

Dividing things up:
The semantics of *or* and the modal/*or* interaction

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NOT THE FINAL VERSION

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Abstract

In this paper, the meanings of sentences containing the word *or* and a modal verb are used to arrive at a novel account of the meaning of *or* coordinations. It is proposed that *or* coordinations denote sets whose members are the denotations of the disjuncts; and that the truth conditions of sentences containing *or* coordinations require the existence of some set made available by the semantic environment which can be ‘divided up’ in accordance with the disjuncts. The relevant notion of ‘dividing things up’ is made explicit in the paper. Detailed attention is given to the question of how the proposed truth conditions are derived from the syntactic input. The account offered allows for the derivation of both the disjunctive and the non-disjunctive readings of modal/*or* sentences, including the much-discussed *free choice* readings of such sentences.

1. Introduction

In this paper, the meanings of sentences containing the word *or* and a modal verb are used to arrive at a novel account of the meaning of *or* coordinations. It has long been known that such sentences – which I will call “modal/*or*” sentences, for short – pose a problem for the standard treatment of *or* as a Boolean connective equivalent to set union. The problem is that modal/*or* sentences have readings which are not predicted by this Boolean account, given the standard treatment of modals. Moreover, it seems unlikely that the solution to the problem lies in the treatment of modals, as a similar problem arises when *or* occurs embedded in the antecedent of a conditional, under comparative adjectives, and under certain intensional verbs. It thus seems most plausible that the problems are due to the standard analysis of *or*. It is this standard analysis which will be revised here. The justification for the revised analysis will come from the account I provide of the problematic data. But I will also try to show that the proposed treatment of *or* captures other intuitions about the natural interpretations of sentences containing it.

In the paper, we will be considering two different sentence types in which modals and *or* interact. The first type are sentences in which two modal clauses are conjoined by *or*, such as (1) and (2). We will call such sentences *wide or sentences*.

- (1) Jane may sing or she may dance.
- (2) Jane must sing or she must dance.

The second type are sentences in which *or* (at least on the surface) conjoins two phrases under a modal verb, as in (3) and (4). These, we will call *narrow or sentences*.

- (3) Jane may sing or dance.
- (4) Jane must sing or dance.

As you can see from the examples, we will be concerned both with sentences containing weak modals (like *may*) and sentences containing strong modals (like *must*).

Sentences (1)-(4) each have two distinct readings, which are often referred to as the *narrow scope or* (NS) and *wide scope or* (WS) readings.¹ The NS reading is generally the more salient, and is brought out by appending to any of the sentences the continuation, "...whichever she prefers". On the NS interpretation, sentences (1) and (3) mean that Jane has permission to do either of the things mentioned – to sing or to dance – although not necessarily to do both.² Thus, this reading entails that both singing and dancing are permissible activities for Jane. The NS interpretations of (2) and (4) mean that Jane has an obligation which is fulfilled by her doing either of the two things mentioned, but which does not require both.³ This reading again entails that both singing and dancing are permissible activities for Jane, but does not entail that either is obligatory. Clearly, the NS reading is not a disjunctive one, in the logical sense. So I will sometimes just call this reading the *non-disjunctive* reading.

The WS reading is brought out by appending to the sentences the continuation "...but I don't know which".⁴ On their WS reading, sentences (1) and (3) mean that Jane has (at least) one of two permissions: to sing or to dance. On this reading, there is no entailment to permissibility of *either* activity. Note that on this reading, the sentences cannot be used to grant permission to Jane, but only as uncertain reports of permissions granted to Jane by a third party. Similarly, sentences (2) and (4) on the WS reading assert that Jane has (at least) one of two obligations. Like (1) and (3), they would serve as uncertain reports as to Jane's obligations. In contrast to the NS reading, the WS reading of (2) and (4) entails that at least one of these activities *is* obligatory for Jane (but does not entail that both are), but does *not* entail that both activities are permissible. This second reading is the one which is predicted for the wide *or* sentences by the standard account of *or* as inclusive disjunction, and I will call it simply the *disjunctive* reading.

