Dynamic inquisitive semantics

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Goal

1. To develop a logical framework that combines insights from:
   - Dynamic semantics
   - Inquisitive semantics
2. To highlight some of the advantages that such a framework has w.r.t. frameworks that are non-dynamic or non-inquisitive in the analyses of questions.

1. Motivating dynamic inquisitive semantics
2. Dynamic inquisitive semantics
3. Explaining the motivating data

1 Motivating dynamic inquisitive semantics

2 Dynamic inquisitive semantics

3 Explaining the motivating data

Motivation for a dynamic semantics of questions

- A dynamic semantics of questions is needed to capture certain types of anaphora.

  (Groenendijk, 1998; van Rooij, 1998; Haida, 2007)

- It also provides an attractive account of certain kinds of intervention effects.

  (Haida, 2007)

Anaphora

- In dynamic semantics, meaning is not equated with truth-conditions but rather with context change potential.
- This includes the potential to introduce discourse referents.
- Dynamic semantics has been motivated by contrasts like:

(1) a. [One of my ten marbles]^x is not here.
It$_x$ is probably under the sofa.

b. Nine of my ten marbles are here.
#It is probably under the sofa.

- The first sentence in (1-a) introduces a discourse referent picked up by the anaphoric pronoun.
- The first sentence in (1-b) is truth-conditionally equivalent but does not introduce a suitable discourse referent.

Anaphora in questions

- Dynamic semantics for questions is motivated by examples like:

(2) a. Which^x one of your three sons is living in Paris? And is he$_x$ happy there?
    b. Which^x two of your three sons are not living in Paris? #And is he$_x$ happy there?

- The initial questions in (2-a) and (2-b) are equivalent in terms of resolution/answerhood conditions.
- But they differ in their potential to license pronominal anaphora.

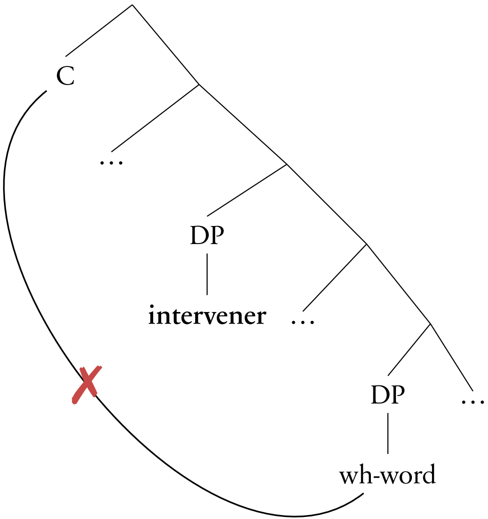
Anaphora in questions

- The fact that wh-questions license anaphora has been the primary piece of motivation for dynamic approaches to question semantics.
  (Groenendijk, 1998; van Rooij, 1998; Haida, 2007)

- Wh-words are taken to introduce discourse referents, just like plain existential indefinites, which can be picked up by anaphoric pronouns.

Intervention effects

Certain operators appearing between a wh-word and the interrogative complementizer lead to ungrammaticality.



Intervention effects: an example

Example from Beck (2006):

(3)

a. Wer hat Luise **wo** angetroffen?
who-Nom has Luise where met
'Who met Luise where?'

b. ??Wer hat niemandem **wo** angetroffen?
who-Nom has nobody-Dat where met
'Who didn't meet anybody where?'

c. Wer hat **wo** niemandem angetroffen?
who-Nom has where nobody-Dat met
'Who didn't meet anybody where?'

Intervention effects: possible interveners

- Possible interveners (Beck, 2006):
  - Focus sensitive operators: *only*, *even*,...
  - Nominal quantifiers: *every*, *no*, *most*, *few*,...
  - Adverbial quantifiers: *always*, *often*, *never*,...
  - Negation: *not*

Intervention effects: two approaches

- How to understand the licensing relation between the wh-word and the complementizer?
- Two prominent approaches (among others):
  - Focus approach (Beck, 2006) – wh-words introduce focus alternatives. Intervention effects arise when C cannot access these focus alternatives because they are consumed by a focus sensitive intervener.
  - Dynamic approach (Haida, 2007) – wh-words introduce discourse referents that C has to access. Intervention effects arise if such access is blocked by operators that do not let discourse referents project from their scope.

Intervention effects: the focus approach

- The focus approach works well for only and even, which are without doubt focus sensitive.
- However, nominal quantifiers every, no, and most are more problematic because they are not necessarily focus sensitive.
  Beck (2006, Section 4), and Haida (2007, Chapter 8)
- Experimental data suggest that the focus-sensitive particle also is not an intervener in German.
  Haida and Repp (2013)

Intervention effects: the dynamic approach

- The dynamic approach works well for quantifiers and negation, which are known to block discourse referents from projecting:

(4)
 a. John didn't consider buying [a car]^i.
 *It$_i$ was too expensive.
 b. Most students considered buying [a car]^i.
 *It$_i$ was very cheap.
 c. John often considered buying [a car]^i.
 *It$_i$ was very cheap.

- Focus sensitive particles do not block discourse referents from projecting, so require a different explanation (Haida, 2007)
- Cross-linguistic variation as to which operators act as interveners is largely an open issue for both approaches.

Motivation for an inquisitive semantics of questions

- Dynamic theories of questions are based on partition semantics (Groenendijk, 1998; van Rooij, 1998; Haida, 2007)
- Partition semantics is suitable to model exhaustive interpretations of questions:

(5) Which of the guests are vegetarian?

Motivation for an inquisitive semantics of questions

- However, non-exhaustive question interpretations are difficult to capture in partition semantics

(6) Who has a bike that I could borrow for 15 minutes?

