at which the enriched FC meaning can be derived, and no such position exists in (38). Since the scope of *no student* only contains strong scalar items—*required* and *and*—no further strengthening can occur at any embedded position. Because the inferences cannot be derived from embedding the mechanism we have for (1), they must be derived at the matrix level, above negation, namely they necessitate a global derivation.

The lesson from Chemla (2009b) is thus that while having only a local derivation may seem unproblematic for the positive case in (37), a global derivation is needed in order to explain the attested inferences of the parallel case in (38). This alone necessitates a global derivation which would presumably be applicable to both cases, given the parallelism between them. In what follows we strengthen the argument in favor of a global derivation, claiming that having only a local derivation is problematic even for the positive universal FC case in (37).

5.3 VP-ellipsis constructions as an argument for a global derivation

A local derivation faces problems for positive universal FC cases like (37) when embedded in VP ellipsis constructions where the elided material is in a DE environment, as in (39) (this argument has been pointed out to us by Luka Crnič, p.c.):

(39) Every girl is allowed to eat ice cream or cake on her birthday. Interestingly, No boy is ~~allowed to eat ice cream or cake on his birthday~~. $\approx$

a. Every girl is allowed to eat ice cream *and* allowed to eat cake on her birthday, and $\forall x \in [\![\text{girl}]\!](\Diamond Px \wedge \Diamond Qx)$

b. no boy is allowed to eat ice cream and (likewise) no boy is allowed to eat cake on his birthday. $\neg \exists x \in [\![\text{boy}]\!]\Diamond(Px \vee Qx)$

The situation here is similar to example (7) of VP-ellipsis with unembedded FC discussed above: the first conjunct in (39) is interpreted as having an enriched FC meaning, (39a), while the elided material inside the second conjunct is interpreted as having a basic disjunctive, non-FC meaning, (39b).

At first glance this may seem unproblematic for a local derivation: we could derive the FC meaning locally for the first conjunct with an embedded $\mathcal{E}xh$, and (similarly to our analysis of (7)) this $\mathcal{E}xh$ would be absent from the elided material. Note, however, that both the antecedent and the elided material in (39) contain a bound variable, the pronoun *her* which is bound by *every girl* in the antecedent. Under the assumption that the binder of any elided bound variable has to be inside the "parallelism domain" for ellipsis (Rooth 1992; Heim 1996), it follows that everything embedded under *every girl* has to be inside the parallelism domain.

If FC for the first sentence were derived locally, then to satisfy parallelism the elided material would be forced to have an FC meaning,[24] thus giving rise to a globally weaker meaning for the second sentence:

(40) No boy is both allowed to eat ice cream *and* allowed to eat cake on his birthday. $\neg \exists x \in [\![\text{boy}]\!](\Diamond Px \wedge \Diamond Qx)$

A global derivation of FC for the first sentence, on the other hand, would allow us to satisfy parallelism without generating embedded FC meaning for the second sentence. Having $\mathcal{E}xh$ above the binder of *her* allows for a parallelism domain containing the binder of *her* while not containing $\mathcal{E}xh$.[25]

5.4 Deriving universal Free Choice globally

As we have seen, a local derivation turns out to be insufficient for universal FC, in positive and negative cases alike, and a global derivation is needed. We want to show now that once we assume Innocent Inclusion, a global derivation for universal Free

24 See Crnič (2015) for arguments showing that truly embedded $\mathcal{E}xh$ is taken into account for parallelism considerations.

25 In fact, even if the parallelism domain contained $\mathcal{E}xh$, the correct result would still be derived (when $\mathcal{E}xh$ has scope over the binder), since it would have scope above *no* in the second sentence and would therefore be vacuous.

Choice is indeed available.

Consider the set of alternatives we generate for (37), in (42). We assume that alternatives where the universal quantifier *every* is replaced with the existential one *some* are generated (contra Fox 2007: fn. 35, but along with Chemla & Spector 2011; Romoli 2012; see Bar-Lev & Fox 2016; Gotzner & Romoli 2017), and thus the set of alternatives is multiplied by 2 compared to the 4 alternatives of unembedded Free Choice disjunction; we end up with the 8 alternatives in (42).

(41) Every boy is allowed to eat ice cream or cake. [=(37)] $\forall x \Diamond (Px \vee Qx)$

(42) **Set of alternatives for universal Free Choice**:
$Alt(\forall x \Diamond (Px \vee Qx)) =$
$$\{\underbrace{\forall x \Diamond (Px \vee Qx)}_{\text{Prejacent}}, \underbrace{\forall x \Diamond Px, \forall x \Diamond Qx}_{\text{Universal-disjunctive alts.}}, \underbrace{\forall x \Diamond (Px \wedge Qx)}_{\text{Universal-conjunctive alt.}},$$
$$\underbrace{\Diamond \exists x (Px \vee Qx)}_{\text{Existential alt.}}, \underbrace{\Diamond \exists x Px, \Diamond \exists x Qx}_{\text{Existential-disjunctive alts.}}, \underbrace{\Diamond \exists x (Px \wedge Qx)}_{\text{Existential-conjunctive alt.}}\}$$

The universal FC inference follows straightforwardly with one application of $\mathcal{E}xh^{\text{IE+II}}$, since $\forall x \Diamond Px$ and $\forall x \Diamond Qx$ are II. Let us show how this result is achieved.

In order to determine which alternatives are II, we first have to determine which are IE. The maximal sets of alternatives that can be assigned false consistently with the prejacent are in (43a), and their intersection which is the set of IE alternatives is in (43b). The IE alternatives are then $\forall x \Diamond (Px \wedge Qx)$ and $\exists x \Diamond (Px \wedge Qx)$.[26]

(43) a. **Maximal sets of alternatives in** $Alt(\forall x \Diamond (Px \vee Qx))$ **that can be assigned false consistently with** $\forall x \Diamond (Px \vee Qx)$:
(i) $\{\forall x \Diamond Px, \forall x \Diamond Qx, \forall x \Diamond (Px \wedge Qx), \exists x \Diamond (Px \wedge Qx)\}$
(ii) $\{\forall x \Diamond Px, \exists x \Diamond Px, \forall x \Diamond (Px \wedge Qx), \exists x \Diamond (Px \wedge Qx)\}$
(iii) $\{\forall x \Diamond Qx, \exists x \Diamond Qx, \forall x \Diamond (Px \wedge Qx), \exists x \Diamond (Px \wedge Qx)\}$

b. $IE(\forall x \Diamond (Px \vee Qx), Alt(\forall x \Diamond (Px \vee Qx)))) = \bigcap$ (43a) $=$
$\{\forall x \Diamond (Px \wedge Qx), \exists x \Diamond (Px \wedge Qx)\}$

We can now ask what is II: we should check what are the maximal sets of alternatives that can be assigned true consistently with the prejacent and the falsity of all IE alternatives. Namely, what are the maximal sets of alternatives that are consistent with the truth of the prejacent $\forall x \Diamond (Px \vee Qx)$ taken together with the falsity of $\exists x \Diamond (Px \wedge Qx)$ (we can ignore the other IE alternative, $\forall x \Diamond (Px \wedge Qx)$, since its falsity is entailed by the falsity of $\exists x \Diamond (Px \wedge Qx)$)? As in the case of unembedded

26 As in the case of unembedded FC disjunction (see fn. 11), the following derivation of universal FC does not depend on the exclusion of any of the IE alternatives. In many cases they would not be relevant and thus would not be assigned false.

FC disjunction, there is only one such set since all the non-IE alternatives together are consistent with the prejacent and the falsity of all IE alternatives, as in (44a). Therefore the set of II alternatives in (44b) contains all the non-IE alternatives.

(44) a. **Maximal sets of alternatives in** $Alt(\forall x\Diamond(Px \vee Qx))$ **that can be assigned true consistently with** $\forall x\Diamond(Px \vee Qx) \wedge \neg\exists x\Diamond(Px \wedge Qx)$:

(i) $\{\forall x\Diamond(Px \vee Qx), \forall x\Diamond Px, \forall x\Diamond Qx,$
$\exists x\Diamond(Px \vee Qx), \exists x\Diamond Px, \exists x\Diamond Qx\}$

b. $II(\forall x\Diamond(Px \vee Qx), Alt(\forall x\Diamond(Px \vee Qx)))) = \bigcap$ (44a) =
$\{\forall x\Diamond(Px \vee Qx), \forall x\Diamond Px, \forall x\Diamond Qx, \exists x\Diamond(Px \vee Qx), \exists x\Diamond Px, \exists x\Diamond Qx\}$

As in the case of unembedded FC disjunction, exhaustification here assigns a truth value to every alternative. Most importantly, the alternatives $\forall x\Diamond Px$ and $\forall x\Diamond Qx$ are members in the set of II alternatives. Applying $\mathcal{E}xh^{\text{IE+II}}$ would then assign them true and derive the desired universal FC inferences, as in (45).

(45) $\mathcal{E}xh^{\text{IE+II}}_{Alt(\forall x(\Diamond(Px\vee Qx))}\forall x\Diamond(Px \vee Qx)$
$\Leftrightarrow \forall x\Diamond Px \wedge \forall x\Diamond Qx \wedge \neg\exists x\Diamond(Px \wedge Qx)$

In fact, once we have computed the IE alternatives, we could have concluded the result in (45) without explicitly computing what is II: since the *Cell* interpretation for $\forall x\Diamond(Px \vee Qx)$ is not contradictory, by Cell Identification it follows that applying $\mathcal{E}xh^{\text{IE+II}}$ yields the *Cell* interpretation, as in (45). The results of Innocent Exclusion and Innocent Inclusion are represented in figure 3.

The notion of Innocent Inclusion which applies directly to the set of alternatives is what is responsible for our derivation of universal FC. If we were to apply Fox's (2007) $\mathcal{E}xh^{\text{IE}}$ recursively in this case, we would only derive the weak inferences $\exists x\Diamond Px$ and $\exists x\Diamond Qx$, but would fail to derive the stronger $\forall x\Diamond Px$ and $\forall x\Diamond Qx$.[27]

27 This is since the set of exhaustified alternatives for the second level of exhaustification is as follows:

(i) $Alt(\mathcal{E}xh^{\text{IE}}(\forall x\Diamond(Px \vee Qx))) =$
$\{\mathcal{E}xh^{\text{IE}}(\forall x\Diamond(Px \vee Qx)) = \forall x\Diamond(Px \vee Qx) \wedge \neg\exists x\Diamond(Px \wedge Qx),$
$\mathcal{E}xh^{\text{IE}}(\forall x\Diamond Px) = \forall x\Diamond Px \wedge \neg\exists x\Diamond Qx,$
$\mathcal{E}xh^{\text{IE}}(\forall x\Diamond Qx) = \forall x\Diamond Qx \wedge \neg\exists x\Diamond Px,$
$\mathcal{E}xh^{\text{IE}}(\forall x\Diamond(Px \wedge Qx)) = \forall x\Diamond(Px \wedge Qx),$
$\mathcal{E}xh^{\text{IE}}(\exists x\Diamond(Px \vee Qx)) = \exists x\Diamond(Px \vee Qx) \wedge \neg\exists x\Diamond(Px \wedge Qx) \wedge \neg\forall x\Diamond(Px \vee Qx),$
$\mathcal{E}xh^{\text{IE}}(\exists x\Diamond Px) = \exists x\Diamond Px \wedge \neg\forall x\Diamond Px \wedge \neg\exists x\Diamond Qx,$
$\mathcal{E}xh^{\text{IE}}(\exists x\Diamond Qx) = \exists x\Diamond Qx \wedge \neg\forall x\Diamond Qx \wedge \neg\exists x\Diamond Px,$
$\mathcal{E}xh^{\text{IE}}(\exists x\Diamond(Px \wedge Qx)) = \exists x\Diamond(Px \wedge Qx) \wedge \neg\forall x\Diamond(Px \vee Qx)\}$

