Comparative Quantifiers and Negation: Implications for Scope Economy

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Abstract

I examine the scopal interactions between comparative quantifier phrases (CQPs) and negation and argue, based on the complex pattern of interactions observed, that negation must be subject to operative constraints on scope shifting, contra a suggestion of Takahashi (2006). I adopt Mayr and Spector's (2012) Generalized Scope Economy Condition (GSEC) and combine it with a decompositional analysis of CQPs (Hackl 2000; Takahashi 2006), showing that the decompositional analysis is empirically and conceptually preferable to the generalized quantifier treatment Mayr and Spector assume. I further propose that the GSEC's ban on scope shifts in which the output configuration entails the input configuration be limited to cases in which the entailment is non-vacuous (in a sense to be defined). The result is an analysis in which the interactions between CQPs and negation are accounted for solely in terms of more general principles and constraints on scope shifting.

1 Introduction

1.1 Overview

Comparative quantifier phrases (CQPs), such as *more than three books* and *fewer than five students*, are known to be relatively restricted in their scope-taking properties. Unlike quantificational DPs such as *every book*, CQPs are unable to take inverse scope above a structurally superior quantifier; with few exceptions, they have been observed to be scopally stuck in their surface positions. This property of CQPs is unexpected on an unconstrained quantifier raising (QR)-based theory of quantifier scope, and a number of researchers have endeavoured to explain these DPs' apparent scopal immobility (Beghelli and Stowell 1997; Liu 1997; Szabolcsi 1997). Of particular interest is the proposal of Takahashi (2006), who seeks to derive the observed restrictions on CQP scope from independently motivated assumptions about the structural composition of CQPs (Hackl 2000) and grammatical constraints on covert scope-shifting operations (SSOs) under the banner of Scope Economy (Fox 2000).

Takahashi leaves open at least one important mystery in his treatment of CQP scope, namely how to account for interactions between CQPs and negation. Mayr and Spector

(2012) observe a configurational asymmetry in the licensing of SSOs that invert CQPs and negation; their proposed extension of Fox's Scope Economy derives the asymmetry as part of a more general account of the regulation of SSOs. Mayr and Spector, however, forgo the decompositional analysis of CQPs advocated by Hackl and Takahashi in favor of a unitary generalized quantifier treatment, a move which makes incorrect predictions elsewhere.

Here I forge an integration of the proposals of Takahashi (2006) and Mayr and Spector (2012), maintaining the valuable core insights of each while eliminating their respective weaknesses in the areas of negation and CQP structure. In offering a solution to the prima facie puzzling behavior of CQPs and negation, the analysis presented here more fully integrates CQPs into our understanding of natural language quantification.

1.2 A Puzzle, and Attempted Solutions

The basic pattern to be accounted for is sketched in (1): a CQP in object position can undergo scopal inversion with negation but a CQP in subject position cannot, unless it occurs in an embedded clause selected by a non–order-preserving (i.e., downward-entailing [DE] or non-monotone) operator. (Unless otherwise noted, configurations and judgments are provided in the order SURFACE SCOPE, INVERSE SCOPE.)

(1) a. John didn't read more than three books. (NEG > CQP, CQP > NEG)
b. More than three students didn't do the reading. (CQP > NEG, #NEG > CQP)
c. Whenever more than three students don't do the reading, I cancel class. (CQP > NEG, NEG > CQP)

Mayr and Spector (2012) propose to account for the pattern in (1) by extending Fox's (2000) Scope Economy. Whereas Scope Economy bars SSOs in which the input and output configurations have identical truth conditions, Mayr and Spector propose to bar all SSOs in which the output configuration entails the input configuration: i.e., they bar SSOs that involve no change in truth conditions as well as those that result in logical strengthening, where the output asymmetrically entails the input. This system offers a natural explanation for scope inversion asymmetries that parallel syntactic configurational asymmetries, as with (1a) vs. (1b). In particular, Mayr and Spector treat CQPs as generalized quantifiers with existential force, and observe that the scope configuration NEG > ∃ asymmetrically entails the scope configuration ∃ > NEG;[1] scope inversion in (1a) then results in logical weakening and is permitted, while inversion in (1b) results in logical strengthening and is barred. Their account of (1c), where the CQP subject can indeed invert with negation, relies on the rescuing effect of the higher DE operator *whenever* (see section 2.1 for details). Mayr and Spector dub their proposal the Generalized Scope Economy Condition (GSEC), defined in (2).

(2) Generalized Scope Economy Condition (GSEC): A covert SSO is licensed in a sentence S only if there exists a constituent C of S (possibly S itself) such that the

[1] This entailment follows on the "Aristotelian" assumption that quantifiers are presupposed to have non-empty restrictions; for discussion, see Mayr and Spector (2012:5). A somewhat stronger presupposition may in fact be needed, as discussed by Mayr and Spector (2012:12); see footnote 4.

covert SSO does not make the semantic value of C stronger than or equivalent to what it would be without the covert SSO (Mayr and Spector 2012:20).

While Mayr and Spector account successfully for the interactions between CQPs and negation seen in (1), their generalized quantifier treatment of CQPs overpredicts CQP scope inversion possibilities elsewhere. As detailed below in section 2.1, their analysis leads to the erroneous expectation that a CQP object should be able to take inverse scope above a universally quantified subject in a DE environment, as in (3).

(3) Whenever every student registers for more than one class, the advisors are pleased. ($\forall >$ CQP, #CQP $> \forall$)

This problem can be overcome by abandoning the generalized quantifier treatment of CQPs in favor of the decompositional analysis of Hackl (2000). On Hackl's approach, CQPs are structurally complex, consisting of an individual quantifier (*many*) and a structurally superior degree quantifier (*-er*). Takahashi (2006) adopts Hackl's decompositional treatment and shows that the degree quantifier *-er* is scopally commutative with both the existential and universal quantifiers. The unavailability of scopal inversion in (3) then falls out as a violation of Scope Economy, the semantically vacuous SSO that inverts universal *every* and the degree quantifier *-er* forestalling the application of any further SSOs. (I discuss an additional problematic prediction of Mayr and Spector's account involving CQPs headed by *fewer*, and its resolution in Hackl's decompositional system, in section 5.2.)

Takahashi's analysis, in turn, encounters problems with negation. Alongside Fox's Scope Economy, Takahashi proposes a constraint he calls the Cross-Polar Anomaly Constraint, whose effect is to bar derivations in which a non–order-preserving operator passes through the scope position between *-er* and *many*, the two CQP components; this is sketched in (4).

(4) * *-er* $>$ non-UE Op $>$ *many* Cross-Polar Anomaly Constraint

The constraint is meant to explain the inability of CQP objects to take inverse scope above DE and non-monotone subjects. As defined, it also ensnares negation; but as we have seen in (1), scopal inversion of CQPs and negation is not systematically barred. Takahashi (2006:89n23), recognizing the problem, tentatively suggests that constraints regulating SSOs (i.e., the Cross-Polar Anomaly Constraint and Scope Economy) might effectively ignore negation; but then we are at a loss to explain the unavailability of scopal inversion in (1b).

1.3 An Integrated Approach

The proposals of Takahashi (2006) and Mayr and Spector (2012) have complementary strengths and weaknesses in the areas of CQP structure and CQPs' interactions with negation. In what follows, I offer an analysis that combines the strengths of the two proposals: namely, an integration of the GSEC with the decompositional approach to CQP structure. In order to integrate these two proposals, two major issues need to be addressed: (i) the nature of the interactions between negation and *-er* and (ii) the scopal immobility of CQP objects with DE and non-monotone subjects, which inspired Takahashi's Cross-Polar Anomaly Constraint. With regard to (i), I propose in section 3 that the GSEC bars only those SSOs that yield what I call non-vacuous entailment from the output configuration to

the input configuration; I show that the strengthening SSO implicated in the CQP object cases, which converts NEG > *-er* into *-er* > NEG, necessarily involves strengthening either from a tautology or to a contradiction. As for (ii), in section 4 I adopt Iatridou and Sichel's (2011) decompositional account of negative quantifiers, which allows us to circumvent the problem that the Cross-Polar Anomaly Constraint was employed to solve.

The outline of the paper is as follows. In section 2, I discuss the proposals of Mayr and Spector and Takahashi in detail. In sections 3 and 4, I present the non-vacuous entailment amendment to Mayr and Spector's GSEC and then implement the elimination of Takahashi's Cross-Polar Anomaly Constraint. In section 5, I investigate the behavior of CQPs headed by *fewer* and show that the combination of the amended GSEC and the decompositional analysis of CQPs makes a number of correct predictions. Section 6 provides further discussion of the GSEC, the non-vacuous entailment condition, and the way in which they assess and regulate SSOs. Section 7 concludes.