- In inquisitive semantics both exhaustive and non-exhaustive question interpretations can be captured straightforwardly
- Moreover, dynamic inquisitive semantics allows us to relate exhaustive and non-exhaustive questions to another contrast, strong and weak readings of donkey anaphora

Summary

	Anaphora	Intervention	Non-exhaustive
Static inquisitive	No	No	Yes
Dynamic partition	Yes	Yes	No
Dynamic inquisitive	**Yes**	**Yes**	**Yes**

- There are other potential advantages to dynamic inquisitive semantics which we do not explicitly discuss here

1 Motivating dynamic inquisitive semantics

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3 Explaining the motivating data

Dynamic inquisitive semantics

Our compositional dynamic inquisitive semantics ($\mathsf{Inq_D}$) integrates the basic static inquisitive system, $\mathsf{Inq_B}$, with

- the dynamic system GSV (Groenendijk *et al.*, 1996), and
- the compositional dynamic system CDRT(Muskens, 1996)

The system presented here simplified.

$\mathsf{Inq_D}$ for a first-order language:
to appear in Dotlačil and Roelofsen, Proceedings of SuB.

Contexts in GSV

Contexts in GSV encapsulate:

1. information about the world
2. information about the discourse referents
3. information about dependencies between the world and the discourse referents

Contexts in GSV

Formally, a context in GSV is a set s of $\langle w, g\rangle$ pairs (possibilities), such that w is a possible world and g a (partial) assignment function

This encodes:

- Information about the world:

$$\mathbf{worlds}(s) := \{w \mid \langle w, g\rangle \in s \text{ for some } g\}$$

- Information about the discourse referents:

$$\mathbf{assigns}(s) := \{g \mid \langle w, g\rangle \in s \text{ for some } w\}$$

- Information about dependencies between the world and the discourse referents

Contexts in GSV

Limits on the notion of context in GSV:

- Contexts represent information, but not contextual issues
- Contexts in GSV are not rich enough to analyze questions

Contexts in inquisitive semantics

Contexts in inquisitive semantics encapsulate:

1. information about the world
2. issues raised about the world

Contexts in inquisitive semantics

Context C – a set of information states, each a set of possible worlds

Conditions on C:

- each information state contains enough information to resolve the raised issues
- no information state contains any worlds that have been ruled out by the available information
- set of information states is downward closed and non-empty
- the information available in a context, $\textsc{info}(c) := \bigcup c$.
- A context c is inquisitive just in case $\textsc{info}(c) \notin c$.

Contexts in inquisitive semantics

Limits on the notion of context in Inq_B:

- Contexts represent information and issues about the world
- They do not model information and issues about discourse referents
- They do not model dependencies between the world and the discourse referents

Contexts in dynamic inquisitive semantics

Context C – a set of information states, each a set of possibilities ($\langle w, g \rangle$ pairs)
(w – world; g – dref assignment function)

Conditions on C:

- each information state contains enough information to resolve the raised issues
- no information state contains any possibilities that have been ruled out by contextual information
- set of information states is downward closed and non-empty

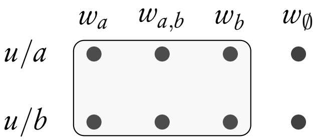
Contexts in dynamic inquisitive semantics – conventions

- $\mathsf{Inq_D}$ has four basic types: e for individuals, s for possible worlds, t for truth values, and r for discourse referents

Object	Type	Type abbreviation	Variable convention
dref assignment f.	(re)	a	-
possibility	$(s \times a)$	-	p
information state	$((s \times a)t)$	i	s
context	(it)	k	c, c'
update function	(kk)	T	A, B

Table: Types and abbreviation conventions

Contexts in dynamic inquisitive semantics – graphical representation



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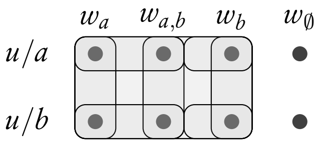

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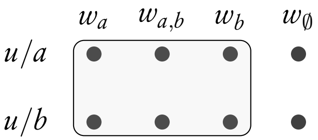

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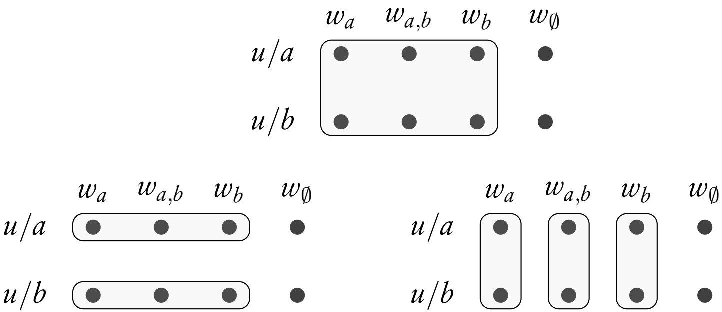

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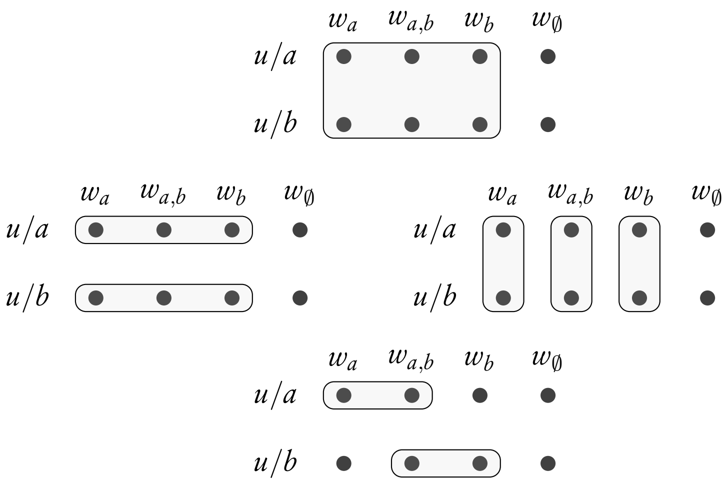


