Common Nouns as Modally Non-Rigid Restricted Variables

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1. Introduction

Common nouns are frequently analyzed semantically as predicates. Beginning logic students are trained to represent sentences like (1)a. using formulas like (1)b., in which the 1-place predicate M seems to correspond directly to the common noun *man*, just as the 1-place predicate S corresponds to the verb *smiles*:

(1) a. Every man smiles.
b. $\forall x[M(x) \rightarrow S(x)]$

M and S here are syntactically and semantically similar in every way: Both combine with an argument to form a formula; both are the sort of thing which can be truthfully or falsely predicated of an individual; and therefore both are naturally analyzed as denoting the set of individuals of which they can be truthfully predicated (or almost equivalently, the characteristic function of this set).

Montague (1973) treated common nouns as belonging to logical type $\langle\langle s, e\rangle, t\rangle$ — that is, as 1-place predicates of "individual concepts." Following Bennett (1975), most subsequent literature in the Montague-derived tradition (including standard textbooks such as Dowty, Wall and Peters (1981), Chierchia and McConnell-Ginet (1990), Gamut (1991) Heim and Kratzer (1998), and others) has analyzed them as belonging to type $\langle e, t\rangle$ — as 1-place predicates of individuals. In both approaches, common nouns are treated identically to intransitive verbs. Exceptions are sometimes made for relational nouns such as *mother*, *brother*, *top*, *bottom*, etc., which are then treated as being of type $\langle e, \langle e, t\rangle\rangle$; but this still gives such nouns the status of predicates — just 2-place predicates rather than 1-place.

Proper names, in contrast, are most often treated as being of type e. That is, each proper name is analyzed in such a way that it denotes some particular individual, rather than "holding true" of all the individuals in some class. This results in a sharp division of semantic function with proper names on one side but verbs and common nouns together on the other. This division is preserved even in most analyses which do not treat proper names as individual-denoting — for example by assimilating proper names to quantifiers, as in Russell's (1910) theory of proper names as "disguised definite descriptions" or in more modern treatments of names and quantifiers as second order predicates (whether of type $\langle\langle e, t\rangle, t\rangle$ or Montague's more baroque $\langle\langle s, \langle\langle s, e\rangle, t\rangle\rangle, t\rangle$).

Occasionally, proper names have been analyzed as predicates (Fara 2015, Quine 1960), but then, of course, common nouns are treated as predicates too: verbs, common nouns and proper names are all treated similarly, with no clear semantic correlate to the morphosyntactic distinction between verbs and nouns (whether proper or common).

What I will argue in this paper is that common nouns are similar in semantic type to proper names, and differ in type from verbs. The morphosyntactic distinction between nouns and verbs thus corresponds directly to a difference in semantic type. More particularly, I will argue that common nouns are *variables*, in roughly the same sense as the variables of predicate logic. *Man* is more like the *x* in (1)b. than the *M*.

This is not an entirely new idea. Lepore and Ludwig (2007: 61), for example, say "In 'All men', 'men' functions as if it were a variable restricted to taking on as values only men…" But this suggestion appears only in their informal discussion. In their formalization, common nouns are not treated this way — or analyzed at all, really: Lepore and Ludwig give formalized rules only for interpreting a simplified artificial version of English which does not contain sentences like (1)a. but only predicate-logic-like formulas such as (2):

(2) [Every x : x is a man](x smiles)

No interpretation rule is given for the single word *man* (or any other common noun), but only for the whole open formula 'x is a man'. If any semantic analysis of the single word *man* is intended, it is not made explicit; and the notation here, with separate elements 'x' and 'man', does not suggest that the noun itself is a variable, despite Lepore and Ludwig's informal discussion.

To my knowledge, the idea that common nouns are variables has never been developed or defended in detail. The idea faces a number of technical and theoretical challenges: How can we deal with relational nouns? With quantification? With intensionality? Can such an analysis be made compositional? Even if all these challenges are met, is there any *advantage* to treating common nouns as variables, or does this idea turn out to be equivalent (or inferior) to a treatment of common nouns as predicates?

I will develop a formal grammar to show how each of these challenges can be answered, and will argue that there are in fact several advantages to treating common nouns as variables, over treating them as predicates: it predicts the conservativity of nominal quantification, it simplifies the analysis of articleless languages (and articleless styles and constructions in languages that do have articles), it allows a treatment of donkey anaphora that avoids the proportion problem, it improves the analyses of the temperature paradox and of bare plurals, and it allows for a closer correlation between semantic types and morphosyntactic categories.

The presentation will be clearer, I think, if the grammar is presented in stages, and if successive stages do not merely extend the analysis of the preceding stage, but alter it. This will allow us to begin from a familiar starting point, and make clear the motivations for the various departures from it which I will suggest. But this strategy also necessitates a warning: certain features of the formalization presented in the early sections of this paper will not survive to the end, and must not be taken as part of the analysis proposed here.

2. First sketch

What does it mean to say that common nouns are variables? This will depend on exactly what a "variable" is, of course. Unfortunately, there is no universally agreed-on, standard definition of variables, so our main thesis is somewhat obscure at the outset. We shall therefore have to begin with a technique for the semantic analysis of variables which I think most readers will at least find familiar, and show how common nouns can be treated as variables using this technique. In the end, I will adopt a somewhat different approach to the semantics of variables; but starting with a more familiar technique should at least clarify the intuition underlying the claim that common nouns are variables.

In this familiar technique, expressions are assigned denotations relative to a series of parameter values, including an *assignment of values to variables*. Semantic rules are given in such a way that one can derive equations of the form in (3), where α is a linguistic expression, *g* is an assignment of values to variables, and the three dots abbreviate whatever other parameters denotations are relativized to:

(3) $[\![\alpha]\!]^{\ldots,g} = a$

Variables are expressions whose denotations are fixed directly by the assignment of values to variables. That is, α is a variable iff for all *g* (and all ways of filling in the three dots):

(4) $[\![\alpha]\!]^{\ldots,g} = g(\alpha)$

Variable *binding* is analyzed as the assignment of denotations relative to a given assignment *g* based on denotations relative to assignments which agree with *g* in what they assign to all variables other than the one being bound.[1] For example, we can define standard variable binding operators like ∀ and ∃ as in (5):

(5) a. $[\![\forall\alpha\varphi]\!]^{\ldots,g} = 1$ iff $[\![\varphi]\!]^{\ldots,h} = 1$ for all *h* agreeing with *g* on all variables other than α.
b. $[\![\exists\alpha\varphi]\!]^{\ldots,g} = 1$ iff $[\![\varphi]\!]^{\ldots,h} = 1$ for at least one *h* agreeing with *g* on all variables other than α.

With this understanding of what variables and binding are as background, let us develop a "toy" grammar for a fragment of English, in which common nouns are treated as variables. Both the syntax and semantics will be severely simplified, in order to allow a quick illustration of the basic technique.

[1] This phrasing does not imply that the relevant assignments differ from *g* in what they assign to the variable being bound. Any assignment *g* agrees with itself on all variables other than α (for any variable α).

The lexicon of this toy grammar will include the following words, with the grammatical categories listed. I use category labels in the style of Categorial Grammar, where $((X_1,\ldots,X_i)/(Y_1,\ldots,Y_j))$ is the category of an expression which combines with an expression belonging to all of categories $Y_1,\ldots,Y_j$ on its right to form an expression belonging to all of categories $X_1,\ldots,X_i$, and $((Y_1,\ldots,Y_j)\backslash(X_1,\ldots,X_i))$ is the category of an expression which combines with an expression belonging to all of categories $Y_1,\ldots,Y_j$ on its left to form an expression belonging to all of categories $X_1,\ldots,X_i$ (for any categories $X_1,\ldots,X_i$, $Y_1,\ldots,Y_j$):

(6) a. *John*, *Mary*, *Bill*, *Susan*: DP
b. *professor*, *student*: NP
c. *every*: ((DP,Q)/NP)
d. *smiles*, *frowns*: (DP\TP)
e. *sees, hears*: ((DP\TP)/DP)

Semantic interpretation will be read off a level of "LF" rather than surface representation. This is formed in the usual way, by adjoining each phrase of category Q to a phrase of category TP containing it, and leaving a trace (notated *e*) in its original position. We write 'TRACE(ξ) = ε' to mean that ε marks the original position of ξ.

By the *antecedent* of a trace, let us mean the noun phrase of the quantifier phrase whose trace it is. That is, where δ is of category ((DP,Q)/NP) and κ is of category NP:

(7) ANTECEDENT(TRACE(δκ)) = κ

Note that this is a slightly non-standard use of the term *antecedent*. In a sentence like [[*every professor*][*e sees John*]], the antecedent of *e* is *professor*, not [*every professor*].

It will be important in what follows that denotations are assigned to *occurrences* of linguistic expressions, rather than to linguistic expressions as abstract types.[2] This will allow the two occurrences of *professor* in *Every professor sees every professor* to vary independently of one another, for example. Let us write 'OCCURRENCE(α, A)' to mean that α is an occurrence of expression A, and 'CATEGORY(α, C)' to mean that α is an occurrence of an expression belonging to category C.

In this preliminary grammar, we will not relativize denotations to any parameters other than assignments of values to variables. We define an *assignment of values to variables* as follows:

(8) *g* is an assignment of values to variables iff *g* is a function whose domain is included in {α | CATEGORY(α, NP) or CATEGORY(ANTECEDENT(α), NP)} such that:
a. If α ∈ DOMAIN(*g*) and OCCURRENCE(α, *professor*) then *g*(α) is a professor;

[2] Ultimately, denotations should probably be assigned to "uses" of expressions (where uses may be modeled as expression-context pairs); or equivalently, denotations should be assigned to expressions relative to contexts, with each constituent in an uttered sentence interpreted relative to its own separate context. For this technique, see Lasersohn (2017). We forego using it here mainly in the interests of simplicity, and because indexicality is not a central concern of this paper.

b. If $\alpha \in$ DOMAIN(g) and OCCURRENCE(α, *student*) then $g(\alpha)$ is a student.

Note that traces may be assigned values by an assignment of values to variables, but there are no restrictions on those values analogous to the restrictions illustrated in (8)a. and b. for common nouns.

By a *variable* let us (for now) mean a common noun or trace. The denotations of variables (that is, their contributions to the semantic composition) will simply be the values they receive from the assignments:

(9) For all assignments of values to variables g: if α is in the domain of g, then $[\![\alpha]\!]^g = g(\alpha)$.

To illustrate, let **professor** be an occurrence of *professor* and **student** an occurrence of *student*.[3] Assume that John and Mary are professors, and Bill and Susan are students. Let g_1 be a function which maps **professor** onto John and **student** onto Bill, and let g_2 be a function which maps **professor** onto Mary and **student** onto Susan. Then $[\![\textbf{professor}]\!]^{g_1}$ = John, $[\![\textbf{professor}]\!]^{g_2}$ = Mary, $[\![\textbf{student}]\!]^{g_1}$ = Bill, and $[\![\textbf{student}]\!]^{g_2}$ = Susan. Notice that the denotation of a common noun as assigned by these rules is an individual, not a set of individuals or a function mapping individuals to truth values.

Proper names and verbs are assigned denotations via simple stipulations:[4]

(10) For all assignments of values to variables g:

a. If OCCURRENCE(α, *John*) then $[\![\alpha]\!]^g$ = John;
b. If OCCURRENCE(α, *Mary*) then $[\![\alpha]\!]^g$ = Mary;
c. If OCCURRENCE(α, *Bill*) then $[\![\alpha]\!]^g$ = Bill;
d. If OCCURRENCE(α, *Susan*) then $[\![\alpha]\!]^g$ = Susan;
e. If OCCURRENCE(α, *smiles*) then $[\![\alpha]\!]^g = [\lambda x : x$ is an individual . x smiles];
f. If OCCURRENCE(α, *frowns*) then $[\![\alpha]\!]^g = [\lambda x : x$ is an individual . x frowns];
g. If OCCURRENCE(α, *sees*) then $[\![\alpha]\!]^g = [\lambda x : x$ is an individual . $[\lambda y : y$ is an individual . y sees x]];
h. If OCCURRENCE(α, *hears*) then $[\![\alpha]\!]^g = [\lambda x : x$ is an individual . $[\lambda y : y$ is an individual . y hears x]].

These expressions are assigned denotations relative to assignments of values to variables, even though they contain no variables. The relativization is a "third wheel" — technically present, but with no semantic effect.

[3] Here and throughout I use boldface to indicate occurrences and italics to indicate expressions. Both typefaces are also used for various other purposes; I trust the context will make the clear how to interpret them.

[4] For the moment, we ignore tense in giving the semantics of verbs.

It will be useful to assign logical types to expressions. We will use standard Montagovian types,[5] limiting ourselves for now to extensional types:

(11) a. e is a type;
b. t is a type;
c. For any σ, τ: if σ is a type and τ is a type, then $\langle\sigma, \tau\rangle$ is a type.

Let us assume that individuals (such as John, Mary, Bill and Susan) form a set **D**. We assign a *denotation domain* $\mathbf{D}_\sigma$ based on **D** to each type σ in the usual way:

(12) a. $\mathbf{D}_e = \mathbf{D}$
b. $\mathbf{D}_t = \{0, 1\}$
c. For any types σ, τ: $\mathbf{D}_{\langle\sigma, \tau\rangle} = \{f \mid f \text{ is a function from } \mathbf{D}_\sigma \text{ to } \mathbf{D}_\tau\}$

Now for any expression occurrence α, we may identify its type by its denotations:

(13) α is of type σ iff for all assignments of values to variables g, $[\![\alpha]\!]^g \in \mathbf{D}_\sigma$.

It should be evident that our rules so far treat occurrences of *John*, *Mary*, *Bill*, *Susan*, *man* and *woman* as all of type e. In contrast, occurrences of *smiles* and *frowns* are of type $\langle e, t\rangle$, and occurrences of *sees* and *hears* are of type $\langle e, \langle e, t\rangle\rangle$.

It will be useful to have notation to indicate that one assignment of values to variables agrees with another on all arguments other than some particular common noun occurrence κ, and any trace which is anaphoric to κ:

(14) For all assignments of values to variables g, h and all κ such that CATEGORY(κ, NP):
$g \sim_\kappa h$ iff
a. There exists some x such that $h(\kappa) = x$, and for all ε such that ANTECEDENT(ε) = κ, $h(\varepsilon) = x$; and
b. for all $\nu \in$ DOMAIN(g), if $\nu \neq \kappa$ and ANTECEDENT(ν) ≠ κ, then $h(\nu) = g(\nu)$.

In this "first pass" formalization, we will not assign denotations to occurrences of the quantifier *every*, or to quantifier phrases, but instead will give a rule for whole sentences in which *every* appears, in (15). In its current form, this rule is in some sense non-compositional; but we will replace it with a compositional rule in Section 9.

(15) If OCCURRENCE(δ, *every*), CATEGORY(κ, NP) and CATEGORY(φ, TP), then for all assignments of values to variables g: $[\![\delta\ \kappa\ \varphi]\!]^g =$
1 if $\forall h\,[g \sim_\kappa h \rightarrow [\![\varphi]\!]^h = 1]$;
0 if $\exists h[g \sim_\kappa h\ \&\ [\![\varphi]\!]^h = 0]$.

The rule in (15) may be equivalently reformulated as in (16). This formulation is less compact, but will facilitate later revisions.

[5] Montagovian type labels produce an unfortunate notational clash, since e is used both for traces and for the type of expressions which denote individuals. I trust the context will make clear which is intended.

(16) If OCCURRENCE(δ, *every*), CATEGORY(κ, NP) and CATEGORY(φ, TP), then for all assignments of values to variables g: $[\![\delta\ \kappa\ \varphi]\!]^g =$
1 if $\forall x \in \mathbf{D}_e[\exists h\,[g \sim_\kappa h\ \&\ h(\kappa) = x] \rightarrow \exists h\,[g \sim_\kappa h\ \&\ h(\kappa) = x\ \&\ [\![\varphi]\!]^h = 1]]$;
0 if $\exists x \in \mathbf{D}_e\, \exists h[g \sim_\kappa h\ \&\ h(\kappa) = x\ \&\ [\![\varphi]\!]^h = 0]$.

Subject-predicate sentences are assigned truth values relative to assignments by function application:

(17) If CATEGORY(α, DP) and CATEGORY(β, (DP\TP)), then for all assignments of values to variables g: $[\![\alpha\beta]\!]^g = [\![\beta]\!]^g([\![\alpha]\!]^g)$.

Verb phrases consisting of a transitive verb and a term phrase also receive their denotations by simple function application:

(18) If CATEGORY(β, ((DP\TP)/DP)) and CATEGORY(α, DP), then for all assignments of values to variables g: $[\![\beta\alpha]\!]^g = [\![\beta]\!]^g([\![\alpha]\!]^g)$.

Finally, we define truth and falsity as truth and falsity relative to the empty assignment ∅, as in Heim and Kratzer (1998):

(19) If CATEGORY(φ, TP) then:
a. φ is *true* iff $[\![\varphi]\!]^\emptyset = 1$; and
b. φ is *false* iff $[\![\varphi]\!]^\emptyset = 0$.

It may be helpful to work through an example. Let **Every professor e smiles** be an occurrence of the sentence *Every professor e smiles*. Suppose John, Mary, Bill and Susan are all the individuals there are. As before John and Mary are the professors, and Bill and Susan are the students. John, Bill and Mary all smile, but Susan does not.

(20) a. $[\![\textbf{Every professor e smiles}]\!]^\emptyset = 1$ iff for $\forall x \in \mathbf{D}_e\, [\exists h[\emptyset \sim_{\textbf{professor}} h\ \&\ h(\textbf{professor}) = x] \rightarrow \exists h[\emptyset \sim_{\textbf{professor}} h\ \&\ h(\textbf{professor}) = x\ \&\ [\![\textbf{e smiles}]\!]^{\mathrm{h}} = 1]]$.
b. There is an h such that $\emptyset \sim_{\textbf{professor}} h$ and $h(\textbf{professor})$ = John, namely g_1, where g_1 is that function with domain {**professor**, **e**} such that $g_1(\textbf{professor})$ = John, $g_1(\textbf{e})$ = John.
c. There is also an h such that $\emptyset \sim_{\textbf{professor}} h$ and $h(\textbf{professor})$ = Mary, namely g_2, where g_2 is that function with domain {**professor**, **e**} such that $g_2(\textbf{professor})$ = Mary and $g_2(\textbf{e})$ = Mary.
d. Because John and Mary are the only two professors, they are the only individuals for which such assignments exist.
e. Since John and Mary both smile, $[\![\textbf{e smiles}]\!]^{g_1} = 1$ and $[\![\textbf{e smiles}]\!]^{g_2} = 1$.
f. Therefore $[\![\textbf{Every professor e smiles}]\!]^\emptyset = 1$.

3. Conservativity

Our grammar is far from its final form, but we can already informally identify one advantage to analyzing common nouns as variables, over analyzing them as predicates: If common nouns are analyzed as variables, the conservativity of nominal quantification falls out as an automatic consequence. In contrast, if common nouns are analyzed as predicates, conservativity must be independently stipulated as a constraint on determiner meanings.

This claim requires some clarification. Standard, textbook-style definitions of conservativity generally presuppose a predicational analysis of common nouns, and are formulated essentially as in (21):

(21) Where D is a 2-place relation between sets, D is *conservative* iff for all A, B: $D(A, B)$ iff $D(A, A \cap B)$.

Here D is the denotation of a determiner, each value of A is understood as the set denoted by some common noun with which the determiner combines, and each value of B is the set of things satisfying the scope of the quantifier in some sentence. If we give up the idea that common nouns are predicates, we should not expect them to denote sets, and it becomes much less natural to expect that determiners will denote 2-place relations on sets as in (21). But if determiners do not denote such relations, then they will *never* be conservative in the sense defined in (21).

In saying that an analysis of common nouns as variables predicts conservativity, therefore, I do *not* mean that it predicts that all determiners will be conservative in exactly the sense of (21).

To evaluate this claim, we need a more general notion of conservativity, which is not tied to one particular approach to the analysis of nouns and determiners. A more useful way to conceptualize conservativity for our current purposes is to recognize that the intuitive content of (21) is that A functions as a domain of quantification. That is, in ascertaining whether $D(A, B)$, one need not consider those members of B which are not in A. Put differently (and more sloppily and English-specifically): in determining the truth value of a sentence of the form D N VP, one only need consider the N's: Which N's does VP apply to and which N's doesn't VP apply to? One never needs to consider the truth value that results from applying VP to something which isn't an N.

With this intuitive understanding of (21), we can generalize: In any semantic analysis which assigns truth conditions, whether it treats common nouns as predicates, or variables, or in some other way, some distinction must be drawn between the things which a given noun accurately describes and those which it doesn't. If common nouns are analyzed as denoting sets, these are the members of the set denoted by the noun; if common nouns are analyzed as denoting functions of type $\langle e, t\rangle$, these are the things mapped onto 1 by the function denoted by the noun; if common nouns are analyzed as variables, these are the things assigned to the noun by the various assignments of values to variables. No matter which approach is taken, a theory claims that determiner quantification is conservative iff whenever a determiner-noun combination combines with a predicate π to form a sentence, the truth value of that sentence can be ascertained by considering only how π applies to the things accurately described by the noun, so that the truth values which result from applying π to things *not* accurately described by the noun are irrelevant. This characterization is still very informal, but I trust the idea is clear enough for present purposes.

Informally, we can already see how conservativity falls out from the general approach to determiner quantification sketched in Section 2. Assignments of values to variables, in this approach, are functions from *common nouns* (and traces) to individuals. For any such function, the individual assigned to a given noun N is something which "is an N" — something which would be a member of the extension of N in a more conventional analysis. As one considers a class of assignments which differ at most in what they assign to N (and any trace anaphoric to N), therefore, one is effectively considering the things which are accurately described by the noun. As long as object-language quantification over individuals is analyzed in terms of metalanguage quantification over assignments of values to variables, and as long as determiner-noun combinations are interpreted by quantifying over those assignments which agree on all variables other than the noun with which the determiner combines (and its anaphors), conservativity is automatic.

To give just a little more detail: Let us assume that interpretation rules for quantifiers conform to the general template in (22), based on the rule for *every* in (16):

(22) If CATEGORY(δ, ((DP,Q)/NP)), category(κ, NP) and CATEGORY(φ, TP), then for all assignments of values to variables g: $[\![\delta\kappa\varphi]\!]^g =$
1 if for δ-many assignments of values to variables h such that $g \sim_\kappa h$, $[\![\varphi]\!]^h = 1$;
0 if it is not the case that for δ-many assignments of values to variables h such that $g \sim_\kappa h$, $[\![\varphi]\!]^h = 1$.

Rules conforming to this template derive a truth value for $[\![\delta\kappa\varphi]\!]^g$ based on $[\![\varphi]\!]^h$, for various assignments h which differ from g at most in what they assign to κ (and any trace with κ as its antecedent). But h(κ) will always be something which is accurately described by κ, and so will h(**e**), where ANTECEDENT(**e**) = κ. There simply *are* no assignments of values to variables h such that $g \sim_\kappa h$ which assign **e** a value which is not accurately describable by κ, so it makes no sense to ask whether the truth value of φ is relative to such assignments is relevant to the truth value of the whole sentence δκφ. Quantification is just over those individuals which are accurately described by the noun, which is to say, the quantification is conservative — and this follows from the general architecture of the theory.

In contrast, if we assume simply that determiners denote 2-place relations between sets (or functions of type $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$), nothing guarantees conservativity; it must be independently stipulated.[6]

This account of conservativity is in some ways reminiscent of existing accounts which attempt to derive conservativity from the "Copy Theory" of syntactic movement (Fox 1999, 2002, Romoli 2015, Sauerland 2004). These analyses differ from one another in important ways, and we cannot review them all here; but the fundamental idea in all cases is that the source node for syntactic movement is interpreted as though some of the moved material were still positioned there — as might be expected if syntactic movement is simply a copying operation followed by deletion. This idea is preserved in our analysis here: binding involves quantification over assignments in which the trace is interpreted identically to its antecedent noun. We would obtain identical results if a copy of the noun were left in the source position, rather than a trace *e*.[7]

Most attempts to derive conservativity from the Copy Theory of movement do so by interpreting the source node as a definite description, whose NP is identical to that of the DP moved by QR.[8] For example, (23)a. is interpreted as (23)b., where [the dog]$_1$ is a bindable variable carrying a presupposition that its value is a dog.[9]

(23) a. [every dog]$_1$[[every dog]$_1$ barks]
b. [every dog]$_1$[[the dog]$_1$ barks]

The usual idea, then, is that if the determiner were not conservative, this kind of structure would give a semantically trivial (hence pragmatically anomalous) reading; or — as Romoli argues at length — a reading which could be equivalently expressed by a conservative quantifier.

The analysis in the current paper is similar enough to these others that perhaps it too could be taken as supporting the copy theory of movement.[10] But it differs in taking common nouns as variables of type *e*, rather than as predicates of type $\langle e, t\rangle$ which take such variables as arguments. The result is that as one quantifies over assignments which

[6] See Keenan and Stavi (1986) for a proof that only a proper subset of the set of functions of this type are conservative.

[7] For that matter, we would obtain the same semantic results if the noun phrase were simply left in situ, with QR moving only the determiner, provided we had some way of encoding which determiners were associated with which noun phrases. We explore this possibility briefly in Section 11, but set it aside here in order to keep the focus on semantic matters rather than syntax.

[8] As far as I know, no one argues that the source and target positions for syntactic movement should be interpreted identically, which would be the expectation under the simplest (some might say the naïvest) version of the Copy Theory.

[9] To make this idea more precise, let $[\![\mathit{the}\,\alpha_i]\!]^g$ be defined only in the case where $[\![\alpha]\!]^g(g(i)) = 1$, in which case $[\![\mathit{the}\,\alpha_i]\!]^g = g(i)$. See Sauerland (2004: 67). Romoli (2015) employs the slightly different option of interpreting the lower copy as an open formula whose predicate is the NP, and conjoining this formula with one representing the rest of the clause.

[10] Which is not to say that I personally would advance such an argument.

stand in the "$\sim_\kappa$" relation to a given assignment *g*, no cases which would violate conservativity can arise. It is not that such cases would be semantically anomalous, or equivalent to cases in which the determiner is conservative; there simply *are no such cases*.