2 The Ingredients of CQP Scope

2.1 Generalized Scope Economy

Takahashi (2006), whose overriding concern is to explain the general scopal immobility of CQPs, notes an exception involving negation: CQPs in object position are able to take scope above structurally superior negation, as shown in (5a) (Takahashi 2006:89n23; see also Szabolcsi 2010:84). Mayr and Spector (2012) observe, by contrast, that CQPs in subject position cannot undergo quantifier lowering (QL) to take scope below negation, as in (5b),[2] but that in certain embedded clauses, scopal inversion of CQP subjects and negation is indeed licensed, as in (5c).[3] (The examples in (5) are repeated from (1).)

(5) a. John didn't read more than three books. (NEG > CQP, CQP > NEG)
b. More than three students didn't do the reading. (CQP > NEG, #NEG > CQP)
c. Whenever more than three students don't do the reading, I cancel class. (CQP > NEG, NEG > CQP)

The subject–object asymmetry in the availability of CQP–negation inversion seen in (5a) and (5b) might prompt one to explore a configurational or cartographic account of scope taking along the lines of Beghelli and Stowell (1997); but the contrast between main- and embedded-clause CQP subjects seen in (5b) and (5c) makes any such proposal difficult to maintain. Meanwhile, the fact that scopal inversion of a CQP and negation always

[2]That QL-based inversion of quantificational subjects and negation is not categorically barred is shown by examples like (i).

i. Every student didn't do the reading. (∀ > NEG, NEG > ∀)

[3]To wit, (5c) can be uttered by a very strict instructor who cancels class anytime there are more than three students who haven't done the reading (CQP > NEG); in a class with fifteen students, as many as eleven can do the reading, yet their instructor will walk out in disgust at their four lackadaisical classmates. On the other hand, this sentence might be uttered by a fairly lax instructor who nonetheless has his limits: in this case, class will be cancelled just in case the number of students who have done the reading fails to exceed three (NEG > CQP).

has truth-conditional consequences, regardless of the direction of inversion, means that we cannot invoke Scope Economy (Fox 2000) to rule out cases like (5b), since Scope Economy disallows only those SSOs that have no effect on truth conditions.

Mayr and Spector (2012) propose an extension to Fox's Scope Economy that is designed precisely to handle scope inversion asymmetries that parallel configurational asymmetries like the one above. They propose the Generalized Scope Economy Condition (GSEC), defined above in (2) and repeated in (6) (to be slightly modified below in section 3).

(6) Generalized Scope Economy Condition (GSEC): A covert SSO is licensed in a sentence S only if there exists a constituent C of S (possibly S itself) such that the covert SSO does not make the semantic value of C stronger than or equivalent to what it would be without the covert SSO (Mayr and Spector 2012:20).

The GSEC rules out semantically vacuous SSOs, like Fox's Scope Economy. In addition, it rules out SSOs in which the interpretation of the output configuration asymmetrically entails that of the input configuration, i.e., SSOs that result in logically strengthened meanings. Put more generally, *the GSEC rules out any SSO in which the output configuration entails the input configuration.*

Mayr and Spector show that the general framework for SSO regulation embodied in the GSEC correctly predicts the pattern of interactions between CQPs and negation observed in (5). They treat CQPs as generalized quantifiers with existential force; their lexical entry template for determiners of the form *more than n* is shown in (7). ('#' is a cardinality function, with X a variable over plural individuals; cf. Hackl's (2000) lexical entry for *many* below in (10b).)

(7) $[\![\textit{more than } n]\!] = \lambda P \lambda Q . \exists X [\#X > n \wedge P(X) \wedge Q(X)]$ (Mayr and Spector 2012:11)

This move allows Mayr and Spector to explain the pattern in (5) as an instance of the more general pattern of interactions between existentials and negation. They observe that the scope configuration NEG $>$ $\exists$ asymmetrically entails the scope configuration $\exists >$ NEG.[4] If CQPs are existential quantifiers, inversion of a CQP object with negation involves an SSO from the stronger NEG $>$ $\exists$ surface scope configuration to the weaker $\exists >$ NEG inverse scope configuration. Inversion of a CQP subject with negation involves an SSO in the opposite direction. The latter, but not the former, involves logical strengthening and thus runs afoul of the GSEC, whence the contrast in acceptability between (5a) and (5b). The GSEC thus successfully captures the subject–object asymmetry noted above.

Embedded CQP subjects, as in (5c), differ from their main-clause counterparts in being able to invert with negation. While the GSEC bars SSOs that result in logical strengthening, note that as defined in (6), the GSEC allows the assessment of logical strengthening to be made either locally or globally. This means that an SSO that is barred for inducing

[4] As Mayr and Spector (2012:12) observe, this entailment follows only if quantifiers are "Aristotelian" in a somewhat stronger sense than that discussed in footnote 1: we must presuppose not just a nonempty restriction but a nonempty smallest live-on set. Without this stronger presupposition, the entailment does not follow in situations where the restriction's cardinality is smaller than what is stipulated in the truth conditions of the CQP: e.g., if there are exactly three boys in the domain, then *More than three boys didn't come* will be false under the surface-scope ($\exists >$ NEG) reading but true under the inverse-scope (NEG $>$ $\exists$) reading. (I owe this example to Danny Fox (p.c.).)

logical strengthening in an unembedded environment—e.g., the SSO that takes CQP > NEG and yields NEG > CQP in (5b)—will be permitted when it occurs embedded in a DE environment, where entailments are reversed.[5] As Mayr and Spector (2012:19) observe, the restriction of *whenever* is one such environment. The GSEC thus correctly predicts that the offending SSO in (5b) will be permitted in a *whenever* clause, with the result that CQP subjects in such clauses will be able to invert with negation, as shown in (5c).[6]

Mayr and Spector's GSEC successfully captures the complex pattern of interactions between CQPs and negation detailed in (5). Unfortunately, as mentioned in section 1.2, the unitary generalized quantifier treatment of CQPs that Mayr and Spector adopt leads to some incorrect predictions about interactions between CQPs and other quantifiers. Consider, for example, what the existential generalized quantifier treatment of CQPs, in concert with the GSEC, predicts for clauses that contain an *every* subject and a CQP object. Since the $\forall > \exists$ surface scope configuration is asymmetrically entailed by the $\exists > \forall$ inverse scope configuration, the GSEC correctly predicts that inverse scope should be unavailable in main clauses, as in (8a).[7]

(8) a. Every student registered for more than one class.
($\forall >$ CQP, #CQP $> \forall$)

b. Whenever every student registers for more than one class, the advisors are pleased.
($\forall >$ CQP, #CQP $> \forall$)

c. Whenever every student registers for one class, the advisors are pleased.
($\forall >$ *one*, *one* $> \forall$)

But the GSEC also leads us to expect that this strengthening SSO should be licensed in

[5]Note that, since the assessment of logical strengthening can be made either locally or globally per the definition of the GSEC in (6), the reversal of entailment in a DE environment does not render otherwise licit SSOs—like the weakening SSO that takes NEG > CQP and yields CQP > NEG in (5a)—ungrammatical. In such a case, there exists a constituent C in which the SSO does not result in logical strengthening (or equivalence): C is simply the embedded clause.

[6]An additional prediction of Mayr and Spector's account, not discussed by them, is that the offending strengthening SSOs will be permitted in non-monotone environments, where entailment patterns are not reversed but obliterated. The complements of factive predicates are such environments (for discussion, see Heim 1992 and especially von Fintel 1999). Inversion of CQP subjects with negation is thus possible in factive complement clauses, as in (i).

i. It's too bad that more than three students didn't do the reading. (CQP > NEG, NEG > CQP)

[7]Ordinary existential objects are of course able to take inverse scope above universal subjects, a fact that Mayr and Spector (2012:31–32) attribute to the availability of non-QR scoping mechanisms (e.g., choice-function variable approaches) for such existentials; the GSEC regulates QR but does not prevent existentials from taking wide scope in other ways. As for CQPs, they note that "It is also known that the mechanism whereby simple indefinites can achieve wide scope without movement does not apply to modified numerals such as *more than NP*." However widespread the recognition that ordinary existentials and CQPs behave differently, one would like to have an explanation for this difference; by breaking CQPs down into an existential and a degree quantifier that is scopally commutative with universals and existentials, the decompositional approach to CQP structure introduced in the next section offers one. Finally, note that the failed prediction of the existential generalized quantifier approach to CQPs detailed in (8b) is independent of the matter of non-QR scoping mechanisms: QR should be able (per the GSEC) to raise an existential past a universal in a DE environment, but (8b) shows that this prediction fails for putatively existential CQPs.

a DE embedded clause, where entailments are reversed.[8] As (8b) shows, this expectation is not borne out. Imagine that the advisors want the students to get to know each other better, and thus want there to be more than one class that every student is registered for (CQP $>$ $\forall$); evidently (8b) cannot express the advisors' pleasure at this situation, having only the reading on which they are pleased that the students are taking more than one class apiece ($\forall$ $>$ CQP). This contrasts with what we find with the ordinary existential *one* in (8c), where the "single class in common for every student" reading is available.

To summarize, while Mayr and Spector's GSEC successfully captures the complex interactions between CQPs and negation shown in (5), the authors rely on an analysis of CQPs that leads to problems elsewhere. In the next section, I show how these problems can be overcome by adopting the decompositional analysis of CQPs developed by Hackl (2000) and advocated by Takahashi (2006). This must then be coupled with a modification of the GSEC (that is, a further modification of Scope Economy), a task to which I turn in section 3.