These data pose two different, but obviously related, problems. One is the problem which is posed by any case of (presumably) non-lexical ambiguity: how to associate two distinct sets of truth conditions with a single surface string. This problem must be solved at the syntax/semantics interface. The other problem is specific to these data, and is purely semantic: how to generate the

1. This terminology carries some implications about the analysis of the readings. My analysis ultimately makes use of, among other things, a type of scope difference between the two cases, although this is not the core of the account.

2. I won't have anything to say here as to why this is the case. I assume that this is a pragmatic effect, not a semantic one, as it is cancellable. (This seems to be the consensus in the literature.) I also assume that this effect is related to the tendency to interpret unembedded *or* coordinations as implying (although not entailing) that at most one disjunct is true, another issue which I will not address here.

3. In earlier drafts of this paper, I called the NS interpretation the *options* reading, as it is the one which grants options to the subject. Elsewhere, the NS reading of sentence (1) has been called its *conjunctive* reading. The NS reading of sentence(3) has been called its *free choice* reading (see especially Kamp 1973).

4. Or, as Graeme Forbes points out, "...but I'm not telling you which".

non-disjunctive truth conditions of the NS reading. For the standard Boolean account, it is the NS reading of *may/or* sentences (the well-known *free choice or* sentences) which is particularly problematic in this respect.

This paper started with a hunch as to how the modal/*or* sentences acquire their non-disjunctive meanings. The hunch was that the function of an *or* coordination is to divide up some domain in accordance with the contents of the disjuncts. In the case of a *must/or* sentence, the non-disjunctive, NS reading arises when we use the *or* coordination to divide up the set of deontically accessible worlds into two sets: Jane-sing worlds and Jane-dance worlds. That is, on its non-disjunctive reading, a *must/or* sentence is true just in case the set of deontically accessible worlds can be divided up into Jane-sing worlds and Jane-dance worlds. Similarly, a *may/or* sentence is true just in case some *subset* of the deontically accessible worlds can be so divided. The goal of this paper, then, is to provide a semantics for *or* coordinations which allows for the formulation of truth conditions along these lines, along with parallel truth conditions for the disjunctive reading. In addition, the analysis must account for the observed ambiguity by providing a systematic way of generating two different sets of truth conditions for each modal/*or* sentence.

An additional observation which the account must explain concerns the interpretation of modal/*or* sentences which are read epistemically. Epistemic versions of (1)-(4) are given below:⁵

- (5) Jane might sing or she might dance.
- (6) Jane might sing or dance.

- (7) Jane must have sung or she must have danced.
- (8) Jane must have sung or danced.

Zimmerman (2000) observes that for sentences such as these, the disjunctive reading is highly dispreferred, and for the standard case may not be available at all. If we append "...but I don't know which" to an utterance of (5) or (6), the natural interpretation is that I don't know which she *will* do (not that I don't know which she *might* do). Similarly, by appending this continuation to (7) or (8), the speaker says that he does not know which Jane *did*. The case in which the disjunctive reading becomes possible is that where the epistemic modal is dependent on the epistemic state of someone other than the speaker. Consider the following situation: Suppose I have heard on the radio a report about the suspected location of a fugitive. What I heard was the following: "The investigators believe it possible that the fugitive is in Cambridge." But I don't know whether the Cambridge in question is Cambridge, Mass., or Cambridge, England. I might then say: "The fugitive might be in Massachusetts or in England, but I don't know which." In this situation, the utterance has a disjunctive interpretation.