Extension and subsistence

A state s' extends a state s, $s' \geq s$, iff $s' = s$ or:

- s' extends the information about the world provided in s (by excluding some possibilities), and/or
- s' extends the information about the discourse referents provided in s
  (by adding new discourse referents)

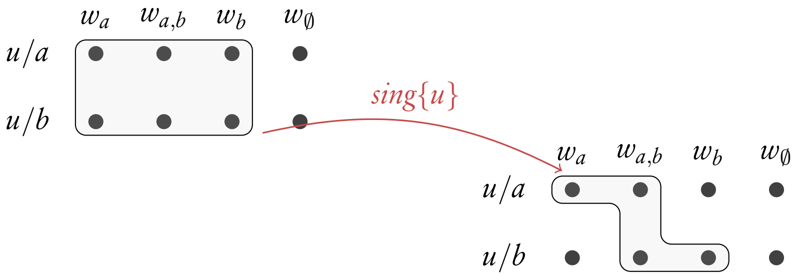
A state s subsists in a state s' iff:

- $s' \geq s$, and
- every possibility in s is in s', modulo the addition of new discourse referents

A state s subsists in a context C iff there is some $s' \in C$ such that s subsists in s'. In this case, s' is called a descendant of s.

Semantics: relations and conjunction

(7) $$R\{u_1, \ldots, u_n\} := \lambda c_k \lambda s_i.\quad s \in c \wedge \forall p \in s\ (R(w_p)(g_p(u_1)) \ldots (g_p(u_n)))$$



(8) $$A_{(kk)}; B_{(kk)} := \lambda c_k.B(A(c))$$

Semantics: dref introduction

- If C is a context and u not in assignments in C, then introducing the dref u creates the largest context C' such that every $s' \in C'$ has some $s \in C$ s.t. s subsists in s' and $\mathbf{assigns}(s') = \mathbf{assigns}(s) \cup \{u\}$.

Semantics: dref introduction

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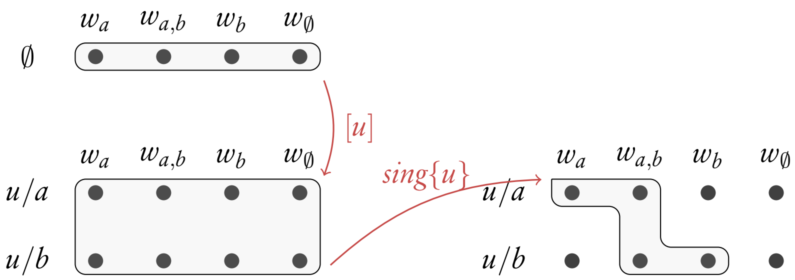
- $[\![g[u]g']\!]^{F,I,\theta} = 1$ iff
  - $[\![\mathbf{dom}(g')]\!]^{F,I,\theta} = [\![\mathbf{dom}(g)]\!]^{F,I,\theta} \cup \{[\![u]\!]^{F,I,\theta}\}$ and
  - $[\![\forall v_r((v \neq u \wedge v \in \mathbf{dom}(g)) \rightarrow g(v) = g'(v))]\!]^{F,I,\theta} = 1$

- $[\![p[u]p']\!]^{F,I,\theta} = 1$ iff $[\![w_p = w_{p'}]\!]^{F,I,\theta} = 1$ and $[\![g_p[u]g_{p'}]\!]^{F,I,\theta} = 1$

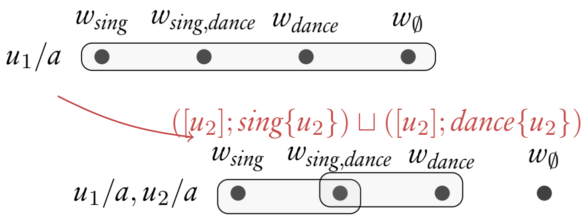
- $[u] := \lambda c_k \lambda s_i. \left\{ \begin{array}{l} \exists s' \in c. \\ \forall p \in s. \;\; \exists p' \in s'. \; (p'[u]p) \wedge \\ \forall p' \in s'. \; \exists p \in s. \;\; (p'[u]p) \end{array} \right\}$

Semantics: dref introduction



Semantics: disjunction

(9) $A_{(kk)} \sqcup B_{(kk)} := \lambda c_k . A(c) \cup B(c)$



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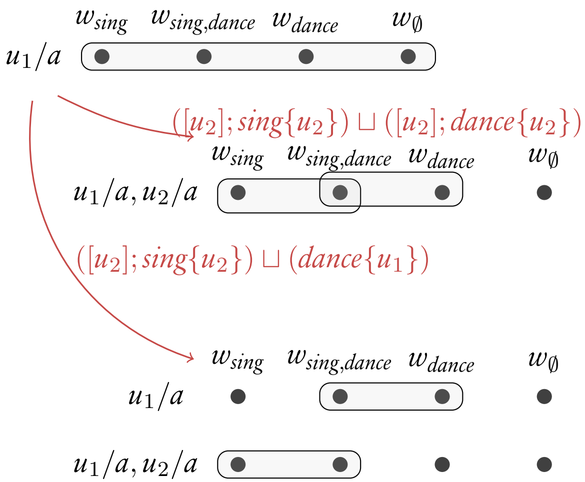

Semantics: disjunction

(10) Bill either rented a^u car or hitchhiked.
*It$_u$ was probably a cabriolet.

(11) Bill either rented a^u blue car or a^u red car.
It$_u$ was probably a cabriolet.

cf. Stone (1992)

Semantics: disjunction

(10) Bill either rented a^u car or hitchhiked.
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(11) Bill either rented a^u blue car or a^u red car.
It$_u$ was probably a cabriolet.