Building conservativity this deeply into the semantics of quantification raises a potential objection: What about *only*, which is often cited as an example of a non-conservative quantifier, or other potential counterexamples such as the "reverse proportional" readings of *many* and *few* (Westerståhl 1985)?

In considering such cases, it is helpful to recognize that under the explanation offered here, conservativity is the result of two main features of the analysis: 1. Object-language quantification is analyzed in terms of metalanguage quantification over assignment functions, particularly over assignment functions standing in the "$\sim_\kappa$" relation to the assignment of evaluation; and 2. In the "$\sim_\kappa$" relation, the variable κ is the common noun with which the quantifier is syntactically combined. Two ways of explaining apparent counterexamples to conservativity therefore suggest themselves: 1. Some object-language quantification may not be interpreted via metalanguage quantification over assignment functions; and 2. Some quantification may bind variables other than the noun with which the quantifier is syntactically combined.

I think the first of these strategies is probably a less productive avenue to explore. Certainly there are ways of formalizing the semantics of quantification without using assignment functions,[11] but because these have a similar expressive power to techniques that do employ assignment functions, it seems reasonable to expect that they will allow a treatment of common nouns which is analogous in relevant ways to the one suggested here, and which therefore also predicts the conservativity of nominal quantification.

The second strategy, on the other hand, seems quite likely to yield an explanatory account of apparent counterexamples. It ties apparent counterexemplification directly to the status of bound variables as nouns or not. If a quantifier binds some other variable than a noun, we should expect on this second strategy that it will give the appearance of a counterexample to conservativity.

How can we tell whether the variable bound by a quantifier is a noun? It is useful in thinking about this question to look at the relation between restricted and unrestricted quantification in predicate logic, and the relation between restricted quantifiers in predicate logic and English quantification in the analysis offered in Section 2 above. It is an elementary fact that predicate logic formulas with unrestricted quantifiers such as (24)b., representing (24)a., can be equivalently reformulated using restricted quantifiers as in (24)c.

[11] See the variable-free semantics developed and promoted by Polly Jacobson in a long series of works (Jacobson 1992, Jacobson 1993, Jacobson 1999, Jacobson 2000, Jacobson 2007, Jacobson 2008), or substitutional quantification in the manner of Frege (1879), etc.

(24) a. Every professor smiles.
b. $\forall x[\text{PROFESSOR}(x) \rightarrow \text{SMILE}(x)]$
c. $(\forall x : \text{PROFESSOR}(x))\text{SMILE}(x)$

We interpret formulas like c. using the rule in (25), so that the truth value of the whole formula depends on the truth values of SMILE(x) only relative to those assignments which also satisfy PROFESSOR(x) — the formula immediately after the colon in effect restricts what the variable can have as a value.

(25) $[\![(\forall x : \varphi)\psi]\!]^g = 1$ iff for all h such that $[\![\varphi]\!]^h = 1$ and h agrees with g on all variables other than x, $[\![\psi]\!]^h = 1$.

In the semantics for English advocated here, variables are also restricted, but the restriction is "built in" rather than given by a separate restriction clause present in the syntax. Each occurrence of the noun *professor*, for example, is a variable which is restricted to having professors as its values, simply by the lexical semantics of the variable itself. This is analogous to replacing (24)c. with a formula like (26), where p is a variable restricted to professors, not because of a separate clause in the formula, but because of a general rule in the language for how to interpret p:

(26) $\forall p\, \text{SMILE}(p)$

This gives us a heuristic for identifying which cases of quantification involve variables which are nouns, and which involve some other kind of variable. If the truth conditions of a sentence can be represented in predicate logic using restricted quantification, does the restriction clause correspond to a noun,[12] or to something else?

Now we can see that in apparent counterexamples to conservativity, the variable is *not* the noun with which the quantifier combines. Sentences with *only* may be translated into predicate logic on the model illustrated in (27), using an unrestricted quantifier as in (27)b., or equivalently, a restricted quantifier as in (27)c.:

(27) a. Only professors smile.
b. $\forall x[\text{SMILE}(x) \rightarrow \text{PROFESSOR}(x)]$
c. $(\forall x\text{: SMILE}(x))\text{PROFESSOR}(x)$

The crucial point is that in c., the restriction clause on the quantifier does not correspond to the noun *professor*, but rather to the predicate *smile* (or perhaps to the clause [*e smile*], where *e* is a trace left by quantifier raising). In a framework where variables come with "built-in" restrictions, then, the variable bound by *only* would be one restricted to smiling things — not the noun *professor*.

[12] Or to a complex noun phrase. These will be addressed in Section 6.

To make things more precise, we may introduce a variant on our tilde notation as in (28), and give a rule for *only* like (29):[13]

(28) For all assignments of values to variables g, h and all traces ε: $g \sim_{\varepsilon,\varphi} h$ iff $[\![\varphi]\!]^h = 1$ and for all $\nu \in$ DOMAIN(g), if $\nu \neq \varepsilon$ then $h(\nu) = g(\nu)$.

(29) If OCCURRENCE(α, *only*), CATEGORY(κ, NP) and CATEGORY(φ, TP), then for all assignments of values to variables g: $[\![\alpha\ \kappa\ \varphi]\!]^g =$
1 if $\forall h\ [g \sim_{\text{trace}(\alpha\kappa),\varphi} h \rightarrow \exists i[h \sim_\kappa i\ \&\ h(\varepsilon) = i(\kappa)]]$;
0 if $\exists h[g \sim_{\text{trace}(\alpha\kappa),\varphi} h\ \&\ \forall i[h \sim_\kappa i \rightarrow h(\varepsilon) \neq i(\kappa)]]$.

According to this rule, *only* involves a kind of "double shift" in variable values: the main shift is in the value of the trace, restricting attention to values which satisfy the clause from which the *only*-phrase is extracted. That is, this clause — which serves as the *scope* of quantifiers like *every* — serves instead as the *restriction* with *only*. The second shift is the more usual shift in the value of the noun, with the sentence true iff for every such shift of the trace, there is a way of shifting the noun so that their values are equal. For example **Only professors e smile** is true iff for every way of fixing a value to **e** which satisfies **[e smile]**, there is a way of fixing the same value to **professor**.

Does this make *only* a counterexample to conservativity or not? There can be no answer without giving a much more precise and formal definition of conservativity than we have been using, and then the answer will depend on the choices we make in formulating the definition. I think the more helpful thing to note in this case is simply that the semantics for *only* is not stated exclusively by quantifying on possible values for *noun* with which it combines, but rather by quantifying on possible ways of satisfying the *clause* from which the *only*-phrase is extracted.[14] That is, quantification with *only* is in some sense not really *nominal* quantification at all. In our approach, which appeals directly to the semantics of nouns in explaining the conservativity of quantification with determiners like *every*, it should be no surprise that a non-nominal quantifier like *only* would behave differently.

This point deserves a little more elaboration, because conservativity is traditionally explained as a lexical restriction on determiners, rather than as something to do with the analysis of nouns. The classical account of the exceptional behavior of *only* is simply that it is not a determiner, hence not subject to the same lexical restriction ((Barwise and Cooper 1981)). But this cannot be the whole story, because adverbial quantification also normally appears to be conservative (von Fintel 1994):

(30) a. Dogs always like frisbees ≡ Dogs are always dogs that like frisbees

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[13] This rule is given just as an illustration of the technique. It is not intended as a complete analysis of *only*, which can appear in a much wider variety of syntactic contexts than the rule presupposes, and often selects its variable based on intonational focus or other matters. We do not at this point bother to give all the rules necessary so that our grammar will generate sentences with plural nouns and verbs, even though (27)a. involves both of these; see Section 7.

[14] Put more technically, it appeals to the "$\sim_{\varepsilon,\varphi}$" relation, not just the "$\sim_\kappa$" relation.

b. Dogs usually like frisbees ≡ Dogs are usually dogs that like frisbees
c. Dogs never like frisbees ≡ Dogs are never dogs that like frisbees

But in cases like those in (30), it is important to note that the quantification is restricted to values of a noun, just like in determiner quantification — and unlike quantification with *only*. As soon as we take the step of analyzing these examples by letting the adverbs bind the common noun *dog*, conservativity will fall out from the general approach advocated here.

Should we then expect that in cases of quantification not involving nouns, non-conservativity should be common? Once again, the answer will depend on the formal details of what we mean by *conservativity*. If all we mean by conservativity is that quantification is restricted, then even *only* is conservative: the restriction just comes from its clausal argument rather than its nominal argument.[15] But if by *conservativity* we mean something closer to the traditional notion, so that quantifiers do not count as conservative simply because they are restricted, then the current framework does predict that in all cases of non-conservative quantification, the variable being bound is something other than a noun.[16]

4. Modally non-rigid variables

The semantics for common nouns given in Section 2 faces some apparent problems, once we extend our grammar to include intensional contexts. To see these problems, let us assume a possible-worlds approach to intensionality. In particular, let us relativize denotation assignment not just to assignments of values to variables, but also to possible worlds. We now write $[\![\alpha]\!]^{w,g}$ for the denotation of α relative to world w and assignment g. We revise our type system to include types beginning with s, adding (31)a. to (11) and (31)b. to (12). Here, W is the set of all possible worlds.

(31) a. For any σ, if σ is a type, then $\langle s, \sigma\rangle$ is a type.
b. For all types σ, $\mathbf{D}_{\langle s, \sigma\rangle} = \{f \mid f \text{ is a function from } W \text{ to } \mathbf{D}_\sigma\}$

Adding relativization to worlds is really only useful if our language includes intensional constructions, so let us add some clausal complement verbs. For any individual

[15] Put differently, *only* may be thought of in a traditional Generalized Quantifier framework as denoting the subset relation, just like *every*:

(i) ONLY(A, B) iff A ⊆ B

but as choosing differently which part of the syntactic tree corresponds to A, and which to B. On this view, ONLY is conservative, since ⊆ is conservative.

[16] This leaves open the deeper and more difficult question of whether all natural language quantification is restricted, and if so, why? Any attempt to deal with this question must address the sizable philosophical literature on the issue of whether the idea of completely unrestricted quantification is even coherent. (See Rayo and Uzquiano (2007) for a sampling.)

x and world w, let $\mathbf{B}_{x,w}$ be the set of worlds compatible with x's belief state in w, and $\mathbf{K}_{x,w}$ the set of worlds compatible with x's knowledge state in w:

(32) a. *believes*, *knows*: ((DP\TP)/CP)
b. *that*: (CP/TP)

(33) For all $w \in W$ and all assignments of values to variables g:
a. If OCCURRENCE(α, *believes*) then $[\![\alpha]\!]^{w,g} = [\lambda p : p \in \mathbf{D}_{\langle s,t\rangle} . [\lambda x : x \in \mathbf{D}_e . \forall w' \in W[w' \in \mathbf{B}_{x,w} \rightarrow p(w') = 1]]$;
b. If OCCURRENCE(α, *knows*) then $[\![\alpha]\!]^{w,g} = [\lambda p : p \in \mathbf{D}_{\langle s,t\rangle} . [\lambda x : x \in \mathbf{D}_e . \forall w' \in W[w' \in \mathbf{K}_{x,w} \rightarrow p(w') = 1]]$

We interpret the combination of a complementizer and a sentence as denoting the intension of the sentence. Verb phrases containing clausal complement verbs and a complementizer phrase are interpreted by simple function application:

(34) a. If CATEGORY(α, CP/TP) and CATEGORY(φ, TP), then $[\![\alpha\varphi]\!]^{w,g} = [\lambda w' : w' \in W . [\![\varphi]\!]^{w',g} = 1]$.
b. If CATEGORY(β, ((DP\TP)/CP) and CATEGORY(ψ, CP), then for all worlds w and assignments of values to variables g, $[\![\beta\ \psi]\!]^{w,g} = [\![\beta]\!]^{w,g}([\![\psi]\!]^{w,g})$.

Our existing semantic rules will also need some revision. Different individuals smile in different possible worlds (and likewise for frowning), so the denotations of verbs will vary with the world index (cf. (10)e.–h.) Here and throughout, I indicate changes to our rules and definitions by highlighting them in red:

(35) For all possible worlds w and all assignments of values to variables g:
a. If OCCURRENCE(α, *smiles*) then $[\![\alpha]\!]^{w,g} = [\lambda x : x \in \mathbf{D}_e . x$ smiles in $w]$;
b. If OCCURRENCE(α, *frowns*) then $[\![\alpha]\!]^{w,g} = [\lambda x : x \in \mathbf{D}_e . x$ frowns in $w]$;
c. If OCCURRENCE(α, *sees*) then $[\![\alpha]\!]^{w,g} = [\lambda x : x \in \mathbf{D}_e . [\lambda y : y \in \mathbf{D}_e . y$ sees x in $w]]$;
d. If occurrence(α, *hears*) then $[\![\alpha]\!]^{w,g} = [\lambda x : x \in \mathbf{D}_e . [\lambda y : x \in \mathbf{D}_e . y$ hears x in $w]]$

Although proper names are rigid designators, whose denotations do not normally vary from world to world, it will be helpful to the smooth operation of the formalism to include a world index in the rules assigning their denotations:

(36) For all possible worlds w and all assignments of values to variables g:
a. If OCCURRENCE(α, *John*) then $[\![\alpha]\!]^{w,g} =$ John;
b. If OCCURRENCE(α, *Mary*) then $[\![\alpha]\!]^{w,g} =$ Mary;
c. If OCCURRENCE(α, *Bill*) then $[\![\alpha]\!]^{w,g} =$ Bill;
d. If OCCURRENCE(α, *Susan*) then $[\![\alpha]\!]^{w,g} =$ Susan.

Our compositional rules also need to be updated to include a world index. As a first guess, let us simply add a w superscript just as we did with proper names, but not give it any role in the operation of the rules themselves. For all worlds w and all assignments of values to variables g:

(37) If OCCURRENCE(α, *every*), CATEGORY(κ, NP) and CATEGORY(φ, TP), then for all $w \in W$ and all assignments of values to variables g: $[\![\alpha\ \kappa\ \varphi]\!]^{w,g} =$
1 if $\forall x \in \mathbf{D}_e[\exists h\,[g \sim_\kappa h\ \&\ h(\kappa) = x] \rightarrow \exists h\,[g \sim_\kappa h\ \&\ h(\kappa) = x\ \&\ [\![\varphi]\!]^{w,h} = 1]]$;
0 if $\exists x \in \mathbf{D}_e\, \exists h[g \sim_\kappa h\ \&\ h(\kappa) = x\ \&\ [\![\varphi]\!]^{w,h} = 0]$.

(38) If CATEGORY(α, DP) and CATEGORY(β, (DP\TP)), then for all $w \in W$ and all assignments of values to variables g: $[\![\alpha\beta]\!]^{w,g} = [\![\beta]\!]^{w,g}([\![\alpha]\!]^{w,g})$.

(39) If CATEGORY(β, ((DP\TP)/DP)) and CATEGORY(α, DP), then for all $w \in W$ and all assignments of values to variables g: $[\![\beta\alpha]\!]^{w,g} = [\![\beta]\!]^{w,g}([\![\alpha]\!]^{w,g})$.

Truth and falsity relative to a world are defined in terms of truth and falsity relative to a world and assignment. Truth and falsity (*tout court*) are defined as truth and falsity relative to the actual world $w_@$:

(40) If CATEGORY(φ, TP) then:
a. φ is *true relative to w* iff $[\![\varphi]\!]^{w,\emptyset} = 1$;
b. φ is *false relative to w* iff $[\![\varphi]\!]^{w,\emptyset} = 0$;
c. φ is *true* iff φ is true relative to $w_@$;
d. φ is *false* iff φ is false relative to $w_@$.

Now, what about common nouns? The simplest way to adapt our rules to a possible worlds framework would be to simply add a world index to our rule in (9):[17]

(41) For all $w \in W$ and all assignments of values to variables g: if α is in the domain of g, then $[\![\alpha]\!]^{w,g} = g(\alpha)$.

The implications of (41) will depend on how we understand (8), the definition of an assignment of values to variables for this grammar; but under the most natural interpretation, (8)and (41) will interact to give wrong results.

As formulated, (8) says that g(**professor**) is a professor and g(**student**) is a student (for any occurrence **professor** of *professor*, and occurrence **student** of *student*, included in the domain of any assignment of values to variables g). But whether or not a particular individual is a professor or a student can vary from world to world, and (8) does not explicitly say in which world g(**professor**) is a professor and g(**student**) is a student. As (40)c. indicates, we normally take a sentence to be true (*tout court*) iff it is true in the *actual* world. Of course, (40)c. is about our object language, but we may interpret our metalanguage according to the same principle. If no other world is explicitly mentioned in a statement, the conditions expressed by that statement must obtain in the actual world if the statement is to qualify as true. Under this interpretation, (8) will give the effect that g(**professor**) is a professor in $w_@$ and g(**student**) is a student in $w_@$ (provided **professor**, **student** $\in$ DOMAIN(g)). With this understanding of (8), we can now see that according to

[17] This is, in fact, exactly how variables are typically treated in intensional logic; $[\![x]\!]^{w,g} = g(x)$, so the denotation of x varies with g but not with w. See Montague (1973: 231), and the inclusion of variables in the class of "intensionally closed expressions" in Gamut (1991: 130).

(41), for any w it will be the case that $[\![$**professor**$]\!]^{w,g}$ is a professor in $w_@$ — but nothing requires $[\![$**professor**$]\!]^{w,g}$ to be a professor in w (and similarly for **student**).

In effect, our rules treat common nouns as rigid designators. This is *not* the result we want. As may be easily confirmed, it gives the effect that **Some professor e smiles** is true relative to w iff there is at least one individual who is a professor in $w_@$ who smiles in w. Hence, **John believes that some professor e smiles** will be true in $w_@$ iff for all worlds w compatible with John's belief state in $w_@$, someone who is a professor in $w_@$ smiles in w. Perhaps the sentence does have such a reading, but this is not the only reading — or even, I think, the most prominent reading — of the sentence. The sentence can also mean that in all worlds w compatible with John's belief state in $w_@$, someone who is a professor in w smiles in w. Under the semantics for common nouns just sketched, this reading cannot be derived.

The source of this problem is that (8) defines an assignment of values to variables simply as a function from occurrences of common nouns (and traces) to individuals, with no sensitivity to worlds. If we are to maintain the idea of common nouns as variables, but allow common nouns not to be rigid designators, we need to revise (8) to allow for *modally non-rigid variables*:[18]

(42) g is an assignment of values to variables iff g is a function with domain included in $\{\alpha \mid \text{CATEGORY}(\alpha, \text{NP}) \text{ or } \text{CATEGORY}(\text{ANTECEDENT}(\alpha), \text{NP})\} \times W$ such that:

a. for all $w \in W$, if OCCURRENCE(α, *professor*) and $\langle \alpha, w\rangle$ is in the domain of g, then $g(\alpha, w)$ is a professor in w; and

b. for all $w \in W$, if OCCURRENCE(α, *student*) and $\langle \alpha, w\rangle$ is in the domain of g, then $g(\alpha, w)$ is a student in w.

Revising our definition of assignments of values to variables this way necessitates some revisions to our other rules, to make them compatible with the fact that assignments now take a world argument in addition to a variable argument. We replace (9) with (43):

(43) For all assignments of values to variables g and $w \in W$: if $\langle \alpha, w\rangle$ is in the domain of g, then $[\![\alpha]\!]^{w,g} = g(\alpha, w)$.

We also must revise our "$\sim_\kappa$" notation (c.f. (14)). Note that traces, unlike common nouns, are kept modally rigid in assignments occupying the right-hand argument place of the $\sim_{\kappa,w}$ relation:

(44) For all assignments of values to variables g, h, all κ such that CATEGORY(κ, N) and all $w \in W$: $g \sim_{\kappa,w} h$ iff

a. there exists some x such that $h(\kappa, w) = x$, and for all ε such that ANTECEDENT(ε) $= \kappa$ and all $w' \in W$, $h(\varepsilon, w') = x$; and

[18] Modally non-rigid variables have rarely been suggested before, but see Hughes and Cresswell (1968: 195–201).

b. for all $\nu \in$ DOMAIN(g) and all $w' \in W$, if $\nu \neq \kappa$ and ANTECEDENT(ν) $\neq \kappa$, then $g(\nu, w') = h(\nu, w')$.

Now we must revise our *every* rule (c.f. (37)):

(45) If OCCURRENCE(δ, *every*), CATEGORY(κ, NP) and CATEGORY(φ, TP), then for all $w \in W$ and all assignments of values to variables g: $[\![\delta\ \kappa\ \varphi]\!]^{w,g} =$
1 if $\forall x \in \mathbf{D}_e[\exists h[g \sim_{\kappa,w} h\ \&\ h(\kappa, w) = x] \rightarrow \exists h[g \sim_{\kappa,w} h\ \&\ h(\kappa, w) = x\ \&\ [\![\varphi]\!]^{w,h} = 1]]$;
0 if $\exists x \in \mathbf{D}_e\, \exists h[g \sim_{\kappa,w} h\ \&\ h(\kappa, w) = x\ \&\ [\![\varphi]\!]^{w,h} = 0]$.

The operation of these rules is straightforward and gives the expected results. It is easily confirmed, for example, that under these rules, **[John believes that [every professor [e smiles]]]** is true (in $w_@$) iff in every world w compatible with John's belief state (in $w_@$), every individual x such that x is a professor in w smiles in w.

At this point it may be interesting also to consider bound variable pronouns, as in the reading of (46)a. which corresponds to the formula in (46). We add *he*, *she*, *him* and *her* to our lexicon in (47), listing them as belonging to category PRONOUN, as well as appropriate case categories:

(46) a. Every professor believes that she smiles.
b. ∀x[professor(x) → believe(x, smile(x))]

(47) a. *he*, *she*: PRONOUN, NOM
b. *him*, *her*: PRONOUN, ACC

We are concerned here only with anaphoric pronouns, not indexical pronouns, so let us assume that for each pronoun occurrence π, there some noun phrase occurrence κ such that ANTECENDENT(π) = κ.

The introduction of case-marked items into our grammar forces us to do some minor "clean up" of our earlier rules, to keep everything consistent. To assure that the correct case forms appear in the correct positions, we need to revise the categorization of verbs:

(48) a. *smiles*, *frowns*: (NOM\TP)
b. *sees, hears*: ((NOM\TP)/ACC)
c. *believes*, *knows*: ((NOM\TP)/CP)

Proper names and determiners must now be assigned to case categories as well. Since in English no overt case is marked, we simply assign them to both cases:

(49) a. *John*, *Mary*, *Bill*, *Susan*: NOM, ACC
b. *every*: ((NOM, ACC, Q)/NP)

This eliminates the category DP from our syntactic rules, which in turn forces a revision of some of our semantic rules:

(50) a. If CATEGORY(α, NOM) and CATEGORY(β, (NOM\TP)), then for all assignments of values to variables g: $⟦\alpha\beta⟧^{w,g} = ⟦\beta⟧^{w,g}(⟦\alpha⟧^{w,g})$.

b. If CATEGORY(β, ((NOM\TP)/ACC)) and CATEGORY(α, ACC), then for all assignments of values to variables g: $⟦\beta\alpha⟧^{w,g} = ⟦\beta⟧^{w,g}(⟦\alpha⟧^{w,g})$.

Now we can turn to the interpretation of pronouns. Bound pronouns are interpreted like the traces marking the surface positions of quantifier phrases. We let pronouns, like traces, count as variables. Now consider what truth conditions our rules assign to (46)a., assuming ANTECEDENT(**she**) = **professor**:

(51) a. $⟦$**Every professor e believes that she smiles**$⟧^{w_@,g} = 1$ iff $\forall x \in D_e[\exists h[g \sim_{\textbf{professor}, w_@} h \,\&\, h(\textbf{professor}, w_@) = x] \rightarrow \exists h[g \sim_{\textbf{professor}, w_@} h \,\&\, h(\textbf{professor}, w_@) = x \,\&\,$ $⟦$**e believes that she smiles**$⟧^{w_@,h} = 1]]$;

b. $⟦$**e believes that she smiles**$⟧^{w_@,h} = 1$ iff $\forall w \in W[w \in \textbf{B}_{h(\textbf{e}, w_@), w_@} \rightarrow ⟦$**she smiles**$⟧^{w,h} = 1]$;

c. $⟦$**she smiles**$⟧^{w,h} = 1$ iff $h(\textbf{she}, w)$ smiles in w;

d. For all $w' \in W$, $h(\textbf{she}, w') = h(\textbf{she}, w)$, by (44)a.;

e. Hence $⟦$**she smiles**$⟧^{w,h} = 1$ iff $h(\textbf{she}, w_@)$ smiles in w;

f. $h(\textbf{she}, w_@) = h(\textbf{professor}, w_@)$.

That is, for every x such that x is a professor in $w_@$, in every world w compatible with x's belief state in $w_@$, x smiles in w — the correct truth condition.

5. The definite and indefinite articles

In this section we expand our grammar to include the articles *the* and *a*. It is a matter of debate whether these should be treated in the same way as quantificational determiners such as *every*. If so, the technique developed above may be applied to them. But I think it is also worth showing how our current system can accommodate alternative treatments. A variety of alternative analyses have been suggested; we cannot review all of them, of course, but as an illustration, I will sketch one way of treating phrases with *the* and *a* as similar in semantic type to proper names and pronouns, rather than quantificational phrases with *every*.

The first step is to add *the* and *a* to our lexicon:

(52) *the*, *a*: ((NOM, ACC)/NP)

Let us suppose that a phrase of the form *the* κ simply denotes the unique pragmatically relevant object which is accurately described by κ, if there is one; and fails to denote otherwise. A preliminary rule giving this effect is given in (53):[19]

[19] To be fully explicit about the relativity of uniqueness to pragmatic context, we would need to relativize denotation assignment to contexts; but I forego that here in the interest of simplicity.

(53) If OCCURRENCE(δ, *the*) and CATEGORY(κ, NP), then for all $w \in W$ and all assignments g:
$[\![\delta\ \kappa]\!]^{w,g} = g(\kappa, w)$, provided that for all assignments h such that $g \sim_{\kappa,w} h$, it holds that $g(\kappa, w) = h(\kappa, w)$. (If there exists some w-assignment h such that $g \sim_{\kappa,w} h$ but $g(\kappa, w) \neq g(\kappa, w)$, then $[\![\textit{the}\ \kappa]\!]^{w,g}$ is undefined.)