The last five alternatives contradict the prejacent and hence can be trivially excluded. The only non-trivially IE alternatives are $\mathcal{E}xh^{\text{IE}}(\forall x\Diamond Px)$ and $\mathcal{E}xh^{\text{IE}}(\forall x\Diamond Qx)$, the negation of both yields $(\forall x\Diamond Px \rightarrow \exists x\Diamond Qx) \wedge (\forall x\Diamond Qx \rightarrow \exists x\Diamond Px)$. Taken together with the prejacent, this yields the result of the second application of $\mathcal{E}xh^{\text{IE}}$ in (ii):

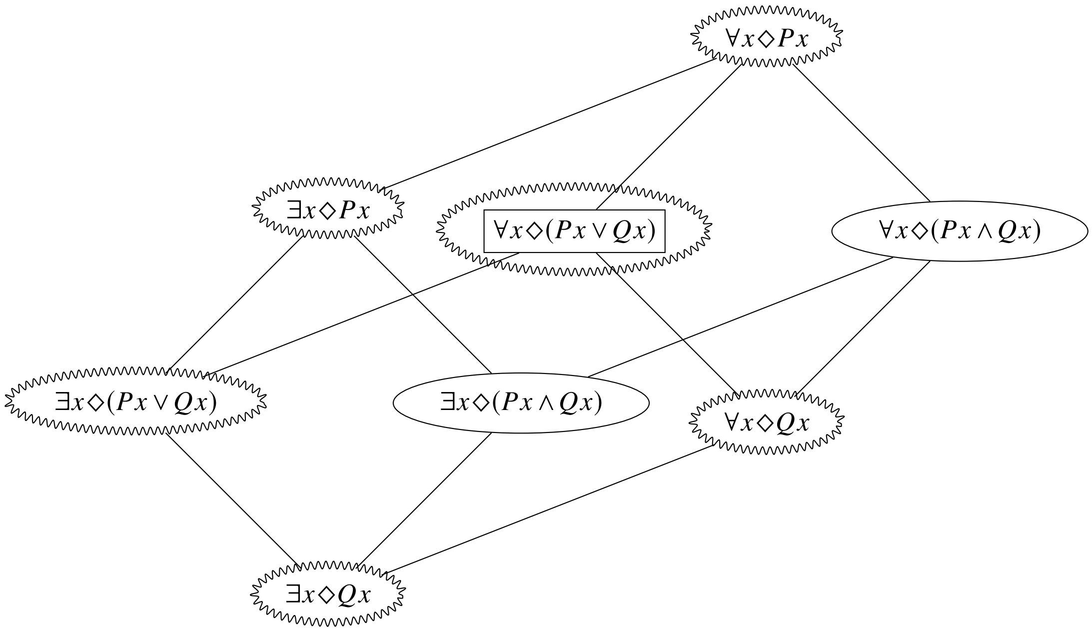

Figure 3 Results of Innocent Exclusion and Innocent Inclusion for universal Free Choice (notation as in figure 2 on p. 16).

This is so even though, as we have mentioned, the *Cell* interpretation in this case is non-contradictory. The case of universal FC then illustrates our claim from §3.3.2, that $\mathcal{E}xh^{\text{IE}}$ doesn't predict Cell Identification in (23), and moreover that Cell Identification seems to hold empirically.

Recall the motivation we presented for providing a global account of universal FC based on the negative universal FC case in (38). Since the same kind of entailment relations hold between the alternatives in the negative case (38) and the positive case (37) (assuming they give rise to parallel alternatives, as in (47)), the result is parallel as can be seen in (48).[28]

(46) No student is required to solve both problem A and problem B [=(38)]
$\neg\exists x\Box(Px \wedge Qx)$

(ii) $\forall x\Diamond(Px \vee Qx) \wedge \neg\exists x\Diamond(Px \wedge Qx) \wedge \exists x\Diamond Px \wedge \exists x\Diamond Qx$

28 Chemla's (2009b) results show a significant difference in robustness between the universal FC inferences in (37) and (38). This might follow from the existence of another route to embedded FC in the positive case of universal FC, namely with a local derivation of FC, which is unavailable in the negative case. The presence of negation might also play a role here by potentially introducing more alternatives; the universal FC inferences will not be derived if, as we propose in §8.2, alternatives where negation is replaced with $\mathcal{E}xh$ are generated.

(47) $Alt(\neg\exists x \Box(Px \wedge Qx)) =$
$$\{\neg\exists x \Box(Px \wedge Qx), \neg\exists x \Box Px, \neg\exists x \Box Qx, \neg\exists x \Box(Px \vee Qx),$$
$$\neg\forall x \Box(Px \wedge Qx), \neg\forall x \Box Px, \neg\forall x \Box Qx, \neg\forall x \Box(Px \vee Qx)\}$$

(48) $\mathcal{E}xh^{\text{IE+II}}_{Alt(\neg\exists x(\Box(Px \wedge Qx))}\neg\exists x \Box(Px \wedge Qx)$
$\Leftrightarrow \neg\exists x \Box Px \wedge \neg\exists x \Box Qx \wedge \forall x \Box(Px \vee Qx)$

6 Free Choice for sentences of the form $\Diamond\forall x(Px \vee Qx)$ (Nouwen 2017)

A similar problem to that of universal Free Choice has been discussed in Nouwen (2017). While the issue with universal FC is that of deriving from $\forall x \Diamond(Px \vee Qx)$ the inferences $\forall x \Diamond Px$ and $\forall x \Diamond Qx$, Nouwen's (2017) concern is with deriving the analogous inferences for sentences where an existential modal takes scope above a universal quantifier; namely deriving from sentences of the form $\Diamond\forall x(Px \vee Qx)$ the inferences $\Diamond\forall x Px$ and $\Diamond\forall x Qx$.

Providing direct evidence for this is however more difficult than might seem at first, due to the ability of disjunction to take non-surface scope. One could think that (49) is such a case:

(49) John allowed every kid to eat ice cream or cake. Possibly: $\Diamond\forall x(Px \vee Qx)$
   a. $\rightsquigarrow$ John allowed every kid to eat ice cream. $\Diamond\forall x Px$
   b. $\rightsquigarrow$ John allowed every kid to eat cake. $\Diamond\forall x Qx$

However, as Nouwen points out, it is difficult to rule out other scope possibilities for (49). Besides the scope construal we are after, *allowed* > *every* > *or*, there are two other possible LFs which are expected to derive the desired inferences: *every* > *allowed* > *or*, which is essentially a universal FC construal, and *allowed* > *or* > *every*, which would make it a basic FC disjunction construction.

How can we show that $\Diamond\forall x(Px \vee Qx)$ is the underlying structure? Here is an attempt. First, let us make sure we use a structure in which the existential modal takes scope above *every*; (50) seems to only admit such a reading:

(50) The teacher is OK with every student talking now.

Based on (50), we take it that the existential modal in (51) (repeated from (4)) takes scope above *every*; and following Larson's (1985) observation that dislocated *either* fixes the scope of disjunction, we assume that disjunction here takes scope below *every*.

(51) The teacher is OK with every student either talking to Mary or to Sue.
   a. $\rightsquigarrow$ The teacher is OK with every student talking to Mary.
   b. $\rightsquigarrow$ The teacher is OK with every student talking to Sue.

We have then an example of the form $\Diamond\forall x(Px \vee Qx)$ which indeed admits the inferences $\Diamond\forall x Px$ and $\Diamond\forall x Qx$.[29]

6.1 FC with ability modals

Given the difficulty in providing direct evidence for this kind of inference illustrated above, Nouwen's evidence comes from ability modals.[30] It has been claimed that ability modals like *can* are a combination of both existential and universal quantifiers, in the following sense (see Nouwen 2017, ex. (11)-(12), and references therein):

(52) x can do A *iff* **there is** an action available to x that would **reliably** bring about A.

Assuming this analysis, (53) gives rise to the meaning in (53a); as (53b) illustrates, this is the kind of logical structure we are after.

(53) Betty can balance a fishing rod on her nose or on her chin. (Geurts 2010)
   a. There is a proposition p (characterizing an action by Betty) such that in all worlds where p is true, either Betty balances a fishing rod on her nose or on her chin.
   b. $\exists p(\forall w \in p(Pw \vee Qw))$

As Geurts (2010) observes, (53) has an FC inference. Given the analysis sketched above for the semantics of *can*, we get the inference pattern exemplified with the formulas on the right, which is the inference pattern we are after:

(54) Betty can balance a fishing rod on her nose or on her chin. $\exists p(\forall w \in p(Pw \vee Qw))$
   a. $\rightsquigarrow$ Betty can balance a fishing rod on her nose. $\exists p(\forall w \in p(Pw))$
   b. $\rightsquigarrow$ Betty can balance a fishing rod on her chin. $\exists p(\forall w \in p(Qw))$

29 As pointed out to us by Gennaro Chierchia, (i) can be taken to further support Nouwen's descriptive claim. Specifically, if we assume that *any* is an existential quantifier it must be strengthened to a universal quantifier in this environment, and this can be done by exhaustification as pointed out in Chierchia (2013); Crnič (2017).

(i) The teacher allowed every girl to talk to any of her friends.

30 Though here too one might respond by claiming that there is a parse with disjunction taking scope above the universal quantifier if one considers a decomposition of *can* into existential and universal components.

6.2 Deriving FC for $\Diamond\forall x(Px \vee Qx)$

We have seen empirical evidence that a sentence with the form $\Diamond\forall x(Px \vee Qx)$ should be strengthened to entail $\Diamond\forall x Px$ and $\Diamond\forall x Qx$. We will see now that this follows from Innocent Inclusion.

Note, first, that these entailments will not follow from exhaustification at an embedded level, since strengthening of $\forall x(Px \vee Qx)$ could only give us irrelevant inferences. The desired result would have been derived by embedded exhaustification if we could strengthen $\forall x(Px \vee Qx)$ to $\forall x(Px \wedge Qx)$, but that of course never happens: (55a)-(55b) cannot be taken to follow from (55). Moreover, even in our case it would yield a result which is too strong: it would yield for (54) an inference that Betty can balance a fishing rod on her nose and chin at the same time.

(55) Every kid ate ice cream or cake.
  a. $\not\rightsquigarrow$ Every kid ate ice cream.
  b. $\not\rightsquigarrow$ Every kid ate cake.

As in the case of universal FC, then, the desired strengthening must result from global exhaustification. Nouwen claims that Fox (2007) incorrectly predicts the disjunctive alternatives $\Diamond\forall x Px$ and $\Diamond\forall x Qx$ to be IE and therefore that their falsity would be incorrectly derived. But this holds only insofar as we ignore the alternatives $\Diamond\exists x Px$ and $\Diamond\exists x Qx$. Admitting the latter alternatives makes the former non-IE.[31] Suppose then that we derive the following set of alternatives:

(56)
$$Alt(\Diamond\forall x(Px \vee Qx)) = \{\underbrace{\Diamond\forall x(Px \vee Qx)}_{\text{Prejacent}}, \underbrace{\Diamond\forall x Px, \Diamond\forall x Qx}_{\text{Universal-disjunctive alts.}}, \underbrace{\Diamond\forall x(Px \wedge Qx)}_{\text{Universal-conjunctive alt.}},$$
$$\underbrace{\Diamond\exists x(Px \vee Qx)}_{\text{Existential alt.}}, \underbrace{\Diamond\exists x Px, \Diamond\exists x Qx}_{\text{Existential-disjunctive alts.}}, \underbrace{\Diamond\exists x(Px \wedge Qx)}_{\text{Existential-conjunctive alt.}}\}$$

As desired, the disjunctive alternatives are not IE due to the presence of the existential alternatives.