The paper is organized as follows. I begin (section 2) by formulating truth conditions for the non-disjunctive readings of *may/or* and *must/or* sentences, initially setting aside the question of how these truth conditions are associated with the syntactic input. I turn to this question in sections 3 and 4. Section 3 explores the syntactic part of the answer to this question, and section 4, the semantic part. In section 5, I turn to the truth conditions and derivation of disjunctive readings. At the end of this section, I provide an explanation for the absence of disjunctive readings of epistemic modal/*or* sentences. Finally, in section 6, I explore some pragmatic constraints on *or* coordinations.

5. (1) and (3) can be interpreted epistemically as well as deontically; but many speakers seem to find *might* more natural for the expression of weak epistemic modality. So I will use *may* for examples I want to be interpreted deontically, and *might* for examples intended epistemically.

2. Truth conditions for non-disjunctive readings

2.1. First steps

The standard truth conditions for a *may* sentence are given in (9).^{6,7}

$$(9) \quad \text{MAY}[\phi] \text{ is true at } w^* \text{ iff } \exists w \in \text{ACC}_{d,w^*} \text{ s.t. } w \models [\phi]$$

where ACC_{d,w^*} is the set of worlds deontically accessible from w^* , i.e. the set of worlds which make true all propositions expressing what is required at w^* . (From here on, I will suppress the subscripts for readability.)

If the condition in (9) is satisfied, then so too is the condition in (10):

$$(10) \quad \exists S \subseteq \text{ACC} \text{ s.t. } S \models [\phi]$$

This is so since, if there is a world which satisfies the existential in (9), then any set containing that world will satisfy the existential (10). Thus the condition in (10) is a set-based variant on the standard modal truth conditions. What this shows us, then, is that the standard truth conditions impose a condition on a subset of ACC. The idea to be developed, then, is that an *or* coordination requires there to be a subset of ACC which can be divided up as specified by the disjuncts, i.e.:

$$(11) \quad \text{MAY}[\phi \text{ or } \psi] \text{ is true iff } \exists S \subseteq \text{ACC} \text{ s.t. } S \text{ is divided up into } \phi\text{-worlds and } \psi\text{-worlds.}$$

2.2. Getting more precise about ‘dividing up’

One familiar formal notion which provides a way of dividing up a given set is that of a partition:

Definition 1: Partition

A set of sets P is a *partition* of a set S iff:

- (i) Every member of P is a subset of S
- (ii) Every member of S belongs to some member of P
- (iii) The members of P are non-intersecting
- (iv) The empty set is not in P

6. Problems with the standard Kripke-style accessibility account of natural language modals are well known from the work of Angelika Kratzer (Kratzer 1977, 1991). I take Kratzer’s revised semantics for modals to be far more adequate, but for the purposes of this paper, it will be simpler to talk in standard accessibility terms. In fact, given the Limit Assumption, everything I say here can be rendered compatible with Kratzer’s account. Simply substitute references to the set of (deontically or epistemically) accessible worlds with reference to the set of g -closest worlds, where the latter is that subset of the relevant modal base whose members are closer than any other world in the modal base to the ideal established by the relevant ordering source. If we give up the Limit Assumption, things are more complicated.

7. There is a difficulty with this standard view, which carries over both to Kratzer’s account and to the account I will propose here. The difficulty is that on standard accounts, the syntactic residue which remains when the modal is extracted from sentences of this form lacks a tense feature, and thus cannot strictly speaking be a proposition-denoting expression. As this is a general problem for the treatment of the syntax/semantics interface of modals, I will set it aside here. But see Condoravdi (2001) for discussion, and Werner (2003) for further discussion and an alternative syntactic proposal.

If P is a partition of S , then $\cup P=S$

Suppose then that we take this notion to be the relevant notion of ‘dividing up’. This suggests a formulation of truth conditions as follows:

(12) $\text{MAY}[\phi \text{ or } \psi]$ is true iff $\exists S \subseteq \text{ACC}$ s.t. $\{[\phi], [\psi]\}$ is a partition of S .

Let’s consider the suitability of these truth conditions for one of our specific examples, sentence (3), repeated below. Let’s assume that this is to have the truth conditions in (13):

(3) Jane may sing or dance.