Note that this rule treats the whole phrase as identical in denotation to its noun (when it has a denotation at all). Since the noun is a variable, the whole phrase is also effectively a variable.

Indefinites may be treated similarly, but dropping the uniqueness condition. It has been clear since Lewis (1975) that indefinites are bindable (by adverbs of quantification and perhaps by other operators as well). Hence they must contribute something like a free variable to the sentences in which they occur.

(54) If OCCURRENCE(δ, *a*) and CATEGORY(κ, NP), then for all $w \in W$ and all assignments g:
$[\![\delta\ \kappa]\!]^{w,g} = g(\kappa, w)$

The question now arises why sentences like *A professor smiled* are interpreted as existentially quantified. Let us assume that this is due to a general principle of existential closure, roughly as in Heim (1982).[20] We define truth or falsity for "texts" (sequences of sentence-occurrences) and as a first attempt, revise our truth definition in (40) to (55), using existential quantification instead of truth relative to the empty assignment:

(55) If $\varphi_1,\ldots,\varphi_n$ is a text, then:
a. $\varphi_1,\ldots,\varphi_n$ is *true relative to w* iff there is an assignment g such that $[\![\varphi_1]\!]^{w,g} = 1,\ldots, [\![\varphi_n]\!]^{w,g} = 1$;
b. $\varphi_1,\ldots,\varphi_n$ is *false relative to w* iff there is no assignment g such that $[\![\varphi_1]\!]^{w,g} = 1,\ldots, [\![\varphi_n]\!]^{w,g} = 1$;
c. $\varphi_1,\ldots,\varphi_n$ is *true* iff $\varphi_1,\ldots,\varphi_n$ is true relative to $w_@$;
d. $\varphi_1,\ldots,\varphi_n$ is *false* iff $\varphi_1,\ldots,\varphi_n$ is false relative to $w_@$.

We may account for intersentential anaphora to indefinite antecedents simply by allowing for the case where *antecedent*(α) = κ and κ is a noun attached to the indefinite article rather than a quantifier. The text in (56)a. receives a reading which is truth-conditionally equivalent to (56)b., in the usual Kamp-Heim fashion:[21]

(56) a. A professor smiles. She frowns.

[20] The alternative would be to assume that the existential quantification is contributed by the indefinite article, and that indefinites are bindable due to an operation of existential *dis*closure as in Dekker (1993). But our stated purpose in this section is to explore the semantics of definites and indefinites on the assumption that the articles are *not* quantifiers.

[21] This is not to say that the English text is equivalent to the first-order formula in its compositional structure, of course. Under our analysis, *professor* is not treated as a predicate, as it is in the formula, nor does the English text contain a syntactically represented operator analogous to $\exists$.

b. $\exists x$[professor(x) & smile(x) & frown(x)]

In addition to whole texts, existential closure must take place on the restriction and scope of each quantifier; but let us delay consideration of this until after we have rules in place for noun complementation and modification.

An interesting feature of this analysis is that both definites and indefinites are of the same logical type as their nouns. This gives us a plausible explanation for why so many languages lack definite and/or indefinite articles: bare nouns are already of the right type (type *e*) to serve as arguments to a verb, even without an article. In the Latin sentence in (57), for example, the noun *librum* may serve directly as an argument to the verb *scrībō*:

(57) Librum scrībō
book.acc write.1sg.pres.indic
"I write a/the book"

In contrast, in an analysis in which common nouns are of type $\langle e, t\rangle$, we must appeal to hidden determiners, or type-shifting operations, or something similar, in order to derive an expression of type *e* from the type $\langle e, t\rangle$ noun, so that it can fill the type *e* argument place of the verb. A very high proportion of languages do not have overt articles — very likely the majority. In light of this, it seems unnatural to have to treat nouns as unsuitable to serve as arguments to verbs unless some hidden structure or special operation is first applied to them. Even in languages like English, where articles are obligatory (at least with singular count nouns) in most stylistic registers, they are easily dropped in special styles such as recipes and instructions, headlines, tweets, etc. By treating nouns as variables of type *e* rather than as predicates of type $\langle e, t\rangle$, we have a reasonable explanation for why articles are so easily omitted.

6. Compositionally defined assignments

Noun complementation and modification present an apparent challenge for the view that common nouns are variables. In a conventional semantics, where common nouns are predicates which take variables as arguments, rather than being variables themselves, noun complementation and modification present no particular difficulties: nouns which take complements may be analyzed as multiplace predicates, and noun modification may be modeled as the compositional derivation of a complex predicate. For example, *Every friend of John smiles* may be represented as in (58)a., with *friend* as a 2-place predicate; and *Every happy professor smiles* as in b., reduced from c., where the complex predicate $\lambda y[\mathit{happy}(y)\ \&\ \mathit{professor}(y)]$ is straightforwardly constructible from *happy* and *professor*:

(58) a. $\forall x[\mathit{friend}(x, j) \rightarrow \mathit{smile}(x)]$
b. $\forall x[[\text{happy}(x)\ \&\ \text{professor}(x)] \rightarrow \text{smile}(x)]$

c. $\forall x[\lambda y[\text{happy}(y)\ \&\ \text{professor}(y)](x) \rightarrow \text{smile}(x)]$

But how are we to deal with such examples if *friend* and *professor* are not predicates, but variables? If *friend* is a variable, can it take *j* as an argument? Or should we treat the whole complex phrase *friend of John* as a variable? — and if so, how can we assign denotations to variables compositionally? Likewise, the whole complex phrase *happy professor* must presumably be analyzed as playing a similar role in the semantic composition as the simple noun *professor*. If the simple noun is a variable, does that mean the complex phrase must be one too? If so, how do we derive its semantics compositionally?

The first of these questions — whether a variable can take an argument — is the simplest to answer. Of course variables can take arguments, if we allow function variables, as in second-order logic or other higher-order logics such as Montague's IL. We may analyze relational nouns like *friend* as variables of type $\langle e, e\rangle$, for example. Just as we restrict assignments of values to variables so that for any such assignment g, $g($**professor**$, w)$ is a professor in w, we restrict them so that for any g, $g($**friend**$, w)$ is some function $f \in \mathbf{D}_{\langle e, e\rangle}$ such that for all $x \in \mathbf{D}_e$, x is a friend of $f(x)$ in w.

But now we face a complication, which becomes apparent as soon as we consider traces or pronouns bound by quantifiers attached to phrases like *friend of John*. In a sentence like **Every friend of John e believes that she sees him**, we presumably should not treat **e** and **she** as being of type $\langle e, e\rangle$. That is, we should not analyze **e** and **she** as "occurrences of the same variable" as **friend**, taking the same values. Rather these variables must range over friends of John — that is, over values for the whole phrase **friend of John**, not over values of **friend.** This suggests that the whole phrase *friend of John* should be treated as a variable.

The idea that a complex phrase, whose semantics is determined compositionally, could be a variable seems at first rather odd. I don't believe I have ever encountered a semantic theory in which complex phrases, and not just lexical items, receive their denotations from assignments of values to variables. Such assignments are not generally conceived as providing their values compositionally, but rather as arbitrary mappings from a distinguished class of basic expressions (the "variables") to values of the appropriate types. But in fact, there is no technical obstacle to compositional assignment of values to variables, nor, as far as I can tell, any conceptual difficulty in this idea. Just as the function represented by the double brackets $[\![\cdot]\!]$ may be defined compositionally, so may an assignment of values to variables g.

To illustrate, let us add *friend* and *of* to our lexicon:

(59) *friend*: NP/PP

(60) *of*: PP/ACC

We let *of* denote the identity function in type $\langle e, e\rangle$:

(61) If OCCURRENCE(α, *of*) then for all $w \in W$ and all assignments g: $[\![\alpha]\!]^{w,g} = [\lambda x : x \in \mathbf{D}_e \,.\, x]$

Now we revise our definition of *w*-assignments from (42) to (62), adding clauses c. and d:

(62) Where $w \in W$, g is an assignment of values to variables iff g is a partial function with domain $\{\alpha \mid$ CATEGORY(α, NP) or CATEGORY(ANTECEDENT(α), NP)$\} \times W$ such that:

a. for all $w \in W$, if OCCURRENCE(α, *professor*) and $\langle \alpha, w \rangle$ is in the domain of g, then $g(\alpha, w)$ is a professor in w;

b. for all $w \in W$, if OCCURRENCE(α, *student*) and $\langle \alpha, w \rangle$ is in the domain of g, then $g(\alpha, w)$ is a student in w;

c. for all $w \in W$, and all $x \in \mathbf{D}_e$, if OCCURRENCE(α, *friend*) and there exists a y such that $y = g(\alpha, w)(x)$, then $g(\alpha, w)(x)$ is a friend of x in w;

d. if CATEGORY(α, NP/PP) and CATEGORY(β, PP), then for all $w \in W$: $g(\alpha\beta, w) = g(\alpha, w)([\![\beta]\!]^{w,g})$.

Our rules will now handle sentences like *Every friend of John e believes that she smiles*. To illustrate, let us consider how the truth value of an occurrence of this sentence is assigned relative to the actual world $w_@$, and an arbitrary assignment g, assuming ANTECEDENT(**she**) = **friend of John**:

(63) a. $[\![$**Every friend of John e believes that she smiles**$]\!]^{w_@,g} = 1$ iff $\forall x \in \mathrm{D_e}[\exists h\, [g \sim_{\text{friend of John}, w_@} h \,\&\, h(\textbf{friend of John}, w_@) = x] \rightarrow [\exists h\, [g \sim_{\text{friend of John}, w_@} h \,\&\, h(\textbf{friend of John}, w_@) = x \,\&\, [\![\textbf{e believes that she smiles}]\!]^{w_@,h} = 1]]]$;

b. iff $\forall x \in \mathrm{D_e}[\exists h\, [g \sim_{\text{friend of John}, w_@} h \,\&\, h(\textbf{friend of John}, w_@) = x] \rightarrow [\exists h\, [g \sim_{\text{friend of John}, w_@} h \,\&\, h(\textbf{friend of John}, w_@) = x \,\&\, \forall w \in W[w \in \mathbf{B}_{h(\mathbf{e}, w_@), w_@} \rightarrow h(\textbf{she}, w)$ smiles in $w]]]$.

c. For any assignment h: $h(\textbf{friend of John}, w_@) = h(\textbf{friend}, w_@)([\![\textbf{of John}]\!]^{w_@,h})$.

d. $[\![\textbf{of John}]\!]^{w_@,h} = [\![\textbf{John}]\!]^{w_@,h} =$ John.

e. For any $y \in \mathrm{D}_e$, $h(\textbf{friend}, w_@)(y)$ is a friend of y in $w_@$.

f. So, $h(\textbf{friend of John}, w_@)$ is a friend of John in $w_@$.

g. Given that $g \sim_{\text{friend of John}, w_@} h$, it is the case that for all $w \in W$, $h(\mathbf{e}, w_@) = h(\textbf{she}, w) = h(\textbf{friend of John}, w_@)$.

h. So, $[\![$**Every friend of John e believes that she smiles**$]\!]^{w_@,g} = 1$ iff $\forall x \in \mathbf{D}_e[x$ is a friend of John in $w_@ \rightarrow \forall w \in W[w \in \mathbf{B}_{x, w_@} \rightarrow x$ smiles in $w]]$.

It should be noted that in this example, **e**, **she**, and **friend of John** all covary, as though they were "occurrences of the same variable" — even though **friend of John** is a complex phrase whose denotation is determined compositionally from its parts.

Modified nouns may be handled in much the same way; we simply define assignments of values to variables so that they compositionally assign values to whole complex noun phrases, and not just to lexical nouns. As an illustration, let us add the relative pronoun *who* to our grammar:

(64) *who*: R, PRONOUN, NOM, ACC

To allow relative clauses to attach to the nouns they modify, let us assume the "Comp" position of a relative clause is filled by a hidden operator, notated *Rel*:

(65) *Rel*: ((NP\NP)/TP)

Just as we required any phrase of category Q to adjoin to a clause containing it in the derivation of LF, let us now assume a level of PF and require phrases of category R to adjoin to a c-commanding occurrence of *Rel* in the derivation of PF. We count as ungrammatical any sentence whose PF contains an occurrence of *Rel* to which a phrase of category R has not been adjoined. At LF, relative pronouns remain *in situ*.

Rather than adopting an analysis in which relative pronouns act as lambda-binders (as in most of the post-Montague literature), I will here adopt an analysis closer to that of traditional grammar, in which relative pronouns, like personal pronouns, stand in an antecedence relation to the nouns which their clauses modify, and are interpreted as coreferring with those nouns. (Nothing in the rest of the analysis turns on this choice, however.) Where ρ is an occurrence of a relative pronoun (that is where OCCURRENCE(ρ, R)), ρ must appear as part of a relative clause attached to a noun phrase occurrence κ; in this case let us require that ANTECEDENT(ρ) = κ. We consider relative pronouns to be pronouns, hence variables, which receive values from assignments of values to variables. By the definition of a the $\sim_{\kappa,w}$ relation, they will covary with their antecedents under quantification.

Relative clauses, as clauses, may now be treated as type *t*, with truth values like other clauses.

We define the complementizer *Rel* as follows:

(66) For all $w \in W$ and all assignments of values to variables *g*, if OCCURRENCE(ρ, *Rel*), then $[\![\rho]\!]^{w,g} = [\lambda p : p \in \mathbf{D}_t \;\&\; p = 1 \,.\, [\lambda x : x \in \mathbf{D}_e \,.\, x]]$

We also need a rule for interpreting the combination of *Rel* with a clause. This is done by simple function application:

(67) If OCCURRENCE(ρ, *Rel*) and CATEGORY(φ, TP), then for all $w \in W$ and all assignments of values to variables *g*: $[\![\rho\ \varphi]\!]^{w,g} = [\![\rho]\!]^{w,g}([\![\varphi]\!]^{w,g})$.

Finally, we add a clause to the definition of assignments values to variables, to make sure that they assign values to combinations of a noun and a relative clause:

(68) Where $w \in W$, *g* is an assignment of values to variables iff *g* is a partial function with domain {α | CATEGORY(α, NP) or CATEGORY(ANTECEDENT(α), NP)} × *W* such that:

a. for all $w \in W$, if OCCURRENCE(α, *professor*) and $\langle \alpha, w \rangle$ is in the domain of g, then $g(\alpha, w)$ is a professor in w;
b. for all $w \in W$, if OCCURRENCE(α, *student*) and $\langle \alpha, w \rangle$ is in the domain of g, then $g(\alpha, w)$ is a student in w;
c. for all all $w \in W$, and all $x \in \mathbf{D}_e$, if OCCURRENCE(α, *friend*) and there exists a y such that $y = g(\alpha, w)(x)$, then $g(\alpha, w)(x)$ is a friend of x in w;
d. if CATEGORY(α, NP/PP) and CATEGORY(β, PP), then for all $w \in W$: $g(\alpha\beta, w) = g(\alpha, w)([\![\beta]\!]^{w,g})$;
e. if CATEGORY(κ, NP) and CATEGORY(ψ, NP\NP), then for all $w \in W$: $g(\kappa, \psi) = [\![\psi]\!]^{w,g}([\![\kappa]\!]^{w,g})$.

Under these rules, the combination of a noun with a relative clause has the same denotation relative to an assignment g as the noun does, on the condition that the relative clause is true relative to g; otherwise the whole construction fails to denote relative to g.

As a quick example, consider the phrase *professor who sees Susan*. At LF, this phrase will be represented as [*professor* [*Rel* [*who sees Susan*]]]]. Suppose that in $w_@$, John and Mary are the professors, that John sees Susan and Mary does not see anyone. Suppose g_1(**professor**, $w_@$) = John:

(69) a. g_1(**professor Rel who sees Susan**, $w_@$) = [[**Rel who sees Susan**]]$^{w_@,g_1}$([[**professor**]]$^{w_@,g_1}$)
b. = [[**Rel who sees Susan**]]$^{w_@,g_1}$(g_1(**professor**, $w_@$))
c. = [[**Rel who sees Susan**]]$^{w_@,g_1}$(John)
d. = $[\lambda p : p \in \mathbf{D}_t \,\&\, p = 1 \,.\, [\lambda x : x \in \mathbf{D}_e \,.\, x]]$([[**who sees Susan**]]$^{w_@,g_1}$)(John)
e. = $[\lambda p : p \in \mathbf{D}_t \,\&\, p = 1 \,.\, [\lambda x : x \in \mathbf{D}_e \,.\, x]]$([[**sees**]]$^{w_@,g_1}$([[**Susan**]]$^{w_@,g_1}$)([[**who**]]$^{w_@,g_1}$))(John)
f. = $[\lambda p : p \in \mathbf{D}_t \,\&\, p = 1 \,.\, [\lambda x : x \in \mathbf{D}_e \,.\, x]]$([[**sees**]]$^{w_@,g_1}$(Susan)($g_1$(**who**)))(John)
g. = $[\lambda p : p \in \mathbf{D}_t \,\&\, p = 1 \,.\, [\lambda x : x \in \mathbf{D}_e \,.\, x]]$([[**sees**]]$^{w_@,g_1}$(Susan)(John))(John)
h. = $[\lambda p : p \in \mathbf{D}_t \,\&\, p = 1 \,.\, [\lambda x : x \in \mathbf{D}_e \,.\, x]](1)$(John)
i. = $[\lambda x : x \in \mathbf{D}_e \,.\, x]$(John)
j. = John

It should be evident that relative to a different assignment g_2, where g_2(**professor**) = Mary, and Mary does not see Susan, the whole phrase **professor Rel who sees Susan** will not receive a value. (That is to say, the phrase will not be in the domain of g_2.) This is the result we want; as a variable, **professor Rel who sees Susan** should range over professors who see Susan, and Mary is not among them.

Our rules will need some adjustment in order to give the correct results when a complex noun phrase contains an indefinite. Consider an example like **Every friend of a professor e smiles**, for example. Suppose that in $w_@$, John and Mary are professors. John has exactly one friend: Susan. Mary has exactly one friend: Bill. The phrase **friend of a professor** should range over Susan and Bill. But suppose g_1(**professor**) = John. In this case whenever $g_1 \sim_{\textbf{friend of a professor}, w_@} h$, it will be the case that h(**friend of a professor**, $w_@$) = Susan, and never that h(**friend of a professor**, $w_@$) = Bill. The result is that **Every friend of a professor e smiles** will be assigned a value of 1 relative to g_1 and $w_@$ iff Susan smiles, without requiring Bill to smile. Because truth is defined in terms of existential closure, the sentence will be counted as true iff Susan smiles or Bill smiles, but is not assigned a reading which requires both of them to smile. This is incorrect.[22]

I think the source of this problem is our definition of the "$\sim_{\kappa, w}$" relation. Our current definition sticks too close to the traditional approach, which was developed without any envisaged application to variables whose values are assigned compositionally. Intuitively, a complex variable should range over all the different values it can have by varying the values of its parts — holding constant only those which are bound from the outside. In finding values of **friend of a professor**, for example, we vary the function denoted by **friend** and the individual denoted by **professor** so that the whole phrase ranges over the set of all those individuals x such that there is a professor of whom x is a friend. But in [**Bill believes that [[every friend of him][Mary sees e]]]**, with ANTECEDENT(**him**) = **Bill**, we want **friend of him** to range over friends of Bill — that is, we want it to range over the values we get by varying the function denoted by **friend** but keeping constant the individual denoted by **him**.

The issue now becomes one of how to identify those variables in a noun phrase which must be bound from the outside. These are the pronouns which do not have antecedents in the noun phrase itself. In finding the values which can be assigned to a noun phrase, we may shift the value of any noun in that noun phrase, along with any pronoun which is anaphoric to it, but must hold constant the values of other pronouns.

The nouns in a noun phrase are exactly the nouns which are in the domain of every assignment which yields a value for that noun phrase. To identify this set more formally, first let us define a predicate which identifies all the assignments relative to which a given expression has a denotation at a given world:

(70) $\textsc{g-def}_w(\alpha) = [\lambda g : g$ is an assignment of values to variables . $\alpha \in \textsc{domain}([\![\cdot]\!]^{w,g})]$

Next, where NP-PROJECTING(κ) iff CATEGORY(κ, NP) or CATEGORY(κ, NP/PP),[23] we define a function to identify which nouns are assigned values (at a given world) by all the assignments in some set:

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[22] The point here is that the analysis so far fails to predict the most natural reading of the sentence, under which it requires that anyone who is a friend of any professor smiles. Perhaps the sentence also has a reading like the one which our current rules derive, but this does not mean our current rules are adequate.

[23] More generally, for any category C, define the set of "potential head categories" of C inductively as the smallest set meeting the following conditions:

(i) C is a potential head category of C;

(ii) For any categories $D_1,\ldots,D_n, E_1,\ldots,E_m$: if D_i ($1 \leq i \leq n$) is a potential head category of C, then $((D_1,\ldots,D_n)/(E_1,\ldots,E_m))$ and $((E_1,\ldots,E_m)\backslash(D_1,\ldots,D_m))$ are potential head categories of C.

(71) Where G is a function from $\{g \mid g$ is an assignment of values to variables$\}$ into $\{0, 1\}$: $\textsc{nouns}_w(G) = \{\kappa \mid \text{NP-}\textsc{projecting}(\kappa)\ \&\ \forall g[G(g) = 1 \rightarrow \langle\kappa, w\rangle \in \textsc{domain}(g)]\}$

Now if κ is a complex common noun phrase which receives a value relative to some assignment g and world w, $\textsc{nouns}_w(\text{G-}\textsc{def}_w(\kappa))$ will be the set of nouns in κ which receive values relative to w.

For example, let us assume as before that in $w_@$, John is a professor, Mary is a professor, Susan is the sole friend of John, and Bill is the sole friend of Mary. Let κ be **friend of a professor**:

(72) a. $\text{G-}\textsc{def}_{w@}$(**friend of a professor**) $= [\lambda g: g$ is an assignment of values to variables $.\ \langle$**friend of a professor**$, w_@\rangle \in \textsc{domain}(g)]$

b. For any g such that ⟨**friend of a professor**, $w_@$⟩ ∈ $\textsc{domain}(g)$: g(**friend of a professor**, $w_@$⟩ = g(**friend**, $w_@$)(⟦**of a professor**⟧$^{w@,g}$) = g(**friend**, $w_@$)(g(**professor**, $w_@$))

c. So if ⟨**friend of a professor**, $w_@$⟩ ∈ $\textsc{domain}(g)$, then ⟨**friend**, $w_@$⟩ ∈ $\textsc{domain}(g)$ and ⟨**professor**, $w_@$⟩ ∈ $\textsc{domain}(g)$; but other pairs need not be in the domain of g.

d. That is, {κ | NP-$\textsc{projecting}$(κ) & ∀g[G-$\textsc{def}_w$(**friend of a professor**)(g) = 1 → ⟨κ, w⟩ ∈ $\textsc{domain}(g)$]} = {**friend of a professor**, **friend**, **professor**} = $\textsc{nouns}_w$(G-$\textsc{def}_w$(**friend of a professor**))

In contrast, $\textsc{nouns}_w$(G-$\textsc{def}_w$(**friend of him**)) = {**friend of him**, **friend**}. This set does not include **him** because **him** does not belong to category NP or NP/PP.

Next we need to redefine the "$\sim_{\kappa,w}$" relation to allow that $\varphi \sim_{\kappa,w} \psi$ if ψ differs from φ on $\textsc{nouns}_w(\kappa)$, not just on κ itself:

(73) For all assignments of values to variables g, h, all κ such that $\textsc{category}$(κ, NP) and all $w \in W$: $g \sim_{\kappa,w} h$ iff

a. there exists some x such that $h(\kappa, w) = x$, and for all ε such that $\textsc{antecedent}(\varepsilon) = \kappa$ and all $w' \in W$, $h(\varepsilon, w') = x$; and

b. for all $\nu \in \textsc{domain}(g)$ and all $w' \in W$, if $\nu \notin \textsc{nouns}_w(\text{G-}\textsc{def}_w(\kappa))$ and $\textsc{antecedent}(\nu) \notin \textsc{nouns}_w(\text{G-}\textsc{def}_w(\kappa))$, then $g(\nu, w') = h(\nu, w')$.

Our grammar will now assign correct truth conditions to sentences like *Every friend of a professor smiles*, or *John sees every student who hears a professor*. However, it does not yet give the right readings to sentences like *Every professor sees a student*, because our rule for *every* does not existentially close the scope of the quantifier. Fortunately, this is not hard to fix, using the tools we just developed for the redefinition of '$\sim_{\kappa,w}$'. Where φ is the

Now for any occurrence α and categories C, D: C-$\textsc{projecting}$(α) iff $\textsc{category}$(α, D) and D is a potential head category of C.

scope of a quantifier, we simply quantify on NOUNS$_w$(G-DEF$_w$(φ)). This can be accomplished by defining a relation like $\sim_{\kappa,w}$, but not limited to nouns:

(74) For all assignments of values to variables g, h, all occurrences α, and all $w \in W$: $g \rightharpoondown_{\alpha,w} h$ iff for all $\nu \in$ DOMAIN(g) and all $w' \in W$, if $\nu \notin$ NOUNS$_w$(G-DEF$_w$(α)) and ANTECEDENT(ν) $\notin$ NOUNS$_w$(G-DEF$_w$(α)), then $g(\nu, w') = h(\nu, w')$.

Intuitively, "$g \rightharpoondown_{\alpha,w} h$" means that h is like g except in the values it assigns to nouns in α, and to traces or pronouns anaphoric to such nouns. Now we can define *every* as in (75):

(75) If OCCURRENCE(α, *every*), CATEGORY(κ, NP) and CATEGORY(φ, TP), then for all $w \in W$ and all assignments of values to variables g: $[\![\alpha\ \kappa\ \varphi]\!]^{w,g} =$
1 if $\forall x \in \mathbf{D}_e[\exists h[g \sim_{\kappa,w} h\ \&\ h(\kappa, w) = x] \rightarrow \exists h[g \sim_{\kappa,w} h\ \&\ h(\kappa, w) = x\ \&\ \exists i[h \rightharpoondown_{\varphi,w} i\ \&$
$[\![\varphi]\!]^{w,i} = 1]]$;
0 if $\exists x \in \mathbf{D}_e \exists h[g \sim_{\kappa,w} h\ \&\ h(\kappa, w) = x\ \&\ \forall i[h \rightharpoondown_{\varphi,w} i \rightarrow [\![\varphi]\!]^{w,i} = 0]]$.