(57) a. **Maximal sets of alternatives in** $Alt(\Diamond\forall x(Px \vee Qx))$ **that can be assigned false consistently with** $\Diamond\forall x(Px \vee Qx)$:
  (i) $\{\Diamond\forall x Px, \Diamond\forall x Qx, \Diamond\forall x(Px \wedge Qx), \Diamond\exists x(Px \wedge Qx)\}$
  (ii) $\{\Diamond\forall x Px, \Diamond\exists x Px, \Diamond\forall x(Px \wedge Qx), \Diamond\exists x(Px \wedge Qx)\}$
  (iii) $\{\Diamond\forall x Qx, \Diamond\exists x Qx, \Diamond\forall x(Px \wedge Qx), \Diamond\exists x(Px \wedge Qx)\}$

31 It is admittedly not straightforward to see where the existential alternatives should come from in the case of ability modals. We will need to say that there is a covert universal quantifier in the structure, which can be replaced with an existential one.

b. $IE(\Diamond\forall x(Px \vee Qx), Alt(\Diamond\forall x(Px \vee Qx))) = \bigcap$ (57a) $=$
$\{\Diamond\forall x(Px \wedge Qx), \Diamond\exists x(Px \wedge Qx)\}$

Once again, the recursive application of $\mathcal{E}xh^{\text{IE}}$ falls short of deriving the desired inferences (for essentially the same reasons that it failed in the universal FC case in (37) where the universal quantifier had wide scope).

With Innocent Inclusion, on the other hand, we derive the right result. As might be obvious at this point, the derivation is completely parallel to that of universal FC, and so is the output of $\mathcal{E}xh^{\text{IE+II}}$: since the *Cell* interpretation is non-contradictory applying $\mathcal{E}xh^{\text{IE+II}}$ yields that interpretation, i.e., the falsity of all the IE alternatives and the truth of all the non-IE ones (which, recall, are II whenever the *Cell* interpretation is consistent):

(58) $\mathcal{E}xh^{\text{IE+II}}_{Alt(\Diamond\forall x(Px \vee Qx))} \Diamond\forall x(Px \vee Qx)$
$\Leftrightarrow \Diamond\forall x Px \wedge \Diamond\forall x Qx \wedge \neg\Diamond\exists x(Px \wedge Qx)$

To conclude, we have shown that assuming Innocent Inclusion we can account for FC with ability modals, which as Nouwen (2017) pointed out is problematic for previous implicature approaches to FC. And more generally we account for cases where disjunction takes scope below a universal quantifier which in turn takes scope below an existential one.[32] We now move on to argue that Innocent Inclusion accounts for Simplification of Disjunctive Antecedents.

7 Simplification of Disjunctive Antecedents (SDA)

7.1 The puzzle of SDA as an FC puzzle

Conditionals with disjunctive antecedents usually lead to the inference that the disjunctive alternatives are true, as in (59) (repeated from (5)). This is known as

32 Nouwen's more general claim is that standard implicature-based analyses of Free Choice rely on *distribution over disjunction* as a necessary condition for deriving FC: Within such approaches, $\phi(a) \wedge \phi(b)$ can only be derived from $\phi(a \vee b)$ if ϕ distributes over disjunction.

(i) **Distribution over disjunction**: ϕ distributes over disjunction *iff* $\phi(a \vee b) \Leftrightarrow \phi(a) \vee \phi(b)$

However, the context surrounding disjunction does not distribute over disjunction in both universal FC, (iia), and the Nouwen (2017) case discussed in this section, (iib):

(ii) a. $\forall x\Diamond(Px \vee Qx) \not\Leftrightarrow (\forall x\Diamond Px) \vee (\forall x\Diamond Qx)$
b. $\Diamond\forall x(Px \vee Qx) \not\Leftrightarrow (\Diamond\forall x Px) \vee (\Diamond\forall x Qx)$

As we have shown, Innocent Inclusion derives FC for both cases. Unlike the analyses Nouwen discusses, then, ours does not have *distribution over disjunction* as a necessary condition for deriving FC inferences.

Simplification of Disjunctive Antecedents (SDA):

(59) If you eat ice cream or cake, you will feel guilty. $(p \vee q) \Box\!\!\rightarrow r$

a. $\rightsquigarrow$ If you eat ice cream, you will feel guilty. $p \Box\!\!\rightarrow r$

b. $\rightsquigarrow$ If you eat cake, you will feel guilty. $q \Box\!\!\rightarrow r$

Whether SDA should be semantically valid or not has been a topic of much debate. While under a strict-conditional analysis SDA is valid, it isn't valid within a variably-strict semantics for conditionals (Stalnaker 1968; Lewis 1973):[33]

(60) **Variably-strict conditional**:
$p \Box\!\!\rightarrow q$ is true *iff* the closest p-worlds are q-worlds.

Fine (1975); Nute (1975) have argued against a varibaly strict analysis on these grounds (namely, its inability to derive what seems to be a valid inference). But as Fine (1975) notes, accepting SDA as semantically valid as in (61a) (together with some plausible assumptions) leads to the unwelcome validity of (61b) (since p is equivalent to $p \vee (p \wedge q)$):

(61) a. Semantic validation of SDA: $(p \vee q) \Box\!\!\rightarrow r \models p \Box\!\!\rightarrow r$

b. $p \Box\!\!\rightarrow r \models (p \wedge q) \Box\!\!\rightarrow r$

Loewer (1976) was probably the first to suggest an analogy with Free Choice disjunction.[34] Loewer points out that SDA should not follow from the basic semantics for similar reasons to those presented above for FC; assuming (62a) (together with some plausible assumptions) would lead to (62b):

(62) a. Semantic validation of FC: $\Diamond(p \vee q) \models \Diamond p$

b. $\Diamond p \models \Diamond p \wedge \Diamond q$

The similarity between FC and SDA calls for a unified theory. However, as Franke (2011) points out, a recursive application of $\mathcal{E}xh^{\text{IE}}$ does not derive SDA from a variably-strict basic meaning.[35] In what follows we demonstrate that this problem for exhaustification is resolved with $\mathcal{E}xh^{\text{IE+II}}$.

33 We remain agnostic regarding the proper variably-strict analysis. In what follows we occasionally refer to "the closest world", as if we assumed a Stalnakerian analysis, but this is only done for ease of presentation. As the reader may verify, the results in this section hold assuming a Lewisian system as well.

34 Loewer (1976) writes: "Notice the similarity between the two situations. In both cases the surface form of an English sentence is 'Modal operator (A or B)', but its logical form seems to be 'Modal operator A and modal operator B'" (p. 534).

35 This is since the exhaustified disjunctive alternatives end up falsifying the prejacent (which, unlike in the case of FC, is not entailed by them); as a result their falsity on the second layer of exhaustification becomes vacuous. We illustrate this here for the disjunctive alternative $p \Box\!\!\rightarrow r$:

7.2 Deriving SDA with Innocent Inclusion

The set of alternatives we derive for the basic SDA example in (59), which is of the form $(p \vee q) \mathrel{\Box\!\!\rightarrow} r$, is the set of alternatives derived by replacing disjunction with the disjuncts and with conjunction, assuming that the conditional itself does not generate any other alternatives.

(63) $$Alt((p \vee q) \mathrel{\Box\!\!\rightarrow} r) = \{\underbrace{(p \vee q) \mathrel{\Box\!\!\rightarrow} r}_{\text{Prejacent}}, \underbrace{p \mathrel{\Box\!\!\rightarrow} r, q \mathrel{\Box\!\!\rightarrow} r}_{\text{Disjunctive alts.}}, \underbrace{(p \wedge q) \mathrel{\Box\!\!\rightarrow} r}_{\text{Conjunctive alt.}}\}$$

Note that, just like in the basic FC case, the conjunction of the disjunctive alternatives, $(p \mathrel{\Box\!\!\rightarrow} r) \wedge (q \mathrel{\Box\!\!\rightarrow} r)$, is absent from the set of alternatives, i.e., the set is not closed under conjunction.

Let us see now what is IE. What's important here is (a) that, as in the case of FC and simple disjunction, the truth of the prejacent is consistent with the falsity of one disjunctive alternative, and (b) that it is inconsistent with the falsity of both disjunctive alternatives (since, as we will see, it entails the disjunction of the disjunctive alternatives). Take for example $p \mathrel{\Box\!\!\rightarrow} r$. The truth of the prejacent alone doesn't ensure the truth of $p \mathrel{\Box\!\!\rightarrow} r$ since it is possible that the closest $p \vee q$-world is a $q \wedge \neg p$ world, and the closest p world is a $\neg r$ world. But if we take the prejacent to be true and $q \mathrel{\Box\!\!\rightarrow} r$ to be false, then $p \mathrel{\Box\!\!\rightarrow} r$ cannot be false anymore (the closest $p \vee q$-world is the closest p-world or the closest q-world). For this reason, this alternative is not IE and obviously the same holds for $q \mathrel{\Box\!\!\rightarrow} r$. Just like the case of FC disjunction, the only IE alternative is then the conjunctive alternative.[36]

(64) a. **Maximal sets of alternatives in $Alt((p \vee q) \mathrel{\Box\!\!\rightarrow} r)$ that can be as-**

(i) a. $\mathcal{E}xh^{\text{IE}}_{Alt((p \vee q) \Box\!\rightarrow r)} p \mathrel{\Box\!\!\rightarrow} r \Leftrightarrow (p \mathrel{\Box\!\!\rightarrow} r) \wedge \neg(q \mathrel{\Box\!\!\rightarrow} r) \wedge \neg((p \vee q) \mathrel{\Box\!\!\rightarrow} r) \wedge \neg((p \wedge q) \mathrel{\Box\!\!\rightarrow} r)$

b. $((p \vee q) \mathrel{\Box\!\!\rightarrow} r) \wedge \neg \mathcal{E}xh^{\text{IE}}_{Alt((p \vee q) \Box\!\rightarrow r)} p \mathrel{\Box\!\!\rightarrow} r \Leftrightarrow ((p \vee q) \mathrel{\Box\!\!\rightarrow} r)$

36 The excludability of the conjunctive alternative, which is shared between our account and Franke's, might seem objectionable since the falsity of $(p \wedge q) \mathrel{\Box\!\!\rightarrow} r$ does not seem to be a necessary implicature of $(p \vee q) \mathrel{\Box\!\!\rightarrow} r$. Note first that the falsity of $(p \wedge q) \mathrel{\Box\!\!\rightarrow} r$ is at least consistent with $(p \vee q) \mathrel{\Box\!\!\rightarrow} r$:

(i) If you drink now a bottle of beer or a shot of whisky you'll feel great, but if you drink both you'll feel really bad.

Recall moreover that a similar objection was discussed in fn. 11 regarding the excludability of the conjunctive alternative in FC disjunction. Our response there applies here as well: since the set of alternatives is not closed under conjunction and the conjunctive alternative is not the conjunction of the disjunctive ones, it is possible for the disjunctive alternatives to be relevant without making the conjunctive alternative relevant. In contexts where it's not relevant, it will indeed not be excluded.

signed false consistently with $(p \vee q) \boxright r$:
(i) $\{p \boxright r, (p \wedge q) \boxright r\}$
(ii) $\{q \boxright r, (p \wedge q) \boxright r\}$
b. $IE((p \vee q) \boxright r, Alt((p \vee q) \boxright r))) = \bigcap$(64a) $= \{(p \wedge q) \boxright r\}$

Since the *Cell* interpretation is non-contradictory, by applying $\mathcal{E}xh^{\text{IE+II}}$ we get the falsity of the IE alternative and the truth of the non-IE ones. We, thus, derive SDA by Inclusion (see also left side of figure 4 on page 39):

(65) $\mathcal{E}xh^{\text{IE+II}}_{Alt((p \vee q) \boxright r)}(p \vee q) \boxright r$
$\Leftrightarrow (p \boxright r) \wedge (q \boxright r) \wedge \neg((p \wedge q) \boxright r)$

Deriving SDA by Innocent Inclusion further obviates several problems for previous implicature accounts of SDA. As Franke points out, his account fails to derive SDA directly when the antecedent contains more than two disjuncts (just as in FC, see Fox & Katzir 2018 for details):

(66) If you eat an apple, an orange or a pear, you will be healthy. $(p \vee q \vee r) \boxright s$
a. $\rightsquigarrow$ If you eat an apple you will be healthy. $p \boxright s$
b. $\rightsquigarrow$ If you eat an orange you will be healthy. $q \boxright s$
c. $\rightsquigarrow$ If you eat a pear you will be healthy. $r \boxright s$

With Innocent Inclusion, on the other hand, none of the disjunctive alternatives is IE in this case, since the truth of the prejacent together with the falsity of any two (one-disjunct) disjunctive alternatives (e.g., $q \boxright s$ and $r \boxright s$) ensures the truth of the remaining disjunctive alternative (e.g., $p \boxright s$). So none of them is IE, and the falsity of all the IE (conjunctive) alternatives is consistent with their truth, hence they are assigned true.