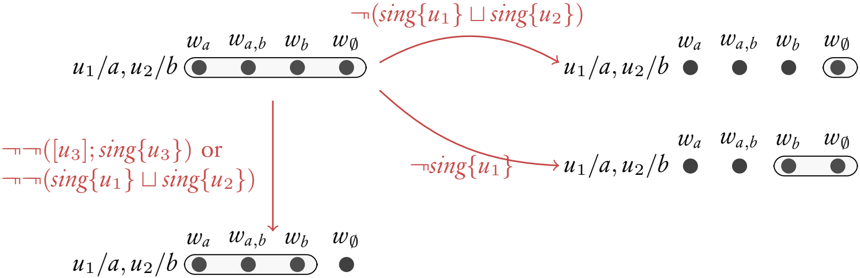
cf. Stone (1992)

(12) **A:** Bill either rented a^u car or hitchhiked.
B: The former, of course. It$_u$ was a cabriolet.

(13) **A:** Did Bill rent a^u car↑ or did he hitchhike?↓
B: The former, of course. It$_u$ was a cabriolet.

Semantics: negation

(14) $\neg A_{(kk)} := \lambda c_k \lambda s_i . s \in c \wedge \neg \exists t \subseteq s(t \neq \emptyset \wedge t \text{ subsists in } A(c))$



Semantics: discharging inquisitiveness

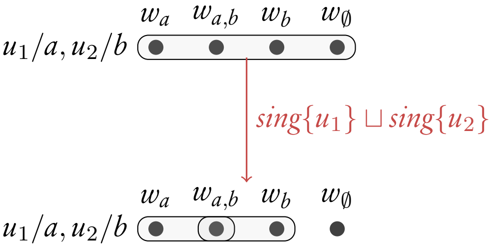
(15) $!A_{(kk)} := \lambda c_k \lambda s_I . s \in !(A(c)) \wedge \exists s' \in c(s \text{ is an extension of } s')$

- For any context C:

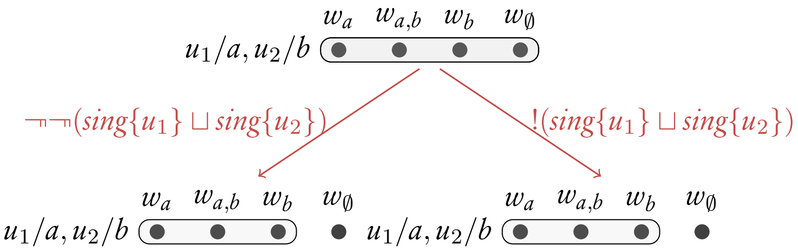
  $!C := \{\bigcup C\}^{\downarrow}$ is the non-inquisitive projection of C

- $!A$ discharges any inquisitiveness introduced by A

Semantics: negation and exclamation operator (graphical comparison)



Semantics: negation and exclamation operator (graphical comparison)



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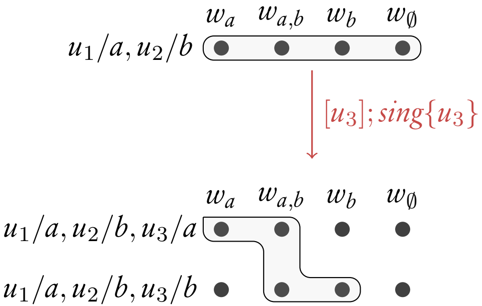

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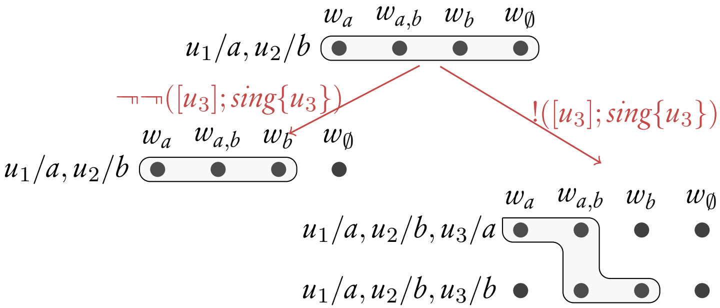
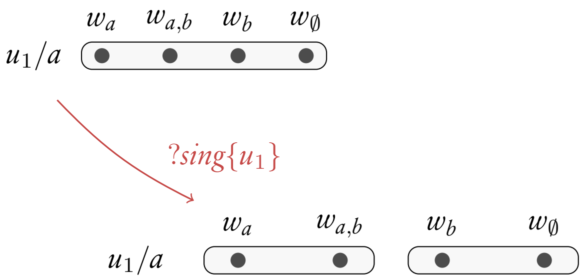


Semantics: ensuring inquisitiveness

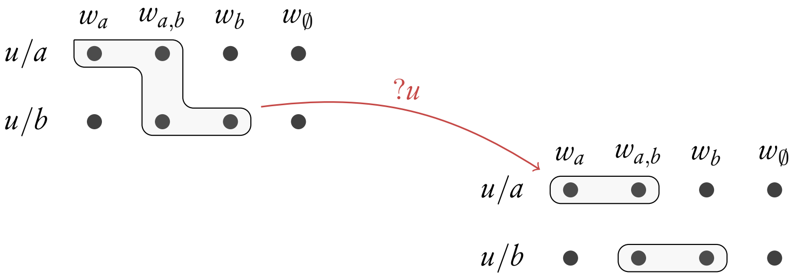
(16) $?A_{(kk)} := A \sqcup \neg A$

(17) $\langle ? \rangle A := \begin{cases} ?A & \text{if } A \text{ is not inquisitive} \\ A & \text{otherwise} \end{cases}$



Semantics: identity of drefs

(18) $?u := \lambda c_k \lambda s_i . s \in c \wedge \exists x_e . \forall p \in s . g_p(u) = x$