Our grammar will now assign an appropriate reading to occurrences of *Every professor sees a student*, as may be easily confirmed.[24]

7. Donkey anaphora

This is not the place to give a full-scale presentation and defense of a theory of donkey anaphora, but it should be noted that the rules we now have in place give an interesting account of it. In particular, the "weak" reading of donkey sentences (under which *Every farmer who owns a donkey beats it* means that every farmer who owns a donkey beats at least one of his or her donkeys) is predicted by our grammar, with no additional stipulations. In addition, by treating whole complex phrases as variables, and letting quantifiers bind those variables, we avoid the "proportion problem" posed by examples like *Most farmers who own a donkey beat it*.

To see how donkey anaphora is represented in our current grammar, let us work through an example: **Every professor Rel who sees a student e hears him**, assuming ANTECEDENT(**him**) = **student**. Let us suppose that in $w_@$, Mary and Susan are the professors, and John and Bill are the students. Mary sees John and Bill, and hears John. Susan sees Bill and hears Bill. Counting the sentence (occurrence) as a very short text, it is true iff there is an assignment g such that [\![**Every professor Rel who sees a student e hears him**]\!]$^{w_@,g} = 1$.

(76) a. [\![**Every professor Rel who sees a student e hears him**]\!]$^{w_@,g} = 1$ iff $\forall x \in \mathbf{D}_e[\exists h\ [g$ $\sim_{\textbf{professor Rel who sees a student}, w_@}$ $h\ \&\ h($**professor Rel who sees a student**, $w_@) = x]]$

[24] The grammar as currently formulated assigns only the reading which is consistent with each professor seeing a different student, and does not assign a reading which entails that all the professors see the same student. There are a variety of ways we might extend the grammar to account for the omitted reading, but to consider this issue here would take us too far afield.

$\rightarrow \exists h[g \sim_{\textbf{professor Rel who sees a student}, w@}\ h\ \&\ h(\textbf{professor Rel who sees a student}, w@) = \text{x}\ \&\ \exists i\,[h \rightharpoondown_{\textbf{e hears him}, w@}\ i\ \&\ [\![\textbf{e hears him}]\!]^{w@,i} = 1]]]$

b. $\text{NOUNS}_{w@}(\text{G-DEF}_{w@}(\textbf{e hears him})) = \emptyset$, because **e hears him** contains no N-projecting constituents.

c. Therefore, if $h \rightharpoondown_{\textbf{e hears him}, w@}\ i$, then $h = i$.

d. So $\exists i\,[h \rightharpoondown_{\textbf{e hears him}, w@}\ i\ \&\ [\![\textbf{e hears him}]\!]^{w@,i} = 1]]$ iff $[\![\textbf{e hears him}]\!]^{w@,h} = 1$.

e. $[\![\textbf{e hears him}]\!]^{w@,h} = 1$ iff $h(\textbf{e}, w@)$ hears $h(\textbf{him}, w@)$.

f. If $g \sim_{\textbf{professor Rel who sees a student}, w@}\ h$, then $h(\textbf{e}, w@) = h(\textbf{professor}, w@)$, and $h(\textbf{him}, w@) = h(\textbf{student}, w@)$.

g. So $[\![\textbf{e hears him}]\!]^{w@,h} = 1$ iff $h(\textbf{professor}, w@)$ hears $h(\textbf{student}, w@)$.

h. If $g \sim_{\textbf{professor Rel who sees a student}, w@}\ h$, then $h(\textbf{professor Rel who sees a student}, w@) = h(\textbf{professor}, w@)$, and $h(\textbf{professor}, w@)$ is a professor, and $h(\textbf{student}, w@)$ is a student, and $h(\textbf{professor}, w@)$ sees $h(\textbf{student}, w@)$.

i. So $g \sim_{\textbf{professor Rel who sees a student}, w@}\ h$ and $h(\textbf{professor Rel who sees a student}, w@) = x$ and $[\![\textbf{e hears him}]\!]^{w@,h} = 1$ iff $h(\textbf{professor}, w@)$ is a professor and $h(\textbf{student}, w@)$ is a student and $h(\textbf{professor}, w@)$ sees $h(\textbf{student}, w@)$ and $\text{h}(\textbf{professor}, w@) = x$ and $\text{h}(\textbf{professor}, w@)$ hears $h(\textbf{student}, w@)$.

j. So there will be an h such that $g \sim_{\textbf{professor Rel who sees a student}, w@}\ h$ and $h(\textbf{professor Rel who sees a student}, w@) = x$ and $[\![\textbf{e hears him}]\!]^{w@,h} = 1$ iff there is a y such that y is a professor and a z such that z is a student and y sees z and $y = x$ and y hears z.

k. To say that this condition is met for every x such that $\exists h[g \sim_{\textbf{professor Rel who sees a student}, w@}\ h$ and $h(\textbf{professor Rel who sees a student}, w@) = x]$, is to say that for all x such that there is a y such that y is a professor and there is a z such that z is a student and y sees z and $y = x$, there is a y such that y is a professor and there is a z such that y sees z and $y = x$ and y hears z.

l. Or more simply, for all x such that x is a professor and there is a student whom x sees, there is a student whom x sees and hears.

It is worth considering at a more intuitive level what allows for the anaphora in cases like this. We are treating the whole phrase **professor Rel who sees a student** as a variable — so as we quantify on this variable, we are quantifying over those assignments which yield a value for it. Because this variable has other variables as parts, an assignment will yield a value for it only if it also yields values for those parts, including **student.** So when we consider the scope **e hears him** relative to different assignments of a value to **e**, those assignments also automatically yield values for **student.** Since **him** has **student** as its antecedent, it will receive those same values. This is so, even though **a student** does not ccommand **him**, **every** does not bind **a student** (in the sense of quantifying over professor-student pairs), and no appeal was made to dynamic operators.[25]

Allowing complex phrases whose denotations are assigned compositionally to be variables allows an interesting solution to the "proportion problem." This problem arises under analyses of donkey anaphora which appeal to unselective binding, so that a sentence like *Most farmers that own a donkey beat it* is represented as having a logical structure like '$\text{most}_{x,y}$([farmer(x) & donkey(y) & own(x, y)], beat(x, y))'. Assuming that a structure of the form '$\text{most}_{x,y}(\varphi, \psi)$' is true iff more than half the assignments of values to x and y which satisfy φ also satisfy ψ, the sentence is predicted to be true iff more than half the pairs of a farmer and a donkey such that the farmer owns the donkey are also pairs such that the farmer beats the donkey. But this prediction is erroneous; in a situation where there are ten donkey-owning farmers, nine of whom own just a single donkey and do not beat it, and one of whom owns a hundred donkeys and beats them all, the sentence *Most farmers who own a donkey beat it* is false, not true.

We avoid this problem by letting the whole complex phrase *farmers who own a donkey* function as a single variable. We regard determiners as simply binding their noun phrases, not as unselectively binding all free variables contained in those noun phrases. In this way, *most* will "count" farmers, not farmer-donkey pairs.

Since *most* combines with plural nouns, we will need to draw a singular/plural distinction before proceeding to a consideration of *most*. All the common nouns in our existing entries should be marked as singular:

(77) a. *professor*, *student*: NP, SINGULAR
b. *friend*: ((NP, SINGULAR) /PP),

Suffixing *-s* to these forms plural nouns:[26]

(78) a. *professors*, *students*: NP, PLURAL
b. *friends*: ((NP, PLURAL)/PP)

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[25] This last point differentiates our analysis from that of Chierchia (1995). Like Chierchia, we exploit the conservativity of quantification to "carry over" values for indefinites into the scope of the quantifier, so that they can properly antecede the pronoun. Unlike Chierchia, we do not appeal to dynamic quantification or dynamic conjunction to accomplish this, and do not need to stipulate conservativity.

[26] We can give a general rule for pluralization using the notion of "potential head categories" from footnote 23 as follows: For all lexical items α, if α belongs to a potential head category of N, and to some category C which is a potential head category of SINGULAR, then α-*s* belongs to PLURALIZE(C) and to all categories other than C to which α belongs, where:

a. PLURALIZE(SINGULAR) = PLURAL;
b. If C = $((D_1,\ldots,D_i,\ldots,D_n)/(E_1,\ldots,E_m))$, where D_i is a potential head category of SINGULAR, then PLURALIZE(C) = $((D_1,\ldots,\text{PLURALIZE}(D_i),\ldots,D_n)/(E_1,\ldots,E_m))$;
c. If C = $((E_1,\ldots,E_m)\backslash(D_1,\ldots,D_i,\ldots,D_n))$ where D_i is a potential head category of SINGULAR, then PLURALIZE(C) = $((E_1,\ldots,E_m)\backslash(D_1,\ldots,\text{PLURALIZE}(D_i),\ldots,D_n))$.

The determiners *every* and *a* must now be recategorized so that they combine only with singulars, and so that the phrases they head also count as singular. *The* can combine with both singulars and plurals, but to avoid digressing into the details of how to analyze plural *the*, we limit attention here to singulars, and therefore categorize it like *a*:

(79) a. *every*: ((NOM, ACC, Q, SINGULAR)/(NP, SINGULAR))
b. *a*, *the*: ((NOM, ACC, SINGULAR)/(NP, SINGULAR))

Verbs must also be recategorized so that they agree in number with their subjects. In a more complete and detailed grammar, singular and plural verbs should be derived morphologically from their uninflected base forms; but here we simplify, taking plural verbs as morphologically basic and deriving (third person) singular verbs from their plural counterparts by suffixation of -*s*:

(80) a. *smile*, *frown*: ((NOM, PLURAL)\TP)
b. *see, hear*: (((NOM, PLURAL)\TP)/ACC)
c. *believe*, *know*: (((NOM, PLURAL)\TP)/CP)

(81) For all lexical items α: If α belongs to some category ((NOM, PLURAL)\X), where X is a potential head category of TP, then α-*s* belongs to ((NOM, SINGULAR)\X), and to all categories other than ((NOM, PLURAL)\X) to which α belongs.

We also need to assign number to proper names and pronouns:

(82) a. *John*, *Mary*, *Bill*, *Susan*: NOM, ACC, SINGULAR
b. *he*, *she*: PRONOUN, NOM, SINGULAR
c. *him*, *her*: PRONOUN, ACC, SINGULAR

And finally, we can add *most* to our lexicon:

(83) *most*: ((NOM, ACC, Q, PLURAL)/(NP, PLURAL))

Semantically, plural nouns — like singulars — will function as variables. We assume that the value of a singular noun, relative to any given assignment of values to variables *g*, is an individual; but that the value of a plural noun may be either an individual or a "group" with individuals as its members.[27] We now assume that our domain **D** includes groups as well as individuals, and that some sort of membership relation holds between a group and its members. We count each individual as the sole member of itself. We may now expand our definition of assignments of values to variables:[28]

(84) *g* is an assignment of values to variables iff *g* is a function with domain included in {α | CATEGORY(α, NP) or CATEGORY(ANTECEDENT(α), NP)} × *W* such that:

[27] We defer to another occasion an attempt to formulate a semantics for the current grammar in which each plural expression denotes multiple individuals, rather than denoting a group, as in Schein (1993) and related work.

[28] Clauses c. and e. could easily be collapsed by considering individuals to be 0-place functions.

a. for all $w \in W$, if OCCURRENCE(α, *professor*) and $\langle \alpha, w\rangle$ is in the domain of g, then $g(\alpha, w)$ is a professor in w;

b. for all $w \in W$, if OCCURRENCE(α, *student*) and $\langle \alpha, w\rangle$ is in the domain of g, then $g(\alpha, w)$ is a student in w.

c. for all $w \in W$ and all $x \in \mathbf{D}_e$, if CATEGORY(α, NP) and CATEGORY(α, SINGULAR), then if x is a member of $g(\alpha\text{-}s, w)$, there exists an assignment h such that $h(\alpha, w) = x$;

d. for all $w \in W$, and all $x \in \mathbf{D}_e$, if OCCURRENCE(α, *friend*) and there exists a y such that $y = g(\alpha, w)(x)$, then $g(\alpha, w)(x)$ is a friend of x in w;

e. for all $w \in W$, and all $x, y \in \mathbf{D}_e$, if CATEGORY(α, ((NP, SINGULAR)/PP)), then if x is a member of $g(\alpha\text{-}s, w)(y)$, there exists an assignment h such that $h(\alpha, w)(y) = x$;

f. if CATEGORY(α, NP/PP) and CATEGORY(β, PP), then for all $w \in W$: $g(\alpha\beta, w) = g(\alpha, w)([\![\beta]\!]^{w,g})$.

Plural verbs are predicates that can hold of groups and/or individuals — but some of the verbs in our fragment are all semantically distributive, in which case the groups are in some sense redundant. Singular verbs hold only of the individuals:

(85) a. If OCCURRENCE(α, *smile*) then $[\![\alpha]\!]^{w,g} = [\lambda x: x \in \mathbf{D}_e\,.\, \forall y \in \mathbf{D}_e[y$ is a member of $x \rightarrow y$ smiles in $w]$;

b. If OCCURRENCE(α, *frown*) then $[\![\alpha]\!]^{w,g} = [\lambda x: x \in \mathbf{D}_e\,.\, \forall y \in \mathbf{D}_e[y$ is a member of $x \rightarrow y$ frowns in $w]$;

c. If OCCURRENCE(α, *see*) then $[\![\alpha]\!]^{w,g} = [\lambda x: x \in \mathbf{D}_e\,.\, [\lambda y: y \in \mathbf{D}_e\,.\ y$ sees x in $w]]$;

d. If OCCURRENCE(α, *hear*) then $[\![\alpha]\!]^{w,g} = [\lambda x: x \in \mathbf{D}_e\,.\, [\lambda y: y \in \mathbf{D}_e\,.\ y$ hears x in $w]]$

e. If OCCURRENCE(α, *believe*) then $[\![\alpha]\!]^{w,g} = [\lambda p: p \in \mathbf{D}_{\langle s,t\rangle}\,.\, [\lambda x: x \in \mathbf{D}_e\,.\, \forall y \in \mathbf{D}_e\,[y$ is a member of $x] \rightarrow \forall w' \in W[w' \in \mathbf{B}_{y,w} \rightarrow p(w') = 1]]$;

f. If OCCURRENCE(α, *know*) then $[\![\alpha]\!]^{w,g} = [\lambda p: p \in \mathbf{D}_{\langle s,t\rangle}\,.\, [\lambda x: x \in \mathbf{D}_e\,.\, \forall y \in \mathbf{D}_e[y$ is a member of $x] \rightarrow \forall w' \in W[w' \in \mathbf{K}_{y,w} \rightarrow p(w') = 1]]$;

g. If OCCURRENCE(α-*s*, (NOM, SINGULAR)\X)), where X is a potential head category of S, then $[\![\alpha\text{-}s]\!]^{w,g} = [\lambda x: x$ is an individual . $[\![\alpha]\!]^{w,g}(x)]$.

Despite combining with a plural noun, *most* — like many other plural quantifiers — typically quantifies only over individuals.[29] We may give the following rule:

(86) If OCCURRENCE(α, *most*), CATEGORY(κ, NP) and CATEGORY(φ, TP), then for all $w \in W$ and all assignments of values to variables g: $[\![\alpha\ \kappa\ \varphi]\!]^{w,g} =$

[29] *Most* resists combination with collective predicates. For example, (i) sounds noticeably odd. For discussion see Lasersohn (2011). *Most* does allow collective predicates in the partitive construction, as in (ii), but we will not attempt an account of that here:

(i) *?Most students gathered in the hallway*

(ii) *Most of the students gathered in the hallway*

1 if $\{x \mid x$ is an individual & $\exists h[g \sim_{\kappa,w} h$ & $h(\kappa, w) = x$ & $\exists i[h \rightharpoondown_{\varphi,w} i$ & $[\![\varphi]\!]^{w,i} = 1]\}$ is of greater cardinality than $\{x \mid x$ is an individual & $\exists h[g \sim_{\kappa,w} h$ & $h(\kappa, w) = x$ & $\exists i[h \rightharpoondown_{\varphi,w} i$ & $[\![\varphi]\!]^{w,i} = 0]\}$;
0 if $\{x \mid x$ is an individual & $\exists h[g \sim_{\kappa,w} h$ & $h(\kappa, w) = x$ & $\exists i[h \rightharpoondown_{\varphi,w} i$ & $[\![\varphi]\!]^{w,i} = 1]\}$ is of equal or lesser cardinality than $\{x \mid x$ is an individual & $\exists h[g \sim_{\kappa,w} h$ & $h(\kappa, w) = x$ & $\exists i[h \rightharpoondown_{\varphi,w} i$ & $[\![\varphi]\!]^{w,i} = 0]\}$.

To see the effect of this definition, consider the sentence occurrence **Most professors Rel who see a student e hear him**, with ANTECEDENT(**him**) = **student**, and assuming that in $w_@$ the facts are as follows: Mary, Susan and Wanda are the professors. John, Bill, Fred, Ralph and Seymour are the students. Mary sees and hears John. Susan sees and hears Bill. Wanda sees Fred, Ralph and Seymour, but does not hear any of them. The sentence occurrence (regarded as a short text) is true iff there is at least one g such that $[\![$**Most professors Rel who see a student e hear him**$]\!]^{w_@,g} = 1$. Our semantics correctly predicts this is the case, even though there are more professor-student pairs where the professor sees but does not hear the student than professor-student pairs where the professor sees and hears the student:

(87) $[\![$**Most professors Rel who see a student e hear him**$]\!]^{w_@,g} = 1$ iff $\{x \mid x$ is an individual & $\exists h[g \sim_{\textbf{professors Rel who see a student}, w} h$ & $h($**professors Rel who see a student**, $w_@) = x$ & $\exists i[h \rightharpoondown_{\textbf{e hear him}, w_@} i$ & $[\![$**e hear him**$]\!]^{w_@,i} = 1]\}$ is of greater cardinality than $\{x \mid x$ is an individual & $\exists h[g \sim_{\textbf{professors Rel who see a student}, w_@} h$ & $h($**professors Rel who see a student**, $w_@) = x$ & $\exists i[h \rightharpoondown_{\textbf{e hear him}, w_@} i$ & $[\![$**e hear him**$]\!]^{w_@,i} = 0]\}$

This gives a complicated appearance, but from our rules and previous examples, it should be apparent that it simply means that the set of all individuals x such that x is a professor, and there is some y such that y is a student, x sees y, and x hears y, has greater cardinality that the set of all individuals x such that x is a professor, and there is some y such that y is a student, x sees y and x does not hear y. This is the case in the scenario just outlined.

To summarize, our analysis avoids the "proportion problem" for quantifiers like *most*. It does this by treating entire common noun phrases, like *professors who see a student* (or *farmers who own a donkey*) as variables. The determiner *most* simply quantifies on the values of this variable, rather than quantifying over pairs as in an unselective-binding analysis.

If common noun phrases are variables, it seems natural that determiners would quantify on the values of the noun phrases with which they combine. In contrast, in a more conventional analysis treating these phrases as something like open formulas, some special stipulation must be made to ensure that the quantifier "counts" only values of the variable serving as argument to the head noun and not other free variables contained in the noun phrase.

8. The temperature paradox

Treating common nouns as variables solves a problem in how to analyze the "temperature paradox." I assume some familiarity with the temperature paradox, which was classically discussed in Montague (1973)[30]; nonetheless a brief summary seems appropriate.

Intuitively, (88) is not a valid argument, even though it appears to have the logical structure in (89),[31] which by standard principles should be valid.

(88) The temperature is rising.
The temperature is ninety.
Therefore, ninety is rising.

(89) $\exists x[\mathbf{temperature}(x)\ \&\ \forall y[\mathbf{temperature}(y) \rightarrow x = y]\ \&\ \mathbf{rise}(x)]$
$\exists x[\mathbf{temperature}(x)\ \&\ \forall y[\mathbf{temperature}(y) \rightarrow x = y]\ \&\ x = n]$
$\therefore \mathbf{rise}(n)$

Montague's solution to this problem is regard *rise* not as a type $\langle e, t\rangle$ predicate of individuals, but as a type $\langle\langle s, e\rangle, t\rangle$ predicate of functions from world-time pairs to individuals. He includes two operators notated '^' and '˅' in his logical notation, with the interpretations given in (90):[32]

(90) a. $[\![{}^\wedge\alpha]\!]^{M,w,t,g}$ = that function f with domain $W \times T$ such that for all $w' \in W$, $t' \in T$, $f(w', t') = [\![\alpha]\!]^{M,w',t',g}$
b. $[\![{}^\vee\alpha]\!]^{M,w,t,g} = [\![\alpha]\!]^{M,w,t,g}(w, t)$

In other words, $^\wedge\alpha$ denotes the intension of α, and — where α already denotes the sort of thing which can serve as an intension — $^\vee\alpha$ denotes (relative to a given world and time) the extension of α (at that world and time).

Now (88) can be represented as in (91):

(91) $\exists x[\mathbf{temperature}(x)\ \&\ \forall y[\mathbf{temperature}(y) \rightarrow x = y]\ \&\ \mathbf{rise}(x)]$
$\exists x[\mathbf{temperature}(x)\ \&\ \forall y[\mathbf{temperature}(y) \rightarrow x = y]\ \&\ {}^\vee x = {}^\vee n]$
$\therefore \mathbf{rise}(n)$

That is, *The temperature is rising* is taken to mean that there is a unique function which picks out the temperature at each time and world; *The temperature is ninety* is taken to mean that the value of this function at the current time and actual world is identical to the value of the function which picks out 90 at each time and world; *Ninety is rising* is taken to mean that this function — the constant function which picks out 90 at all world-time pairs

[30] Montague attributes the example to Barbara Partee.
[31] I have made some minor adjustments to Montague's original notation.
[32] T is the set of all times. Here also, I have made some minor changes to Montague's notation.

— is rising. It is easily seen that the first two formulas can be true without the third formula being true.

However, this analysis has a problem, originally pointed out by Anil Gupta,[33] which Lasersohn (2005) attempted to solve. Intuitively, (92) is a valid argument:

(92) Necessarily, the temperature is the price.
The temperature is rising.
Therefore, the price is rising.

But these sentences translate to the formulas in (93), and (93) is *not* a valid argument, given Montague's semantics:

(93) $\Box\exists x[\mathbf{temperature}(x)\ \&\ \forall y[\mathbf{temperature}(y) \rightarrow x = y]\ \&\ \exists z[\mathbf{price}(z)\ \&\ \forall y[\mathbf{price}(y) \rightarrow z = y]\ \&\ {}^{\vee}x = {}^{\vee}z]$
$\exists x[\mathbf{temperature}(x)\ \&\ \forall y[\mathbf{temperature}(y) \rightarrow x = y]\ \&\ \mathbf{rise}(x)]$
$\therefore \exists x[\mathbf{price}(x)\ \&\ \forall y[\mathbf{price}(y) \rightarrow x = y]\ \&\ \mathbf{rise}(x)]$

To see this, note first that Montague's analysis allows a noun like *temperature* to denote a different function of type $\langle\langle s, e\rangle, t\rangle$ at each world-time pair. Assume a model with just one possible world (w_1) and three times (t_1 t_2, t_3). Let T_1, T_2, T_3 be the functions shown in (94). Relative to $\langle w_1, t_1\rangle$ let *temperature* denote the function which maps T_1 onto 1 but T_2 and T_3 onto 0; relative to $\langle w_1, t_2\rangle$ let it denote the function which maps T_2 onto 1 but T_1 and T_3 onto 0; and relative to $\langle w_1, t_3\rangle$ let it denote the function which maps T_3 onto 1 but T_1 and T_2 onto 0:

(94)

$T_1(w_1, t_1) = 99$	$T_2(w_1, t_1) = 89$	$T_3(w_1, t_1) = 79$
$T_1(w_1, t_2) = 100$	$T_2(w_1, t_2) = 90$	$T_3(w_1, t_2) = 80$
$T_1(w_1, t_3) = 101$	$T_2(w_1, t_3) = 91$	$T_3(w_1, t_3) = 81$

Assuming that $t_1 < t_2 < t_3$, it is clear that at all three world-time pairs *The temperature is rising* is true: At $\langle w_1, t_1\rangle$ there is exactly one temperature function, namely T_1, and it is rising, since it progresses from 99 to 100 to 101. At $\langle w_1, t_2\rangle$ there is also exactly one temperature function, namely T_2, and it is also rising, since it progresses from 89 to 90 to 91. And at $\langle w_1, t_3\rangle$ there is exactly one temperature function, namely T_3, which is rising, since it progresses from 79 to 80 to 81. So the first premise of (93) is true.

Now suppose that at $\langle w_1, t_1\rangle$ the noun *price* denotes the function which maps P_1 onto 1 but P_2 and P_3 onto 0; at $\langle w_1, t_2\rangle$ it denotes the function which maps P_2 onto 1 but P_1 and P_3 onto 0; and at $\langle w_1, t_3\rangle$ it denotes the function which maps P_3 onto 1 but P_1 and P_2 onto 0:

[33] This problem appears to have first been discussed in Dowty, Wall and Peters (1981), but they credit Gupta with discovering it.

(95) $P_1(w_1, t_1) = 99$ $P_2(w_1, t_1) = 91$ $P_3(w_1, t_1) = 83$
$P_1(w_1, t_2) = 98$ $P_2(w_1, t_2) = 90$ $P_3(w_1, t_2) = 82$
$P_1(w_1, t_3) = 97$ $P_2(w_1, t_3) = 89$ $P_3(w_1, t_3) = 81$

Notice that at each world-time pair, the unique temperature function yields the same value as the unique price function: At $\langle w_1, t_1\rangle$, T_1 is the unique temperature function and P_1 is the unique price function, and both of these functions yield 99 for the pair $\langle w_1, t_1\rangle$. At $\langle w_1, t_2\rangle$, T_2 is the unique temperature function and P_2 is the unique price function, and both these functions yield 90 for the pair $\langle w_1, t_2\rangle$. And at $\langle w_1, t_3\rangle$, T_3 is the unique temperature function and P_3 is the unique price function, and they both yield 81 for the pair $\langle w_1, t_3\rangle$. So the second premise of (93) is true.