Another problem arises for Franke by embedding a conditional with a disjunctive antecedent in the scope of a universal quantifier, as in (67). Put differently, the problem of universal FC is replicated with universal SDA, as in (67). Franke (2011) doesn't account for those inferences for the same reason he doesn't account for universal FC (and a local derivation is not available in his framework).

(67) Everyone will feel guilty if they eat ice cream or cake.
$\forall x((Px \vee Qx) \boxright Rx)$
a. $\rightsquigarrow$ Everyone will feel guilty if they eat ice cream. $\forall x(Px \boxright Rx)$
b. $\rightsquigarrow$ Everyone will feel guilty if they eat cake. $\forall x(Qx \boxright Rx)$

The universal SDA example in (67) is accounted for along the same lines of our account of universal FC. Assuming that *every* generates *some*-alternatives, the universal disjunctive alternatives end up II, yielding the inferences in (67a)-(67b).

Finally, Klinedinst's (2007) approach derives SDA locally, namely by strengthening the antecedent itself. Santorio (2016) claims that this assumption is problematic when considering conditionals with disjunctive antecedents in DE contexts, as in (68). To capture the inferences of such sentences this putative local implicature will have to be derived under negation since, given a variably-strict analysis, (68a)-(68b) don't follow from the basic semantics (in this respect SDA is different from FC).

(68) It's not true that you will feel guilty if you eat ice cream or cake. $\neg((p \vee q) \,\Box\!\!\rightarrow r)$

a. ⇝ It's not true that you will feel guilty if you eat ice cream. $\neg(p \,\Box\!\!\rightarrow r)$

b. ⇝ It's not true that you will feel guilty if you eat cake. $\neg(q \,\Box\!\!\rightarrow r)$

While we are not convinced that this is a decisive argument against Klinedinst's account,[37] we want to point out that with Innocent Inclusion there is no issue to begin with, since our derivation is at the global level, above negation.[38] The derivation is similar to the positive case in (59), assuming that the only alternatives

37 The reason why we do not think this argument is too strong is that (as Klinedinst had already pointed out) this local implicature would be derived in a non-monotonic context rather than a downward entailing one (given the non-monotonicity of conditionals within a variably strict semantics). Importantly, deriving implicatures in DE contexts normally leads to a weakening of the meaning at the global level; in the current case on the other hand it leads to strengthening. It is thus essentially different than computing an implicature in a DE context.

38 Another argument by Santorio against implicature accounts, however, holds also for globalist accounts such as Franke (2011) and our own proposal. The argument comes from *probably*-conditionals:

(i) If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

a. ⇝ If the winning ticket is between 1 and 70, probably Sarah won.

b. ⇝ If the winning ticket is between 31 and 100, probably Sarah won.

As Santorio argues, the sentence could be true if both simplifications are true but what is taken on our approach to be the prejacent is false. We suspect however that the problem is not unique to conditionals and thus the solution should not be hardwired into the semantics of conditionals as Santorio does. The same effect can be seen with *most*. Suppose there are 7 kids, such that 3 of them are both on team A and team B, 2 of them are only on team A and the remaining two are only on team B, and there are no other people on either team. The situation can be described as in (ii).

(ii) Most kids on team A or team B are on both teams.

a. ⇝ Most kids on team A are on both teams.

b. ⇝ Most kids on team B are on both teams.

Here too, it seems that the sentence can be true in virtue of (iia)-(iib) being true, even though it is false that most of the 7 kids are on both teams (which is equivalent to (ii)). We thank Paolo Snatorio for pointing out this problem and wish there was more we could say. We return to simplification with *most* in §8.3.

of $\neg((p \vee q) \mathrel{\Box\!\!\rightarrow} r)$ are the disjunctive and conjunctive ones.[39] Since $\neg((p \vee q) \mathrel{\Box\!\!\rightarrow} r)$ entails the disjunction of $\neg(p \mathrel{\Box\!\!\rightarrow} r)$ and $\neg(q \mathrel{\Box\!\!\rightarrow} r)$, none of them is IE and the only IE alternative is the conjunctive one $\neg((p \wedge q) \mathrel{\Box\!\!\rightarrow} r)$.[40] The *Cell* interpretation is non-contradictory, and therefore the disjunctive alternatives are assigned true.

The idea that what's responsible for SDA is the disjunctive alternatives is not unique to implicature accounts of SDA; it is shared with alternative semantics-based approaches, most notably Alonso-Ovalle (2009). Our account (along with Klinedinst 2007; Franke 2011) differs from such approaches in not hard-wiring quantifiation over such alternatives into the system. Instead, SDA follows from the independently needed exhaustification mechanism we have argued for based on Free Choice phenomena. Moreover, by not hard-wiring quantification over the disjunctive alternatives into the system, our approach makes distinct predictions which we investigate in §8.

Our Inclusion-based account of simplification is highly dependent on the form of the alternatives generated and on the entailment relations among them. In §8.1 we focus on predictions pertaining to the entailment relations and argue that they are verified by the absence of simplification inferences in an environment identified by McKay & van Inwagen (1977). We then move on in §8.2 to discuss predictions pertaining to the form of the alternatives, focusing on a puzzle discovered by Ciardelli, Zhang & Champollion (2018). We introduce a proposal due to Schulz (2018) and explain how it can be taken to verify our prediction. Finally, our account predicts simplification inferences to be found outside the domain of conditionals; §8.3 discusses simplification of *most* with a disjunctive restrictor as a relevant case.

8 Further predictions of simplification by Inclusion

8.1 Failure of simplification

There is a puzzle about SDA that has been known for quite a while: while SDA seems to hold in general, there are certain cases where it doesn't.[41] It has been observed that sometimes conditionals with disjunctive antecedents are true even though one of their simplifications is false. This has been pointed out by McKay & van Inwagen (1977), based on the following example:

(69) If Spain had fought with the Axis or with the Allies, it would have been with the Axis.

39 In §8.2 we assume that negation triggers $\mathcal{E}xh$ as an alternative. As the reader may verify, adding such alternatives in the case at hand would change nothing.

40 Here too the exclusion of the conjunctive alternative is not a necessary inference of (68).

41 The bulk of the ideas in this section came up in a discussion with Itai Bassi, to whom we are greatly indebted.

a. $\rightsquigarrow$ If Spain had fought with the Axis it would have been with the Axis. (trivially)

b. $\not\rightsquigarrow$ # If Spain had fought with the Allies it would have been with the Axis.

McKay & van Inwagen (1977) argue against the semantic validity of SDA based on the acceptability of (69).[42] Nute (1980) contrasts (69) with (70), for which SDA seems to be valid, thus leading to oddity given world knowledge that Hitler would have been pleased only if Spain had fought with Axis, not with the Allies.[43]

(70) #If Spain had fought with the Axis or with the Allies, Hitler would have been pleased.

a. $\rightsquigarrow$ If Spain had fought with the Axis, Hitler would have been pleased.

b. $\rightsquigarrow$ # If Spain had fought with the Allies, Hitler would have been pleased.

Let us now show that our account predicts a failure of simplification for (69), solely based on the fact that (69) has the form $(p \vee q) \mathrel{\Box\!\!\rightarrow} p$, i.e., the consequent is equivalent to one of the disjuncts in the antecedent.[44] As we will immediately demonstrate, the attested difference between the non-SDA example (69) and the SDA example (70)

42 See Lassiter (2018) for a more elaborate argument against the semantic validity of SDA based on examples of the following sort:

(i) If Spain had fought with the Axis or the Allies it's likely, but not certain, that it would have fought with the Axis.

In the following discussion we set aside such cases and the complications they give rise to in determining both the basic semantics and the alternatives generated.

43 Nute (1980) further observed that the status of (70) improves if it appears as a continuation to (69), as in (i). Regrettably we have nothing to say about this effect.

(i) If Spain had fought with the Axis or with the Allies, it would have been with the Axis. So if Spain had fought with the Axis or with the Allies, Hitler would have been pleased.

44 More generally, following Nute (1980), we assume that SDA fails whenever the conditional has the form $(p \vee q) \mathrel{\Box\!\!\rightarrow} p^+$ (where p^+ logically entails p). This generalization captures the acceptability of (69) as well as (i), also discussed by Nute, in which the consequent is strictly stronger than the first disjunct in the antecedent. We will not explicitly discuss such cases; the interested reader may verify that the explanation to be provided here for (69) extends to (i), the key fact being that $(p \vee q) \mathrel{\Box\!\!\rightarrow} p^+$ entails $p \mathrel{\Box\!\!\rightarrow} p^+$. See Bar-Lev (2018: §1) for complete derivations.

(i) If Spain had fought with the Axis or with the Allies, it would have been with the Axis and Hitler would have been pleased.

is predicted on our account since the entailment relations between the alternatives change and consequently the result we get by applying $\mathcal{E}xh^{\text{IE+II}}$ does too.

As we have seen, in a sentence of the form $(p \vee q) \boxright r$, such as (70), none of the disjunctive alternatives is IE. This is because there are two maximal sets of alternatives that can be assigned false consistently with the prejacent, as in (71) (repeated from (64a)):

(71) a. $\{p \boxright r, (p \wedge q) \boxright r\}$
b. $\{q \boxright r, (p \wedge q) \boxright r\}$

Since none of the disjunctive alternatives is in both sets, none of them is IE. Not being IE, they might be II, and as we have seen they indeed are.

(69) however has a slightly different structure, that of $(p \vee q) \boxright p$. The alternatives we derive for such a structure would then be $p \boxright p$ which is a tautology, the contingent proposition $q \boxright p$, and the conjunctive alternative $(p \wedge q) \boxright p$, which is a tautology as well. There is therefore only one maximal set of alternatives that can be assigned false consistently with the prejacent:[45]

(72) $\{q \boxright p\}$

The proposition in this set would then be IE and there would be nothing left for Inclusion to do. The results for the two cases are then as follows (see also figure 4):[46]

(73) a. Result for conditionals of the form $(p \vee q) \boxright r$, as in (70):
$\mathcal{E}xh^{\text{IE+II}}_{Alt((p\vee q)\boxright r)}(p \vee q) \boxright r \Leftrightarrow (p \boxright r) \wedge (q \boxright r) \wedge \neg((p \wedge q) \boxright r)$ [=(65)]
b. Result for conditionals of the form $(p \vee q) \boxright p$, as in (69):
$\mathcal{E}xh^{\text{IE+II}}_{Alt((p\vee q)\boxright p)}(p \vee q) \boxright p \Leftrightarrow ((p \vee q) \boxright p) \wedge \neg(q \boxright p)$

The solution to the puzzle presented by the contrast between McKay & van Inwagen's example in (69) in which SDA does not go through and the example in (70) in which it does is due to the fact that only the former generates a disjunctive alternative which is entailed by the prejacent. The existence of this alternative breaks the symmetry that usually holds between disjunctive alternatives and thereby derives a different result.