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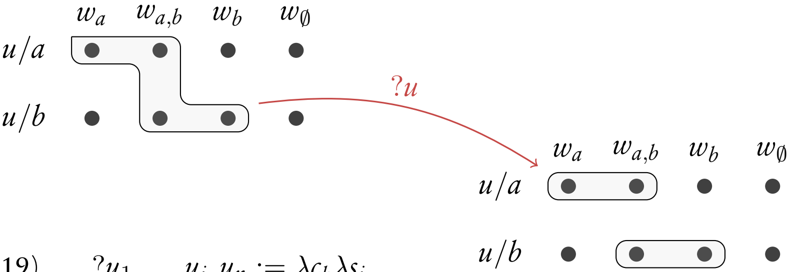
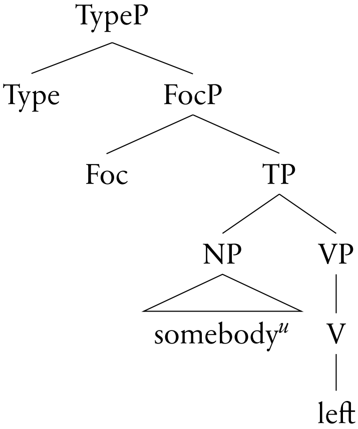


(19) $?u_1, \ldots u_j, u_n := \lambda c_k \lambda s_i .$

$$\left\{ \begin{array}{l} s \in c \ \wedge \\ \exists f .\ \forall p \in s.\ g_p(u_n) = f(g_p(u_1), \ldots g_p(u_j)) \end{array} \right\}$$

Translation of declaratives



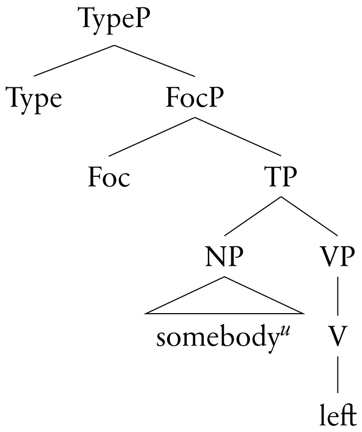
$[\![\text{Foc}]\!] = \lambda A_{(kk)}!(A)$
removes inquisitiveness of complement

$[\![\text{leave}]\!] = \lambda v_r.\mathit{leave}\{v\}$

$[\![\text{somebody}^u]\!] = \lambda P_{rT}.[u]; P(u)$

(20) $[\![[_{FocP}\text{Somebody}^u \text{ left}]]\!] =!([u]; \mathit{left}\{u\})$

Translation of declaratives

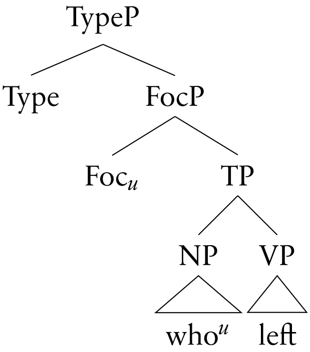


$[\![\text{Type}]\!] = \lambda A_{(kk)} !(A)$
removes inquisitiveness of complement

(21) $[\![\text{Somebody}^u \text{ left}]\!] = !!([u]; \mathit{left}\{u\})$

- (21) introduces a dref that left but (21) does not raise issues

Translation of wh examples

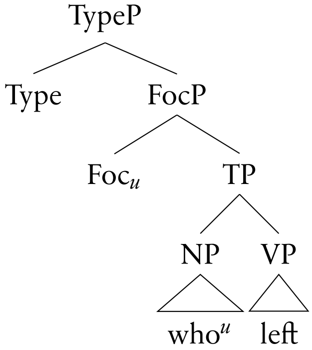


$[\![\text{Foc}_u]\!] = \lambda A_{(kk)} !(A); ?u$
removes inquisitiveness of complement
raises an issue about the identity of drefs
introduced by wh-words

$[\![\text{who}^u]\!] = \lambda P_{rT}.[u]; P(u)$

(22) $[\![[_{FocP} \text{Who}^u \text{ left}]]\!] = !([u]; \mathit{left}\{u\}); ?u$

Translation of wh examples



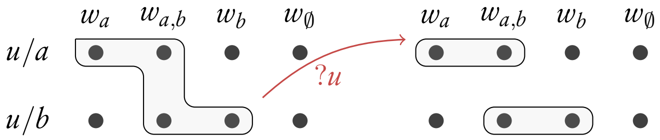
$[\![\text{Type}]\!] = \lambda A_{(kk)} \langle ? \rangle (A)$
ensures inquisitiveness of complement

(23) $[\![\text{Who}^u \text{ left}]\!] = !([u]; \mathit{left}\{u\}); ?u$

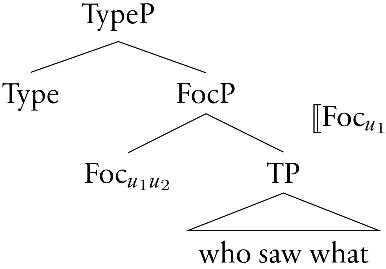
- (23) introduces a dref that left and (23) raises an issue about the identity of the dref

Translations of wh examples

(24) $[\![\text{Who left}]\!] = !([u]; \mathit{left}\{u\}); ?u$



Translations of wh examples



$$[\![\text{Foc}_{u_1,u_2}]\!] = \lambda A_{(kk)}!(A); ?u_1; ?u_1 u_2$$

(25) $[\![\text{Who saw what}]\!] = !([u_1]; [u_2]; saw\{u_1, u_2\}); ?u_1; ?u_1 u_2$

Explaining anaphora

(26) $[\![\text{Someone}^u \text{ left. He}_u \text{ was furious.}]\!]$
$= !([u]; \mathit{left}\{u\}); !(\mathit{furious}\{u\})$

(27) $[\![\text{Who}^u \text{ left? (I don't know but) he}_u \text{ was furious.}]\!]$
$= !([u]; \mathit{left}\{u\}); ?u; !(\mathit{furious}\{u\})$

- Wh-words are indefinites.
- Their inquisitiveness is introduced by the Foc head that they agree with.
- This does not affect their binding possibilities.
- Their licensing of anaphora is captured.