2.2 CQP Decomposition

Hackl (2000) proposes that CQPs like *more than three books* be analyzed not as traditional generalized quantifiers but as complex elements composed of a degree quantifier (*-er than three*) and an individual quantifier (*d-many books*). Hackl's CQP syntax is sketched in (9); the LF in (9b) shows both of these quantifiers raising to take scope. I adopt Hackl's truth conditions for *many* alongside Heim's (2006) truth conditions for comparative *-er* in (10).[9]

(9) a. John read [[-er than three]-many books].
b. LF: [-er than three]$_1$ [d_1-many books]$_2$ John read t_2

(10) a. $[\![\text{-}er]\!] = \lambda A_{\langle d,t\rangle}\lambda B_{\langle d,t\rangle}.A \subset B$ (Heim 2006:43; Takahashi 2006:69)
b. $[\![many]\!] = \lambda d_d \lambda P_{\langle e,t\rangle}\lambda Q_{\langle e,t\rangle}.\exists X[\#X = d \wedge P(X) \wedge Q(X)]$ (Hackl 2000:83)

Hackl's lexical entry for *many* in (10b) is similar to Mayr and Spector's proposal for *more than n* in (7), involving existential quantification over plural individuals, with '#' a cardinality function (Hackl 2000:199ff.). If, with Takahashi (2006), we take the phrase *than three* to denote the degree interval $\{d : 0 \leq d \leq 3\}$, then the treatment of CQPs adopted here yields the following truth conditions for the sentence *John read more than three books*:

[8] We can likewise illustrate this with a non-monotone embedded environment rather than a DE one (cf. footnote 6); this may help us avoid potential complications related to verifying the non-availability of asymmetrically entailing readings; see Mayr and Spector's (2012:21ff.) discussion of truth dominance (Meyer and Sauerland 2009). The interactions between CQPs and negation (cf. (5)) and between CQPs and *every* (cf. (8)) are the same in non-monotone environments as they are in DE environments. Non-monotone examples are shown for negation in footnote 6 and for *every* in (i) and (ii).

i. It's too bad that every student registered for more than one class. ($\forall$ $>$ CQP, #CQP $>$ $\forall$)
ii. It's too bad that every student registered for one class. ($\forall$ $>$ *one*, *one* $>$ $\forall$)

[9] Heim's treatment of *-er* is more readily generalizable to CQPs headed by *fewer* than is Hackl's, which relies on a maximalization operator. I discuss *fewer* in section 5.

(11) $[\![$*John read more than three books*$]\!]$
$= [\![$*-er than three*$]\!](\lambda d_d.[[\![$*d-many books*$]\!](\lambda x_e.[\![$*John read* $x]\!])])$
$= [\![$*-er than three*$]\!](\lambda d_d.\exists X[\#X = d \wedge \textbf{books}(X) \wedge \textbf{John-read}(X)])$
$= 1$ iff $\{d : 0 \leq d \leq 3\} \subset \{d : \exists X[\#X = d \wedge \textbf{books}(X) \wedge \textbf{John-read}(X)]\}$

The truth conditions derived in (11) state that John read (every element of)[10] some plural individual X consisting of books with cardinality greater than three; this is what makes the set of degrees d for which there exists some such plural individual a proper superset of the interval $\{d : 0 \leq d \leq 3\}$.[11]

Takahashi (2006:79ff.) proves that the degree quantifier *-er* that is a component of the decomposed CQP is scopally commutative with both the existential and universal quantifiers. Any SSO that inverts an existential or a universal with *-er* will thus be barred by Fox's Scope Economy, which prohibits SSOs that have no truth-conditional effect.[12] Together with the assumptions that QR moves the individual components of a CQP separately (a matter I discuss further in section 6.1) and that QR is subject to Shortest Move, scopal inversion of a CQP object with an existential or universal subject thus depends on semantically vacuous inversion of the degree quantifier *-er* and the existentially or universally quantified subject. That inverse scope is indeed unavailable in such configurations, as shown in (12), follows naturally on the decompositional analysis of CQPs as a violation of Scope Economy, which blocks the vacuous SSOs sketched in (13). (I adopt the convention of underlining the non-CQP scope-bearing element in examples that depict SSOs, like (13); this intended as a legibility aid.)[13]

(12) a. Some student read more than three books. ($\exists >$ CQP, #CQP $> \exists$)
b. Every student read more than three books. ($\forall >$ CQP, #CQP $> \forall$)

(13) a. $\underline{\exists/\forall} >$ *-er* $>$ *many* (surface scope)

[10]For space reasons, I have omitted the semantic contribution of a distributivity operator from the expression **John-read**(X) in (11). See footnote 13 for further discussion of distributivity and CQP scope.

[11]I assume that degree expressions are monotone, that is, that if John read n books, then it is also true that he read m books, for any $m \leq n$ (Takahashi 2006:79).

[12]As Takahashi (2006:89n23) observes, modals appear to be exempt from this restriction. I have no suggestion at present for how to amend the theory of scope shifting embodied in the GSEC in order to provide a natural account of the interaction between *-er* and modals.

[13]Mayr and Spector (2012:15ff.) question the claim that CQPs can never take inverse scope above an existentially quantified subject, citing examples like (i) below (their (25)). They attribute the availability of this reading to the intercession of a distributivity operator with which the (existential) CQP is not scopally commutative; for them, the inverse scope reading depends on QL of the existential subject past the raised CQP and the distributivity operator in a single step. The decompositional analysis of CQPs, together with the assumption of Shortest Move, certainly fails to predict this possibility, as semantically vacuous QL of the existential subject past *-er* would forestall any further SSO. With this in mind, I note that CQP inverse scope appears to be unavailable if the existential determiner is changed from *a* to *some* or *one*, as in (ii) and (iii). This in turn suggests that the range of available readings in (i) might not be representative of the broader pattern licensed by the GSEC. I leave a fuller investigation of the issue to future research. (I use '%' below to indicate pragmatic awkwardness, in contrast to '#', which indicates semantic anomaly/unavailability.)

i. A soldier is standing on more than ten government buildings. (%$\exists >$ CQP, CQP $> \exists$)
ii. % Some soldier is standing on more than ten government buildings. (%$\exists >$ CQP, #CQP $> \exists$)
iii. % One soldier is standing on more than ten government buildings. (%$\exists >$ CQP, #CQP $> \exists$)

b. * *-er* > $\underline{\exists/\forall}$ > *many* (vacuous SSO blocked by Scope Economy)

c. . . . inverse scope unavailable due to blocked intermediate scope

Takahashi's account of the scopal immobility of CQPs extends naturally to the cases seen above in (8a) and (8b), which proved problematic for Mayr and Spector. Once we adopt the decompositional analysis of CQPs and recognize Takahashi's proofs of the scopal commutativity of *-er* with existentials and universals, we predict that a CQP object should never be able to take inverse scope above an *every* subject, regardless of the clausal environment in which it occurs: since truth-conditionally vacuous SSOs are barred in all circumstances (by Fox's Scope Economy and by the GSEC), the DE environment in (8b) will not rescue the vacuous SSO that inverts *every* and *-er*. We likewise gain a principled explanation for the lack of inverse scope in (8a), no longer having to rely on the stipulation that the non-QR scope-taking mechanisms available to other existentials are unavailable to putatively existential CQPs (see footnote 7).

For all its advantages, Takahashi's account of CQP scope is not equipped to deal with negation. The problem stems from Takahashi's Cross-Polar Anomaly Constraint, defined in (14), a constraint he proposes in order to explain the inability of CQP objects to take inverse scope above DE and non-monotone subjects, as shown in (15). Since such quantifiers are not scopally commutative with *-er* (or *many*), we cannot rely on Scope Economy alone to explain CQP objects' scopal immobility here. The Cross-Polar Anomaly Constraint is violated whenever an SSO causes a DE or non-monotone element to intervene between *-er* and *many* (as it must do in order to effect full inversion of said quantifier with the CQP, per Shortest Move). Takahashi (2006:89) suggests that this "violates principles of grammar, [and] we cannot apply any further SSO", i.e., that violations of the Cross-Polar Anomaly Constraint are tantamount to violations of Scope Economy. The offending SSOs are sketched in (16).