45 With this line of reasoning one would expect to find other cases which are unrelated to simplification where the tautological nature of one of the alternatives leads to other alternatives becoming IE. However, we did not yet find such cases which are not predicted to be bad for independent reasons.

46 The exclusion inference $\neg(q \boxright p)$ in (73b) is contextually redundant for (69) given world knowledge that fighting with one side (usually) entails not fighting with the rival side. It would however be detectable if it wasn't contextually entailed, as in (i) (uttered in a context where it is common ground that Mary studies Physics).

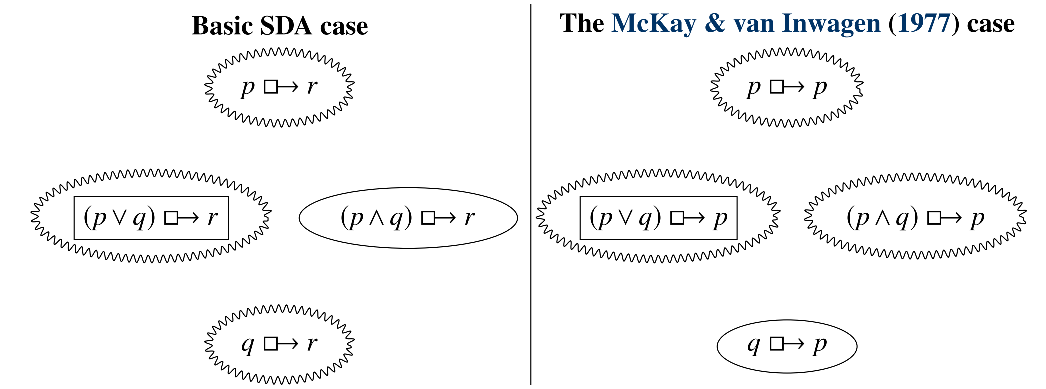

Figure 4 Results of Innocent Exclusion and Innocent Inclusion for conditionals with disjunctive antecedents (notation as in figure 2 on p. 16). On the left: results when the consequent is logically independent from the antecedent, e.g., (70). On the right: results when the consequent is equivalent to one of the disjuncts in the antecedent, e.g., (69). [Indication of entailment relations suppressed.]

For the sake of completeness we should mention that even though Franke only mentions the McKay & van Inwagen (1977) case in passing, his IBR paradigm makes the same predictions as ours as long as there are only two disjuncts in the antecedent, and shares the correct result that this case should behave differently than regular SDA cases.[47]

(i) If Mary had studied Linguistics or History, she would have studied Linguistics.

It is quite difficult to imagine (i) being uttered truthfully by a speaker who believes that if Mary had studied Linguistics or History she would have definitely studied Linguistics, but possibly also History. The exclusion inference predicted by applying $\mathcal{E}xh^{\text{IE+II}}$ (together with the assumption that $\neg(q \mathrel{\Box\!\!\to} p) \Rightarrow q \mathrel{\Box\!\!\to} \neg p$, which follows for example if Conditional Excluded Middle holds, as it does on Stalnaker's analysis) correctly precludes this possibility. This inference can be avoided though; Kai von Fintel (p.c.) pointed out that (iia) is felicitous. This behavior turns out to be predicted assuming that *Linguistics or both* is parsed as *$\mathcal{E}$xh(Linguistics) or both* (see §8.2). While the infelicity of (iib) may make the exclusion inference look like an obligatory one, we believe this is an independent issue having to do with felicity conditions on the use of *at least*, as is suggestive by the felicity of (iic). We thank Benjamin Spector for providing (iic) and this perspective on (iib).

(ii) a. If Mary had studied Linguistics or History, she would have studied Linguistics or both.
b. #If Mary had studied Linguistics or History, she would have at least studied Linguistics.
c. If Mary had studied Linguistics or History or Philosophy, she would have at least studied Linguistics AND Philosophy.

47 This is not the case for Klinedinst (2007): for him the implicature leading to simplification is derived

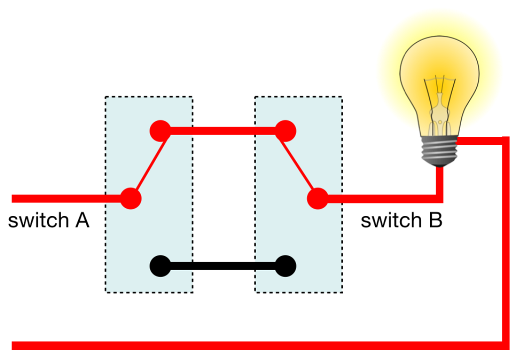

Figure 5 Ciardelli et al.'s scenario (the picture is taken from their work).

8.2 Turning switches

In a recent paper, Ciardelli et al. (2018) have discussed an intriguing difference in interpretation between the following two sentences (keeping with their notation, we notate p, q for *A is up*, *B is up* respectively, and $\bar{p}, \bar{q}$ for *A is down*, *B is down*, respectively).

(74) If switch A or switch B were down, the light would be off. $(\bar{p} \vee \bar{q}) \Box\!\!\rightarrow r$

(75) If switch A and switch B were not both up, the light would be off. $(\neg(p \wedge q)) \Box\!\!\rightarrow r$

Ciardelli et al. discuss the situation depicted in figure 5. In this situation both switches are up right now. If only switch A was down or only switch B was down, the light would be off. But if both were down, the light would be on. They show experimentally that while (74) is mostly judged true in this scenario, (75) is mostly judged false or neither-true-nor-false. This difference is surprising since the antecedents in the two sentences are truth conditionally equivalent (given that the switches cannot be neither up nor down). We wish to sharpen the puzzle by considering another example, which is minimally different than (74):

(76) If switch A or switch B or both were down, the light would be off.

According to our judgments, (76) is clearly false in Ciardelli et al.'s scenario. Moreover, this judgment seems clearer than the judgment for (75) which we waver on (a feeling consistent with Ciardelli et al.'s data where (75) is not unanimously

at an embedded level, within the antecedent, and subsequently cannot be affected by the identity of the consequent.

judged not true). Note however that the antecedents in (74), (75) and (76) are all semantically equivalent, and yet the truth judgments are clearly different. Most plausibly, the falsity of (76) is due to the simplification inference that *if both switches were down, the light would be off*. But how does this inference come about for (76), and why isn't it derived for (74)?

The expectation that we get a parallel result for all of those cases, given our view of simplification, relies on the assumption that they all have completely parallel sets of alternatives. But if the alternatives were different, we might get different results. Specifically, the conjunctive alternative equivalent to *if both switches were down, the light would be off* might cease to be IE given different alternatives, and possibly become II. In what follows we explore this route.

We will account for the facts as follows: first, we will claim that given independently needed assumptions, we expect the set of alternatives of (76) to be different than that of (74), and as a result the conjunctive alternative is II in this case. Second (following the spirit of Schulz 2018), we will employ the same machinery to explain Ciardelli et al.'s (75), and furthermore aim to explain why the facts are not as clear as in (76).

Chierchia et al. (2012) have argued that disjunctions of the form *P or Q or both*, as in the antecedent of (76), involve obligatory exhaustification of the first disjunct *P or Q* (due to Hurford's constraint against disjunctions in which one of the disjuncts entails the other). In other words, such disjunctions are obligatorily parsed as *$\mathcal{E}xh$(P or Q) or both*. Assuming this, (76) would have the form of $(\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \mathrel{\Box\!\!\rightarrow} r$. Note that this move does not make any difference for the truth conditions of the prejacent: it is still equivalent to $(\bar{p} \vee \bar{q}) \mathrel{\Box\!\!\rightarrow} r$. It will however change the alternatives we generate for (76), some of which would now have an embedded $\mathcal{E}xh$:[48]

(77) $$Alt((\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \mathrel{\Box\!\!\rightarrow} r) = \{\underline{(\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \mathrel{\Box\!\!\rightarrow} r}, (\mathcal{E}xh(\bar{p}) \vee (\bar{p} \wedge \bar{q})) \mathrel{\Box\!\!\rightarrow} r, (\mathcal{E}xh(\bar{q}) \vee (\bar{p} \wedge \bar{q})) \mathrel{\Box\!\!\rightarrow} r, \underline{(\bar{p} \wedge \bar{q}) \mathrel{\Box\!\!\rightarrow} r}, \mathcal{E}xh(\bar{p} \vee \bar{q}) \mathrel{\Box\!\!\rightarrow} r, \underline{\mathcal{E}xh(\bar{p}) \mathrel{\Box\!\!\rightarrow} r}, \underline{\mathcal{E}xh(\bar{q}) \mathrel{\Box\!\!\rightarrow} r}\}$$

To facilitate the presentation, we will only consider some of the alternatives generated and pretend that the set of alternatives is smaller, as in (78) which contains the underlined alternatives in (77) (simplified); the reader may verify that the result doesn't change given the full set of alternatives in (77).

48 We omit the alternative with the contradictory antecedent $(\mathcal{E}xh(\bar{p} \vee \bar{q}) \wedge (\bar{p} \wedge \bar{q})) \mathrel{\Box\!\!\rightarrow} r$, since being a non-contingent proposition it will not affect the result: if it is taken to be a tautology as in Stalnaker (1968), for example, it will be trivially II. It is possible that more alternatives are generated, ones with no $\mathcal{E}xh$: $\bar{p} \mathrel{\Box\!\!\rightarrow} r$ and $\bar{q} \mathrel{\Box\!\!\rightarrow} r$. Since they are equivalent to other alternatives they would change nothing.

(78) $Alt((\mathcal{E}xh(\bar{p}\vee\bar{q})\vee(\bar{p}\wedge\bar{q}))\,\Box\!\!\rightarrow r) =$
$\{(\bar{p}\vee\bar{q})\,\Box\!\!\rightarrow r, (\bar{p}\wedge\neg\bar{q})\,\Box\!\!\rightarrow r, (\bar{q}\wedge\neg\bar{p})\,\Box\!\!\rightarrow r, (\bar{p}\wedge\bar{q})\,\Box\!\!\rightarrow r\}$

The effect of having exhaustified alternatives is that the conjunctive alternative $(\bar{p}\wedge\bar{q})\,\Box\!\!\rightarrow r$ is no longer IE as in the basic case of (74). Namely there is a maximal set of alternatives that can be assigned false consistently with the prejacent which doesn't contain the conjunctive alternative, from which it follows that it is non-IE. This is since there are now alternatives symmetric to it: the falsity of $(\bar{p}\wedge\neg\bar{q})\,\Box\!\!\rightarrow r$ and $(\bar{q}\wedge\neg\bar{p})\,\Box\!\!\rightarrow r$ is consistent with the truth of the prejacent, and together they entail the truth of $(\bar{p}\wedge\bar{q})\,\Box\!\!\rightarrow r$.[49] In fact, no alternative is IE:

(79) a. **Maximal sets of alternatives in** $Alt((\mathcal{E}xh(\bar{p}\vee\bar{q})\vee(\bar{p}\wedge\bar{q}))\,\Box\!\!\rightarrow r)$ **that can be assigned false consistently with** $(\mathcal{E}xh(\bar{p}\vee\bar{q})\vee(\bar{p}\wedge\bar{q}))\,\Box\!\!\rightarrow r$:
(i) $\{(\bar{p}\wedge\neg\bar{q})\,\Box\!\!\rightarrow r, (\bar{p}\wedge\bar{q})\,\Box\!\!\rightarrow r\}$
(ii) $\{(\bar{q}\wedge\neg\bar{p})\,\Box\!\!\rightarrow r, (\bar{p}\wedge\bar{q})\,\Box\!\!\rightarrow r\}$
(iii) $\{(\bar{p}\wedge\neg\bar{q})\,\Box\!\!\rightarrow r, (\bar{q}\wedge\neg\bar{p})\,\Box\!\!\rightarrow r\}$

b. $IE((\mathcal{E}xh(\bar{p}\vee\bar{q})\vee(\bar{p}\wedge\bar{q}))\,\Box\!\!\rightarrow r, Alt((\mathcal{E}xh(\bar{p}\vee\bar{q})\vee(\bar{p}\wedge\bar{q}))\,\Box\!\!\rightarrow r) =$
$\bigcap$(79a) $= \varnothing$