Explaining anaphora

(26) ⟦Someone^u left. He$_u$ was furious.⟧
$= !([u]; \mathit{left}\{u\}); !(\mathit{furious}\{u\})$

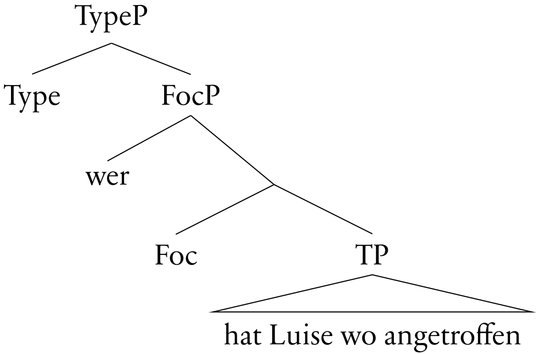
(27) ⟦Who^u left? (I don't know but) he$_u$ was furious.⟧
$= !([u]; \mathit{left}\{u\}); ?u; !(\mathit{furious}\{u\})$

- Wh-words are indefinites.
- Their inquisitiveness is introduced by the Foc head that they agree with.
- This does not affect their binding possibilities.
- Their licensing of anaphora is captured.

(28) ⟦Who^u left? Was he$_u$ furious?⟧
$= !([u]; \mathit{left}\{u\}); ?u; ?!(\mathit{furious}\{u\})$

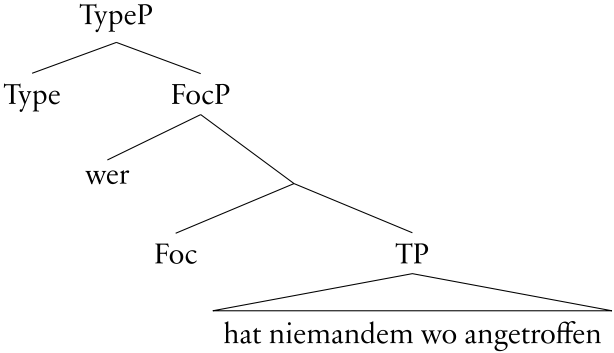
Intervention effects

- Baseline: a grammatical example



(29) ⟦ Who met Luise where? (German) ⟧
$= !([u_1]; [u_2] \wedge \mathit{meet}(u_1, L, u_2)); ?u_1; ?u_1 u_2$

Intervention effects



(30) ⟦ Who met nobody where? (German) ⟧

$$= !([u_1]; \neg([u_3] \wedge [u_2] \wedge M(u_1, u_3, u_2))); ?u_1; ?u_1 u_2$$

- Negation blocks discourse referents from projecting

Intervention effects, II

- Mayr (2014): plural quantifiers intervene only when interpreted distributively

(31) a. Wo haben sich mehr als drei Maler wann eine
where have self more than three painters when a
Pizza geteilt?
pizza shared
'Where did more than three painters share a pizza when?'

b. *Wo haben sich mehr als drei Maler wann eine
where have self more than three painters when a
Arbeitshose angezogen?
dungaree put.on

Intervention effects, II

- Mayr (2014): plural quantifiers intervene only when interpreted distributively
- This contrast is predicted in our account because only distributive readings of quantifiers block the projection of dynamic information (cf. Kamp and Reyle 1993):

(32)
a. More than three students shared a pizza. It was tasty.
b. More than three students sipped a coffee. #It was cold.

?u – ever visible?

Q-particles are ?*u*

- words that can function both as indefinites and as wh-words (quexistentials)
- a separate morpheme (a Q(uestion)-particle) obligatorily accompanies quexistentials and its syntactic position can disambiguate the reading

(33) a. Daa *(**sá**) aawaxáa i éesh?
what Q he.ate.it your father
'What did your father eat?' (Tlingit)

b. Tlél goodéi *(**sá**) xwagoot.
not where.to Q I.went
'I didn't go anywhere.' (Tlingit)

Cable (2010)

Q-particle – overt realization of the identity operator

- Q-particles can be separated from quexistentials
- Whenever a Q-particle can appear in two structurally different positions and the two positions differentiate between indefinite and wh-interrogative interpretations, then the low position must be the indefinite interpretation and the high position must be the wh-question interpretation.

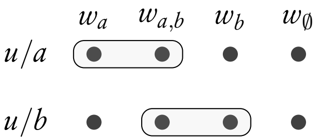
Correct for Japanese, Sinhala, Tlingit

Hagstrom (1998),Cable (2010), Uegaki (2019)

Non-exhaustive readings

Non-exhaustive readings straightforwardly captured:

(34) Who has a bike that I could borrow for 15 minutes?



Donkey anaphora in conditional questions

(35) If a farmer owns a donkey, does he beat it?