(14) Cross-Polar Anomaly Constraint: The partial ordering relation denoted by the comparative operator is defined only over sets of degrees that contain the same end of a scale. (Takahashi 2006:86)

(15) a. No student read more than three books. (*no* > CQP, #CQP > *no*)

b. Exactly five students read more than three books. (!5 > CQP, #CQP > !5)

(16) a. $\underline{no/!5}$ > *-er* > *many* (surface scope)

b. * *-er* > $\underline{no/!5}$ > *many* (blocked by Cross-Polar Anomaly Constraint)

c. . . . inverse scope unavailable due to blocked intermediate scope

Unfortunately, the Cross-Polar Anomaly Constraint erroneously predicts that the DE-ness of negation should prevent CQPs from scopally inverting with negation under any circumstances, contrary to what we have seen in (5). Takahashi (2006:89n23) recognizes the problem and tentatively suggests that negation might escape regulation by the Cross-Polar Anomaly Constraint and Scope Economy in much the same way that modals appear to escape regulation by Scope Economy. (Takahashi notes that negation and modals share the property of being non-DP scope-bearing elements.) The unavailability of scopal inversion in (5b), along with the behavior of *fewer* to be discussed in section 5, show that this suggestion cannot be correct for negation. In order to achieve an explanatory account of the interactions

between CQPs and negation, we must look for an alternative to the Cross-Polar Anomaly Constraint.

2.3 Toward an Integration

I suggest that the solution lies in combining the GSEC with the decompositional approach to CQPs. My proposal proceeds from the observation that the analyses of Mayr and Spector (2012) and Takahashi (2006) have complementary strengths and weaknesses. Mayr and Spector's GSEC offers an account of the configurational asymmetry in the availability of CQP–negation inversion seen in (5) but fails to predict the scopal immobility of the CQP objects in (8). Takahashi, in adopting Hackl's decompositional analysis of CQPs, can explain the latter but not the former. In adopting the GSEC alongside the decompositional analysis of CQPs, I thus aim to combine the strengths of Mayr and Spector's and Takahashi's proposals while eliminating their respective shortcomings vis-à-vis CQP structure and negation.

As mentioned at the outset, the task of integrating these two proposals requires us to address two major issues: (i) the nature of the interactions between negation and *-er* and (ii) the scopal immobility of CQP objects with DE and non-monotone subjects, which inspired Takahashi's Cross-Polar Anomaly Constraint. I turn to these issues in the next two sections.

3 On the GSEC's Regulation of Logical Strengthening

I now turn to the interaction between negation and *-er*. Whereas Mayr and Spector's successful treatment of the scopal interactions between negation and (non-decomposed) CQPs-as-existentials is directly transferable to the scopal interactions between negation and the existential CQP component *many*, the same cannot be said for the other component of the decomposed CQP, the degree quantifier *-er*. In this section I examine the nature of the scopal interactions between negation and *-er* and offer a small amendment to the GSEC that makes it fully compatible with the decompositional analysis of CQPs.

To begin, I sketch the SSOs that are implicated in CQP–negation inversion on the decompositional analysis of CQPs. Focusing first on the interactions between negation and existential *many*, we can see clearly that the GSEC correctly predicts the subject–object asymmetry noted above in (5): since the scope configuration NEG $>$ *many* is logically stronger than the scope configuration *many* $>$ NEG (see footnotes 1 and 4), inversion of negation and *many* will involve logical weakening with the CQP object in (17a) but logical strengthening with the CQP subject in (17b). Inversion is thus permitted in the former case, as in (5a), but barred in the latter case, as in (5b) (absent the rescuing effect of a higher non–order-preserving logical operator).

(17) a. Inversion of CQP object with negation:
   i. $\underline{\text{NEG}} > \textit{-er} > \textit{many}/\exists$ (surface scope)
   ii. $\textit{-er} > \underline{\text{NEG}} > \textit{many}/\exists$ (intermediate scope)
   iii. $\textit{-er} > \textit{many}/\exists > \underline{\text{NEG}}$ (inverse scope)

   b. Inversion of CQP subject with negation:

i. *-er* > *many*/∃ > NEG (surface scope)

ii. * *-er* > NEG > *many*/∃ (intermediate scope; blocked by GSEC)

iii. . . . inverse scope unavailable due to blocked intermediate scope

For the CQP object in (17a), we also need to show that the first SSO, from surface scope to intermediate scope, is licensed by the GSEC. We can easily demonstrate, however, that with *-er* defined as in (10a), the intermediate scope configuration *-er* > NEG asymmetrically entails the surface scope configuration NEG > *-er*. Recall that *-er* relates two sets of degrees: the first is syntactically instantiated as a *than* clause, the second as a predicate abstract derived via type-driven movement of the DegP headed by *-er*. When negation occurs in the immediate scope of *-er*, it applies to the second argument of *-er*. Suppose this second argument (absent negation) is the set B (or, equivalently, the function $\lambda d_d.B(d)$); when negation scopes immediately below *-er*, the second argument will be the complement of B (i.e., $U - B$, where U is the universe of discourse; equivalently, the function $\lambda d_d.\neg B(d)$). The syntax–semantics mapping is sketched in (18).

(18) a. LF: [-er than A]$_1$ NEG [d_1-B]

b. Truth conditions: $[\![\text{-}er]\!](\lambda d_d.A(d))(\lambda d_d.\neg B(d))$
$= 1$ iff $A \subset (U - B)$

When negation takes scope above *-er*, we get the LF and truth conditions shown in (19).

(19) a. LF: NEG [-er than A]$_1$ [d_1-B]

b. Truth conditions: $\neg([\![\text{-}er]\!](\lambda d_d.A(d))(\lambda d_d.B(d)))$
$= 1$ iff $\neg(A \subset B)$

Now, if a non-empty set A is a proper subset of the complement of B, as in (18), then it will invariably fail to be a proper subset of B itself, which is precisely what is required in (19).[14] The scope configuration *-er* > NEG thus entails the scope configuration NEG > *-er*. Looking in the opposite direction, we can observe that there are many different scenarios in which A will fail to be a proper subset of B; to take a simple example, this will be true if A and B have partial overlap. But in that case (19) will be true while (18) is false, which means that NEG > *-er* does not entail *-er* > NEG. The scope configuration *-er* > NEG thus asymmetrically entails—i.e., is logically stronger than—the scope configuration NEG > *-er*.

This appears problematic for our account of CQP inversion with negation: in (17a), the first SSO, from surface scope to intermediate scope, involves a change from NEG > *-er* to the logically stronger *-er* > NEG. It thus seems that the GSEC should block this SSO and all subsequent SSOs, with the result that the inverse scope reading with CQP objects should be unavailable, contrary to fact.

I suggest that the solution to this problem lies in the nature of the sets of degrees related by comparative *-er*. All degree-based theories of comparison, whatever their differences, share the assumption that sets of degrees are not ordinary unordered sets, but rather ordered

[14] This entailment does not hold if A is the empty set. Since A here is an argument of *-er* denoting a degree interval, it is plausible to assume that the grammar treats A as non-empty, i.e., that the GSEC evaluates *-er* as if its arguments denote non-empty degree intervals; for discussion of the modularity of the GSEC, see section 6.

sets arranged to form a scale. Subsets of such a scale, which may serve as arguments to operators like *-er*, typically are further constrained to pick out contiguous intervals that include one of the scale's endpoints, as with the set $\{d : 0 \leq d \leq 3\}$ seen above in (11).[15] The complement of such an interval includes the other scalar endpoint; for the interval mentioned immediately above, this is $\{d : 3 < d \leq \infty\}$. Two subsets of an ordered set of degrees—i.e., two scalar intervals—that include opposite endpoints of the scale will perforce never stand in the subset relation to each other (a fact that Takahashi (2006) exploits in his Cross-Polar Anomaly Constraint).

With this in mind, I propose a small but significant modification of the GSEC. Whereas Mayr and Spector propose that the GSEC bars SSOs that are semantically vacuous or that involve logical strengthening—i.e., SSOs in which the output configuration entails the input configuration—I propose instead that the GSEC forbids SSOs in which the output configuration non-vacuously entails (N-entails) the input configuration, in a sense to be defined immediately below. On this view, if an SSO is such that either the input configuration expresses a tautology (which is vacuously entailed by every proposition) or the output configuration expresses a contradiction (which vacuously entails every proposition), the GSEC does not regulate its application. This condition on the GSEC's power to block an SSO, which I call the N-entailment condition, can be incorporated into Mayr and Spector's GSEC as in (20) (cf. (6)).[16]

(20) Generalized Scope Economy Condition (GSEC; final version): A covert SSO is licensed in a sentence S only if there exists a constituent C of S (possibly S itself) such that the covert SSO does not make the semantic value of C non-vacuously entail (N-entail) what it would be without the covert SSO.

p N-entails q iff

a. p entails q

b. there is a proposition p' such that p' does not entail q; and

c. there is a proposition q' such that p does not entail q'

The GSEC with the N-entailment condition blocks both logically strengthening and semantically vacuous SSOs, just like Mayr and Spector's GSEC (note that every proposition N-entails itself, per clause (a)). The only difference is that it allows strengthening SSOs in which the input is entailed by every proposition or the output entails every proposition.

With the N-entailment condition in place, let us return to the problematic SSO in (17a), from NEG > *-er* to *-er* > NEG, with the truth conditions shown in (18) and (19). There are two cases. (i) The sets A and B include opposite scalar endpoints: in this case, the input of the SSO, $\neg(A \subset B)$, is a tautology, and the output of the SSO, $A \subset (U - B)$, is a contingency. (ii) The sets A and B include the same scalar endpoint: in this case, the input $\neg(A \subset B)$ is a contingency, but the output $A \subset (U - B)$ is a contradiction. In both scenarios, the SSO involves logical strengthening, but the strengthening is non–N-entailing, in the sense defined above: in the first case, there is no proposition that fails to entail

[15] Not all analyses share this assumption; for a proposal that countenances endpointless scalar intervals, see Schwarzschild and Wilkinson (2002).