The *Cell* interpretation is non-contradictory, namely all the alternatives are II. We get then (see also figure 6):

(80) $\mathcal{E}xh^{\text{IE+II}}_{Alt((\mathcal{E}xh(\bar{p}\vee\bar{q})\vee(\bar{p}\wedge\bar{q}))\Box\!\rightarrow r)}(\mathcal{E}xh(\bar{p}\vee\bar{q})\vee(\bar{p}\wedge\bar{q}))\,\Box\!\!\rightarrow r \Leftrightarrow$
$(\bar{p}\wedge\neg\bar{q})\,\Box\!\!\rightarrow r \wedge (\bar{q}\wedge\neg\bar{p})\,\Box\!\!\rightarrow r \wedge (\bar{p}\wedge\bar{q})\,\Box\!\!\rightarrow r$

Importantly, we derive the truth of the alternative $(\bar{p}\wedge\bar{q})\,\Box\!\!\rightarrow r$, which explains the clear difference between (74) and (76). While the exhaustified meaning of the former entails the falsity of *if both switches were down the light would be off*, the exhaustified meaning of the latter entails its truth.[50] Since this conditional statement is indeed false in Ciardelli et al.'s scenario, the difference in judgments is predicted. The independently motivated assumption that (76) contains an embedded $\mathcal{E}xh$ has led to the derivation of the desired simplification inference $(\bar{p}\wedge\bar{q})\,\Box\!\!\rightarrow r$ for this case. We turn now to the case of (75), and propose a similar derivation to that of (76).

Schulz (2018) suggested to account for Ciardelli et al.'s contrast between (75) and (74) by blaming negation, which is present in (75) but not in (74), for generating

49 This can be understood more easily if we consider what the closest $\bar{p}\vee\bar{q}$-world might be. It can be either a $\bar{p}\wedge\neg\bar{q}$-world, or a $\bar{q}\wedge\neg\bar{p}$ world, or a $\bar{p}\wedge\bar{q}$-world. If we assign false to $(\bar{p}\wedge\neg\bar{q})\,\Box\!\!\rightarrow r$ and $(\bar{q}\wedge\neg\bar{p})\,\Box\!\!\rightarrow r$, then the closest $\bar{p}\vee\bar{q}$-world cannot be a $\bar{p}\wedge\neg\bar{q}$-world or a $\bar{q}\wedge\neg\bar{p}$-world while still satisfying the prejacent; it must be a $\bar{p}\wedge\bar{q}$-world, namely the alternative $(\bar{p}\wedge\bar{q})\,\Box\!\!\rightarrow r$ must be true.

50 The ultimate story is a bit more nuanced. As we know from fn. 36, the falsity of the conjunctive alternative is derived for (74) only if it's relevant. The truth of the conjunctive alternative in the case of (76) is in contrast an obligatory inference; see §9.1.

Simplification of disjunctive antecedents containing *or both*

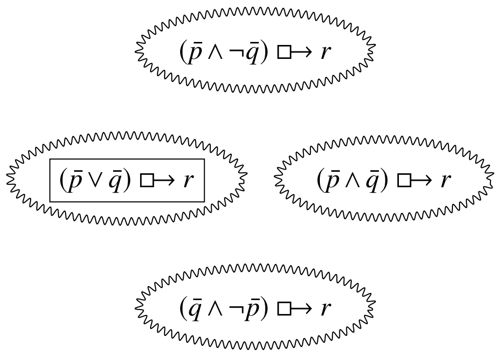

Figure 6 Result of Innocent Exclusion and Innocent Inclusion for conditionals with a disjunctive antecedent containing *or both*, e.g., (76) (notation as in figure 2 on p. 16). [Indication of entailment relations suppressed.]

more alternatives. Her solution is couched in a framework in which the alternatives generated by the antecedent are quantified over by the conditional operator (see, e.g., Alonso-Ovalle 2009; Santorio 2016). However, Schulz essentially stipulates that negated conjunction triggers more alternatives than disjunction, and it is not clear how this result could be achieved in a more principled fashion within her framework. We thus propose to convert Schulz's idea into the current framework, and carry it out in the following way: we assume that negation triggers $\mathcal{E}xh$ as an alternative.[51] This assumption yields a very similar result to what we have just derived for (76), since the set of alternatives will be (almost) the same. The alternatives we get for (75) under this assumption (and the assumption that $Alt(p \wedge q) = Alt(p \vee q)$, as we assumed for the derivation of negative universal FC at the end of §5.4) are as follows:

(81) $$\begin{aligned} &Alt((\neg(p \wedge q)) \,\Box\!\!\rightarrow r) = \\ &\{(\neg(p \wedge q)) \,\Box\!\!\rightarrow r, (\neg p) \,\Box\!\!\rightarrow r, (\neg q) \,\Box\!\!\rightarrow r, (\neg(p \vee q)) \,\Box\!\!\rightarrow r, \\ &\quad (\mathcal{E}xh(p \vee q)) \,\Box\!\!\rightarrow r, (\mathcal{E}xh(q)) \,\Box\!\!\rightarrow r, (\mathcal{E}xh(p)) \,\Box\!\!\rightarrow r, (\mathcal{E}xh(p \wedge q)) \,\Box\!\!\rightarrow r\} \end{aligned}$$

As the reader may verify (bearing in mind that $\neg p = \bar{p}$ and $\neg q = \bar{q}$, i.e., *not up*=*down*), the alternatives in (81) and (77) yield the same propositions, except for the last

51 Of course, this assumption should follow from a general theory of alternative generation such as Katzir (2007). A more in-depth discussion of this issue is something that we hope to return to in the future. Another issue is that so-called 'indirect implicatures', e.g., from *not all students came* to *some students came*, are not derivable if alternatives where negation is replaced with $\mathcal{E}xh$ are generated. As we mention towards the end of this section we assume that whether such alternatives are generated depends on whether negation is dominated by focus or not; we'd hence expect indirect implicatures only if it's not. See Chierchia (2004) and Romoli (2012) for conflicting opinions as to the status of indirect implicatures relative to standard ones.

alternative in (81) which has no parallel in (77). For essentially the same reasons as in (76) then, the conjunctive alternative $((\neg p) \wedge (\neg q)) \;\Box\!\!\rightarrow r$ is no longer IE. As it turns out (modulo the last alternative in (81) which ends up IE but changes nothing otherwise), the two cases are the same.

(82) $$\mathcal{E}xh^{\text{IE+II}}_{Alt((\neg(p\wedge q))\Box\!\rightarrow r)}(\neg(p \wedge q)) \;\Box\!\!\rightarrow r \Leftrightarrow \\ (p \wedge \neg q) \;\Box\!\!\rightarrow r \wedge (q \wedge \neg p) \;\Box\!\!\rightarrow r \wedge ((\neg p) \wedge (\neg q)) \;\Box\!\!\rightarrow r \wedge \neg((p \wedge q) \;\Box\!\!\rightarrow r)$$

Importantly, we derive the truth of the alternative $((\neg p) \wedge (\neg q)) \;\Box\!\!\rightarrow r$, which explains the significant decrease in true judgments for (75) relative to (74). Recall, however, that while (75) was mostly judged false or neither-true-nor-false, its non-truth was not as clear as that of (76). We submit that this is due to the possible effects of focus on the alternatives (Rooth 1992; Fox & Katzir 2011; Katzir 2014): if negation is dominated by focus, then it will be replaced with $\mathcal{E}xh$, the result we derived above will hold and the sentence will be false. If negation is not dominated by focus, then no replacement of negation with $\mathcal{E}xh$ will be triggered, the set of alternatives for $(\neg(p \wedge q)) \;\Box\!\!\rightarrow r$ will be identical to that of $(\bar{p} \vee \bar{q}) \;\Box\!\!\rightarrow r$, and the sentence will be true.[52]

8.3 Simplification with *most*

Our Inclusion-based account of SDA isn't tied to the semantics of conditionals and hence predicts simplification inferences to surface outside this domain. Our goal in this section is to investigate this prediction for a construction in which *most* has a disjunctive restrictor. As we will demonstrate below, we predict (83) to give rise to the inferences in (83a)-(83b).[53]

(83) Most students in Linguistics or Philosophy took Advanced Syntax. $Most(P \cup Q)(R)$

a. $\rightsquigarrow$ Most students in Linguistics took Advanced Syntax. $Most(P)(R)$

b. $\rightsquigarrow$ Most students in Philosophy took Advanced Syntax. $Most(Q)(R)$

According to our intuitions, these are indeed inferences of (83). Consider the following scenario: 35 out of the 40 Linguistics students and 2 out of the 30 Philosophy

52 Another reason might be that unlike in the case of (76) where the conjunctive alternative can be arrived at by merely deleting parts of the prejacent, its parallel in the case of (75) cannot. See §9.1 for the relevance of this difference.

53 Another potentially relevant case of a simplification problem is with superlatives:

(i) I climbed the highest mountains in North America or South America.

We have not yet fully investigated the applicability of Innocent Inclusion to such cases.

students took Advanced Syntax (and there are no Linguistics-and-Philosophy students). In this case (83) seems odd even though more than half of the members in the union of Linguistics students and Philosophy students took Advanced Syntax (37 out of 70). This, presumably, is since the inference in (83b) is false.[54,55] Admittedly, the intuition that the sentence is odd in the above scenario is not shared by all speakers, in contrast with SDA. While further investigation into the status of such inferences is needed, their existence requires an explanation which we will now show is provided by Innocent Inclusion.

For expository purposes we assume the following semantics for *most*, the crucial ingredient being that it is non-monotonic with respect to its restrictor:

(84) $Most(P)(Q) = 1 \text{ } \mathit{iff} \text{ } \frac{|P \cap Q|}{|P|} > \frac{1}{2}$

The set of alternatives we assume for $most(P \cup Q)(R)$ is in (85), considering not only the disjunctive and conjunctive alternatives but also alternatives where *most* is replaced with *some*, similarly to our discussion of universal FC above (without the *some*-alternatives $most(P)(R)$ and $most(Q)(R)$ would end up IE rather than II).[56,57]

(85) $Alt(most(P \cup Q)(R)) =$
$\{most(P \cup Q)(R), most(P)(R), most(Q)(R), most(P \cap Q)(R),$
$some(P \cup Q)(R), some(P)(R), some(Q)(R), some(P \cap Q)(R)\}$

The IE alternatives are as follows:

(86) a. **Maximal sets of alternatives in $Alt(\boldsymbol{most}(P \cup Q)(R))$ that can be**

54 For reasons we do not understand, embedding disjunction in a relative clause seems to make the sentence less odd in this scenario:

(i) Most students who study Linguistics or Philosophy took Advanced Syntax.

55 Here too we can generate a McKay & van Inwagen-style effect as in (i). As with simplification in conditionals, we infer that it is false that *most Philosophy students are Linguistics students* (see fn. 46). For elaboration on this see Bar-Lev (2018: §1).

(i) Most students in Linguistics or Philosophy are Linguistics students. $Most(P \cup Q)(P)$

56 *Most* would also trigger *all*-alternatives, of course, which we ignore since they only add more IE alternatives but have no effect on what's II, which is the focus of our discussion.