But P_1, P_2 and P_3 are are all *falling* functions: P_1 progresses from 99 to 98 to 97; P_2 from 91 to 90 to 89; and P_3 from 83 to 82 to 81. So the conclusion of (93) is false. Since (93) has true premises and a false conclusion, it is not a valid argument, which means it does not accurately represent the argument in (92), which is intuitively valid.

Lasersohn (2005) suggested a solution to this problem which relied on the idea that nouns like *price* and *temperature* were not predicates of type $\langle\langle s, e\rangle, t\rangle$, but just ordinary predicates of type $\langle e, t\rangle$, in contrast to *rise* and *fall*, which remained as type $\langle\langle s, e\rangle, t\rangle$. The latter type seems appropriate for *rise* and *fall*: it is impossible to determine whether a given function is rising or falling at a time t simply by examining the value which the function yields for t. Instead one must also know the values which the function yields for neighboring times, to see if the earlier ones are lower than the later ones. Put more concretely: you can't tell whether the temperature is rising at 3:00 simply by examining a single photograph of a thermometer taken at 3:00; you need multiple photographs, taken at different times. In contrast, a single photograph suffices to demonstrate what the temperature is at a given time, and photographs taken at other times are essentially irrelevant to the question.

But if *price* and *temperature* are of type $\langle e, t\rangle$, while *rise* and *fall* are of type $\langle\langle s, e\rangle, t\rangle$, an issue arises how they can combine. We cannot use formulas like $\exists x[\mathbf{temperature}(x)\ \&\ \forall y[\mathbf{temperature}(y) \rightarrow x = y]\ \&\ \mathbf{rise}(x)]$, because this will now impose contradictory requirements on the type of x — is it a variable of type e, or of type $\langle s, e\rangle$? Lasersohn proposed we could resolve this problem by giving up the Russellian quantificational analysis of the definite article, and instead assuming a presuppositional analysis under which definite noun phrases are of type e. We translate *the temperature* as $\iota x\ \mathbf{temperature}(x)$, and for any $\langle w, t\rangle$, we let $[\![\iota x\ \mathbf{temperature}(x)]\!]^{M,w,t,g}$ be the unique element u such that $[\![\mathbf{temperature}]\!]^{M,w,t,g}(u) = 1$ if there is such a unique element, undefined otherwise. In this case, by the standard semantics of the ^-operator, $^\wedge\iota x\ \mathbf{temperature}(x)$ will denote, relative to any world-time pair $\langle w, t\rangle$, the function which maps any pair $\langle w', t'\rangle$ onto $[\![\iota x\ \mathbf{temperature}(x)]\!]^{M,w',t',g}$.

For example, suppose $[\![\iota x\, \mathbf{temperature}(x)]\!]^{M,w_1,t_1,g} = 89$, $[\![\iota x\, \mathbf{temperature}(x)]\!]^{M,w_1,t_2,g} = 90$, and $[\![\iota x\, \mathbf{temperature}(x)]\!]^{M,w_1,t_3,g} = 91$. In this case $[\![{}^\wedge\iota x\, \mathbf{temperature}(x)]\!]^{M,w_1,t_1,g} = [\![{}^\wedge\iota x\, \mathbf{temperature}(x)]\!]^{M,w_1,t_2,g} = [\![{}^\wedge\iota x\, \mathbf{temperature}(x)]\!]^{M,w_1,t_3,g} = \mathrm{T}$, where T is as shown in (96):

(96) $\mathrm{T}(w_1, t_1) = 89$
$\mathrm{T}(w_1, t_2) = 90$
$\mathrm{T}(w_1, t_3) = 91$

That is, in this system, there is only one temperature function — the same one at every world-time pair — rather than potentially different temperature functions as in Montague's formalism.

This allows us to represent the argument in (92) as in (97):

(97) $\Box\iota x\, \mathbf{temperature}(x) = \iota x\, \mathbf{price}(x)$
$\mathbf{rise}({}^\wedge\iota x\, \mathbf{temperature}(x))$
$\therefore \mathbf{rise}({}^\wedge\iota x\, \mathbf{price}(x))$

This is easily confirmed to be valid, as desired.

Unfortunately, this analysis also turned out to be problematic, as shown in Romero (2008). It wrongly predicts that we should not get quantificational subjects in sentences with predicates like *rise*, or at least that if we do, nothing like the temperature paradox should arise. Indeed, Montague himself may have noticed this problem; he gave (98) to show that we do get such examples:

(98) A price rises.
Every price is a number.
Therefore, a number rises.

The analysis in Lasersohn (2005) cannot even represent this argument, let alone correctly predict its invalidity. The first premise of (98) involves existential quantification[34], but if we translate it as (99), we impose contradictory requirements on the type of x:

(99) $\exists x[\mathbf{price}(x)\ \&\ \mathbf{rise}(x)]$

In order to serve as the argument of **price**, the analysis of Lasersohn (2005) requires x to be of type e, but in order to serve as the argument of **rise** it must be of type $\langle s, e\rangle$. More generally, this example suggests that in order to deal with sentences containing predicates like *rise*, we must allow for variables over individual concepts, and not just variables over individuals; so Lasersohn was wrong to claim that the temperature paradox provided evidence for a non-quantificational analysis of the definite article.

We could easily deal with this problem by sticking with Montague's original typing of both *price* and *rise* as $\langle\langle s, e\rangle, t\rangle$, but it would be better if we could find a way not to do so.

[34] This is compatible with the possibility that the quantification is due to a general operation of existential closure, rather than to the internal semantics of the indefinite article.

As already pointed out, nouns like *price* and *temperature* are not temporally intensional in their criteria of application — these are words for which "a single snapshot suffices."

If we analyze common nouns as variables, and allow variables to be temporally non-rigid in the same way as we allowed them to be modally non-rigid, such a solution is available. First, let us add *ninety*, *temperature* and *price* to our lexicon. These are of category NP/PP, but in our main examples they appear without an *of*-phrase; we therefore add the derivational morphological rule in (102), allowing an *of*-noun also to appear as an ordinary common noun of category NP. As our notation indicates, these will be interpreted by existentially quantifying the *of*-argument:[35]

(100) *ninety*: NOM, ACC, SINGULAR
(101) *temperature*, *price*: NP/PP
(102) For all lexical items α: If α belongs to category NP/PP, then $\alpha_\exists$ belongs to category NP.

Our set of NP's will now include *temperature*$_\exists$ and *price*$_\exists$.

To allow for temporal intensionality, we now relativize denotation assignment to times. For the most part this is a simple matter of adding a t superscipt:[36]

(103) For all $w \in W$, all $t \in T$, and all assignments of values to variables g:
  a. If OCCURRENCE(α, *John*) then $[\![\alpha]\!]^{w,t,g} =$ John;
  b. If OCCURRENCE(α, *Mary*) then $[\![\alpha]\!]^{w,t,g} =$ Mary;
  c. If OCCURRENCE(α, *Bill*) then $[\![\alpha]\!]^{w,t,g} =$ Bill;
  d. If OCCURRENCE(α, *Susan*) then $[\![\alpha]\!]^{w,t,g} =$ Susan;
  e. If OCCURRENCE(α, *ninety*) then $[\![\alpha]\!]^{w,t,g} = 90$;
  f. If OCCURRENCE(α, *smile*) then $[\![\alpha]\!]^{w,t,g} = [\lambda x : x \in \mathbf{D}_e \,.\, \forall y \in \mathbf{D}_e[y$ is a member of $x \rightarrow y$ smiles at t in $w]$;
  a. If OCCURRENCE(α, *frown*) then $[\![\alpha]\!]^{w,t,g} = [\lambda x : x \in \mathbf{D}_e \,.\, \forall y \in \mathbf{D}_e[y$ is a member of $x \rightarrow y$ frowns at t in $w]$;
  b. If OCCURRENCE(α, *see*) then $[\![\alpha]\!]^{w,t,g} = [\lambda x : x \in \mathbf{D}_e \,.\, [\lambda y : y \in \mathbf{D}_e \,.\, y$ sees x at t in $w]]$;
  c. If OCCURRENCE(α, *hear*) then $[\![\alpha]\!]^{w,t,g} = [\lambda x : x \in \mathbf{D}_e \,.\, [\lambda y : y \in \mathbf{D}_e \,.\, y$ hears x at t in $w]]$;
  d. If OCCURRENCE(α, *believe*) then $[\![\alpha]\!]^{w,t,g} = [\lambda p : p \in \mathbf{D}_{\langle s,t\rangle} \,.\, [\lambda x : x \in \mathbf{D}_e \,.\, \forall y \in \mathbf{D}_e\, [y$ is a member of $x] \rightarrow \forall w' \in W[w' \in \mathbf{B}_{y,w,t} \rightarrow p(w', t) = 1]]$;

[35] Relational nouns may also be detransitivized by interpreting the *of*-argument indexically, rather than as existentially quantified. This is the more prominent reading for certain nouns, including *friend*; but I do not bother to represent it here.

[36] For all types σ, we redefine $\mathbf{D}_{\langle s,\sigma\rangle}$ as $\{f \mid f$ is a function from $W \times T$ into $\mathbf{D}_\sigma\}$, where T is the set of times. We also begin using 't' as a variable over times, which introduces a notational clash with the type label for expressions denoting elements of {0,1}. I trust the context will make clear what sense is intended when. We let $\mathbf{B}_{x,w,t}$ be the set of worlds compatible with x's belief state at t in w, and $\mathbf{K}_{x,w,t}$ be the set of worlds compatible with x's knowledge state at t in w.

e. If OCCURRENCE(α, *know*) then $[\![\alpha]\!]^{w,t,g} = [\lambda p: p \in \mathbf{D}_{\langle s,t\rangle} . [\lambda x: x \in \mathbf{D}_e . \forall y \in \mathbf{D}_e[y$ is a member of $x] \rightarrow \forall w' \in W[w' \in \mathbf{K}_{y,w,t} \rightarrow p(w', t) = 1]]$;

f. If OCCURRENCE(α-*s*, (NOM, SINGULAR)\X)), where X is a potential head category of TP, then $[\![\alpha\text{-}s]\!]^{w,t,g} = [\lambda x: x$ is an individual . $[\![\alpha]\!]^{w,t,g}(x)]$;

g. If OCCURRENCE(ρ, *Rel*), then $[\![\rho]\!]^{w,t,g} = [\lambda p: p \in \mathbf{D}_t \,\&\, p = 1 . [\lambda x: x \in \mathbf{D}_e . x]]$;

h. If OCCURRENCE(α, *of*) then $[\![\alpha]\!]^{w,t,g} = [\lambda x: x \in \mathbf{D}_e . x]$

Next revise the definition of assignment functions in (62) so that assignments give values to variable-world-time *triples*, and add clauses for *temperature* and *price* and for the lexical rule in (102):

(104) g is an assignment of values to variables iff g is a function with domain included in {α | CATEGORY(α, NP) or CATEGORY(ANTECEDENT(α), NP)} × W × T such that:

a. for all $w \in W$, and all $t \in T$: if OCCURRENCE(α, *professor*) and ⟨α, w, t⟩ is in the domain of g, then $g(\alpha, w, t)$ is a professor in w at t;

b. for all $w \in W$, and all $t \in T$: if OCCURRENCE(α, *student*) and ⟨α, w, t⟩ is in the domain of g, then $g(\alpha, w, t)$ is a student in w at t;

c. for all $w \in W$, all $t \in T$, and all $x \in \mathbf{D}_e$, if CATEGORY(α, N) and CATEGORY(α, SINGULAR), then if x is a member of $g(\alpha\text{-}s, w, t)$, there exists an assignment h such that $h(\alpha, w, t) = x$;

d. for all $w \in W$, all $t \in T$ and all $x \in \mathbf{D}_e$, if OCCURRENCE(α, *friend*) and there exists a y such that $y = g(\alpha, w, t)(x)$, then $g(\alpha, w, t)(x)$ is a friend of x in w at t;

e. for all $w \in W$, all $t \in T$ and all $x \in \mathbf{D}_e$, if OCCURRENCE(α, *temperature*) and there exists a y such that $y = f(\alpha, w, t)(x)$, then $g(\alpha, w, t)(x)$ is the temperature of x in w at t (in the pragmatically relevant scale — °F, °C, etc.);

f. for all $w \in W$, all $t \in T$ and all $x \in \mathbf{D}_e$, if OCCURRENCE(α, *price*) and there exists a y such that $y = f(\alpha, w, t)(x)$, then $g(\alpha, w, t)(x)$ is the price of x in w at t (in the pragmatically relevant units — \$, €, etc.);

g. for all $w \in W$, all $t \in T$, and all $x, y \in \mathbf{D}_e$, if CATEGORY(α, NP/PP) and CATEGORY(α, SINGULAR), then if x is a member of $g(\alpha\text{-}s, w, t)(y)$, there exists an assignment h such that $h(\alpha, w, t)(y) = x$;

h. if CATEGORY(α, NP/PP) and CATEGORY(β, PP), then for all $w \in W$ and all $t \in T$: $g(\alpha\beta, w, t) = g(\alpha, w)([\![\beta]\!]^{w,t,g})$;

i. if CATEGORY(α, NP/PP), then for all $w \in W$, and all $t \in T$: if ⟨$\alpha_\exists$, w, t⟩ is in the domain of g, then there exists some pragmatically relevant $x \in \mathbf{D}_e$ such that for all $w' \in W$, $t' \in T$: x exists in w' at t' iff $g(\alpha_\exists, w', t') = g(\alpha, w', t')(x)$.

Notice that according to (104)i., for each assignment function g, there is one particular object whose temperature at various times and worlds is tracked by the values which g assigns to the noun *temperature*$_\exists$ (relative to those times and worlds). The value which g assigns to *temperature*$_\exists$ relative to a world w' and time t' is just a number — an entity of type e — not a function or set of functions. So for any given object, there will be just one function which picks out the temperature of that object in that world at each time — not different such functions at different times. This eliminates the source of Gupta's problem.

Verbs like *rise* must be assigned denotation in type $\langle\langle s, e\rangle, t\rangle$, as in (105):

(105) If OCCURRENCE(α, *rise*) then $[\![\alpha]\!]^{w,t,g} = [\lambda x : x \in \mathbf{D}_{\langle s, e\rangle} \,.\, x$ rises at t in $w]$

But now we have a problem: If quantification is over assignments in which traces co-denote with their antecedents, and nouns like *temperature*$_\exists$ are type e, then the trace left by phrases like *every temperature*$_\exists$ must also be of type e, so it will not be of the right type to serve as argument to *rise*. Therefore, we now allow traces (and pronouns) of type $\langle s, e\rangle$. If a trace or pronoun is in position to fill an $\langle s, e\rangle$ argument place, the trace/pronoun must be of type $\langle s, e\rangle$. If it is in position to fill a type e argument place, it must be of type e.[37] If a trace or pronoun is of type $\langle s, e\rangle$, its denotation must be the same as the *intension* of its antecedent, rather than its extension. More formally, we replace (73) with (106):

(106) For all assignments of values to variables g, h, all κ such that CATEGORY(κ, NP), all $w \in W$ and all $t \in T$: $g \sim_{\kappa, w, t} h$ iff

a. there exists some x such that $h(\kappa, w, t) = x$, and for all ε such that ANTECEDENT(ε) = κ, all $w' \in W$ and all $t' \in T$, either $h(\varepsilon, w', t') = x$ or $h(\varepsilon, w', t')$ = that function f: $W \times T \to \mathbf{D}_e$ such that for all $w'' \in W$ and all $t'' \in T$, $f(w'', t'') = h(\text{ANTECEDENT}(\varepsilon), w'', t'')$, according as ε is of type e or of type $\langle s, e\rangle$;

b. for all $\nu \in$ DOMAIN(g), all $w' \in W$, and all $t' \in T$: if $\nu \notin$ NOUNS$_w$(G-DEF$_w$(κ)) and ANTECEDENT(ν) $\notin$ NOUNS$_w$(G-DEF$_{w,t}$(κ)), then $g(\nu, w', t') = h(\nu, w', t')$.

(107) G-DEF$_{w,t}$(α) = $[\lambda g : g$ is an assignment of values to variables . $\alpha \in \text{DOMAIN}([\![\cdot]\!]^{w,t,g})]$

Now we can revise our remaining rules simply by adding t superscripts in the appropriate places and making other obvious adjustments:

(108) For all assignments of values to variables g, all $w \in W$ and all $t \in T$: if $\langle\alpha, w, t\rangle$ is in the domain of g, then $[\![\alpha]\!]^{w,t,g} = g(\alpha, w, t)$.

(109) Where G is a function from $\{g \mid g$ is an assignment of values to variables$\}$ into $\{0, 1\}$: NOUNS$_{w,t}$(G) = $\{\kappa \mid$ NP-PROJECTING(κ) & $\forall g[G(g) = 1 \rightarrow \langle\kappa, w, t\rangle \in$ DOMAIN(g)$]\}$

(110) For all assignments of values to variables g, h, all occurrences α, all $w \in W$ and all $t \in T$: $g \leftharpoonup_{\alpha, w, t} h$ iff for all $\nu \in$ DOMAIN(g), all $w' \in W$, and all $t' \in T$, if $\nu \notin$ NOUNS$_{w,t}$(G-DEF$_{w,t}$(α)) and ANTECEDENT(ν) $\notin$ NOUNS$_{w,t}$(G-DEF$_{w,t}$(α)), then $g(\nu, w', t') = h(\nu, w', t')$.

(111) For all $w \in W$, all $t \in T$, and all assignments of values to variables g:

a. If CATEGORY(α, NOM) and CATEGORY(β, (NOM\TP)), then $[\![\alpha\beta]\!]^{w,t,g} = [\![\beta]\!]^{w,t,g}([\![\alpha]\!]^{w,t,g})$;

[37] Perhaps some people will regard it as a compositionality violation to let the type of an expression depend on its position in the syntactic structure. But since traces and pronouns are syntactically simplex, with no proper constituents, this kind of sensitivity to syntactic position does not come into conflict with the idea that the interpretation of a complex expression depends functionally on the interpretations of its parts.

b. If CATEGORY(β, ((NOM\TP)/ACC)) and CATEGORY(α, ACC), then $[\![\beta\alpha]\!]^{w,t,g} = [\![\beta]\!]^{w,t,g}([\![\alpha]\!]^{w,t,g})$;

c. If OCCURRENCE(δ, *the*) and CATEGORY(κ, NP), then $[\![\delta\ \kappa]\!]^{w,t,g} = g(\kappa, w, t)$, provided that for all assignments h such that $g \sim_{\kappa,w,t} h$, it holds that $g(\kappa, w, t) = h(\kappa, w, t)$. (If there exists some w-assignment h such that $g \sim_{\kappa,w,t} h$ but $g(\kappa, w, t) \neq g(\kappa, w, t)$, then $[\![\textit{the}\ \kappa]\!]^{w,t,g}$ is undefined.)

d. If OCCURRENCE(δ, *a*) and CATEGORY(κ, NP), then $[\![\delta\ \kappa]\!]^{w,t,g} = g(\kappa, w, t)$;

e. If OCCURRENCE(ρ, *Rel*) and CATEGORY(φ, TP), then $[\![\rho\ \varphi]\!]^{w,t,g} = [\![\rho]\!]^{w,t,g}([\![\varphi]\!]^{w,t,g})$;

f. If OCCURRENCE(α, *every*), CATEGORY(κ, NP) and CATEGORY(φ, TP), then $[\![\alpha\ \kappa\ \varphi]\!]^{w,t,g} =$
1 if $\forall x \in \mathbf{D}_e[\exists h[g \sim_{\kappa,w,t} h\ \&\ h(\kappa, w, t) = x] \rightarrow \exists h[g \sim_{\kappa,w,t} h\ \&\ h(\kappa, w, t) = x\ \&\ \exists i[h \multimap_{\varphi,w,t} i\ \&\ [\![\varphi]\!]^{w,t,i} = 1]]$;
0 if $\exists x \in \mathbf{D}_e\, \exists h[g \sim_{\kappa,w,t} h\ \&\ h(\kappa, w, t) = x\ \&\ \forall i[h \multimap_{\varphi,w,t} i \rightarrow [\![\varphi]\!]^{w,t,i} = 0]]$;

g. If OCCURRENCE(α, *most*), CATEGORY(κ, NP) and CATEGORY(φ, TP), then $[\![\alpha\ \kappa\ \varphi]\!]^{w,t,g}$ $= 1$ if $\{x \mid x$ is an individual $\&\ \exists h[g \sim_{\kappa,w,t} h\ \&\ h(\kappa, w, t) = x\ \&\ \exists i[h \multimap_{\varphi,w,t} i\ \&\ [\![\varphi]\!]^{w,t,i} = 1]]\}$ is of greater cardinality than $\{x \mid x$ is an individual $\&\ \exists h[g \sim_{\kappa,w,t} h\ \&\ h(\kappa, w, t) = x\ \&\ \exists i[h \multimap_{\varphi,w,t} i\ \&\ [\![\varphi]\!]^{w,t,i} = 0]\}$;
0 if $\{x \mid x$ is an individual $\&\ \exists h[g \sim_{\kappa,w,t} h\ \&\ h(\kappa, w, t) = x\ \&\ \exists i[h \multimap_{\varphi,w,t} i\ \&\ [\![\varphi]\!]^{w,t,i} = 1]]\}$ is of equal or lesser cardinality than $\{x \mid x$ is an individual $\&\ \exists h[g \sim_{\kappa,w,t} h\ \&\ h(\kappa, w, t) = x\ \&\ \exists i[h \multimap_{\varphi,w,t} i\ \&\ [\![\varphi]\!]^{w,t,i} = 0]\}$;

h. If CATEGORY(α, CP/TP) and CATEGORY(φ, TP), then $[\![\alpha\varphi]\!]^{w,t,g} = [\lambda w',t': w' \in W\ \&\ t' \in T.\ [\![\varphi]\!]^{w',t',g} = 1]$;

i. If CATEGORY(β, ((NOM\TP)/CP) and CATEGORY(ψ, CP), then for all worlds w and assignments of values to variables g, $[\![\beta\ \psi]\!]^{w,t,g} = [\![\beta]\!]^{w,t,g}([\![\psi]\!]^{w,t,g})$.

(112) If $\varphi_1,\ldots,\varphi_n$ is a text, then:

a. $\varphi_1,\ldots,\varphi_n$ is *true relative to w, t* iff there is an assignment g such that $[\![\varphi_1]\!]^{w,t,g} = 1, \ldots, [\![\varphi_n]\!]^{w,t,g} = 1$;

b. $\varphi_1,\ldots,\varphi_n$ is *false relative to w, t* iff there is no assignment g such that $[\![\varphi_1]\!]^{w,t,g} = 1, \ldots, [\![\varphi_n]\!]^{w,t,g} = 1$;

c. $\varphi_1,\ldots,\varphi_n$ is *true* iff $\varphi_1,\ldots,\varphi_n$ is true relative to $w_@$, $t_{\varphi_1,\ldots,\varphi_n}$, where $t_{\varphi_1,\ldots,\varphi_n}$ is the time when $\varphi_1,\ldots,\varphi_n$ is asserted[38];

d. $\varphi_1,\ldots,\varphi_n$ is *false* iff $\varphi_1,\ldots,\varphi_n$ is false relative to $w_@$, $t_{\varphi_1,\ldots,\varphi_n}$.

Now our semantics (unlike that of Lasersohn (2005)) will assign a coherent interpretation to sentences like **[Every temperature$_\exists$ [e rises]]**. In order to serve as argument to **rises**, the trace **e** must be of type $\langle s, e\rangle$. Therefore, we evaluate the scope **[e rises]** relative to various assignments h, relative to which **e** denotes the *intension* of its antecedent, **temperature$_\exists$**, relative to h. The intension of **temperature$_\exists$** relative to a given assignment h is

[38] I assume here that each occurrence has a unique time when it is uttered; if the same sentence is twice, two separate occurrences of it are asserted. This is a slightly different terminology than that adopted in Lasersohn (2017).

a function tracking the temperature of one particular object — but relative to another assignment, the intension of the **temperature**$_{\exists}$ might a function tracking the temperature of some other relevant object. Thus, the sentence (occurrence) is true if the temperature function of each relevant object is a rising function.

The original temperature paradox argument correctly comes out invalid, even if we adopt a quantificational analysis of definites. Suppose we were to replace (111)c. with (113):

(113) If OCCURRENCE(α, *the*), CATEGORY(κ, NP) and CATEGORY(φ, TP), then $[\![\alpha\ \kappa\ \varphi]\!]^{w,t,g} =$
1 if $\exists x \in \mathbf{D}_e \exists h[g \sim_{\kappa,w,t} h \;\&\; h(\kappa, w, t) = x \;\&\; \forall i[g \sim_{\kappa,w,t} i \rightarrow i(\kappa, w, t) = x] \;\&\; \exists j[h \rightharpoonup_{\varphi,w,t} j \;\&\; [\![\varphi]\!]^{w,t,j}]]$;
0 if $\sim\exists x \in \mathbf{D}_e \exists h[g \sim_{\kappa,w,t} h \;\&\; h(\kappa, w, t) = x \;\&\; \forall i[g \sim_{\kappa,w,t} i \rightarrow i(\kappa, w, t) = x] \;\&\; \exists j[h \rightharpoonup_{\varphi,w,t} j \;\&\; [\![\varphi]\!]^{w,t,j}]]$.

Consider the truth condition assigned to (114) under this rule:

(114) **[The temperature$_{\exists}$ [e rises]]**

In order for (114) to be true relative to an assignment g, world w, and time t, there must be exactly one individual x which is the value assigned to $\langle$**temperature**$_{\exists}$, w, $t\rangle$ by every h assigning a value to this triple, such that h agrees with g on all noun-world-time triples including a noun other than **temperature**$_{\exists}$. If h agrees with g in this way, there must be some pragmatically relevant object y such that (for all w', t') h assigns the temperature of y at w', t' to $\langle$**temperature**$_{\exists}$, w', $t'\rangle$. Since x is unique, there can be only one function which maps a pragmatically relevant object onto its temperature at each world and time. That is to say, despite the fact that **temperature** is of type e, not $\langle s, e\rangle$, according to our rules (114) requires a unique temperature *function*, not just a unique temperature *value*. Of course this unique temperature function must also be a rising function in order for (114) to be true.