(36)
$$A_{(kk)} \twoheadrightarrow B_{(kk)} := \lambda c_k \lambda s_i. \quad s \in c \land \forall t \subseteq s \; (t \text{ subsists in } A(c) \rightarrow t \text{ subsists in } B(A(c))))$$

$$[\![(35)]\!] = ([u_1]; [u_2]; \mathit{farmer}\{u_1\}; \mathit{donkey}\{u_2\}; \mathit{own}\{u_1, u_2\}) \twoheadrightarrow ?!\mathit{beat}\{u_1, u_2\}$$

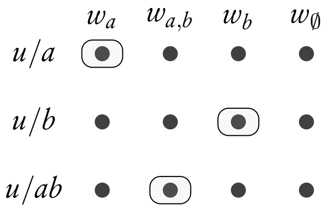
Exhaustive readings

To derive exhaustive readings, a **max** operator can be added to $\mathsf{Inq_D}$

(37) $\mathbf{max}\{u\} := \lambda c_k \lambda s_i . s \in c \wedge \forall p \in s \forall p' \in \bigcup c (w_p = w_{p'} \rightarrow g_{p'}(u) \leq g_p(u))$

(38) Who did not register yet?

(39) $!([u]; \mathit{notregistered}\{u\}; \mathbf{max}\{u\}); ?u$



Exhaustive readings and donkey anaphora

Brasoveanu (2008) derives weak / strong readings of donkey anaphora by postulating an ambiguity of indefinites (max $\Rightarrow$ strong reading)

(40) a. If a farmer owns a donkey, he beats it.
b. If Bill has a dime, he puts it in the parking meter.

While singular indefinites are compatible with both readings, plural indefinites force strong readings:

(41) If Bill has dimes, he puts them in the parking meter.

The same effect is observed with exhaustive/non-exhaustive questions:

(42) Which students have a bike that I could borrow?

Conclusion

- Inq_D integrates insights from dynamic and inquisitive semantics in a single, compositional framework
- It is well-suited to capture both exhaustive and non-exhaustive question interpretations
- It can capture the anaphoric potential of wh-words
- It can be used to derive anaphora-related intervention effects
- It can connect the existence of exhaustive and non-exhaustive readings of questions to the existence of strong and weak readings of donkey anaphora.

THANK YOU

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Appendix – lifting $\mathsf{Inq}_{\mathsf{D}}$ working with sets of assignments

Dependencies

- Dependencies between assignments within one possibility:
- express quantificational dependency

(43) Each boy saw a movie. They liked it.

(44) Which boy saw which movie? Did they like it?

Dependencies

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- express quantificational dependency

(43) Each boy saw a movie. They liked it.

(44) Which boy saw which movie? Did they like it?

- Dependencies between world and assignments:
- express uncertainty with respect to the value of the dref

(45) Someone dances.

Extension and subsistence

A possibility $\langle w', G' \rangle$ is an extension of another possibility $\langle w, G \rangle$, $\langle w', G' \rangle \geq \langle w, G \rangle$, iff $w' = w$ and $G' \geq G$.

(46) Dref matrix extension:

$$G' \geq G := \begin{cases} \forall g \in G.\ \exists g' \in G'.g \subseteq g' \wedge \\ \forall g' \in G'.\exists g \in G.\ g \subseteq g' \end{cases}$$

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(47) State extension: $s' \geq s := \ \forall p' \in s'.\exists p \in s.\ p' \geq p$

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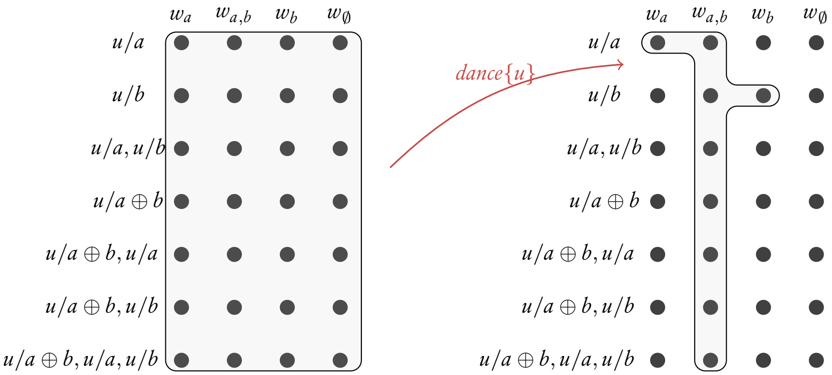
A state s subsists in a state s' iff:

- $s' \geq s$, and
- $\forall p \in s.\exists p' \in s'.\ p' \geq p$

Semantics: relations and conjunction

(48) $R\{u_1, \ldots, u_n\} := \lambda c_k \lambda s_i.\quad s \in c \wedge \forall p \in s(\forall g \in G_p\ (R(w_p)(g(u_1)) \ldots (g(u_n))))$



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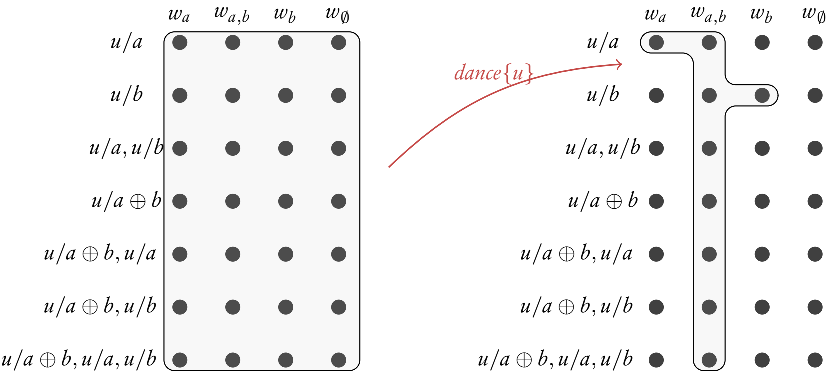

(49) $$A_{(kk)}; B_{(kk)} := \lambda c_k . B(A(c))$$

Semantics: dref introduction

- If C is a context and u not in assignments in C, then introducing the dref u creates the largest context C' such that every $s' \in C'$ has some $s \in C$ s.t. s subsists in s' and $u \in \mathbf{assigns}(s')$.