[16] I thank Danny Fox (p.c.) for suggesting the wording of the definition in (20).

the input configuration, violating clause (b) of (20), and in the second case there is no proposition that the output configuration fails to entail, violating clause (c). As a result, the GSEC does not regulate this SSO; it is permitted under the definition in (20).

I offer some further discussion of the nature of the GSEC and the N-entailment condition in section 6.

4 DE and Non-Monotone Quantifiers

Having established that the GSEC can be amended so as to accommodate the interactions between negation and *-er*, I now turn to the task of doing away with Takahashi's problematic Cross-Polar Anomaly Constraint. Recall that this constraint was proposed in order to explain the inability of CQP objects to take inverse scope above DE and non-monotone subjects. The examples from (15) above are repeated in (21).

(21) a. No student read more than three books. (*no* > CQP, #CQP > *no*)
b. Exactly five students read more than three books. (!5 > CQP, #CQP > !5)

DE and non-monotone quantifiers are not scopally commutative with *-er* or *many*, the two components of the decomposed CQP. Moreover, if the GSEC bars only N-entailment as proposed in section 3, then inversion of the CQP object with *no* in (21a) should be just as felicitous as inversion of a CQP object with negation; meanwhile, inversion with a non-monotone quantifier, as in (21b), will never involve N-entailment. It thus appears that we cannot rely on the GSEC to rule out the unattested inverse scope readings here. I suggest that the solution lies in amending our assumptions not about the GSEC, but about the DE and non-monotone subjects themselves.

For cases like (21a), a solution may be sought in recent work by Iatridou and Sichel (2011), who offer a decompositional analysis of English negative existentials like *no*. Based on the scopal interactions between *no* DPs and modals, Iatridou and Sichel propose that *no* DPs consist of two components: (i) an existential quantifier that may undergo scope diminishment and (ii) a superior, scopally fixed negation.[17] For our purposes, this means that in order for a CQP object to take inverse scope above a *no* subject, the CQP's scope-bearing components must undergo inversion one by one with those of *no*. As discussed in section 2.2, however, Takahashi (2006:79ff.) has proven that *-er*, the scopally highest component of the CQP object, is scopally commutative with the existential quantifier, the scopally lowest component of the *no* subject. Inversion of these two is blocked—SSOs that effect no change on truth conditions count as N-entailments under the definition in (20) and are thus prohibited—and this prevents any further SSOs, as sketched in (22). Crucial for our purposes is the fact that there is no derivable scope configuration in which *no* (as a whole) takes scope between *-er* and *many*, and thus no need for Takahashi's Cross-Polar Anomaly Constraint: inverse scope is ruled out independently by the GSEC.[18]

[17] Decomposition of negatives along these lines has long been pursued for German and Dutch, and to a lesser extent for English; see Jacobs (1980), Ladusaw (1992), Kratzer (1995), Geurts (1996), Potts (2002), and Penka (2011, 2012), and references therein.

[18] Of course, not all DE quantifiers are negative existentials, and so Iatridou and Sichel's proposal will take us only so far. I note, though, that a large number of other DE quantifiers (possibly a majority, or even

(22) *No* subject (NEG > ∃) and CQP object (*-er* > *many*)

a. NEG > ∃ > *-er* > *many* (surface scope)

b. * NEG > *-er* > ∃ > *many* (vacuous SSO blocked)

c. . . . no further SSOs permitted

For cases like (21b), with non-monotone quantifiers, we might pursue a similar decompositional strategy.[19] A quantifier like *exactly two* might be decomposed along the lines suggested for CQPs, with a scopally superior degree quantifier above the existential individual quantifier *many* (i.e., with *exactly* treated as a degree determiner on the model of *-er*). The failure of non-monotone quantifiers to invert with CQP objects would receive the same explanation as suggested above for *no* and for the existentials in (12): the vacuous SSO inverting existential *many* with *-er* blocks everything else. Such an analysis would also account for Mayr and Spector's (2012:46) observation that non-monotone subjects undergo scopal inversion with *every* objects but not with negation, as shown in (23a) and (23b) (Mayr and Spector's examples (93) and (94), respectively).

(23) a. Exactly two boys danced with every girl. (!2 > *every*, *every* > !2)

b. Exactly two guests didn't show up. (!2 > NEG, #NEG > !2)

c. It's too bad that exactly two guests didn't show up. (!2 > NEG, NEG > !2)

On the decompositional proposal sketched here, inverse scope in (23a) is derived via the weakening SSO that inverts *every* and existential *many*, which is the lowest component of *exactly two*; inverse scope in (23b) is blocked by the strengthening SSO moving negation above existential *many*. As expected, this strengthening SSO is permissible in a non-UE environment, whence the availability of inverse scope in (23c). Once again, we need not rely on the Cross-Polar Anomaly Constraint in order to explain the unavailability of inverse scope.[20]

It thus appears that we can safely do away with the Cross-Polar Anomaly Constraint, a welcome result given the GSEC's clear sensitivity to the presence of negation. While there is surely more to be said about the scopal interactions between CQPs and non-UE quantifiers, we have seen that it is possible to explain the relevant data without appeal to an ad hoc constraint against derivations that move a non-UE element through the scope position between *-er* and *many*. That is, we can explain the troublesome data in (21) without erroneously ruling out attested scopings of negation, all while maintaining Takahashi's underlying assumptions about Shortest Move and Scope Economy (in the updated form of the GSEC).

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all) are scalar quantifiers: positives like *few*, comparatives like *fewer than three*, or superlatives like *at most three*. If these are all subject to a decompositional treatment along the lines proposed by Hackl (2000) for CQPs, then the GSEC may indeed suffice to block inversion of CQP objects with all of these DE subjects. I leave detailed investigation of the matter for future research. (On some important differences between comparative and superlative quantificational DPs, see Geurts and Nouwen 2007.)

[19] I owe this suggestion to Danny Fox (p.c.).

[20] I believe that the decompositional approach sketched here is more promising than the suggestion offered by Mayr and Spector (2012:46), who write that the data in (23) "might suggest that the underlying system does not distinguish between such apparently non-monotonic indefinites and monotone-increasing indefinites, and views them as expressing existential quantification."

5 *Fewer*

5.1 Scopal Commutativity of *little* and Negation

CQPs headed by *fewer*, whether subjects or objects, fail to undergo scopal inversion with negation; examples are shown in (24). Furthermore, as shown in (25), being embedded in a non-UE environment does nothing to ameliorate the relevant readings and the SSOs on which they depend.[21] All of the examples in (24) and (25) are limited to surface scope.

(24) a. Fewer than three students didn't do the reading. (CQP > NEG, #NEG > CQP)

b. John didn't read fewer than three books. (NEG > CQP, #CQP > NEG)

(25) a. Whenever fewer than three students don't do the reading, I let the class continue. (CQP > NEG, #NEG > CQP)

b. Whenever John doesn't read fewer than three books, he writes a good paper. (NEG > CQP, #CQP > NEG)

The fact that inversion of a *fewer* CQP with negation is barred regardless of its surface syntactic position, as shown in (24), suggests that a successful explanation of the facts at hand will not make reference to logical strengthening. The syntactic and logical asymmetry that the GSEC capitalizes on with *more* CQPs will not help us here. It is thus unsurprising to see that inversion of *fewer* CQPs with negation remains unavailable in non-UE embedded environments in (25): the GSEC leads us to expect, apparently correctly, that reversing the direction of entailment (or obliterating it entirely) will not rescue an SSO that is barred for reasons having nothing to do with logical strengthening.

Indeed, a closer look at the internal structure of *fewer* CQPs reveals that the offending SSO barring inversion with negation is not a strengthening SSO, but a semantically vacuous one. I adopt the decompositional analysis of *fewer* CQPs advocated by Heim (2006) and Takahashi (2006), according to which *fewer* consists of the degree quantifier *-er* and the

[21] Examples like (25a) are sometimes used by speakers whose intended meaning can in fact be conveyed by replacing *fewer* with *more*: e.g., in (i) below, the speaker expresses shock that the number of people who realize Leibniz beat Newton to calculus isn't greater (not that it isn't smaller); in (ii), the context makes it clear that the opinion-holder sees shortcomings in the theory of evolution and seeks an explanation for why more (not fewer) people don't see them, as well. The conditioning set of factors for this phenomenon appears to include a *fewer* CQP subject along with negation in a non-UE environment. I take this phenomenon to be attributable to the pragmatic principle *triplex negatio confundit* (Horn 1991; note, in this connection, the further presence of the negative verb *distrust* in (iii), making this a case of Horn's *quadruplex negatio fer'blondiat*), and disregard it for present purposes.

i. I'm still shocked fewer people don't realize Leibniz beat Newton to Calculus. Oh well, great disputes make for great reading.
(http://tech.slashdot.org/article.pl?sid=08/03/27/1657250)

ii. One of the points he made to me was that the scientists in the United States have been very lax in properly presenting evolution to the public, which, in his opinion, is why fewer people don't see the obvious shortcomings of the theory.
(http://samuraisam1.blogspot.com/2005/09/media-complicity-fuels-phony.html)

iii. I'm surprised fewer people don't distrust Microsoft, frankly.
(http://www.internet-nexus.com/2006_04_09_archive.htm)

individual quantifier *many* familiar from our treatment of *more*, along with a scale-reversing operator, *little*, that takes scope between them.[22] The truth conditions for *little* are given in (26) (*-er* and *many* are as above in (10)). The syntax of the sentence *John read fewer than three books* is sketched in (27a), with the truth conditions in (27b).