(13) $\exists S. S \subseteq \text{ACC}$ s.t. $\{[jane \text{ sing}], [jane \text{ dance}]\}$ is a partition of S .

If these conditions are satisfied, then by the definition of a partition, S contains both *Jane sing* worlds and *Jane dance* worlds. As S is included in ACC , then we have both *Jane sing* worlds and *Jane dance* worlds in ACC . So we have that it is permissible for Jane to sing and permissible for Jane to dance. This is the central entailment of the NS reading which we need to capture

The problem is that these truth conditions are unlikely to be satisfiable, for two reasons. The first reason is that $\{[jane \text{ sing}], [jane \text{ dance}]\}$ can be a partition only if the set of worlds at which Jane sings is disjoint from the set of worlds at which Jane dances. If there is any world at which Jane both sings and dances, then the truth conditions in (13) could not be satisfied, and sentence (3) would be predicted to be false.

The second problem with these truth conditions is that they require that every world at which Jane sings and every world at which Jane dances be permissible worlds. But again, unless Jane has some exceptional properties, this cannot be. There must, for example, be logically possible worlds in which Jane dances when she is supposed to be doing her homework, and these are not permissible worlds.

The notion of partition, then, imposes constraints on the dividing up of the set which are too strict. We need to move to a less restrictive notion. An obvious candidate is the notion of a cover. A cover is just like a partition, with the non-intersection condition removed.

Definition 2: Cover

A set of sets C is a *cover* of a set S iff:

- (i) Every member of C is a subset of S .
- (ii) Every member of S belongs to some member of C .
- (iii) The empty set is not in C .

If C is a cover of S , then $\cup C=S$

If we replace the partition condition, in our truth conditions in (12), with a cover condition, we solve one of the problems just raised, but not the other. We eliminate the unwanted requirement that the ‘disjuncts’ be non-overlapping. But we would be left with the requirement that the union of the disjuncts be a subset of the set of permissible worlds. This is because, whether \mathcal{D} is a partition of S or a cover of S , $\cup \mathcal{D}=S$.

To solve this additional problem, I introduce another notion, which I call a *supercover*. A supercover of a set S is just like a cover of S *except* that the subset requirement of clause (i) is weakened to a non-empty intersection requirement. Specifically:

Definition 3: Supercover

A set SC is a supercover of S iff:

- (i) Every member of SC contains some member of S.
- (ii) Every member of S belongs to some member of SC.

If SC is a supercover of S, then $\cup SC \supseteq S$

While the union of a cover of a set S (like the union of a partition of S) is identical to S, the union of a supercover of S is a superset of S. But as with a cover, a supercover is a way of dividing up the members of S into (possibly overlapping) categories. The definition of supercover does not include the specification that it cannot have the empty set as a member; but this follows from clause (i), which requires that every member of SC have at least one member.

Now we can reformulate our truth conditions for the non-disjunctive reading of *may/or* sentences. This is given for the general case in (14), and for the specific case of sentence (3) in (15).

(14) $MAY[\phi \text{ or } \psi]$ is true iff $\exists S \subseteq ACC$ s.t. $\{[\phi], [\psi]\}$ is a supercover of S.

(15) $\exists S. S \subseteq ACC$ & $\{[jane \text{ sing}], [jane \text{ dance}]\}$ is a supercover of S.

For this condition to be satisfied, ACC must contain at least one world of each kind, but it does not have to contain all such worlds. Moreover, as the cells of a supercover are allowed to overlap, the existence of worlds at which Jane sings *and* dances is not an obstacle.

The proposed truth conditions correctly predict that if Jane is permitted only one of the activities named, then the sentence is false. They predict further that there may be permissible courses of action for Jane other than singing or dancing, i.e. *Jane may sing or dance* does not entail *Jane must sing or dance*. But the truth conditions do not *entail* that there are other permissible courses of action, which is as desired: *Jane must sing or dance* **does** entail *Jane may sing or dance*.