57 A potential reason for the optionally of simplification inferences with *most* noted above is that the IIness of the disjunctive alternatives in this case depends on the existence of *some*-alternatives in the set, in contrast with SDA. See §9.1 for why the *some*-alternatives might not be in the set of alternatives given that they are derived by substitution of *most* with *some*.

assigned false consistently with *most*$(P \cup Q)(R)$:
(i) $\{most(P)(R), most(Q)(R), most(P \cap Q)(R), some(P \cap Q)(R)\}$
(ii) $\{most(P)(R), some(P)(R), most(P \cap Q)(R), some(P \cap Q)(R)\}$
(iii) $\{most(Q)(R), some(Q)(R), most(P \cap Q)(R), some(P \cap Q)(R)\}$

b. $IE(most(P \cup Q)(R), Alt(most(P \cup Q)(R))) = \bigcap$ (86a) $= \{most(P \cap Q)(R), some(P \cap Q)(R)\}$

The *Cell* interpretation is non-contradictory, and exhaustification thus assigns true to the disjunctive alternatives. We thus capture the simplification inferences of (83).

(87) $\mathcal{E}xh^{\text{IE+II}}_{Alt(most(P \cup Q)(R))} most(P \cup Q)(R) \Leftrightarrow most(P \cup Q)(R) \wedge most(P)(R) \wedge most(Q)(R) \wedge \neg some(P \cap Q)(R)$

8.4 Interim Summary

We have considered several phenomena where the *Cell* interpretation entails attested inferences, yet previous accounts fail to predict them. We have argued on this basis in favor of our mechanism which combines Innocent Exclusion and Innocent Inclusion and delivers the *Cell* interpretation whenever it is non-contradictory. Before concluding, we discuss some remaining issues in §9: Differences between inferences we lump together as resulting from exhaustification, and some problematic cases of embedded FC disjunction.

9 Some loose ends

9.1 Obligatoriness

Experimental work has revealed significant differences between FC inferences and run-of-the-mill scalar implicatures (Chemla 2009b; Chemla & Bott 2014). Similarly, SDA inferences seem more robust than run-of-the-mill scalar implicatures. To explain these differences which prima facie conflict with implicature accounts of FC and SDA, Bar-Lev & Fox (2017); Bar-Lev (2018: §2) hypothesized that FC and SDA are obligatory implicatures while run-of-the-mill scalar implicatures are optional and subject to considerations of relevance. We can't provide here a satisfactory theory of obligatoriness. We would like, however, to mention two possible directions that could be pursued. One direction (pursued in Bar-Lev & Fox 2017) is that the 'literal meaning' of a sentence is what *only* presupposes (see §4); hence Inclusion inferences are obligatory, being part of the 'literal meaning', whereas Exclusion ones are not. Another direction (pursued in Bar-Lev 2018: §2 following the lead of Chemla & Bott 2014) is that the distinction between obligatory and optional inferences tracks the distinction between inferences resulting from alternatives generated by deletion (i.e., replacing a constituent with a sub-constituent) and ones resulting from alternatives generated by substitution (i.e., replacing a constituent with an item from the lexicon; see Katzir 2007; Fox & Katzir 2011). This distinction has been independently argued to be relevant for explaining children's intricate behavior with scalar implicatures (e.g., by Zhou, Romoli & Crain 2013; Tieu, Romoli, Zhou & Crain 2016; Singh et al. 2016). See also Chemla & Singh (2014) for relevant discussion.

9.2 Other cases of embedded FC disjunction

As things stand right now, our account faces a problem accounting for the inferences of (88), which is a case where FC disjunction is embedded under an existential quantifier, i.e., an 'existential FC' example (cf. the discussion of universal FC in §5).[58] There are two inferences to account for: the embedded FC inference in (88a), and the upper bound 'not every' inference in (88b).[59]

(88) Some boy is allowed to eat ice cream or cake. $\exists x \Diamond (Px \lor Qx)$

a. $\rightsquigarrow$ Some boy is both allowed to eat ice cream and allowed to eat cake. $\exists x (\Diamond Px \land \Diamond Qx)$

b. $\rightsquigarrow$ Some boy is not allowed to eat ice cream and not allowed to eat cake. $\neg \forall x \Diamond (Px \lor Qx)$

The problem is as follows: if we only apply $\mathcal{E}xh^{\text{IE+II}}$ globally as in the case of universal FC, we get an inference which is too weak. Rather than deriving that the same boy is allowed ice cream and allowed cake, we get the inferences that *some boy is allowed ice cream* and *some boy is allowed cake*, which can be true even if (88a) is false. A quick fix to this would be to apply $\mathcal{E}xh^{\text{IE+II}}$ locally, below *some boy* and above *allowed*.[60] This however will take its toll from the upper bound inference, which would become weaker than (88b): it would only yield that not every boy is both allowed ice cream and allowed cake, which is compatible with every boy being allowed, say, ice cream.

This problem is reminiscent of a puzzle discussed in Chierchia (2004), which is deriving for (89) both inferences in (89a) and (89b). Here too, an embedded $\mathcal{E}xh$ is needed in order to derive (89a), but having it is harmful for the derivation of (89b) at the matrix level.

58 Similar examples with similar inference patterns to those discussed in this section can be constructed using conditionals with disjunctive antecedents instead of FC disjunction.

59 We thank an anonymous SALT reviewer for pointing out that FC disjunction under *some* still requires a local derivation.

60 An alternative way to address this problem could be by appealing to the theory of wide scope indefinites ('referential indefinites'; Kasher & Gabbay 1976; Fodor & Sag 1982 and much subsequent work). If *some boy* here is referential, the problem doesn't arise.

(89) Someone smokes or drinks. $\exists x(Px \vee Qx)$

a. $\rightsquigarrow$ Someone smokes or drinks but not both.
$\exists x((Px \vee Qx) \wedge \neg(Px \wedge Qx))$

b. $\rightsquigarrow$ Someone doesn't smoke or drink. $\neg\forall x(Px \vee Qx)$

As a tentative solution in the spirit of Chierchia (2004) we suggest then that in both cases there is an embedded $\mathcal{E}xh$, but the alternatives at the matrix position include ones where the lower $\mathcal{E}xh$ is deleted.[61] As the reader may verify, this yields the desired results.[62]

A similar issue arises when embedding FC disjunction under numerals:

(90) Two girls are allowed to eat ice cream or cake.

a. $\rightsquigarrow$ Two girls are both allowed to eat ice cream and allowed to eat cake.

b. $\rightsquigarrow$ No more than two girls are allowed ice cream or cake.

The problem in this case is however more subtle. If we only apply $\mathcal{E}xh^{\text{IE+II}}$ globally (and assuming the sentence generates alternatives where *two* is replaced with other numerals), we get similarly to the case of *some* the too weak inferences that *two girls are allowed ice cream* and *two girls are allowed cake*. However, given the upper bound inference in (90b) which is also derived on the same parse, the desired inference in (90a) follows.

The problem is then that even when the upper bound inference is cancelled, the embedded FC inference persists:

(91) At least two girls are allowed to eat ice cream or cake.

a. $\rightsquigarrow$ At least two girls are both allowed to eat ice cream and allowed to eat cake.

b. $\not\rightsquigarrow$ No more than two girls are allowed ice cream or cake.

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61 This is in fact in line with the expectation of the structural approach to alternative generation in Katzir (2007); Fox & Katzir (2011).

62 Recall that in §5.3 we have argued against a local derivation for universal FC on the basis of VP-ellipsis considerations. Unfortunately, a similar argument can be made for the existential FC case discussed here (cf. (39)):

(i) Some girl is allowed to eat ice cream or cake on her birthday. Interestingly, No boy is ~~allowed to eat ice cream or cake on his birthday~~. $\approx$

a. Some girl is both allowed ice cream *and* allowed cake on her birthday, and

b. no boy is allowed ice cream and (likewise) no boy is allowed cake on his birthday.

In light of these facts, if we want to maintain the analysis proposed in the main text for (88) we will be forced to say, contrary to what we have assumed so far, that $\mathcal{E}xh$ does not in fact count for parallelism considerations. However, we will have to address Crnič's (2015) arguments that it does. We have to leave this as an unresolved problem, tough see fn. 60 for a possible way to avoid it.

The solution we suggested for (88) applies here too. The problem disappears if we assume that $\mathcal{E}xh^{\text{IE+II}}$ applies locally, in the scope of *(at least) two girls*, but the set of alternatives for the higher $\mathcal{E}xh^{\text{IE+II}}$ contains alternatives where the lower $\mathcal{E}xh^{\text{IE+II}}$ is deleted.

We have only discussed up until now embedding under monotonic quantifiers. What about embedding under non-monotonic quantifiers such as (92)?[63] First, note that its inferences (92a)-(92b) are precisely those in (90a)-(90b). The only difference, presumably, is that for (92) the upper bound inference in (92b) is uncancellable.

(92) Exactly two girls are allowed to eat ice cream or cake.

a. $\rightsquigarrow$ Two girls are both allowed to eat ice cream and allowed to eat cake.
b. $\rightsquigarrow$ No more than two girls are allowed ice cream or cake.

One way to account for the facts is to assume that the derivation carries over from the plain numeral case in (90) to the *exactly*-case in (92). Specifically, we can deny that *exactly two* is a non-monotonic quantifier and adopt Landman's (1998) view in which *exactly two* has the same semantic contribution as *two*, and the upper bound inference is derived by exhaustification higher up in the structure (see also Spector 2014).

A different approach to cases like (92) has been taken by Gotzner, Romoli & Santorio (2018). They maintain that *exactly two* is a non-monotonic quantifier and assume a different set of alternatives for achieving the correct results. Specifically, they assume that *exactly two* does not generate alternatives where the numeral is replaced with other numerals, and furthermore that *exactly two* generates *some* as an alternative. Under these assumptions, the alternatives *exactly two girls are allowed ice cream* and *exactly two girls are allowed cake* end up II and the correct interpretation is derived. We do not aim to argue in favor of any of the proposed solutions, and leave this issue along with the previous data discussed in this section for future research.[64]

63 With *exactly one* the desired result can be derived at the global level assuming a standard non-monotonic semantics; the problem arises with higher numerals, as in (92), for which such a derivation is not straightforwardly available.

64 Gotzner et al. (2018) further discuss sentences like (i) and their intriguing inference in (ib):

(i) Exactly two girls are not allowed to eat ice cream or cake.

a. $\rightsquigarrow$ Two girls are not allowed to eat ice cream or cake.
b. $\rightsquigarrow$ No more than two girls are not allowed to eat ice cream or not allowed to eat cake.

Gotzner et al. argue that (ib) is not derived assuming an Exclusion-based approach to FC. But this is only true if *exactly two* is a non-monotonic quantifier. If it is equivalent to *two*, the derivation of such cases boils down to the derivation of the same inferences for *two girls are not allowed to eat ice cream or cake*. And for the latter case (ib) is derived as an exclusion inference, since both alternatives

10 Summary

We presented a novel theory of exhaustification in which the goal of $\mathcal{E}xh$ is to deliver (whenever possible) a cell in the partition induced by the set of alternatives. To let $\mathcal{E}xh$ achieve this goal, we suggested that it must make use not only of Innocent Exclusion (Fox 2007) but of Innocent Inclusion as well. We have argued for Innocent Inclusion by considering a range of data that show that whenever Innocently Includable alternatives exist, their truth is inferred.

References

Aloni, Maria. 2007. Free choice, modals, and imperatives. *Natural Language Semantics* 15(1). 65–94. doi:10.1007/s11050-007-9010-2.

Aloni, Maria. 2016. FC disjunction in state-based semantics. Talk given at Logical Aspects of Computational Linguistics 2016, Nancy, France.

Alonso-Ovalle, Luis. 2005. Distributing the disjuncts over the modal space. In L. Bateman & C. Ussery (eds.), *North east linguistic society (NELS)*, vol. 35, 1–12.

Alonso-Ovalle, Luis. 2009. Counterfactuals, correlatives, and disjunction. *Linguistics and Philosophy* 32(2). 207–244.