In contrast, (115) is true (relative to w, t, g) iff the value of the unique temperature function at w, t is 90; the trace here is of type e, and therefore denotes the extension, not the intension of its antecedent.

(115) **[The temperature$_{\exists}$ [e is ninety]]**

Assuming that the intension of *ninety* is the constant function mapping each world-time pair onto 90, (116) will always be false, even if (114) and (115) are true:

(116) **[Ninety rises]**

It is also easy to see that Gupta's argument comes out valid.[39] Sentence (117)a. equates, at all world-time pairs, the value of the unique temperature function with the value of the unique price function. If the unique temperature function yields the same value at every pair as the unique price function, then they are the same function; so if the temperature function is rising, the price function is rising.

(117) a. Necessarily, the temperature is the price.

[39] Of course, we would need to add *necessarily* to our lexicon, and give a rule to the effect that $[\![$**necessarily** $\varphi]\!]^{w,t,g} = 1$ iff for all w',t', $[\![\varphi]\!]^{w',t',g} = 1$.

b. The temperature is rising.
c. Therefore, the price is rising.

A comparison to Romero's analysis is in order. Romero assigns nouns like *temperature* to type $\langle s, \langle\langle s, e\rangle, t\rangle\rangle$, much as in Montague's analysis. She assumes a situation-based semantics, in which possible worlds have smaller situations as parts, some of which may be temporally ordered with respect to one another; this temporal ordering allows situations to take on the function of world-time pairs in Montague's analysis (or the analysis presented here). An expression of type $\langle s, \langle\langle s, e\rangle, t\rangle\rangle$ therefore has two situation argument places — an "outer" argument place corresponding to the first *s*, and an "inner" argument place corresponding to the second *s*. When a quantifier phrase whose head noun is of this type undergoes QR, it leaves a trace of type $\langle s, e\rangle$, where the *s* corresponds to the inner *s* of the noun. Both the outer situation argument place of a noun, and the situation argument place of a trace, may be filled by situation variables, which may be bound by modal or temporal operators.[40]

Two key features of Romero's analysis interact to give correct results for examples like (117). First, modal operators normally may take scope over DPs which have been raised by QR, and bind situation variables in their outer argument places. Second, temporal operators may take scope under these DPs, and bind situation variables in the argument places of their traces (corresponding to the inner argument places of the nouns). When a sentence containing a modal operator involves temporal quantification, the temporal quantification is attributed not to the modal, but to a hidden temporal operator, with narrower scope, by default a universal quantifier like *always*. Hence (117)a., on its most natural interpretation, may be paraphrased as (118), where the boldfaced "at all times" corresponds to the hidden operator:

(118) In all accessible worlds, the unique temperature function and the unique price function are such that **at all times**, they yield the same value.

More precisely, Romero gives the denotations in (119), where Acc(i) is the set of modally accessible situations from i and ≤ is the part-whole relation on situations:[41]

(119) a. $[\![\textit{necessarily}]\!] = \lambda P_{\langle s,t\rangle}\lambda i\,.\,\forall i' \in \mathrm{Acc}(i)\,[P(i')]$
b. $[\![\textit{always}]\!] = \lambda P_{\langle s,t\rangle}\lambda i\,.\,\forall i'' \leq i\,[P(i'')]$
c. $[\![t_1\ \textit{is}\ t_2]\!] = \lambda i'.\ g(1)(i') = g(2)(i')$

The sentence in (120)a. is assigned the logical translation in (120)b. (on the relevant reading):

[40] Romero uses Gallin's (1975) Ty2 as a translation language rather than Montague's IL, so it contains variables of type *s*.

[41] Romero appears to have suppressed a *g*-superscript on the semantic value brackets in these equations.

(120) a. Necessarily, the temperature in Chicago is the same as the temperature in St. Louis.

b. $\forall i' \in Acc(i)\ [\exists x_{\langle s,e\rangle}\ [\forall y[\text{temp-Chicago}(y, i') \leftrightarrow x = y]\ \&\ \exists z_{\langle s,e\rangle}\ [\forall v[\text{temp-StLouis}(v,i') \leftrightarrow z = v]\ \&\ \forall i'' \leq i'\ [x(i'') = z(i'')]]]]]$

It is easy now to see how Romero's analysis solves the problem presented by examples like (117). Under Montague's original analysis, (117)a. means that at any given world-time pair, the temperature function and the price function at that pair yield the same value at that world and at *that* time. But in Romero's analysis, it means that at any given (accessible) world, the temperature function and the price function at that world are such that they both yield the same value at *all* times. Hence a case like that illustrated in (94) and (95) will not verify (117)a., because in (94) and (95), the functions do not yield the same value at all times (in a given world). Because such cases do not verify (117)a., one of the premises of the argument in (117), they do not affect the validity of the argument.

However, Romero's analysis retains a feature of Montague's which seems to me to be very unintuitive: it allows nouns like *temperature* to denote different sets of functions relative to temporally distinct situations. That is, we can have temporal variation not just in what values get returned by the temperature function (or functions), but in which functions count as temperature functions in the first place.

Intuitively, that can't happen. Suppose we are talking about the temperature of one particular object or location, in one particular world. If at time t_1, the temperature function for that object in that world maps t_1 to 90, t_2 to 91, and t_3 to 92, then it cannot happen that at t_2, the temperature function for that object in that world maps t_1 to 85, t_2 to 84, and t_3 to 83. But nothing in Romero's analysis rules out assigning a function like the one illustrated in (121) as the denotation of *temperature*, where situations s_1, s_2 and s_3 are all parts of the same world w_1, and $s_1 < s_2 < s_3$:

(121) $$[\![\textit{temperature}]\!] = \left[\begin{array}{ll} w_1 \to & \left[\begin{array}{ll} \left[\begin{array}{lll} s_1 & \to & 60 \\ s_2 & \to & 60 \\ s_3 & \to & 60 \end{array}\right] & \to 1 \\ \qquad\vdots & \to 0 \end{array}\right] \\ s_1 \to & \left[\begin{array}{ll} \left[\begin{array}{lll} s_1 & \to & 90 \\ s_2 & \to & 91 \\ s_3 & \to & 92 \end{array}\right] & \to 1 \\ \qquad\vdots & \to 0 \end{array}\right] \\ s_2 \to & \left[\begin{array}{ll} \left[\begin{array}{lll} s_1 & \to & 85 \\ s_2 & \to & 84 \\ s_3 & \to & 83 \end{array}\right] & \to 1 \\ & \end{array}\right] \end{array}\right]$$

$$\vdots \qquad\qquad \rightarrow\; 0$$

$$\vdots$$

According to Romero's semantics, the sentence *The temperature is always 60* is true at w_1, even though s_1 and s_2 are parts of w_1, the unique temperature function at s_1 is rising, and the unique temperature function at s_2 is falling. The reason is that the sentence is assigned a translation like that in (122), which fixes x as the unique temperature function at i. If we set i to w_1, then the x will be the constant function mapping s_1, s_2 and s_3 all onto 60. The function which maps s_1 onto 90, etc., and the function which maps s_2 onto 85, etc., simply do not enter into the calculation.

(122) $\lambda i \,.\, \exists x_{\langle s,e\rangle} [\forall y[\text{temperature}(y, i) \leftrightarrow x = y] \;\&\; \forall i'' \leq i\, [x(i'') = 60]]$

On the other hand, *The temperature is always 60* will be **false** at s_1 and s_2, even though it is **true** at the world of which these situations are part.

Is this a serious problem? It depends in part on issues connected with the nature of assertion, about which Romero is silent. Suppose one is in situation s_1, and asserts *The temperature is always 60*. Is the assertion true, because s_1 is part of w_1, and the sentence is true at w_1? Or is it false, because the sentence is false at s_1?

My own feeling is that issues like this should not even come up, because (121) does not represent any coherent state of affairs. The fact that the issue even arises is a sign that the denotation illustrated in (121) should not be allowed. Such denotations are easy enough to rule out by meaning postulate or some similar lexical stipulation — either in Romero's framework or in Montague's; but it would be preferable, if possible, to find a more "architectural" solution, which prevented a case like this from coming up in the first place.

The analysis proposed here provides such a solution. No denotations analogous to (121) can be constructed under this analysis. Because ordinary common nouns are of type e, rather than $\langle e, t\rangle$, their intensions are of type $\langle s, e\rangle$, and so are the traces which they antecede. This type contains a single s, and therefore allows just a single layer of temporal variation. But if common nouns are basically of type $\langle e, t\rangle$, then there are two levels at which intensionality can enter: the level of the noun's argument place (yielding type $\langle\langle s, e\rangle, t\rangle$) and the level of the noun as a whole (yielding type $\langle s, \langle e, t\rangle\rangle$). If it enters at both levels, the type is Montague's and Romero's $\langle s, \langle\langle s, e\rangle, t\rangle\rangle$, with two layers of potential variation, and some sort of constraint must be imposed to rule out denotations like (121).

9. Making it compositional

Our rules so far have treated quantification syncategorematically. No denotations have been assigned to quantificational determiners or to quantifier phrases. This is unsatisfactory from the point of view of the theory of semantic compositionality; the denotation of every complex phrase should depend functionally on the denotations of its immediate syntactic constituents.

In considering this problem, it is important to be clear about what, exactly, we mean by *compositional* denotation assignment, and to recognize that even the standard semantics for variable binding exposited in textbooks such as Dowty, Wall and Peters (1981), Chierchia and McConnell-Ginet (1990), Gamut (1991), Heim and Kratzer (1998), etc. is in some sense non-compositional. To obtain a fully compositional semantics for variables and variable binding, we will have to make some adjustments to this technique, whether or not common nouns are analyzed as variables.

There are various non-equivalent definitions of compositionality, but here let us assume a semantic theory **T** is compositional iff it meets the condition in (123):

(123) According to **T**, there is some operation O such that for all α: if α has immediate syntactic constituents $\beta_1,\ldots,\beta_n$, then $[\![\alpha]\!]^{p_1,\ldots,p_m} = O([\![\beta_1]\!]^{p_1,\ldots,p_m},\ldots, [\![\beta_n]\!]^{p_1,\ldots,p_m})$ for all parameter values $p_1,\ldots,p_m$ relative to which denotations are assigned according to **T**.

Notice that familiar variable binding rules like those in (5), repeated here with a minor alteration as (124), do not conform to this conception of compositionality, since they make $[\![\forall\alpha\varphi]\!]^g$ and $[\![\exists\alpha\varphi]\!]^g$ dependent not on $[\![\varphi]\!]^g$ but on $[\![\varphi]\!]^h$, for various different assignments h:

(124) a. $[\![\forall\alpha\varphi]\!]^g = 1$ iff $[\![\varphi]\!]^h = 1$ for all h agreeing with g on all variables other than α.
b. $[\![\exists\alpha\varphi]\!]^g = 1$ iff $[\![\varphi]\!]^h = 1$ for at least one h agreeing with g on all variables other than α.

This is not hard to fix, and has not usually been seen as a significant problem for the theory of compositionality precisely because familiar techniques allow a reformulation of (124) so that it is consistent with (123). For example, following Montague (1968), we may take expressions containing free variables (or expressions in general) as denoting functions from assignments to denotations of the more usual sort. For example, we would replace the rule in (125)a. with the one in (125)b., where G is the set of assignments of values to variables:

(125) a. For all assignments g: if α is a variable, then $[\![\alpha]\!]^g = g(\alpha)$
b. If α is a variable, then $[\![\alpha]\!] = [\lambda g: g \in G\,.\, g(\alpha)]$

Combined with other rules on the same model, this gives the effect that each formula has a single denotation, which is a function from assignments to truth values, rather than having a whole series of denotations, each one a truth value, and each one keyed to an assignment. Now we can reformulate (124) as :

(126) a. $[\![\forall\alpha\varphi]\!] = [\lambda g: g \in G.\ [\![\varphi]\!](h) = 1$ for all h agreeing with g on all variables other than $\alpha]$

b. $[\![\exists\alpha\varphi]\!] = [\lambda g: g \in G.\ [\![\varphi]\!](h) = 1$ for at least one h agreeing with g on all variables other than $\alpha]$

The standard generalized quantifier analysis of determiners, under which they denote relations between sets, appeals to a violation of (123) no less than (124) does. For example, the truth value of (127) relative to a given assignment g is determined by whether the set denoted by *dog* relative to g stands in the relation denoted by *every* relative to g to the set of all x such that John sees x — and this set is calculated by finding the truth value of [*John sees e*] relative to a series of assignments h which differ from g in what they assign to *e*. But here again we can easily reformulate the analysis so it conforms to (123) simply by letting expressions denote functions from assignments to denotations of the more usual sort. In this case [*John sees e*] will denote the function indicated in (128)a. The set of things which John sees is straightforwardly recoverable from this function as shown in (128)b.:

(127) [[Every dog][John sees *e*]]

(128) a. $[\![John\ sees\ e]\!] = [\lambda g: g \in G.$ John sees $g(e)]$

b. $\{x \in \mathbf{D}_e \mid$ John sees $x\} = \{x \in \mathbf{D}_e \mid \exists g[g(e) = x\ \&\ [\![John\ sees\ e]\!](g) = 1]\}$

It should be noted that denotations like those assigned in (126) and (128)a. do not fit into our current type system. As a first step in adapting this technique to our current purposes, then, let us add a new series of types beginning with *r*, whose domains will include partial functions whose domain is the set of all assignments $G = \{g \mid g$ is an assignment of values to variables$\}$:

(129) a. $\mathbf{D}_e = \mathbf{D}$

b. $\mathbf{D}_t = \{0, 1\}$

c. For any types σ, τ: $\mathbf{D}_{\langle\sigma,\tau\rangle} = \{f \mid f$ is function from $\mathbf{D}_\sigma$ to $\mathbf{D}_\tau\}$

d. For all types σ, $\mathbf{D}_{\langle s,\sigma\rangle} = \{f \mid f$ is a function from $W \times T$ to $\mathbf{D}_\sigma\}$

e. For all type σ, $\mathbf{D}_{\langle r,\sigma\rangle} = \{f \mid f$ is a partial function from G to $\mathbf{D}_\sigma\}$

Next, we reformulate our lexical entries and compositional rules to eliminate the *g*-superscript, and assign expressions denotations in types beginning with *r*:

(130) For all $w \in W$, all $t \in T$, and all assignments of values to variables g:

g. If OCCURRENCE(α, *John*) then $[\![\alpha]\!]^{w,t} = [\lambda g: g \in G.$ John$]$;

h. If OCCURRENCE(α, *Mary*) then $[\![\alpha]\!]^{w,t} = [\lambda g: g \in G.$ Mary$]$;

i. If OCCURRENCE(α, *Bill*) then $[\![\alpha]\!]^{w,t} = [\lambda g: g \in G.$ Bill$]$;

j. If OCCURRENCE(α, *Susan*) then $[\![\alpha]\!]^{w,t} = [\lambda g: g \in G.$ Susan$]$;

k. If OCCURRENCE(α, *ninety*) then $[\![\alpha]\!]^{w,t} = [\lambda g : g \in G\,.\,90]$;

l. If OCCURRENCE(α, *smile*) then $[\![\alpha]\!]^{w,t} = [\lambda g : g \in G\,.\,[\lambda x : x \in \mathbf{D}_e\,.\,\forall y \in \mathbf{D}_e[y$ is a member of $x \rightarrow y$ smiles at t in $w]]]$;

a. If OCCURRENCE(α, *frown*) then $[\![\alpha]\!]^{w,t} = [\lambda g : g \in G\,.\,[\lambda x : x \in \mathbf{D}_e\,.\,\forall y \in \mathbf{D}_e[y$ is a member of $x \rightarrow y$ frowns at t in $w]]]$;

b. If OCCURRENCE(α, *see*) then $[\![\alpha]\!]^{w,t} = [\lambda g : g \in G\,.\,[\lambda x : x \in \mathbf{D}_e\,.\,[\lambda y : y \in \mathbf{D}_e\,.\,y$ sees x at t in $w]]]$;

c. If OCCURRENCE(α, *hear*) then $[\![\alpha]\!]^{w,t} = [\lambda g : g \in G\,.\,[\lambda x : x \in \mathbf{D}_e\,.\,[\lambda y : y \in \mathbf{D}_e\,.\,y$ hears x at t in $w]]]$;

d. If OCCURRENCE(α, *believe*) then $[\![\alpha]\!]^{w,t} = [\lambda g : g \in G\,.\,[\lambda p : p \in \mathbf{D}_{\langle s,t\rangle}\,.\,[\lambda x : x \in \mathbf{D}_e\,.\,\forall y \in \mathbf{D}_e\,[y$ is a member of $x] \rightarrow \forall w' \in W[w' \in \mathbf{B}_{y,w,t} \rightarrow p(w', t) = 1]]]]$;

e. If OCCURRENCE(α, *know*) then $[\![\alpha]\!]^{w,t} = [\lambda g : g \in G\,.\,[\lambda p : p \in \mathbf{D}_{\langle s,t\rangle}\,.\,[\lambda x : x \in \mathbf{D}_e\,.\,\forall y \in \mathbf{D}_e[y$ is a member of $x] \rightarrow \forall w' \in W[w' \in \mathbf{K}_{y,w,t} \rightarrow p(w', t) = 1]]]]$;

f. If OCCURRENCE(α-*s*, (NOM, SINGULAR)\X)), where X is a potential head category of S, then $[\![\alpha\text{-}s]\!]^{w,t} = [\lambda g : g \in G\,.\,[\lambda x : x$ is an individual $.\,[\![\alpha]\!]^{w,t}(x)]]$;

g. If OCCURRENCE(ρ, *Rel*), then $[\![\rho]\!]^{w,t} = [\lambda g : g \in G\,.\,[\lambda p : p \in \mathbf{D}_t \,\&\, p = 1\,.\,[\lambda x : x \in \mathbf{D}_e\,.\,x]]]$;

h. If OCCURRENCE(α, *of*) then $[\![\alpha]\!]^{w,t} = [\lambda g : g \in G\,.\,[\lambda x : x \in \mathbf{D}_e\,.\,x]]$

(131) For all assignments of values to variables g, all $w \in W$ and all $t \in T$: if $\langle \alpha, w, t\rangle$ is in the domain of g, then $[\![\alpha]\!]^{w,t} = [\lambda g : g \in G\,.\,g(\alpha, w, t)]$.

Next we need to reformulate our compositional rules. Some of our existing rules, such as (111)a. and (111)b., appeal to an operation of function application. But now the immediate constituents of a complex expression will denote functions with domain G, so neither will be able to take the denotation of its sister phrase as an argument. We therefore define an operation, notated APPLY, which "holds open" any argument places for assignments if necessary, but otherwise works like ordinary function application:[42]

(132) a. If for some types σ, τ: $f \in \mathbf{D}_{\langle r, \langle \sigma, \tau\rangle\rangle}$ and $a \in \mathbf{D}_{\langle r, \sigma\rangle}$, then APPLY$(f, a) = [\lambda g : g \in$ *domain*$(f) \cap$ *domain*$(a)\,.\,f(g)(a(g))]$.

b. If for some types σ, τ: $f \in \mathbf{D}_{\langle r, \langle \sigma, \tau\rangle\rangle}$ and $a \in \mathbf{D}_{\sigma}$, then APPLY$(f, a) = [\lambda g : g \in$ *domain*$(f)\,.\,f(g)(a)]$.

Now we can revise the rules in (111)a., (111)b. and (111)i. so that they use APPLY instead of ordinary function application:[43]

(133) For all $w \in W$, all $t \in T$, and all assignments of values to variables g:

[42] Ultimately, it is probably necessary to define APPLY to cover four possible cases: the two in (132), as well as the case where $f \in \mathbf{D}_{\langle \sigma, \tau\rangle}$ and $a \in \mathbf{D}_{\langle r, \sigma\rangle}$, and the case where $f \in \mathbf{D}_{\langle \sigma, \tau\rangle}$ and $a \in \mathbf{D}_{\sigma}$. We omit the definitions for these last two cases here, as they play no role in the examples discussed in this paper; but see Lasersohn (2017) for details.

[43] The g superscripts are also dropped.

a. If CATEGORY(α, NOM) and CATEGORY(β, (NOM\TP)), then $[\![\alpha\beta]\!]^{w,t}$ = APPLY($[\![\beta]\!]^{w,t}$, $[\![\alpha]\!]^{w,t}$);

b. If CATEGORY(β, ((NOM\TP)/ACC)) and CATEGORY(α, ACC), then $[\![\beta\alpha]\!]^{w,t}$ = APPLY($[\![\beta]\!]^{w,t}$, $[\![\alpha]\!]^{w,t}$);

c. If CATEGORY(β, ((NOM\TP)/CP) and CATEGORY(ψ, C), then for all worlds w and assignments of values to variables g, $[\![\beta\ \psi]\!]^{w,t}$ = APPLY($[\![\beta]\!]^{w,t}$, $[\![\psi]\!]^{w,t}$).

It may be helpful to consider the denotation of common noun phrases in this system. According to (130) If α is a variable — such as a common noun occurrence — then $[\![\alpha]\!]^{w,t} = [\lambda g : g \in G\, .\, g(\alpha, w, t)]$. For example, $[\![$**professor**$]\!]^{w,t} = [\lambda g : g \in G\, .\, g($**professor**$, w, t)]$. We continue to use (104) as our definition of assignments of values to variables, according to which $g($**professor**$, w, t)$ is a professor in w at t. So $[\![$**professor**$]\!]^{w,t}$ is a function mapping assignments onto people who are professors in w at t. More generally, if α is a common noun, then $[\![\alpha]\!]^{w,t}$ is a function mapping each assignment g onto whichever individual g maps $\langle\alpha, w, t\rangle$ onto.

This means that the identity of a noun phrase is recoverable from its denotation: Suppose f is a function from assignments to individuals, and that for all g in the domain of f, $f(g) = g(\kappa, w, t)$. Then $f = [\![\kappa]\!]^{w,t}$. It will be useful to have some notation so that we can exploit this fact in our rules. Whenever f is a noun phrase denotation, let DENOTANS(f, w, t) be the noun which denotes f relative to w, t:

(134) DENOTANS(f, w, t) = κ iff for all $g \in domain(f)$: $f(g) = g(\kappa, w, t)$.

This fact is useful, because our existing quantifier definitions directly mention the noun phrase with which the quantifier combines. In a more compositional analysis, the quantifier should denote a function, one of whose arguments is the *denotation* of this noun phrase. The recoverability of the noun phrase from its denotation gives us a way to adapt our existing rules without directly mentioning the noun phrase itself.

(135) For all assignments of values to variables g, h, all f: G → $\mathbf{D}_e$, and all $w \in W$ and $t \in T$: $g \sim_{f,w,t} h$ iff

a. there exists some x such that $h(f) = x$, and for all ε such that ANTECEDENT(ε) = DENOTANS(f, w, t), all $w' \in W$ and all $t' \in T$, either $h(\varepsilon, w', t') = x$ or $h(\varepsilon, w', t')$ = that function f: $W \times T \to \mathbf{D}_e$ such that for all $w'' \in W$ and all $t'' \in T$, $f(w'', t'') = h($ANTECEDENT$(\varepsilon), w'', t'')$, according as ε is of type e or of type $\langle s, e\rangle$;

b. for all ν ∈ DOMAIN(g), all $w' \in W$, and all $t' \in T$: if ν ∉ $\text{NOUNS}_w(\text{G-DEF}_w(\kappa))$ and ANTECEDENT(ν) ∉ $\text{NOUNS}_w(\text{G-DEF}_{w,t}(\kappa))$, then $g(\nu, w', t') = h(\nu, w', t')$.

We will also need to make an adjustment to our '$\leadsto_{\varphi,w,t}$' notation. To keep the semantics compositional, the lexical entry for a quantifier should not directly refer to the sentence-occurrence which serves as its scope, but only to its denotation, which is a function from assignments to truth values. We therefore revise (110) to (136):

(136) For all g, $h \in G$, all $p \in \mathbf{D}_{\langle r,t\rangle}$, all $w \in W$ and all $t \in T$: $g \leftharpoondown_{p,w,t} h$ iff for all $\nu \in$ DOMAIN(g), all $w' \in W$, and all $t' \in T$, if $\nu \notin$ NOUNS$_{w,t}(p)$ and ANTECEDENT(ν) $\notin$ NOUNS$_{w,t}(p)$, then $g(\nu, w', t') = h(\nu, w', t')$.

Now we can define *every*, *most* and *the* as in (137):

(137) a. If OCCURRENCE(α, *every*) then $[\![\alpha]\!]^{w,t} = [\lambda g : g \in G \,.\, [\lambda y : y \in \mathbf{D}_{\langle r,e\rangle} \,.\, [\lambda p : p \in \mathbf{D}_{\langle r,t\rangle} \,.\, \forall x \in \mathbf{D}_e[\exists h[g \sim_{y,w,t} h \;\&\; h(y, w, t) = x] \rightarrow \exists h[g \sim_{y,w,t} h \;\&\; h(y, w, t) = x \;\&\; \exists i[h \leftharpoondown_{\text{NOUNS}_{w,t}(p),w,t} i \;\&\; p(i) = 1]]$.

b. If OCCURRENCE(α, *most*) then $[\![\alpha]\!]^{w,t} = [\lambda g : g \in G \,.\, [\lambda y : y \in \mathbf{D}_{\langle r,e\rangle} \,.\, [\lambda p : p \in \mathbf{D}_{\langle r,t\rangle} \,.\, \{x \mid x \text{ is an individual } \& \;\exists h[g \sim_{y,w,t} h \;\&\; h(y, w, t) = x \;\&\; \exists i[h \leftharpoondown_{\text{NOUNS}_{w,t}(p),w} i \;\&\; p(i) = 1]\} \text{ is of greater cardinality than } \{x \mid x \text{ is an individual } \& \;\exists h[g \sim_{y,w,t} h \;\&\; h(y, w, t) = x \;\&\; \exists i[h \leftharpoondown_{\text{NOUNS}_{w,t}(p),w,t} i \;\&\; \text{p}(i) = 0]\}]]]$.

c. If OCCURRENCE(α, *the*) then $[\![\alpha]\!]^{w,t} = [\lambda g \in G[\lambda y : y \in \mathbf{D}_{\langle r,e\rangle} \;\&\; \forall h \in G[g \sim_{y,w,t} h \rightarrow g(y, w, t) = h(y, w, t) \,.\, y]]]$

It is perhaps worth pointing out that the type of *every* and *most* in this system is $\langle r, \langle\langle r, e\rangle, \langle\langle r, t\rangle, t\rangle\rangle\rangle$, unlike the more conventional $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$. *The* is of type $\langle r, \langle\langle r, e\rangle, e\rangle\rangle$.