Now that we have the basic form of the truth conditions, let us extend the analysis to sentences with strong modals. I repeat here our example sentences (2) and (4) from above:

(2) Jane must sing or she must dance.

(4) Jane must sing or dance.

A strong modal sentence requires for its truth that the entire relevant set of accessible worlds have some property: generally, that it be a subset of the denotation of the modalized proposition i.e. in general:

(16) $MUST[\phi]$ is true iff $ACC \subseteq [\phi]$

According to the hypothesis under consideration, the non-disjunctive reading of a *must/or* sentence requires for its truth that this set have a different property, namely, that it be divisible in accordance with the disjuncts. Using our new supercover terminology:

(17) $MUST[\phi \text{ or } \psi]$ is true iff $\exists S = ACC$ s.t. $\{[\phi], [\psi]\}$ is a supercover of S.

or, more simply:

(18) $MUST[\phi \text{ or } \psi]$ iff $\{[\phi], [\psi]\}$ is a supercover of ACC

Applying this specifically to (2) and (4), then, we propose the following truth conditions for their non-disjunctive readings:

(19) ... true iff {[*Jane sing*], [*Jane dance*]} is a supercover of ACC.

This condition requires that every accessible (permissible) world be either a *Jane sing* world or a *Jane dance* world i.e. Jane has an obligation to do one or the other, but need not necessarily do both.

In section 6, we will return to some additional issues concerning these cases. But with this much in place, let us turn to the question of how these truth conditions are to be derived from the syntactic input.

3. First steps towards compositionality: LFs

3.1. The problem: Where do the cells of the supercover come from?

Remember that the initial idea is that the NS readings are the result of dividing up ACC (or a subset thereof) in accordance with the disjuncts. If the truth conditions in (14) and (18) above are expressions of this idea, then ϕ and ψ in these truth conditions must correspond to the disjuncts. The problem? The sentences types for which we are giving truth conditions don't, at least on the surface, have syntactic forms in which a single modal takes scope over a coordination of proposition-denoting expressions. This is the case for both the narrow *or* sentences, repeated in (20), and the wide *or* sentences, repeated in (21).

(20) Jane may/must sing or dance.

(21) Jane may/must sing or she may/must dance.

We therefore now face two tasks. First, to 'find' the proposition-denoting disjuncts in these sentences; and second, to say explicitly how these disjuncts determine the truth conditions proposed.

Before we begin, however, it is worth noting that the first of these problems is not a new one. A number of papers have contended with the observation that intuitions about the meaning of *or* coordinations is often at odds with their apparent structure. We often understand *or* coordinations as if there were more material in the disjuncts than appears to be the case. Consider for example the following sentence, originally from Rooth and Partee (1982), and discussed further in Larson (1985):

(22) John believes that Bill said that Mary was drinking or playing video games.

This sentence has three readings:

- a. John believes that Bill said [Mary was drinking or Mary was p.v.g.]
- b. John believes [[Bill said Mary was drinking] or [Bill said Mary was p.v.g.]]
- c. [John believes that Bill said Mary was drinking] or [John believes that Bill said Mary was p.v.g.]

On reading (a), the interpretation of the *or* coordination corresponds to the surface syntax. But on readings (b) and (c), the sentence is interpreted as if the disjuncts were 'bigger' than they actually are. Syntactic solutions to this problem are proposed in Larson (1985), who hooks his syntactic

account to the semantics proposed by Rooth and Partee, and in Schwarz (1999). The solution I will offer to this problem, as it arises in the modal/*or* data, will have both syntactic and semantic aspects.

3.2. NS readings of wide *or* sentences

In a wide *or* sentence, the input consists of two modal sentences joined by *or*. So where do we find the non-modal proposition-denoting expressions whose meanings supposedly give us the members of the supercover? In this section, I will propose that from these sentences, we can quite straightforwardly derive an LF structure with the necessary components. But we must begin by looking briefly at the structure of modal sentences in general.