Alxatib, Sam. 2014. Free choice disjunctions under only. In Jyoti Iyer & Leland Kusmer (eds.), *North east linguistic society (NELS)*, vol. 44, 15–28.

Anvari, Amir. 2018. Logical integrity: from maximize presupposition! to mismatching implicatures. Manuscript.

Bar-Lev, Moshe E. 2018. *Free choice, homogeneity, and innocent inclusion*: Hebrew University of Jerusalem dissertation.

Bar-Lev, Moshe E. & Danny Fox. 2016. On the global calculation of embedded implicatures. Poster presented at MIT workshop on exhaustivity.

Bar-Lev, Moshe E. & Danny Fox. 2017. Universal free choice and innocent inclusion. In Dan Burgdorf, Jacob Collard, Sireemas Maspong & Brynhildur Stefánsdóttir (eds.), *Semantics and linguistic theory (SALT)*, vol. 27, 95–115.

Chemla, Emmanuel. 2009a. Similarity: Towards a unified account of scalar implicatures, free choice permission and presupposition projection. Unpublished manuscript.

Chemla, Emmanuel. 2009b. Universal implicatures and free choice effects: Experimental data. *Semantics and Pragmatics* 2(2). 1–33. doi:10.3765/sp.2.2.

Chemla, Emmanuel & Lewis Bott. 2014. Processing inferences at the semantics/pragmatics frontier: Disjunctions and free choice. *Cognition* 130(3). 380 – 396. doi:10.1016/j.cognition.2013.11.013.

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three girls are not allowed ice cream and *three girls are not allowed cake* are IE.

Chemla, Emmanuel & Raj Singh. 2014. Remarks on the experimental turn in the study of scalar implicature, part ii. *Language and Linguistics Compass* 8(9). 387–399.

Chemla, Emmanuel & Benjamin Spector. 2011. Experimental evidence for embedded scalar implicatures. *Journal of semantics* 28(3). 359–400. doi:10.1093/jos/ffq023.

Chierchia, Gennaro. 2004. *Scalar implicatures, polarity phenomena, and the syntax/pragmatics interface* 39–103. Oxford, UK.

Chierchia, Gennaro. 2013. *Logic in grammar: Polarity, free choice, and intervention*. Oxford University Press.

Chierchia, Gennaro, Danny Fox & Benjamin Spector. 2012. Scalar implicature as a grammatical phenomenon. In Claudia Maienborn, Klaus von Heusinger & Paul Portner (eds.), *Semantics: An international handbook of natural language meaning*, vol. 3, 2297–2331. Mouton de Gruyter. doi:10.1515/9783110253382.2297.

Ciardelli, Ivano, Linmin Zhang & Lucas Champollion. 2018. Two switches in the theory of counterfactuals: A study of truth conditionality and minimal change. To appear in Linguistics and Philosophy.

Cohen, L Jonathan. 1971. Some remarks on grice's views about the logical particles of natural language. In *Pragmatics of natural languages*, 50–68. Springer.

Crnič, Luka. 2017. Free choice under ellipsis. *The Linguistic Review* 34(2). 249–294.

Crnič, Luka. 2015. Ellipsis, parallelism, and polarity. In *Semantics and linguistic theory 25*, 308–327. doi:10.3765/salt.v25i0.3204. http://journals.linguisticsociety.org/proceedings/index.php/SALT/article/view/25.308.

Fine, Kit. 1975. Critical notice. *Mind* 84(335). 451–458.

von Fintel, Kai. 1997. Bare plurals, bare conditionals, and only. *Journal of Semantics* 14(1). 1–56.

Fodor, Janet Dean & Ivan A. Sag. 1982. Referential and quantificational indefinites. *Linguistics and Philosophy* 5(3). 355–398. doi:10.1007/BF00351459. https://doi.org/10.1007/BF00351459.

Fox, Danny. 2004. Implicature calculation, pragmatics or syntax, or both? Class notes on Implicatures and Exhaustivity, USC, November 2004. http://lingphil.mit.edu/papers/fox/class_1.pdf.

Fox, Danny. 2007. Free choice and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and implicature in compositional semantics*, 71–120. London: Palgrave Macmillan UK. doi:10.1057/9780230210752_4.

Fox, Danny. 2017. Exhaustivity and the presupposition of questions. Talk given at Sinn und Bedeutung 22.

Fox, Danny & Martin Hackl. 2006. The universal density of measurement. *Linguistics and Philosophy* 29(5). 537–586. doi:10.1007/s10988-006-9004-4. https://doi.org/10.1007/s10988-006-9004-4.

Fox, Danny & Roni Katzir. 2011. On the characterization of alternatives. *Natural Language Semantics* 19(1). 87–107. doi:10.1007/s11050-010-9065-3.

Fox, Danny & Roni Katzir. 2018. Notes on iterated rationality models of pragmatics. Ms., MIT and Tel Aviv University.

Franke, Michael. 2011. Quantity implicatures, exhaustive interpretation, and rational conversation. *Semantics and Pragmatics* 4(1). 1–82. doi:10.3765/sp.4.1.

Gajewski, Jon & Yael Sharvit. 2012. In defense of the grammatical approach to local implicatures. *Natural language semantics* 20(1). 31–57.

Geurts, Bart. 2010. *Quantity implicatures*. Cambridge University Press.

Gotzner, Nicole & Jacopo Romoli. 2017. The scalar inferences of strong scalar terms under negative quantifiers and constraints on the theory of alternatives. To appear in *Journal of Semantics*.

Gotzner, Nicole, Jacopo Romoli & Paolo Santorio. 2018. More free choice and more inclusion: An experimental investigation of free choice in non-monotonic environments. Abstract for SALT 28.

Heim, Irene. 1996. Lecture notes in ellipsis. Unpublished manuscript, MIT.

Ippolito, Michela. 2008. On the meaning of only. *Journal of Semantics* 25(1). 45–91.

Kamp, Hans. 1974. Free choice permission. *Proceedings of the Aristotelian Society* 74(1). 57–74. doi:10.1093/aristotelian/74.1.57.

Kasher, Asa & Dov M Gabbay. 1976. On the semantics and pragmatics of specific and non-specific indefinite expressions. *Theoretical linguistics* 3(1-3). 145–190.

Katzir, Roni. 2007. Structurally-defined alternatives. *Linguistics and Philosophy* 30(6). 669–690. doi:10.1007/s10988-008-9029-y.

Katzir, Roni. 2014. On the roles of markedness and contradiction in the use of alternatives. In Salvatore Pistoia Reda (ed.), *Pragmatics, semantics and the case of scalar implicatures*, 40–71. Springer. doi:10.1057/9781137333285_3.

Klinedinst, Nathan Winter. 2007. *Plurality and possibility*: University of California, Los Angeles dissertation.

Kratzer, Angelika & Junko Shimoyama. 2002. Indeterminate pronouns: The view from japanese. In Yukio Otsu (ed.), *The third tokyo conference on psycholinguistics*, 1–25. Hituzi Syobo.

Križ, Manuel & Benjamin Spector. 2017. Interpreting plural predication: Homogeneity and non-maximality. Ms., Institut Jean Nicod.

Landman, Fred. 1998. Plurals and maximalization. In *Events and grammar*, 237–271. Springer.

Larson, Richard K. 1985. On the syntax of disjunction scope. *Natural Language & Linguistic Theory* 3(2). 217–264.

Lassiter, Daniel. 2018. Complex sentential operators refute unrestricted simplification of disjunctive antecedents. *Semantics and Pragmatics* 11.

Levinson, Stephen C. 2000. *Presumptive meanings: The theory of generalized*

conversational implicature. MIT press.
Lewis, David. 1973. *Counterfactuals*. Basil Blackwell.
Loewer, Barry. 1976. Counterfactuals with disjunctive antecedents. *The Journal of Philosophy* 73(16). 531–537.
Marty, Paul P. 2017. *Implicatures in the dp domain*: Massachusetts Institute of Technology dissertation.
McKay, Thomas & Peter van Inwagen. 1977. Counterfactuals with disjunctive antecedents. *Philosophical studies* 31(5). 353–356.
Nouwen, Rick. 2017. Free choice and distribution over disjunction: the case of free choice ability. Manuscript, Utrecht University.
Nute, Donald. 1975. Counterfactuals and the similarity of words. *The Journal of Philosophy* 72(21). 773–778.
Nute, Donald. 1980. Conversational scorekeeping and conditionals. *Journal of Philosophical Logic* 9(2). 153–166. doi:10.1007/BF00247746. https://doi.org/10.1007/BF00247746.
Romoli, Jacopo. 2012. *Soft but strong. neg-raising, soft triggers, and exhaustification*: Harvard University dissertation.
Rooth, Mats. 1992. Ellipsis redundancy and reduction redundancy. In *Proceedings of the stuttgart ellipsis workshop*, vol. 29, Citeseer.
Sag, Ivan. 1976. *Deletion and logical form*: MIT dissertation.
Santorio, Paolo. 2016. Alternatives and truthmakers in conditional semantics. Manuscript, University of Leeds.
Sauerland, Uli. 2004. Scalar implicatures in complex sentences. *Linguistics and philosophy* 27(3). 367–391. doi:10.1023/B:LING.0000023378.71748.db.
Schlenker, Philippe. 2009. Local contexts. *Semantics and pragmatics* 2. 3–1.
Schulz, Katrin. 2018. The similarity approach strikes back - negation in counterfactual conditionals. Slides, ILLC, University of Amsterdam.
Simons, Mandy. 2005. Dividing things up: The semantics of or and the modal/or interaction. *Natural Language Semantics* 13(3). 271–316. doi:10.1007/s11050-004-2900-7.
Singh, Raj, Ken Wexler, Andrea Astle-Rahim, Deepthi Kamawar & Danny Fox. 2016. Children interpret disjunction as conjunction: Consequences for theories of implicature and child development. *Natural Language Semantics* 24(4). 305–352. doi:10.1007/s11050-016-9126-3.
Spector, Benjamin. 2007. Scalar implicatures: Exhaustivity and gricean reasoning. *Questions in dynamic semantics* 17. 225–249.
Spector, Benjamin. 2013. Homogeneity and plurals: From the strongest meaning hypothesis to supervaluations. Talk presented at Sinn und Bedeutung 18.
Spector, Benjamin. 2014. Plural indefinites and maximality. Talk given at the UCLA linguistics colloquium.

Spector, Benjamin. 2018. Homogeneity and non-maximality within the rational speech act model. In *Proceedings of the amsterdam colloquium*, vol. 21, 435–444.

Spector, Benjamin & Yasutada Sudo. 2017. Presupposed ignorance and exhaustification: how scalar implicatures and presuppositions interact. *Linguistics and Philosophy* 40(5). 473–517.

Stalnaker, Robert. 1968. A theory of conditionals. In Nicholas Rescher (ed.), *Studies in logical theory*, 98–112. Oxford: Basil Blackwell.

Starr, William. 2016. Expressing permission. In *Semantics and linguistic theory*, vol. 26, 325–349. doi:10.3765/salt.v26i0.3832.

Tieu, Lyn, Jacopo Romoli, Peng Zhou & Stephen Crain. 2016. Children's knowledge of free choice inferences and scalar implicatures. *Journal of Semantics* 33(2). 269–298. doi:10.1093/jos/ffv001. http://dx.doi.org/10.1093/jos/ffv001.

Willer, Malte. 2017. Simplifying with free choice. *Topoi* doi:10.1007/s11245-016-9437-5. https://doi.org/10.1007/s11245-016-9437-5.

Williams, Edwin S. 1977. Discourse and logical form. *Linguistic inquiry* 101–139.

von Wright, Georg Henrik. 1968. *An essay in deontic logic and the general theory of action*. Amsterdam: North-Holland.

Zhou, Peng, Jacopo Romoli & Stephen Crain. 2013. Children's knowledge of free choice inferences. In *Semantics and linguistic theory*, vol. 23, 632–651.

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