(26) $[\![\textit{little}]\!] = \lambda d_d \lambda P_{\langle d,t\rangle}.P(d) = 0$ (Heim 2006:38)

(27) a. John read [[[-er than [little-three]]-little]-many books].
LF: [-er than [little-three]]$_1$ [d_1-little]$_2$ [d_2-many books]$_3$ John read t_3

b. $[\![$*John read fewer than three books*$]\!]$
$= 1$ iff $\{d : 3 < d \leq \infty\} \subset \{d : \neg\exists X[\#X = d \wedge \textbf{books}(X) \wedge \textbf{John-read}(X)]\}$

Little takes scope over *many*, yielding the complement of the set of degrees returned by *many*: in this case, those degrees d for which there is no plural individual of cardinality d whose elements are books that John read. This set, in turn, serves as the second argument of *-er*. The truth conditions derived in (27b) state that this set is a proper superset of $\{d : 3 < d \leq \infty\}$, i.e., that it is minimally the interval $\{d : 3 \leq d \leq \infty\}$, which in turn means that the largest plural individual consisting of books that John read has cardinality < 3. These are indeed the intuitively correct truth conditions for the sentence in question. Note that, when *little* occurs in the main clause, we must likewise assume that it occurs in the *than* clause, thereby yielding $\{d : 3 < d \leq \infty\}$, the complement of the set ordinarily denoted by *than three* (for discussion, see Takahashi 2006:83n20). This ensures that we get a sensible subset relation between the two arguments of *-er*, i.e., that sentences with *fewer* CQPs will in general express contingent propositions.

With our decompositional treatment of *fewer* and Heim's semantics for *little* in (26), we are in a position to see why *fewer* CQPs fail to undergo scopal inversion with negation: *little* and negation are scopally commutative. *Little* is a scale-reversing operator; but, as discussed above in section 3, negation has precisely this same scale-reversing effect when applied to a degree interval. This means that any SSO that inverts *little* with negation will be semantically vacuous, i.e., will have no truth-conditional effect. As a result, the SSO will be barred by the GSEC with the N-entailment condition (and by Fox's Scope Economy).[23] To see this, suppose that the second argument of *little* (absent negation) is the function $\lambda d_d.B(d)$. LFs and truth conditions for the scope configurations in which *little* scopes immediately above and below negation are shown in (28) and (29), respectively.

(28) a. LF: [d_1-little]$_2$ NEG [d_2-B]

b. Truth conditions: $[\![\textit{little}]\!]^g(g(1))(\lambda d_d.\neg B(d))$
$= 1$ iff $\neg B(g(1)) = 0$
$= 1$ iff $B(g(1)) = 1$

[22] Mayr and Spector (2012), in keeping with the generalized quantifier treatment they adopt for *more* CQPs, treat *fewer* CQPs as generalized quantifiers with a prefixed negation (noting that the negative component of *fewer* can take scope separately from the rest of the quantifier). One drawback of this approach, not shared by the decompositional approach adopted here, is that there is no straightforward compositional relationship between *more* and *fewer*. *Fewer* is not *more* plus a negative element, as it is on the decompositional approach with *little*. Rather, "*fewer than ten people* is... analyzed as ¬(*ten people or more*)" (Mayr and Spector 2012:36).

[23] Recall, per the definition in (20), that every proposition N-entails itself.

(29) a. LF: NEG $[d_1$-little$]_2$ $[d_2$-B]

b. Truth conditions: $\neg([\![little]\!]^g(g(1))(\lambda d_d.B(d)))$
$= 1$ iff $\neg(B(g(1)) = 0)$
$= 1$ iff $B(g(1)) = 1$

The truth conditions derived in (28) and (29) show that the scope configuration *little* > NEG is truth-conditionally equivalent to the scope configuration NEG > *little*. This suffices to prevent scopal inversion of *fewer* CQPs with negation under any circumstances. Inversion depends on successive SSOs that invert each component of *fewer* with negation (by Shortest Move). As a result, full inversion of *fewer* with negation will always be blocked by the vacuity of *little*–NEG inversion, regardless of *fewer*'s surface position with respect to negation or the entailingness of the environment in which the operators occur.

5.2 Intermediate Scope Readings

While full inversion of *fewer* CQPs with negation is unavailable, the GSEC, along with the amendment proposed in section 3 limiting its ban on strengthening SSOs to those that involve N-entailment, appears to predict the availability of certain intermediate scope readings. This has the potential to be problematic, since *little* and negation will semantically cancel each other out if one is in the immediate scope of the other, leaving behind *-er* > *many*, which is equivalent to a *more* CQP. There are two relevant kinds of cases: (i) those where negation occurs with a *fewer* CQP object (in any environment) and (ii) those where a *fewer* CQP subject occurs with negation in a non-UE environment. The LFs are sketched in (30).

(30) a. Negation with *fewer* CQP object

i. Surface LF:
NEG > *-er* > *little* > *many*/∃

ii. Intermediate scope LF (non–N-entailing SSO licensed by GSEC):
-er > NEG > *little* > *many*/∃
= *-er* > *many*/∃

b. *Fewer* CQP subject with negation in non-UE environment

i. Surface LF:
-er > *little* > *many*/∃ > NEG

ii. Intermediate scope LF (SSO licensed by GSEC in non-UE environment):
-er > *little* > NEG > *many*/∃
= *-er* > *many*/∃

It is clear that the relevant sentences containing *fewer* CQPs and negation do not have intermediate scope readings that are equivalent to sentences with *more* CQPs. The two cases are shown in (31). There is no reading of (31a) (with *fewer*) on which the possibility of John having read merely three books is excluded, as is required by the *more* paraphrase; the same holds, *mutatis mutandis*, for (31b).

(31) a. John didn't read fewer than three books. ≠ John read more than three books.

b. Whenever fewer than three students don't do the reading, I let class continue.
≠ Whenever more than three students do the reading, I let class continue.

If the GSEC permits the SSOs in (30), then why are the readings in (31) unavailable? I suggest that there is in fact no overprediction here. The readings in (31) are unavailable because the intermediate scope LFs in (30) yield comparisons in which the two arguments of *-er* are intervals that contain opposite scalar endpoints, and this runs afoul of the truth conditions of *-er*, whose two arguments must stand in a proper subset relation (per (10a)). Recall from (27) that, in cases where we have *little* in the main clause, we must likewise assume its presence in the *than* clause. This means that the scalar interval denoted by the first argument of *-er* (the *than* clause) in such cases contains the scalar maximum. But if *little* and negation are adjacent and cancel out as in (30), then the scalar interval denoted by the second argument of *-er* contains the scalar minimum. Under the natural assumption that SSOs in the main clause have no effect on the composition of the *than* clause, the truth conditions of intermediate scope readings involving *fewer* and negation thus require a proper subset relation to hold between intervals containing opposite scalar endpoints; such readings are unavailable because they are contradictory.[24]

A more interesting and enlightening set of predictions of the present analysis involves intermediate scope readings of *fewer* and *every*. This is a case where the decompositional approach advocated here captures the facts better than the generalized quantifier approach of Mayr and Spector. As noted by Takahashi (2006:94), sentences with a *fewer* CQP subject and an object headed by *every* have an intermediate scope reading in which *every* takes scope between *little* and *many*.[25] The LFs and truth conditions of the surface scope and intermediate scope readings are sketched in (32).

(32) *fewer* subject with *every* object: e.g., *Fewer than three students read every book*

a. Surface LF:
-er > *little* > *many*/∃ > *every*
Surface truth conditions:
$\{d : 3 < d \leq \infty\} \subset \{d : \neg\exists X[\#X = d \wedge \textbf{students}(X) \wedge \textbf{read-every-book}(X)]\}$

b. Intermediate scope LF (weakening SSO licensed by GSEC):
-er > *little* > *every* > *many*/∃
Intermediate scope truth conditions:

[24] I readily acknowledge that the contradictory intermediate scope reading of a sentence like *John didn't read fewer than three books*, with scope configuration *-er* > NEG > *little* > *many*, is not introspectively accessible in the manner of, e.g., the contradictory reading of Russell's well-known *I thought your yacht was bigger than it is.* (In the former case, the relevant reading is true just in case the interval containing those degrees d such that John read d-many books is a proper superset of the interval from 3 to infinity, i.e., never.) I remain agnostic as to how severe a problem this is for the approach sketched here.