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- $[\![g[u]g']\!]^{F,I,\theta} = 1$ iff
  - $[\![\textbf{dom}(g')]\!]^{F,I,\theta} = [\![\textbf{dom}(g)]\!]^{F,I,\theta} \cup \{[\![u]\!]^{F,I,\theta}\}$ and
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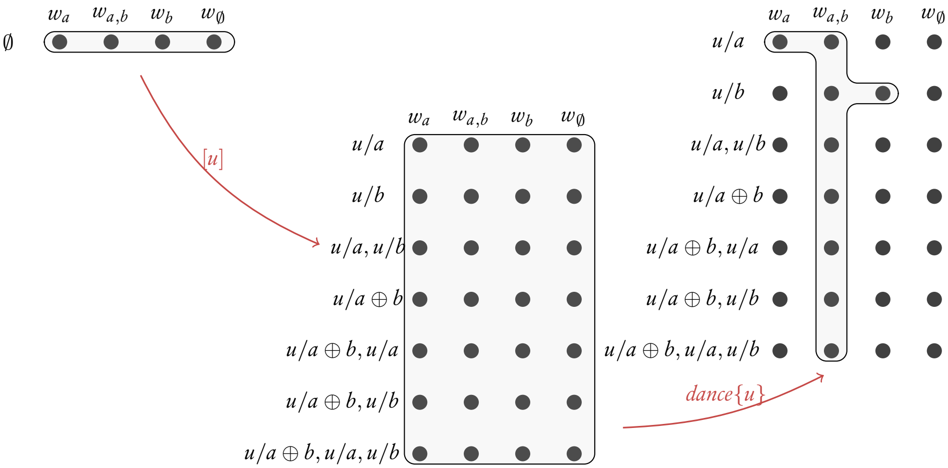
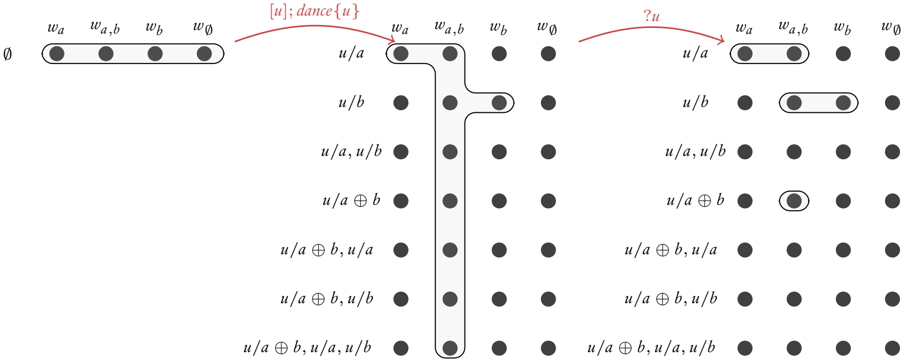
- $[u] := \lambda c_k \lambda s_i. \left\{ \begin{array}{l} \exists s' \in c. \\ \forall p \in s.\ \ \exists p' \in s'.\ (p'[u]p) \wedge \\ \forall p' \in s'. \exists p \in s.\ \ (p'[u]p) \end{array} \right\}$

Semantics: dref introduction



Semantics: identity of drefs

(50) $?u := \lambda c_k \lambda s_I . s \in c \wedge \exists x_e . \forall p \in s . \forall g \in G_p . \, x = g(u)$



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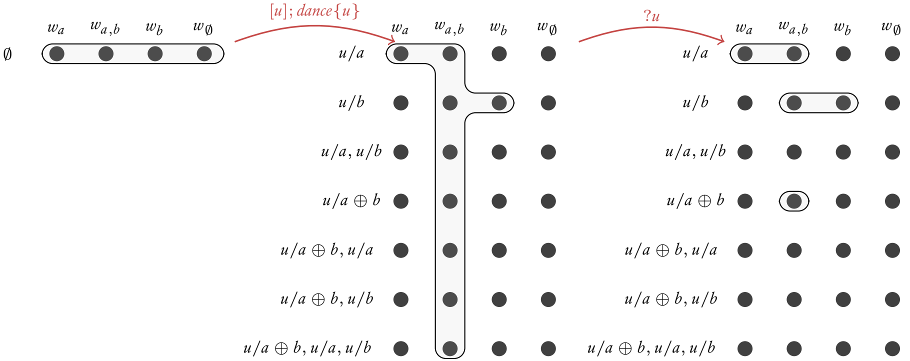
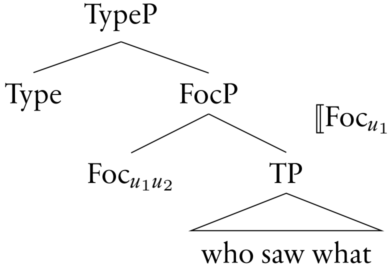


(51) $$?u_1, \ldots u_{n-1}, u_n := \lambda c_k \lambda s_I . \quad s \in c \wedge \exists f_{(e^n, e)} . \forall p \in s . \forall g \in G_p . \\ f(g(u_1), \ldots, g(u_{n-1})) = g(u_n)$$

Translations of wh examples



$$[\![\text{Foc}_{u_1,u_2}]\!] = \lambda A_{(kk)} !(A); ?u_1 u_2$$

(52) $[\![\text{Who saw what}]\!] = !([u_1]; [u_2]; saw\{u_1, u_2\}); ?u_1 u_2$

Multiple wh-questions

(53) $[\![\text{Who saw what}]\!] = !([u_1]; [u_2]; saw\{u_1, u_2\}); ?u_1 u_2$

- The identification operator derives that pair-list reading is functional
  (Higginbotham and May, 1981; Dayal, 1996)

(54) Which student talked to which professor?

a. Alice and Bill both talked to Prof. Carl.

b. #Alice talked to Prof. Carl and Prof. Dan.