Our existing rule in (111)h. does not adapt quite so easily, because it already violates (123) independently of considerations having to do with variables: according to (111)h., the denotation of an expression relative to a world w and time t can depend on its denotations relative to other worlds and times. The rule could be brought into conformity in either of at least two ways: We could let all expressions denote functions from world-time pairs to denotations of a more conventional type, effectively eliminating the intension/extension distinction; or, as I prefer, we could follow Frege (1892) and let expressions "shift" their denotations in certain syntactic contexts, so that in those contexts (only), they denote what would customarily be their intensions.[44] Because this problem is orthogonal to the main concerns of this paper, we leave the rule in some sense non-compositional, but revise it so that it will interact properly with our other rules:

(138) If CATEGORY(α, CP/TP) and CATEGORY(φ, TP), then $[\![\alpha\varphi]\!]^{w,t} = [\lambda g : g \in G\,[\lambda w',t' : w' \in W \;\&\; t' \in T \,.\, [\![\varphi]\!]^{w',t'}(g) = 1]]$.

10. Bare plurals

Treating common nouns as modally non-rigid restricted variables allows an improvement in the analysis of bare plurals. Previous analyses of the semantics of English bare plurals generally fall into two classes. One style of analysis, originating with Carlson (1977a), Carlson (1977b), treats bare plurals as unambiguously denoting "kinds" of things. When a

[44] See Lasersohn (2017) for one way to work this idea out formally, in a framework very similar to the one argued for in this paper.

bare plural combines with a kind-level predicate, as in (139)a., the predicate is applied directly to the kind, analogously to the formula in (139)b.

(139) a. Dodos are extinct.
b. extinct(d)

When bare plural combines with an individual-level, or stage-level predicate, as in (140)a., (141)a., a generic or existential operator quantifies over realizations of the kind, as in (140)b., (141)b.:

(140) a. Elephants are big.
b. (GEN x : realization(x, e))big(x)

(141) a. Bagels are available.
b. $\exists x$[realization(x, b) & available(x)]

In Carlson's original analysis, operators like these[45] are analyzed as part of the internal lexical semantics of the predicate. In all three types of sentences, the bare plural subject is treated as denoting a kind (in these cases, d, e, or b). Let us call this style of analysis "Option 1."

In the second style of analysis, represented by Wilkinson (1991), Diesing (1992), Gerstner-Link and Krifka (1993) and others, bare plurals are treated as ambiguous. When they combine with a kind-level predicate, they denote kinds, just as in Option 1; (139)a. continues to be treated on the model of (139)b. But when a bare plural combines with an individual-level or stage-level predicate, it does not denote a kind, but instead functions like an open formula, containing a free variable. Operators can unselectively bind this variable, which serves directly as argument to the predicate, as in (142)b., (143)b.:

(142) a. Elephants are big.
b. (GEN x : elephant(x))big(x)

(143) a. Bagels are available.
b. $\exists x$[bagel(x) & available(x)]

Note that in (142)b., (143)b. the kind-denoting terms e, b do not appear, but simply the $\langle e, t\rangle$ predicates 'elephant' and 'bagel'. Let us call this style of analysis "Option 2."

Arguments can be given in favor of both options. In support of Option 1, we may note that a single occurrence of a bare plural can combine with a coordinate structure whose conjuncts are predicates from different classes:

(144) Dogs are [widespread, loyal, and asleep on the porch]

Considerations of compositionality lead us to posit a single denotation for *dogs* in this example, which can serve as argument to [*widespread, loyal, and asleep on the porch*]. If

[45] Carlson (1977a), Carlson (1977b) used a somewhat different operator than GEN, which did not bind variables; I have taken the liberty of recasting the analysis using the now widely-adopted GEN operator.

that denotation is the kind, we can treat the whole coordinate structure as a kind-level predicate, as in (145). Operators internal to the second and third conjuncts would then relate the kind to its realizations as required:

(145) λk[widespread(k) & (GEN x: realization(x, k))loyal(x) & $\exists x$[realization(x, k) & asleep(x)]

To deal with this kind of example, an advocate of Option 2 must appeal to "lifting" operations to coerce individual- or stage-level predicates into kind-level predicates:

(146) a. $\text{LIFT}_{ind}(P) = \lambda k$[(GEN x: realization(x, k))$P(x)$]
b. $\text{LIFT}_{stage}(P) = \lambda k[\exists x$[realization($x$, k) & $P(x)$]

The lifted predicates could then conjoin with a kind-level predicate to give the correct interpretation. But making this move amounts to adopting Option 1 when needed, even if we are using Option 2 most of the time; it represents something less than a firm commitment to Option 2.

In support of Option 2, we may note that bare plurals are bindable by adverbs of quantification.

(147) a. Usually, dogs are furry.
b. (usually x: dog(x))furry(x)

This is as expected under Option 2, since bare plurals are treated as analogous to open formulas, containing a free variable which is subject to unselective binding. Nothing in Option 1 leads us to expect that bare plurals should be bindable in this way. The bare plural DP *dogs*, for example, is treated as simply denoting the kind d, with no free variable.

To deal with such examples, an advocate of Option 1 must appeal to a "lowering" operation to coerce a kind-denoting expression to a predicate holding of instantiations of the kind:

(148) $\text{LOWER}_{w,t}(k) = \lambda x[\text{realization}_{w,t}(x, k)]$

This gives the DP *dogs* a secondary denotation λx[realization(x, d)], in addition to its primary denotation d. The combination of this DP with an index i can then be interpreted like an open formula: realization(x_i, d). The DP *dogs*$_i$ now contains a free variable and will permit binding by an adverb of quantification.

But making this move amounts to adopting Option 2 when needed, even if we are using Option 1 most of the time; it represents something less than a firm commitment to Option 1.[46] It appears that no matter whether we choose Option 1 or Option 2, we are forced to use the other option at least some of the time.

[46] It also requires an odd "Duke of York" detour in the derivation, assuming that the kind-denoting DP *dogs* is derived from the type $\langle e, t\rangle$ NP *dogs*. At any given world w and time t, this NP must denote the function $\lambda x[\text{dog}_{w,t}(x)]$. The DP *dogs* receives a denotation in accordance with a rule to the effect that $[\![\text{DP}]\!]^{w,t} =$

Treating common nouns as modally non-rigid restricted variables offers a way out of this dilemma. In this approach, it is possible to recover a kind from the values of a variable relative to different assignments — which is to say, from the denotation of variable relative to different worlds. This will allow us to build reference to kinds into the internal semantics of kind level predicates, instead of the internal semantics of bare plural DPs. Bare plurals can then be treated as unambiguously like plural indefinites.

To see this, first let us define the *maximal* value of a plural noun (in a given world at a given time) as the group which has all and only the values of that noun as subgroups:[47]

(149) $\forall x[x$ is a subgroup of $\mathrm{MAX}(\llbracket \kappa_{\mathrm{pl}} \rrbracket^{w,t}) \leftrightarrow \exists g \llbracket \kappa_{\mathrm{pl}} \rrbracket^{w,t}(g) = x]$

For example, $\mathrm{MAX}(\llbracket \textbf{professors} \rrbracket^{w,t})$ will be the group which has all and only the groups of professors in w at t as subgroups — in other words, the group whose members are all the professors in w at t.

Now the kind corresponding to a bare plural will be the one whose realizations in each world are members of the maximal value of the variable relative to that world:

(150) $K(\llbracket \kappa \rrbracket^{w,t}) = \iota k[\forall w' \forall t' \forall x[\mathrm{realization}_{w',t'}(x, k) \leftrightarrow x$ is a member of $\mathrm{MAX}(\llbracket \kappa \rrbracket^{w',t'})]]$

For example, $K(\llbracket \textbf{professors} \rrbracket^{w,t})$ will be the kind whose realizations in any given world at any given time are members of the maximal group of professors in that world at that time.

The similarity of (150) to the rule mentioned in Footnote 46 for deriving the denotation of a bare plural DP from its NP should be evident. In both cases, a kind is recovered from an NP intension, using essentially the same information. But (150) is not stated directly as a rule for deriving the denotation of a DP — just for recovering the kind. This allows us to build the K operator into the internal lexical semantics of kind-level predicates:

(151) a. *abound*: ((NOM, PLURAL)\TP)
b. If OCCURRENCE(α, *abound*) then $\llbracket \alpha \rrbracket^{w,t} = [\lambda g : g \in G\,.\,[\lambda x : x \in \mathbf{D}_{\langle r,e \rangle}\,.\,K(x)$ abounds in w at $t]]$

In order to allow plural nouns to combine directly with verbs, without a determiner, we simply assign them to case categories:

(152) *professors, students*: NP, PLURAL, NOM, ACC

As an example, let us work through an occurrence of the sentence *Students abound*:

$\iota k[\forall w' \forall t' \forall x[\mathrm{realization}_{w',t'}(x, k) \leftrightarrow \llbracket \mathrm{NP} \rrbracket^{w',t'}(x)]]$. (See Carlson (1977a: 216).) That is, $\llbracket [_{\mathrm{DP}}\, dogs] \rrbracket^{w,t} = \iota k[\forall w' \forall t' \forall x[\mathrm{realization}_{w',t'}(x, k) \leftrightarrow \mathrm{dog}_{w',t'}(x)]] = d$. But now, in order to make this DP bindable, we must apply LOWER to coerce it back to type $\langle e, t \rangle$: $\mathrm{LOWER}_{w,t}(d) = \lambda x[\mathrm{realization}_{w,t}(x, d)] = \lambda x[\mathrm{dog}_{w,t}(x)]$. But this is just what we started with! If we already had something denoting $\lambda x[\mathrm{dog}_{w,t}(x)]$, why detour through $\iota k[\forall w' \forall t' \forall x[\mathrm{realization}_{w',t'}(x, k) \leftrightarrow \mathrm{dog}_{w',t'}(x)]]$ if we are just going to convert it back to $\lambda x[\mathrm{dog}_{w,t}(x)]$?

[47] x is a subgroup of y iff every member of x is a member of y.

(153) $[\![\textbf{Students abound}]\!]^{w,t} = \text{APPLY}([\![\textbf{abound}]\!]^{w,t}, [\![\textbf{students}]\!]^{w,t})$

$= [\lambda g: g \in G\,.\, [\![\textbf{abound}]\!]^{w,t}(g)([\![\textbf{students}]\!]^{w,t})]$

$= [\lambda g: g \in G\,.\, [\lambda x: x \in \mathbf{D}_{\langle r,e\rangle}\,.\, K(x) \text{ abounds in } w \text{ at } t]\ ([\![\textbf{students}]\!]^{w,t})]$

$= [\lambda g: g \in G\,.\, K([\![\textbf{students}]\!]^{w,t}) \text{ abounds in } w \text{ at } t]$

$= [\lambda g: g \in G\,.\, \iota k[\forall w' \forall t' \forall x[\text{realization}_{w',t'}(x, k) \leftrightarrow x \text{ is a member of } \text{MAX}([\![\textbf{students}]\!]^{w',t'})] \text{ abounds in } w \text{ at } t]$

$= [\lambda g: g \in G\,.\, \iota k[\forall w' \forall t' \forall x[\text{realization}_{w',t'}(x, k) \leftrightarrow x \text{ is a student in } w' \text{ at } t'] \text{ abounds in } w \text{ at } t]$

Notice that the bare plural **students** is treated here not as denoting a kind, but simply as a variable ranging over students and groups of students — that is, as a plural indefinite. The predicate **abound** “scans” the values of this variable relative to various worlds and times to recover the kind whose realizations are those values. It then predicates abundance of that kind.

Put differently, **abound** does not directly take a kind as its argument, but rather a variable denotation. This opens the way to conjoin kind-level predicates with individual-level or stage-level predicates, *without* coercing the individual-level or stage-level predicates into kind-level predicates. At the same time, bare plurals, as variables, are predicted to be bindable by adverbs of quantification, without having to undergo any kind of lowering operation.

Why is this solution not available in a more conventional analysis? Because in such an analysis, common nouns are treated not as variables, but as constants of type $\langle e, t\rangle$. A kind can easily be recovered from the intension of such a constant, but the constant cannot be bound, and is not of the right type to serve as argument to a verb. The constant might take a variable as an argument, and this variable would be bindable, and of the right type to serve as argument to the verb; but in a conventional analysis, variables are modally rigid, so one cannot recover the kind corresponding to the noun from the values of a variable which serves as argument to that noun. We eliminated this problem by eliminating the distinction between the noun and the variable.[48]

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[48] Examples like *This kind of animal is widespread* may appear to present a problem for this sort of analysis. On the assumption that the phrase *this kind of animal* denotes a kind, this example would seem to show that *widespread* must take a kind as its argument. However, Wilkinson (1991), Wilkinson (1995) argues that phrases like *this kind of animal* have a reading under which they function as pseudo-partitives, rather than as kind-denoting terms. This idea may be implemented straightforwardly in the present framework. Specifically, we treat *this kind of animal* as a variable ranging over individuals and groups of animals belonging to the indexically indicated kind, similarly to the phrase *animals of this kind*. The predicate *widespread* can recover the kind from these values in different worlds, just as with the bare plural. Taking this approach also explains why *this kind of animal* permits binding by adverbs of quantification, as *This kind of animal is usually fierce*.

11. Some speculative hypotheses

In this section we briefly review several potential advantages to an analysis of common nouns as variables, but only in a tentative and speculative way. The topics discussed here strike me as worthwhile avenues for further exploration, but would all require much more extensive investigation before conclusions could be drawn.

In the interest of readability, in this section we will sometimes revert to the system we had in place before Section 9, in which expressions are assigned denotations relative to assignments of values to variables, rather than denotations which are functions from assignments of values to variables.

11.1. Collective conjunction

One potential advantage to treating common nouns as variables rather than predicates might be that it simplifies the semantics for collectivizing conjunction of common nouns and noun phrases.

By "collectivizing" conjunction, I mean the use of *and* to form a phrase denoting the group whose members are the denotations of the conjuncts. For example, it is natural to analyze the phrase *John and Mary* in (154)a. as denoting the group whose members are John and Mary. This kind of conjunction seems different from the kind of conjunction which reduces to truth-functional negation, as in (155)a. Note that (154)a. is not truth-conditionally equivalent to (154)b., though (155)a. is truth-conditionally equivalent to (155)b.:[49]

(154) a. John and Mary are a happy couple.
b. John is a happy couple and Mary is a happy couple.

(155) a. John smokes and drinks.
b. John smokes and John drinks.

Reverting for the sake of simplicity to system in place before Section 9, we may treat ordinary truth-functional *and* as in (156), essentially following Partee and Rooth (1983):[50]

(156) a. and_t : ((X\X)/X)
b. If OCCURRENCE(α, and_t), then for all $w \in W$ and all assignments g: $[\![\alpha]\!]^{w,t,g}$ = the smallest function f such that:
i. $f(1,1) = 1$; $f(1,0) = 0$; $f(0,1) = 0$; $f(0,0) = 0$;

[49] Various strategies exist for unifying the semantics of the two types of conjunction. See, e.g. Lasersohn (1995), Winter (2001). We treat them here separately in the interest of simplicity.

[50] In (156)a., we let X range over sets of syntactic categories; but under the semantics assigned in (156)b., phrases formed with and_t will not be interpretable unless the conjuncts are of a conjoinable types — that is, types ending in t.

ii. For all types σ, conjoinable types τ, and $h, i \in \mathbf{D}_{\langle\sigma,\tau\rangle}$: $f(h, i) = [\lambda x : x \in \mathbf{D}_\sigma \,.\, f(h(x), i(x))]$.

The reader may now easily confirm that our grammar will interpret **John smiles and frowns** correctly as true iff John smiles and John frowns.

The semantics for collectivizing conjunction of expressions of type e is also fairly straightforward to formulate. We assume for any two individuals x and y, there is a group with x and y as its (sole) members. Let us notate this group "$x+y$". We add and_c to our lexicon and assign a denotation to its occurrences as in (157)b.:[51]

(157) a. and_c : ((X\X)/X)

b. If OCCURRENCE(α, and_c), then for all $w \in W$ and all assignments g: $[\![\alpha]\!]^{w,t,g} = [\lambda x : x \in \mathbf{D}_e \,.\, [\lambda y : y \in \mathbf{D}_e \,.\, x+y]]$.

Subjects formed with and_c normally impose plural agreement on the verb.[52] To obtain this result, we assume the principles in (158), where CG is the "common ground" — the set of worlds compatible with what is presumed as shared public knowledge in the discourse context:

(158) For all occurrences α:

a. if for all $w \in CG$ and all assignments g, $[\![\alpha]\!]^{w,t,g}$ is a group, then CATEGORY(α, PLURAL);

b. if for all $w \in CG$ and all assignments g, $[\![\alpha]\!]^{w,t,g}$ is an individual, then CATEGORY(α, SINGULAR).

The semantic effect of collective conjunction is most easily seen if we add some clearly collective predicates to our lexicon:

(159) a. *meet*, *disperse*: ((NOM, PLURAL)\TP)

b. If OCCURRENCE(α, *meet*) then $[\![\alpha]\!]^{w,t,g} = [\lambda x : x \in \mathbf{D}_e \,.\, x$ meets at t in $w]$;

c. If OCCURRENCE(α, *disperse*) then $[\![\alpha]\!]^{w,t,g} = [\lambda x : x \in \mathbf{D}_e \,.\, x$ disperses at t in $w]$.

Our grammar will now generate and interpret sentence occurrences like **John and Mary meet.**

But now consider sentences like (160)a. and b.:

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[51] Here again our syntax allows and_c to conjoin expressions of any category, but the result will not be interpretable unless the conjuncts are of type e. Note that X need not correspond to the set of *all* categories to which the conjuncts belong. For example, the names *John* and *Mary* both belong to NOM, ACC, and SINGULAR, but letting X = {NOM}, we may combine and_c with *John* and with *Mary* to form *John* and_c *Mary*, which will also be of category NOM. This does not prevent *John* and_c *Mary* from belonging to other categories as well; the principle in (158) will normally place it in category PLURAL.

[52] Exceptions are possible in cases where the conjuncts denote the same individual, as in the famous example in (i), from Quirk, Greenbaum, Leech and Svartik (1972):

(i) His aged servant and the subsequent editor of his papers was/were with him at his deathbed.

We may obtain the correct results for such examples by assuming that for any individual x, $x+x = x$.

(160) a. Every student and professor meet on a regular basis.

b. This man and woman are in love.

If we treat common nouns as being of type $\langle e, t\rangle$, we should expect the conjunction in these examples to be *and*$_t$, not *and*$_c$, since $\langle e, t\rangle$ is a conjoinable type, while *and*$_c$, at least as formulated in (157), yields interpretable results only if its conjuncts are of type e. Sentence (160)a. will be assigned a reading under which it is true iff everyone who is both a student and a professor meets (which makes no sense, since an individual cannot meet), and (160)b. will be assigned a reading under which it is true iff everyone who is both a man and a woman is in love. The more prominent readings, under which (160)a. means that every pair of a professor and a student meets, and (160)b. means that this pair of a man and a woman are in love, are not obtained.

Existing solutions to this problem (e.g. Lasersohn (1995: 278), Heycock and Zamparelli (2005)) generally appeal to some sort of group-forming operation "in the argument places" of predicates. This is not too hard to formulate, especially if we limit attention to one-place predicates[53] — we simply revise (157)b., adding an extra clause for conjuncts belonging to type $\langle e, t\rangle$:

(161) If OCCURRENCE(α, *and*$_c$), then for all $w \in W$ and all assignments g: $[\![\alpha]\!]^{w,t,g}$ = the smallest function f such that:

a. For all $x, y \in \mathbf{D}_e$: $f(x, y) = x+y$;

b. For all $h, i \in \mathbf{D}_{\langle e, t\rangle}$: $f(h, i) = [\lambda x: x \in \mathbf{D}_e \,.\, \exists y, z \in \mathbf{D}_e\, [x = y + z \;\&\; h(y) = 1 \;\&\; i(z) = 1]$.

For example, according to this rule, $[\![$**man and$_c$ woman**$]\!]^{w,t,g}$ will be $[\lambda x : x \in \mathbf{D}_e \,.\, \exists y, z \in \mathbf{D}_e\, [x = y + z \;\&\; [\![\mathbf{man}]\!]^{w,t,g}(y) = 1 \;\&\; [\![\mathbf{woman}]\!]^{w,t,g}(z) = 1]]$ — in other words, the phrase is a predicate which holds of groups consisting of a man and a woman.

But (161) is an odd rule, stated disjunctively. Unlike (156), it doesn't consist of a base clause and an inductive clause. Rather, it simply has two clauses for two cases: the case where *and*$_c$ conjoins expressions denoting individuals, and the case where it conjoins expressions denoting functions.

Analyzing common nouns as being variables of type e (or of other types ending in e) offers a way to simplify the rule in (161): we simply eliminate clause b. If **man** and **woman**

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[53] To deal with multi-place predicates, replace (161)b. with (i):

(i) For all types $\sigma_1,\ldots,\sigma_n$ and $h, i \in \mathbf{D}_{\langle\sigma 1,\ldots\langle\sigma n,\, t\rangle\rangle}$: $f(h, i) = [\lambda z_1 : z_1 \in \mathbf{D}_{\sigma 1} \ldots [\lambda z_n : z_n \in \mathbf{D}_{\sigma n} \,.\, [\exists x_1, y_1 \in \mathbf{D}_1,\ldots,\exists x_n, y_n \in \mathbf{D}_{\sigma n}\, \forall m[1 \leq m \leq n \rightarrow [[\sigma_m = e \rightarrow z_m = x+y] \;\&\; [\sigma_m \neq e \rightarrow z_m = x = y]]] \;\&\; h(x_1)\ldots(x_n) = 1 \;\&\; i(y_1)\ldots(y_n) = 1]]]]$.

This does group-formation in all argument places simultaneously, so $[\![$**mother and$_c$ father**$]\!]^{w,t,g}$, for example, will be $[\lambda z_1 : z_1 \in \mathbf{D}_e \,.\, [\lambda z_2 : z_2 \in \mathbf{D}_e \,.\, \exists x_1, y_1, x_2, y_2 \in \mathbf{D}_e\, [z_1 = x_1 + y_1 \;\&\; z_2 = x_2 + y_2 \;\&\; [\![\mathbf{mother}]\!]^{w,t,g}(x_1)(y_1) = 1 \;\&\; [\![\mathbf{father}]\!]^{w,t,g}(x_2)(y_2) = 1]]]$ — that is, a predicate which holds between two groups if the first group consists of the mother of one member of the second group and the father of the other member of the second group. (The relation will also hold between the group of an individual's parents and that individual, under the assumption of note 52 that $x + x = x$.)

are both of type e, then clause a. can apply, with the effect that $[\![\textbf{man and}_c\ \textbf{woman}]\!]^{w,t,g} = g(\textbf{man}, w, t) + g(\textbf{woman}, w, t)$. Relative to different assignments of values to variables, this phrase will denote different groups consisting of a man and a woman.

Although this idea is attractive, it is far from clear that treating common nouns and noun phrases as type e variables really eliminates the need for rules like (161)b. The reason is that collective conjunction seems to be available for verbs and other predicates, not just for nouns. For example, (162) is true if some of the kids climbed trees and the rest swam in the pond; it does not require it to be true of the whole group that it climbed trees and also true of the whole group that it swam in the pond:

(162) The kids climbed trees and swam in the pond.

This is as predicted by (161)b., but is unexpected from (161)a. alone, under the assumption that verb phases are of type $\langle e, t\rangle$.[54]

Because collective conjunction "in the argument places" of a predicate seems necessary in any case, we leave it as a matter for further investigation whether collective conjunction of common nouns provides significant motivation for treating such nouns as variables of type e.

11.2. Raising just the determiner

As mentioned in Footnote 7 above, analyzing common nouns as restricted variables allows a reformulation of the rule of Quantifier Raising so that it affects only determiners rather than whole DPs, leaving the complement NP of the determiner in situ. In this section we reformulate our rules along these lines, and point out some simplifications which this reformulation makes possible.

In our existing system, *every* was assigned to category ((NOM, ACC, Q, SINGULAR)/(NP, SINGULAR)) (in (79)a.), and *most* was assigned to category ((NOM, ACC, Q, PLURAL)/(NP, PLURAL)) (in (83)). In these category labels, the Q appears on the top side of the slash, with the result that the whole combination of the determiner and its noun phrase belongs to category Q. Since Q is the category of items which are subject to Quantifier Raising, we now revise the categorization of these words as in (163):

(163) a. ((NOM, ACC, SINGULAR)/(N, SINGULAR)), Q
b. ((NOM, ACC, PLURAL)/(N, PLURAL)), Q

Now it is the determiner itself that belongs to category Q, rather than the whole DP; so it will be the determiner, not the whole DP which undergoes Quantifier Raising.

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[54] We do not explore here the possibility of treating verb phrases, not just noun phrases, as variables of type e. No doubt such an analysis could be developed; but it strikes me as unlikely to present many advantages.

Next, we give up the assumption that Quantifier Raising leaves a trace. The purpose of the trace, at least from a semantic point of view, was to make sure a variable appeared in the argument position associated with the quantifier. Because we are now assuming that Quantifier Raising leaves the noun phrase in situ, and that noun phrases are variables, there will always be a variable in the argument position even if no trace is left by QR.

We will need some way of associating raised determiners with the noun phrases they have left behind, to take the place of the TRACE relation used previously to relate quantifier phrases to their traces. Where OCCURRENCE(δ, Q) and OCCURRENCE(κ, NP), let us write 'SOURCE(δ) = κ' to mean that δ has moved by QR from a position as immediate left sister of κ.