[25] This reading exists alongside the surface scope reading, of course. Full inverse scope is available only in non-UE environments, where the strengthening SSO that changes *little* > *every* into *every* > *little* loses its strengthening force (*pace* Takahashi 2006:93, who takes the inverse scope reading to be generally available). Consider (i) and (ii):

i. Fewer than three students read every book. (CQP > *every*, #*every* > CQP)

ii. Whenever fewer than three students read every book, I cancel class. (CQP > *every*, *every* > CQP)

$\{d : 3 < d \leq \infty\} \subset \{d : \neg\forall x[\mathbf{book}(x) \rightarrow \exists X[\#X = d \wedge \mathbf{students}(X) \wedge \mathbf{read}(x)(X)]]\}$

The intermediate scope reading in (32b) states that the book that was read by the smallest number of students was read by fewer than three students; more precisely, the interval of degrees d such that not every book is such that some plural individual consisting of d-many students read it is minimally $\{d : 3 \leq d \leq \infty\}$.[26] Unlike the surface scope reading, it cannot be true in a situation where every book is such that it was read by three or more students. (The surface scope reading can be true in such a situation, given a suitable number of students and provided no more than two of them are such that they read every book.) Takahashi (2006:94n26) notes that some speakers find the intermediate scope reading easier to get with emphasis on *every*. I would add that this reading is made pragmatically more salient in scenarios involving a task with a scalar threshold for completion; consider (33).

(33) a. *[Context: An airport security screening room. Each bag must be checked by at least three inspectors before the plane can be loaded.]*
Fewer than three inspectors have checked every bag, so the plane can't be loaded yet.

b. *[Context: A legislative committee meeting. Each amendment to the bill being considered must be approved by at least three senators in order for the bill to move forward.]*
Fewer than three senators have approved every amendment, so the bill cannot move forward.

The intermediate scope reading of *fewer* and *every* exhibits a configurational asymmetry not remarked on by Takahashi: it is available in sentences with a *fewer* subject and an *every* object, but not in sentences with an *every* subject and a *fewer* object. For example, the sentence *Every student read fewer than three books* has no reading according to which it is (merely) the case that the student who read the smallest number of books read fewer than three books (while allowing for the possibility that other students read three or more). On the decompositional approach to CQPs, this follows naturally from Takahashi's (2006:81) observation that the universal quantifier and *-er* are scopally commutative. Semantically vacuous inversion of *every* and *-er* will be blocked by the GSEC, and this will forestall the subsequent SSO that would be required in order to have *every* take scope between *little* and *many*. By contrast, on Mayr and Spector's generalized quantifier approach—where the meaning of *-er* is folded into the existential quantifier over plural individuals and the role of *little* is played by scopally superior negation—the intermediate scope reading is straightforwardly derivable. To get from surface scope to intermediate scope, all that is required is an SSO converting *every* > NEG to NEG > *every*, a weakening SSO that is ubiquitously licensed by the GSEC. Mayr and Spector thus erroneously predict that the intermediate scope reading of coargument *fewer* and *every* should be available regardless of their surface configuration; the decompositional approach, by contrast, correctly predicts

[26] If you prefer, substitute for the set $\{d : \neg\forall x[\mathbf{book}(x) \rightarrow \exists X[\#X = d \wedge \mathbf{students}(X) \wedge \mathbf{read}(x)(X)]]\}$ the equivalent set $\{d : \exists x[\mathbf{book}(x) \wedge \neg\exists X[\#X = d \wedge \mathbf{students}(X) \wedge \mathbf{read}(x)(X)]]\}$, i.e., the interval of degrees d such that there is some book that no plural individual consisting of d-many students read.

the configurational asymmetry noted above. The two approaches' LFs for the crucial type of example are sketched in (34).

(34) *Every student read fewer than three books.*

a. Decompositional LF:

i. *every* > *-er* > *little* > *many*/∃ (surface scope)

ii. * *-er* > *every* > *little* > *many*/∃ (vacuous SSO blocked; thus...)

iii. * *-er* > *little* > *every* > *many*/∃ (...intermediate scope unavailable)

b. Generalized quantifier LF:

i. *every* > NEG > ∃ (surface scope)

ii. NEG > *every* > ∃ (intermediate scope licensed by GSEC)

Like the interactions between *every* subjects and CQP objects discussed above in (8), the configurational asymmetry in the availability of the intermediate scope reading of *fewer* and *every* constitutes a clear empirical advantage of the decompositional approach to CQPs over the generalized quantifier approach, alongside the theoretical advantages noted earlier.

6 On the Nature of N-Entailment and the GSEC

6.1 Modularity

The GSEC's assessment of non-vacuity in strengthening, like its assessment of entailment itself, takes place within an abstract and modular system. Fox (2000:66ff.) proposes that Scope Economy is governed by a cognitive module he calls the Deductive System (DS). The DS has access only to the logical properties of lexical items, not to the full truth conditions of the expressions in which they occur, which will typically depend on the values of non-logical constants; it regulates their interactions in a strictly local manner.[27] The GSEC inherits this property of Fox's system (Mayr and Spector 2012:44ff.), with the exception that the assessment of logical strengthening may take into account the logical properties of syntactically higher operators, as discussed above. If, as seems plausible, the logical properties of *-er* that are accessible to the DS include the fact that its arguments denote intervals of ordered sets, then the DS will "know" that the strengthening involved in the

[27] Among other things, this means that, even with the N-entailment condition, the GSEC will bar SSOs involving a tautological input or contradictory output LF in cases where the LF's analyticity is due to the values of its non-logical constants. An anonymous reviewer points to sentences like *More than three linguists are not linguists*, in which both the surface scope and inverse scope readings are contradictory, but only the surface scope reading appears to be available. The reviewer correctly observes that the DS does not see either configuration as contradictory (since this determination depends on the values of the two occurrences of the non-logical constant *linguist*). I note that the same goes, *mutatis mutandis*, for examples in which both configurations are tautological but not visible as such to the DS: e.g., *More than three living people are not dead* has only a surface scope reading. I would add that inverse scope in these examples is actually blocked by the SSO from surface scope (*-er* > *many* > NEG) to intermediate scope (*-er* > NEG > *many*), which involves N-entailing strengthening. Inverse scope in fact seems to be available in non-UE embedded environments, just as predicted by the GSEC: consider *It's too bad that more than three linguists aren't linguists* and *It's too bad that more than three living people aren't dead.* For a related point about analyticity and the GSEC, see section 6.2.

SSO from NEG > *-er* to *-er* > NEG is not an instance of N-entailment: we demonstrated in section 3 that this SSO is non–N-entailing for arbitrary degree intervals (i.e., non-logical constants) A and B.

It is unfortunately difficult to find independent sources of evidence for or against the proposal that the GSEC regulates only N-entailment. The reason is that the interaction between *-er* and negation involves two semantic ingredients—operation over ordered sets and set complementation, i.e., scale reversal—that combine to create non–N-entailing strengthening via a strictly local interaction (i.e., without the intercession of other logical operators). This constellation of factors in a single pair of logical operators—i.e., in the highly constrained sort of configuration about which Scope Economy or the GSEC can have a say—is found only with this pair of operators, to the best of my knowledge.

Finally, one might wonder whether there is some option for deriving wide scope for CQP objects that avoids the strengthening SSO in (17a). For example, an anonymous reviewer asks why the CQP cannot move as a unit to a position above negation, with the DegP headed by *-er* only then raising to take scope above *many*. Such a derivation is sketched in (35).

(35) a. NEG [John read [[-er than three]-many books]] (initial merge of CQP)
b. NEG [[-er than three]-many books]$_2$ [John read t_2] (CQP to scope position)
c. [[-er than three]-many books]$_2$ NEG [John read t_2] (CQP moves past NEG)
d. [-er than three]$_1$ [d_1-many books]$_2$ NEG [John read t_2] (DegP takes scope)

The virtue of the derivation in (35) is that it eliminates the problematic strengthening SSO from NEG > *-er* to *-er* > NEG, as *-er* now moves past NEG as part of the larger CQP with *many* (in the change from (35b) to (35c)). Unfortunately, if we make this derivational option available, we risk vitiating the entire project of Scope Economy and the GSEC. In (35), we have replaced a strengthening SSO with a movement operation in which negation undergoes inversion with an uninterpretable constituent: the CQP cannot be interpreted until *-er* raises to take scope. As Takahashi (2006:77) observes, such inversion with a whole CQP is semantically vacuous, as both its input and its output are uninterpretable. It thus must either be barred by Scope Economy and the GSEC (not a promising option, given the availability of inverse scope here) or ignored by them. If the latter, then we are free to move whole CQPs past negation, and we predict a total absence of restrictions (all else being equal) on the scopal interactions between CQPs and negation; but of course such restrictions are present and systematic, as we have seen. Such considerations lead Takahashi (2006:§5) to propose that, for principled reasons, all obligatory movement operations of the sort required for interpretability (e.g., raising of *-er*) must precede all optional SSOs of the sort that yield scope inversion.[28] I follow Takahashi in assuming that derivations like the

[28]Here is an excerpt of the relevant passage from Takahashi (2006:77):

> [O]ptional SSOs are not driven by any semantic requirement since their input structures are already interpretable. Instead, optional SSOs are applied in order to bring forth a designated interpretation.... [D]ue to this very nature of optional SSOs, they cannot be applied unless they have an interpretive motivation for their application. In other words, optional SSOs must have an effect on semantic interpretation; this principle has been known as Scope Economy since Fox (2000). On this view of the division of labor, optional SSOs can never be applied

one in (35) are unavailable as a matter of grammatical principle.[29]

6.2 Analyticity from Non-Logical Constants: A Case Study

The scopal behavior of *fewer* CQPs sheds important light on what the GSEC does and does not regulate, and on the very limited ability of the DS to ascertain the presence of analytic meanings in its assessment of N-entailment. The DS and, by extension, the GSEC see only the logical operators being manipulated in a given SSO. In cases where an input or output configuration is tautological or contradictory but its analyticity depends on the values of its non-logical constants or other factors, the GSEC will see a logically strengthening or semantically vacuous SSO as N-entailing and therefore block it.