Consider a simple modal sentence like (23). This is generally assumed to have the (semantical) logical form in (24).

(23) Jane may sing.

(24) MAY[Jane sing]

The derivation of the structure in (24) is straightforward given some currently standard syntactic assumptions: Assume that the surface subject originates in spec VP, and raises at surface structure to the spec of some higher phrase, which we will simply take to be IP (i.e. some phrase which is the locus of tense and agreement features). Assume that the modal verb heads a Modal Phrase, which dominates VP but is dominated by IP. Finally, assume that the subject reconstructs to its original position at LF, for the purposes of interpretation. Thus, the LF associated with (23), which is also its underlying structure, is as shown in (25):

(25) [IP [MP may [VP Jane sing]]]

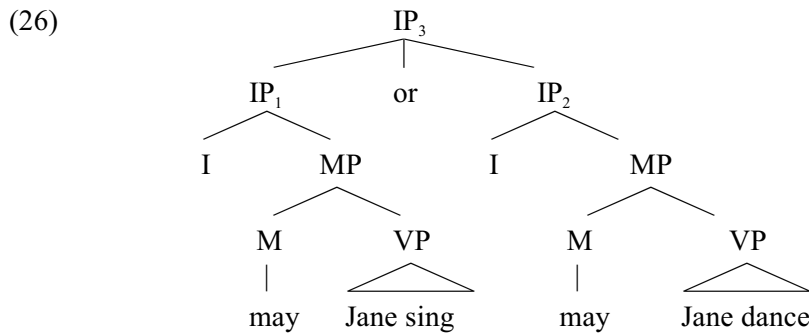
i.e. exactly the structure which we want as input to the semantics.⁸

Now, consider the underlying structure of sentence (1)⁹:

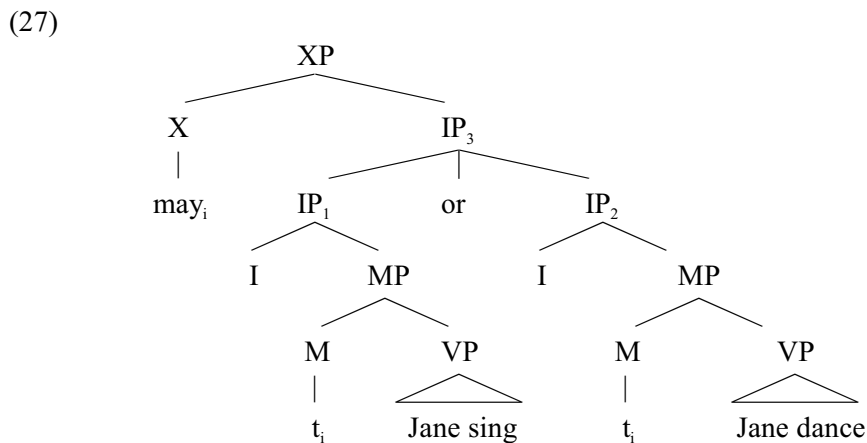
8. But see footnote 7 for a caveat.

9. I here represent the *or* coordination itself as a flat, triple branching structure, although this is probably inaccurate. For a detailed discussion of the structure of coordination, see Munn (1993). The internal structure of the coordination is, however, irrelevant to my current concerns, as long as it does not present an obstacle to movement *out* of the coordination. (See below.) As Johnson (1996), who is the source of the proposed movement analysis, assumes that it does not, I make this assumption too.

(1) Jane may sing or she may dance.