Under these new assumptions, a sentence occurrence like **John sees every professor** will be represented at LF as in (164), with SOURCE(**every**) = **professor**:

(164) [**every** [**John sees professor**]]

Now we need to revise the semantics for *every* and *most*. Our existing rules, repeated here in (165), treat these words as being of type $\langle r, \langle\langle r, e\rangle, \langle\langle r, t\rangle, t\rangle\rangle\rangle$, where the $\langle r, e\rangle$ argument place is for the noun phase with which the determiner combines. Since we are now treating these determiners as combining at LF just with a clause, and not with a separate noun phrase, we may assign them denotations in the simpler type $\langle r, \langle\langle r, t\rangle, t\rangle\rangle$, as in (166):[55]

(165) a. If OCCURRENCE(α, *every*) then $[\![\alpha]\!]^{w,t} = [\lambda g: g \in G\,.\, [\lambda y: y \in \mathbf{D}_{\langle r, e\rangle}\,.\, [\lambda p: p \in \mathbf{D}_{\langle r, t\rangle}\,.\, \forall x \in \mathbf{D}_e[\exists h[g \sim_{y,w,t} h \;\&\; h(y, w, t) = x] \rightarrow \exists h[g \sim_{y,w,t} h \;\&\; h(y, w, t) = x \;\&\; \exists i[h \leftharpoondown_{\textsc{nouns}_{w,t}(p),w,t}\, i \;\&\; p(i) = 1]]$.

b. If OCCURRENCE(α, *most*) then $[\![\alpha]\!]^{w,t} = [\lambda g: g \in G\,.\, [\lambda y: y \in \mathbf{D}_{\langle r, e\rangle}\,.\, [\lambda p: p \in \mathbf{D}_{\langle r, t\rangle}\,.\, \{x \mid x \text{ is an individual} \;\&\; \exists h[g \sim_{y,w,t} h \;\&\; h(y, w, t) = x \;\&\; \exists i[h \leftharpoondown_{\textsc{nouns}_{w,t}(p),w}\, i \;\&\; p(i) = 1]\}$ is of greater cardinality than $\{x \mid x \text{ is an individual} \;\&\; \exists h[g \sim_{y,w,t} h \;\&\; h(y, w, t) = x \;\&\; \exists i[h \leftharpoondown_{\textsc{nouns}_{w,t}(p),w,t}\, i \;\&\; \mathrm{p}(i) = 0]\}]]]$.

(166) a. If OCCURRENCE(α, *every*) then $[\![\alpha]\!]^{w,t} = [\lambda g: g \in G\,.\, [\lambda p: p \in \mathbf{D}_{\langle r, t\rangle}\,.\, \forall x \in \mathbf{D}_e[\exists h[g \sim_{\textsc{source}(\alpha),w,t} h \;\&\; h(\textsc{source}(\alpha), w, t) = x] \rightarrow \exists h[g \sim_{\textsc{source}(\alpha),w,t} h \;\&\; h(\textsc{source}(\alpha), w, t) = x \;\&\; \exists i[h \leftharpoondown_{\textsc{nouns}_{w,t}(p),w,t}\, i \;\&\; p(i) = 1]]$.

b. If OCCURRENCE(α, *most*) then $[\![\alpha]\!]^{w,t} = [\lambda g: g \in G\,.\, [\lambda p: p \in \mathbf{D}_{\langle r, t\rangle}\,.\, \{x \mid x \text{ is an individual} \;\&\; \exists h[g \sim_{\textsc{source}(\alpha),w,t} h \;\&\; h(\textsc{source}(\alpha), w, t) = x \;\&\; \exists i[h \leftharpoondown_{\textsc{nouns}_{w,t}(p),w}\, i \;\&\; p(i) = 1]\}$ is of greater cardinality than $\{x \mid x \text{ is an individual} \;\&\; \exists h[g \sim_{\textsc{source}(\alpha),w,t} h \;\&\; h(\textsc{source}(\alpha), w, t) = x \;\&\; \exists i[h \leftharpoondown_{\textsc{nouns}_{w,t}(p),w,t}\, i \;\&\; \mathrm{p}(i) = 0]\}]]]$.

[55] Some semanticists might consider the rules in (166) not to be strictly compositional, since they assign a denotation to an occurrence α in part by making reference to SOURCE(α), which is a separate phrase, not a constituent of α. But it must be remembered that these rules assign denotations to (occurrences of) the *basic* expressions *every* and *most*, not to any complex expressions. They do not contradict the principle that the denotation of a complex expression depends on the denotations of its parts. Rather, these rules exemplify the comparatively less controversial phenomenon of sensitivity of content to linguistic context.

Raising only the determiner and not its noun phrase means that we cannot use Quantifier Raising to obtain transparent readings for nouns in intensional contexts. For example, determiner-only QR will not deliver the reading of *John believes the president is a fool* which can be paraphrased "the actual president is such that John believes he or she is a fool," with no entailment that John is aware that the person of whom he has this belief is the president.

But this is probably not a problem, since other methods for deriving this reading are available — for example through the use of object-language world variables, or implicit *actually* operators — and since "scope paradoxes" present well-known problems for the idea of accounting for semantic transparency via quantifier scope.[56]

Whether the idea of moving just the determiner and leaving the NP in situ is *syntactically* well-motivated is a separate issue, and one which I will leave to syntacticians. The brief discussion here is intended only to raise the issue in a very preliminary way.

11.3. Eliminating part-of-speech categories

In a semantic theory in which denotations conform to a system of types, and compositional rules are sensitive to those same types, the syntactic distribution of expressions will be at least partly predictable from their denotations. For example, if the only operation used in our compositional rules is one of function application, then we can expect that every complex phrase will consist of an expression denoting a function, together with one or more other expressions whose denotations are of appropriate types to serve as arguments to that function. Any complex phrase not conforming to this pattern would be expected not to occur. For example, the two names *John* and *Mary* would be predicted not to combine with each other, assuming both are type e, since in this case neither one denotes a function which can take the denotation of the other as an argument. On the other hand, *John* is expected to combine with *smiles*, assuming *John* is of type e and *smiles* is of type $\langle e, t\rangle$, since then *smiles* denotes a function which can take the denotation of *John* as an argument. Note that no appeal was made here to syntactic categories, but only to semantic types.

The situation becomes more complex if we allow additional operations besides function application, of course; but the principle remains that those operations, together with the denotations assigned to lexical items, constrain what combinations of those items can sensibly occur as part of the language.

It is natural to wonder how far this kind of type-theoretic explanation for syntactic distributional patterns can be pushed. Could we, for example, do completely without syntactic categories of the traditional sort, instead letting syntactic rules be sensitive to

[56] See Fodor (1970), Cresswell (1990), among many others. Heim and von Fintel (2011) provides a useful summary.

semantic types? If so, could we go further and account for syntactic distribution (except perhaps, for left-to-right ordering) via compositional semantic rules, rather than a separate set of syntactic rules?

These are questions that could only be answered after very extensive investigation, of course; and we have no reason to be very optimistic that they can be answered affirmatively. But the analysis of common nouns offered in this paper does at least remove one easy objection to the idea of eliminating traditional syntactic categories in favor of semantic types: In conventional typing, common nouns and verbs belong to the same semantic types, but their syntactic distribution is very different. Under conventional assumptions, the rules which produce this distribution must therefore be sensitive to something other than semantic type — presumably to syntactic category, more-or-less as traditionally understood. In contrast, in the analysis offered here, common nouns and verbs are of different logical types, so their different syntactic distribution cannot be taken as evidence that such distribution is governed by rules which are sensitive to non-semantic categories.

In general, syntactic categories distinct from semantic types are theoretically necessary only if there are cases where expressions from distinct syntactic categories belong to the same semantic type. An examination of the category and type assignments of lexical items in our current fragment reveals a few such cases: prepositional phrases with *of* are semantically identical to their objects, certain DPs are of the same semantic type as NPs, etc. Further research might reveal ways to eliminate these areas of overlap.

Even if (as seems likely) we find that we must appeal to syntactic category distinctions for which there is no corresponding semantic type distinction, it might still be worthwhile to re-evaluate the theoretical role of certain kinds of grammatical categories in light of the proposal in this paper to introduce a type-theoretic distinction between nouns and verbs. In particular, it seems to me to be worth exploring the possibility of eliminating traditional *part-of-speech* categories from syntactic theory. On this view, syntactic selection, movement, etc., would be driven solely by semantic type and by inflectional features, or other smaller-scale categories than parts of speech, and not by major categories such as *noun*, *verb*, etc.

Part-of-speech categories might still play a role in grammar even under this conception — just not in the syntax. For example, they might be used to state lexical generalizations about which words bear which inflectional features: verbs are inflected for tense, pronouns for person, etc. This is, I think, the central purpose for which part-of-speech categories were introduced by the ancient Greek and Roman grammarians, so constructing a grammatical theory along these lines would represent to some extent a return to the original conception.

11.4. Indexical pronouns

The rule in (55) is not fully satisfactory, because it existentially quantifies all variables which are free at the text level. But pronouns qualify as variables, and antecedentless pronouns are not interpreted as existentially quantified, but rather as indexical. This is not the place to give a detailed theory of indexicality, but it is nonetheless useful in the present context to formulate an existential closure operation which will quantify only free indefinites and pronouns which are anaphoric to them, leaving antecedentless pronouns unquantified.

This requires some way of identifying which indefinites are free in a text. Fortunately, this is straightforward: The free indefinites in a text are exactly those which are in the domain of every assignment relative to which all the sentences in the text have a truth value. More generally, the free indefinites in any set of expressions K are those which are in the domain of every assignment relative to which all the expressions in K have a denotation. Since indefinites are identical in denotation to their nouns, we may give the following definitions:

(167) a. $\text{G-SET}_w(\alpha_1,\ldots,\alpha_n) = \{g \mid g$ is an assignment of values to variables $\mid \exists x_1[\![\alpha_1]\!]^{w,g} = x_1\ \&\ \ldots\ \&\ \exists x_n[\![\varphi_n]\!]^{w,g} = x_n\ \}$

b. Where $G \subseteq \{g \mid g$ is an assignment of values to variables$\}$, $\text{FREE}(G) = \{\kappa \mid \text{CATEGORY}(\kappa, \text{N})\ \&\ \forall g \in G[\kappa \in \text{DOMAIN}(g)]\}$

Now we can define an analog to our "$\sim_{\kappa,w}$" notation which relates assignments which differ at most on a *set* of nouns, rather than a single noun κ:

(168) For all assignments of values to variables g, h, all $\text{K} \subseteq \{\kappa \mid \text{CATEGORY}(\kappa, \text{N})\}$ and all $w \in W$: $g \sim_{\text{K},w} h$ iff

a. For all $\kappa \in \text{K}$ there exists some x such that

i. $h(\kappa, w) = x$, and

ii. for all ε such that $\text{ANTECEDENT}(\varepsilon) = \kappa$ and all $w' \in W$, $h(\varepsilon, w') = x$;

b. for all $\nu \in \text{DOMAIN}(g)$ and all $w' \in W$, if $\nu \not\subseteq \text{K}$ and $\text{ANTECEDENT}(\nu) \not\subseteq \text{K}$, then $g(\nu, w') = h(\nu, w')$.

Now we can interpret texts via existential closure as in (169):

(169) If $\varphi_1,\ldots,\varphi_n$ is a text, then:

a. $[\![\varphi_1,\ldots,\varphi_n]\!]^{w,g} = 1$ iff $\exists h[g \sim_{\text{FREE}(\text{G-SET}_w(\varphi_1,\ldots,\varphi_n)),\ w}\ h\ \&\ [\![\varphi_1]\!]^{w,h} = 1\ \&\ \ldots\ \&\ [\![\varphi_n]\!]^{w,h} = 1\}$

b. $[\![\varphi_1,\ldots,\varphi_n]\!]^{w,g} = 0$ iff $\sim\exists h[g \sim_{\text{FREE}(\text{G-SET}_w(\varphi_1,\ldots,\varphi_n)),\ w}\ h\ \&\ [\![\varphi_1]\!]^{w,h} = 1\ \&\ \ldots\ \&\ [\![\varphi_n]\!]^{w,h} = 1\}$

c. $[\![\varphi_1,\ldots,\varphi_n]\!]^{g} = 1$ iff $\exists h[g \sim_{\text{FREE}(\text{G-SET}_{w@}(\varphi_1,\ldots,\varphi_n)),w}\ h\ \&\ [\![\varphi_1]\!]^{w@,h} = 1\ \&\ \ldots\ \&\ [\![\varphi_n]\!]^{w@,h} = 1\}$

d. $[\![\varphi_1,\ldots,\varphi_n]\!]^g = 0$ iff $\sim\exists h[g \sim_{\mathrm{FREE}(\mathrm{G\text{-}SET}_{w@}(\varphi_1,\ldots,\varphi_n)),\, w}\ h\ \&\ [\![\varphi_1]\!]^{w@,h} = 1\ \&\ \ldots\ \&\ [\![\varphi_n]\!]^{w@,h} = 1\}$

Finally, we relativize truth to contexts, and assume that each context *c* provides an assignment g_c specifying the values of indexical pronouns:

(170) If $\varphi_1,\ldots,\varphi_n$ is a text, then for any context *c*:
   a. $\varphi_1,\ldots,\varphi_n$ is *true relative to c* iff $[\![\varphi_1,\ldots,\varphi_n]\!]^{g_c} = 1$;
   b. $\varphi_1,\ldots,\varphi_n$ is *false relative to c* iff $[\![\varphi_1,\ldots,\varphi_n]\!]^{g_c} = 0$.

As an example, consider the short discourse in (171) relative to a context *c* such that g_c(**him**) = John, assuming that ANTECEDENT(**she**) = **professor** and that **him** has no antecedent:

(171) A professor smiles. She sees him.

(172) a. **A professor smiles. She sees him.** is true relative to *c* iff $[\![$A professor smiles. She sees him.$]\!]^{g_c} = 1$;
   b. iff $[\![$A professor smiles. She sees him.$]\!]^{g_c, w@} = 1$;
   c. iff $\exists h[g \sim_{\mathrm{FREE}(\mathrm{G\text{-}SET}_{w@}(\textbf{A professor smiles, She sees him})),\ w@}\ h\ \&\ [\![\textbf{A professor smiles}]\!]^{w@,h} = 1\ \&\ [\![\textbf{She sees him}]\!]^{w@,h} = 1$;
   d. iff $\exists h[g \sim_{\{\textbf{professor}\}, w@}\ h\ \&\ [\![\textbf{A professor smiles}]\!]^{w@,h} = 1\ \&\ [\![\textbf{She sees him}]\!]^{w@,h} = 1$;
   e. iff $\exists h[g \sim_{\{\textbf{professor}\}, w@}\ h\ \&\ h(\textbf{professor}, w@)$ smiles in $w@\ \&\ h(\textbf{professor}, w@)$ sees $h(\textbf{him}, w@)$ in $w@$;
   f. iff $\exists h[g \sim_{\{\textbf{professor}\}, w@}\ h\ \&\ h(\textbf{professor}, w@)$ smiles in $w@\ \&\ h(\textbf{professor}, w@)$ sees $g_c(\textbf{him}, w@)$ in $w@$;
   g. iff $\exists h[g \sim_{\{\textbf{professor}\}, w@}\ h\ \&\ h(\textbf{professor}, w@)$ smiles in $w@\ \&\ h(\textbf{professor}, w@)$ sees John in $w@$;
   h. iff there is an *x* such that *x* is a professor in $w@$ and *x* smiles in $w@$ and *x* sees John in $w@$.

Now we can see the utility of requiring existential closure on the restriction of a quantifier and not just the scope. A sentence like *Every professor Rel who sees a student e smiles* should be true iff for each *x* such that *x* is a professor and there exists some *y* such that *y* is a student and *x* sees *y*, *x* smiles. This is the effect we obtain:

(173) a. $[\![\textbf{Every professor Rel who sees a student e smiles}]\!]^{w@,g} = 1$ iff $\forall x \in D_e[\exists h\ [g \sim_{\mathrm{FREE}(\mathrm{G\text{-}SET}_{w}(\textbf{professor Rel who sees a student})), w@}\ h\ \&\ h(\textbf{professor Rel who sees a student}, w@) = x] \rightarrow [\exists h\ [g \sim_{\mathrm{FREE}(\mathrm{G\text{-}SET}_{w}(\textbf{professor Rel who sees a student, e smiles})), w@}\ h\ \&\ h(\textbf{professor who Rel sees a student}, w@) = x\ \&\ [\![\textbf{e smiles}]\!]^{w@,h} = 1]]]$
   b. iff $\forall x \in D_e[\exists h\ [g \sim_{\{\textbf{professor Rel who sees a student, professor, student}\}, w@}\ h\ \&\ h(\textbf{professor Rel who sees a student}, w@) = x] \rightarrow [\exists h\ [g \sim_{\{\textbf{professor Rel who sees a student, professor,}}$

student)),$_{w@}$ h & h(**professor who Rel sees a student**, $w@$) = x & ⟦**e smiles**⟧$^{w@,h}$ = 1]]]

c. iff $\forall x \in \mathbf{D}_e[\exists y,z \in \mathbf{D}_e[z$ is a professor and y is a student and z sees y and $z = x]$ $\rightarrow \exists y,z \in \mathbf{D}_e[z$ is a professor and y is a student and z sees y and $z = x$ and z smiles]]

To tie up a loose end before concluding this section, we should revise our semantics for the definite article so that it too involves existential closure on its noun phrase. *The professor Rel who sees a student*, for example, should denote the unique professor x for which it is the case that there is a student whom x sees:

(174) If OCCURRENCE(δ, *the*) and CATEGORY(κ, N), then for all $w \in W$ and all assignments g: ⟦δ κ⟧^{w,g} = g(κ, w), provided that for all assignments h such that $g \sim_{\text{FREE(G-SET}_w(\kappa)),w} h$, it holds that g(κ, w) = h(κ, w). (If there exists some w-assignment h such that $g \sim_{\text{FREE(G-SET}_w(\kappa)),w} h$ but g(κ, w) ≠ g(κ, w), then ⟦*the* κ⟧^{w,g} is undefined.)

12. Conclusion

Treating common nouns as modally non-rigid restricted variables, rather than as predicates which take variables as arguments:

- predicts the conservativity of nominal quantification,
- predicts the weak reading of sentences with donkey anaphora,
- solves the proportion problem presented by quantifiers like *most*,
- improves the analysis of the temperature paradox,
- allows a more unified analysis of bare plurals, and
- regularizes the correspondence between syntactic categories and semantic types.

Potentially, it may also simplify the analysis of collectivizing conjunction, allow a reformulation of Quantifier Raising so that only the determiner raises, and reduce or eliminate the role for part-of-speech categories in syntax.

References

Barwise, J. and Cooper, R. (1981). Generalized quantifiers and natural language. Linguistics and Philosophy, 4, 159–219

Bennett, M.R. (1975). Some extensions of a Montague fragment of English. (Bloomington, Ind.: Indiana University Linguistics Club)

Carlson, G.N. (1977a) Reference to kinds in English. Dissertation, University of Massachusetts

Carlson, G.N. (1977b). A unified analysis of the English bare plural. Linguistics and Philosophy, 1, 413–456

Chierchia, G. (1995). Dynamics of meaning : anaphora, presupposition, and the theory of grammar. (Chicago: University of Chicago Press)
Chierchia, G. and McConnell-Ginet, S. (1990). Meaning and grammar: An introduction to semantics. (Cambridge, Mass.: MIT Press)
Cresswell, M.J. (1990). Entities and indices. (Dordrecht: Kluwer Academic Publishers)
Dekker, P. (1993). Existential disclosure. Linguistics and Philosophy, 16, 561–587
Diesing, M. (1992). Indefinites. (Cambridge, Mass.: MIT Press)
Dowty, D., Wall, R.E. and Peters, S. (1981). Introduction to Montague Semantics. (Dordrecht, Holland ; Boston; Hingham, MA: D. Reidel Pub. Co.)
Fara, D.G. (2015). Names are predicates. Philosophical Review, 124, 59–117
von Fintel, K. (1994) Restrictions on quantifier domains. Dissertation, University of Massachusetts
Fodor, J.D. (1970) The linguistic description of opaque contexts. Dissertation, Massachusetts Institute of Technology
Fox, D. (1999). Reconstruction, binding theory, and the interpretation of chains. Linguistic Inquiry, 30, 157–196
Fox, D. (2002). Antecedent-Contained Deletion and the Copy Theory of Movement. Linguistic Inquiry, 33, 63–96
Frege, G. (1879). Begriffschrift: eine der arithmetischen nachgebildete Formelsprache des reinen Denkens. (Halle: L. Nebert)
Frege, G. (1892). On sense and reference. (In P. Geach and M. Black (Eds.), Translations from the Philosophical Writings of Gottlob Frege (pp. Oxford: Basil Blackwell.)
Gallin, D. (1975). Intensional and higher-order modal logic. (Amsterdam: North-Holland)
Gamut, L.T.F. (1991). Logic, language and meaning, volume 2: intensional logic and logical grammar. (Chicago: University of Chicago Press)
Gerstner-Link, C. and Krifka, M. (1993). Genericity. (In J. Jacobs, A. von Stechow, W. Sternefeld and T. Vennemann (Eds.), Syntax: an international handbook of contemporary research (pp. 966-978). Berlin: DeGruyter.)
Heim, I. (1982). The semantics of definite and indefinite noun phrases. (Amherst, Mass.: Graduate Linguistics Students Association, University of Massachusetts)
Heim, I. and von Fintel, K. (2011) Intensional semantics. Retrieved October 4, 2013 from http://www.kaivonfintel.org/work/
Heim, I. and Kratzer, A. (1998). Semantics in generative grammar. (Malden, Mass.: Blackwell)
Heycock, C. and Zamparelli, R. (2005). Friends and Colleagues: Plurality, Coordination, and the Structure of DP. Natural Language Semantics, 13, 201-270
Hughes, G.E. and Cresswell, M.J. (1968). An introduction to modal logic. (London: Metheun)
Jacobson, P. (1992) Antecedent contained deletion in a variable-free semantics, (in C. Barker and D. Dowty (Eds.), SALT II: Proceedings from the second conference on semantics and linguistic theory (vol. 2, pp. 193–213), Columbus: Ohio State University
Jacobson, P. (1993) i-within-i effects in a variable-free semantics and a categorial syntax, (in P. Dekker and M. Stokhof (Eds.), Proceedings of the ninth Amsterdam colloquium pp. 349–368), Amsterdam: University of Amsterdam
Jacobson, P. (1999). Towards a variable-free semantics. Linguistics and Philosophy, 22, 117–184

Jacobson, P. (2000). Paycheck pronouns, Bach-Peters sentences, and variable-free semantics. Natural Language Semantics, 8, 77–155
Jacobson, P. (2007). Direct compositionality and variable-free semantics: The case of "Principle B" effects. (In P. Jacobson and C. Barker (Eds.), Direct compositionality (pp. 191–236). Oxford: Oxford University Press.)
Jacobson, P. (2008). Direct compositionality and variable-free semantics. (In K. Johnson (Ed.) Topics in ellipsis (pp. 30–68). Cambridge: Cambridge University Press.)
Keenan, E.L. and Stavi, J. (1986). A semantic characterization of natural language determiners. Linguistics and Philosophy, 9, 253–326
Lasersohn, P. (1995). Plurality, conjunction, and events. (Dordrecht ; Boston: Kluwer Academic Publishers)
Lasersohn, P. (2005). The temperature paradox as evidence for a presuppositional analysis of definite descriptions. Linguistic Inquiry, 36, 127–130
Lasersohn, P. (2011). Mass nouns and plurals. (In C. Maienborn, K. von Heusinger and P. Portner (Eds.), Semantics: An international handbook of natural language meaning (pp. 1131–1153). Berlin: de Gruyter.)
Lasersohn, P. (2017). Subjectivity and perspective in truth-theoretic semantics. (Oxford: Oxford University Press)
Lepore, E. and Ludwig, K. (2007). Donald Davidson's truth-theoretic semantics. (Oxford: Oxford University Press)
Lewis, D.K. (1975). Adverbs of quantification. (In E. L. Keenan (Ed.) Formal semantics of natural language (pp. 3–15). Cambridge: Cambridge University Press.)
Montague, R. (1968). Pragmatics. (In R. Klibansky (Ed.) Contemporary philosophy: A survey (pp. 102–122). Florence: La Nuova Italia Editrice.)
Montague, R. (1973). The proper treatment of quantification in ordinary English. (In K. J. J. Hintikka, J. M. E. Moravcsik and P. Suppes (Eds.), Approaches to natural language: Proceedings of the 1970 Stanford workshop on grammar and semantics (pp. 221–242). Dordrecht: D. Reidel Publishing Company.)
Partee, B.H. and Rooth, M. (1983). Generalized quantifiers and type ambiguity. (In R. Bäuerle, C. Schwarze and A. v. Stechow (Eds.), Meaning, Use, and Interpretation of Language (pp. 361–383). Berlin: Walter de Gruyter.)
Quine, W.V.O. (1960). Word and object. (Cambridge, Massachusetts: MIT Press)
Quirk, R., Greenbaum, S., Leech, G. and Svartik, J. (1972). A grammar of contemporary English. (London: Longman)
Rayo, A. and Uzquiano, G. (2007) Absolute generality.
Romero, M. (2008). The Temperature Paradox and Temporal Interpretation. Linguistic Inquiry, 39, 655–667
Romoli, J. (2015). A structural account of conservativity. Semantics-Syntax Interface, 2, 28–57
Russell, B. (1910). Knowledge by acquaintance and knowledge by description. Proceedings of the Aristotelian Society, 11, 108–128
Sauerland, U. (2004). The Interpretation of Traces. Natural Language Semantics, 12, 63–127
Schein, B. (1993). Plurals and events. (Cambridge, Massachusetts: MIT Press)
Westerståhl, D. (1985). Logical constants in quantifier languages. Linguistics and Philosophy, 8, 387–413

Wilkinson, K. (1991) Studies in the semantics of generic noun phrases. Dissertation, University of Massachusetts
Wilkinson, K. (1995). The semantics of the common noun *kind*. (In G. N. Carlson and F. J. Pelletier (Eds.), The generic book (pp. 383–397). Chicago: University of Chicago Press.)
Winter, Y. (2001). Flexibility principles in Boolean semantics: The interpretation of coordination, plurality and scope in natural language. (Cambridge, Mass.: MIT Press)