The interaction between *little* and negation is a case in point. Consider the examples in (36), under the indicated readings.

(36) a. It's too bad that more people don't recycle.
'the number of recyclers should be greater'

b. # It's too bad that fewer people don't smoke.
intended: 'the number of smokers should be smaller'

As indicated in the paraphrases, the readings of interest are those where the CQP subject takes scope below negation in the embedded clause; e.g., (36a) means that it's too bad that it's not the case that more people recycle (than actually do; more on the *than* clause below). The availability of this scope configuration is exactly as predicted by the GSEC in concert with the decompositional approach to CQPs: a *more* subject can take scope below negation in a non-UE environment (such as the complement of factive *too bad*) but, due to the irredeemable vacuity of *little*–NEG inversion, a *fewer* subject cannot.

In the case of (36b), we are left only with the surface scope LF.[30] As it happens, this lone available LF yields a contradictory interpretation, and so the sentence is infelicitous. The contradiction is the product of two factors: the factive presupposition associated with the matrix predicate and the obligatory *de re* construal of the elided *than* clause (Horn 1981:334).[31] To see this, let us assume that, in the absence of any other plausible antecedent, the *than* clause is elided under identity with the other argument of *-er*, i.e., the main clause

before applications of obligatory operations are completed.

[29]Note that the insertion of a distributivity operator below the raised CQP (assumed by Mayr and Spector 2012:15ff.; see also footnote 13) is required for interpretability and thus must precede all optional SSOs. We thus need not worry about potential interactions between, e.g., *many* and DIST in assessing violations of the GSEC.

[30]The intermediate scope configuration *-er* > *little* > NEG > *many*/∃ is licensed by the GSEC in a non-UE environment, but it makes *little* and negation scopally adjacent and thus yields a contradictory comparison of intervals containing opposite scalar endpoints, as discussed above.

[31]For example, Horn observes that the contradictory *de dicto* reading in Russell's ambiguity disappears under ellipsis of the comparative *than* clause: in (ii) below, the elided *than* clause can only be interpreted as denoting the yacht's actual length.

i. I thought your yacht was longer than it is. (*de re*, *de dicto*)

ii. I thought your yacht was longer. (*de re*, **de dicto*)

embedded under *too bad* in the examples above. Let us further assume that predicate negation in the antecedent clause requires us to construe predicate negation in the elided *than* clause. The logical form of the surface scope reading of (36b) will thus be as sketched in (37). (I use subscripts to indicate the world of evaluation of the predicate *smoke*, with w the worlds bound by *too bad* and @ the actual world; I disregard details that are irrelevant for present purposes in the representations here and below.)

(37) # It's too bad that fewer people don't smoke.

a. Assertion: TOO-BAD(λw.fewer people don't smoke$_w$ than don't smoke$_@$)

b. Presupposition: fewer people don't smoke$_@$ than don't smoke$_@$

The factive presupposition associated with *too bad* requires that propositions true at w be true in the actual world, as well. With the elided *than* clause construed *de re*, however, this yields a contradictory presupposition, viz. that the actual number of non-smokers is smaller than itself, as shown in (37b). Infelicity is the result.

In (36a), by contrast, where *more* can take scope below negation, we get the logical form sketched in (38).

(38) It's too bad that more people don't recycle.

a. Assertion: TOO-BAD($\lambda w.\neg$[more people recycle$_w$ than recycle$_@$])

b. Presupposition: $\neg$(more people recycle$_@$ than recycle$_@$)

Here the factive presupposition is a tautology, viz. that it is not the case that the actual number of recyclers is greater than itself. Though surely not an informative presupposition, it spares the listener any burden of accommodation and lets one move right along to evaluating the assertion. Sentences on the model of (36) are thus felicitous only when the CQP subject can take scope below negation, as only then do they yield a non-contradictory presupposition. For reasons discussed above, the requisite SSO is possible with *more* but not with *fewer*, whence the pattern of acceptability seen in (36).[32]

[32]The pattern in (36) is also found with equative quantifier phrases (EQPs) embedded under counterfactuals. Two differences must be borne in mind: (i) counterfactuals presuppose not the truth but the falsity of their complements, and (ii) equative *as* clauses are not fully elided, but rather pronominalized with *so*. Such pronominalized *as* clauses are obligatorily construed *de re*, like their elided *than* clause counterparts (Horn 1981:332). These factors yield the pattern shown below:

i. I wish so many people didn't smoke.

a. Assertion: I-WISH($\lambda w.\neg$[as many people smoke$_w$ as smoke$_@$])

b. Presupposition: $\neg(\neg$[as many people smoke$_@$ as smoke$_@$]); i.e., as many people smoke$_@$ as smoke$_@$

ii. # I wish so few people didn't recycle.

a. Assertion: I-WISH(λw.as few people don't recycle$_w$ as don't recycle$_@$)

b. Presupposition: $\neg$(as few people don't recycle$_@$ as don't recycle$_@$)

EQP subjects with *many* are felicitous in this construction, while those with *few* are not. I suggest that this is because, like their CQP counterparts with *more*, EQPs with *many* are able to undergo scopal inversion with negation; this wide-scope negation then cancels out the negative presupposition associated with counterfactual *wish*, yielding the harmlessly tautological presupposition shown in (i.b). If EQPs with *few*, like their CQP counterparts with *fewer*, contain the scale-reversing operator *little*, then no such inversion with

The GSEC thus forbids semantically vacuous SSOs—such as the SSO inverting *little* and negation—even when such an SSO would rescue an otherwise infelicitous structure like (36b). A natural explanation for this fact is that the contradiction found in examples like (36b) is a product of global, non-logical factors—the factivity of the matrix predicate, the syntactic ellipsis of the *than* clause—to which the DS and, by extension, the GSEC have no access in their strictly local, logical assessment of a given SSO.

7 Summary

The present work is both a modest addendum to, and a synthesis of, two more ambitious projects: those of Takahashi (2006) and Mayr and Spector (2012). I have sought to explain the scopal interactions between CQPs and negation exclusively via fully general principles of scope shifting, all while maintaining a decompositional analysis of CQP structure. To the best of my knowledge, these two goals have never before been successfully integrated in a single analysis.

I have argued, by means of a detailed investigation of both *more* and *fewer*, that there is much to be gained by abandoning Takahashi's Cross-Polar Anomaly Constraint and his conjecture that it ignores negation. This small change notwithstanding, my account preserves the core convictions and guiding spirit of Takahashi's account, namely that the prima facie puzzling scopal behavior of CQPs is the product of independently motivated facts about CQP structure (Hackl 2000) and general constraints on covert scope shifting (Fox 2000). That negation, too, can be brought into the fold only further confirms for me the basic correctness of Takahashi's approach to CQPs.

In adopting Mayr and Spector's GSEC, I have followed much of their reasoning and argumentation concerning the configurational asymmetries observed with *more* and negation. I have endeavoured to expand on their analysis, however, by showing that the GSEC can be made to work with the independently motivated and empirically preferable decompositional treatment of CQPs, once we take the GSEC's prohibition of strengthening SSOs to be limited to cases of N-entailment.

By combining Takahashi's and Mayr and Spector's central insights about CQP structure and the nature of scope shifting, the present paper puts to rest one more mystery of CQP scope, explaining in independently motivated terms the interactions between CQPs and negation. The result is a fuller integration of CQPs into our theoretical understanding of natural language quantification and quantifier scope.

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negation is possible, and we are stuck with the contradictory presupposition in (ii.b), whence the infelicity of the sentence. The parallel behavior of CQPs and EQPs suggests that there is a common and consistent pattern in need of grammatical explanation here. I believe that the decompositional analysis used for CQPs could be successfully adapted to the case of EQPs, a task whose details I leave for future research.

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