To give the observed surface order, we assume that the subjects of both clauses raise to the spec positions of their respective IP's. At LF, presumably, they once again reconstruct. In addition, the modal verbs, because they occur in parallel coordinated structures, are candidates for ATB (Across-the-board) movement.¹⁰ I propose that at LF, these verbs ATB raise to the head of a higher phrase, the precise identity of which is irrelevant for our purposes. This gives us the following structure:



Assuming that the modal is interpreted in its raised position and that its traces are semantically vacuous, we now have a structure in which a single modal takes scope over an *or* coordination of two non-modal proposition-denoting expressions.

(28) MAY [[Jane sing] or [Jane dance]]

We have yet to say just how the truth conditions in (15) above are derived from this LF. But it is at least more plausible that they are derived from this than that they are derived from an LF consisting of a disjunction of two modal clauses. Let me emphasize, though, that the adoption of this LF is not the core of the solution; if we simply applied the standard semantics of *or* to this LF we would not

10. This ATB movement analysis is inspired by Johnson (1996), whose assumptions I will make use of more explicitly below.

arrive at the correct truth conditions. Moreover, as we will see below, the syntactic story has limits; and the semantic solution which will take over when we reach the limit of syntax will depend crucially on the proposed semantics of *or*.

3.2. NS readings of narrow *or* sentences: finding the limit of the syntactic account

Let's consider now the syntactic structure of our narrow *or* sentence (3):

(3) Jane may sing or dance.

Here, it initially appears that *or* conjoins two verbs. As a first step towards an alternative analysis, observe that the second, apparently bare V may be provided with its own subject, as in:

(29) Jane may sing or Harriet dance.

Note that this has a NS reading parallel to that of (3), according to which permission is granted to Jane to sing and to Harriet to dance. On the surface, we appear here to have a disjunction of two clauses or clause-like structures, the leftmost being a finite clause containing a modal, and the rightmost, a tenseless small clause. If this were the structure, we would face a number of puzzles, not the least being the question of how the modal, if it is contained in the first disjunct, can succeed in taking scope over the *or* coordination as a whole.

Observations of this sort were made in Siegel (1984). Johnson (1996) argues on the basis of such data that the modal is not in fact in the leftmost disjunct at all. He assigns to such sentences the following underlying structure, in which a VP disjunction occurs in the scope of the modal¹¹:

(30) [XP [MP may [VP [VP Jane sing] or [VP Harriet dance]]]]

As before, the subjects are assumed to originate inside their VPs. To derive the surface form, Johnson assumes that the subject of the leftmost VP raises to sentence initial position. We can assume further that this DP reconstructs at LF. The result is an LF of just the same form which was claimed above to underlie the NS reading of the wide *or* sentence. So, we explain the fact that the wide *or* sentence and the narrow *or* sentence share a reading by proposing that the two surface structures are associated with isomorphic LFs.

Let's now return to the original sentence (3), where the second verb does not have its own surface subject. Let us assume that this verb in fact *does* have its own subject underlyingly, but that the subjects of the two verbs are identical i.e. that (3) has the following underlying structure:

(31) [XP [MP may [VP [VP Jane sing] or [VP Jane dance]]]]

As the subjects of the two VPs are identical, they are again subject to ATB movement. Let us assume, then, that these DPs ATB raise to matrix subject position, giving us the observed surface form:

(32) [XP Jane_i [MP may [VP [VP t_i sing] or [VP t_i dance]]]]

11. I have simplified and slightly modified Johnson's tree; but the crucial aspects of the structure are preserved.

Finally, assume again that the surface subjects reconstruct at LF, so that the structure which provides the input to the interpretation is the one in (31) where, just as required, the modal has scope over an *or*-coordination of two clause-like (proposition-denoting) expressions.

3.2.1. The limits of the syntactic account?

The strategy indicated by the proposals just made is straightforward: given any modal/*or* sentence, explain the availability of a NS reading by assigning to the sentence an LF in which the modal takes scope over an *or*-coordination of clause-like expressions. Where the surface form of the *or* sentence is a wide *or* sentence, we suppose that the modals ATB raise at LF to produce the appropriate form. Where the surface form of the *or* sentence is a narrow *or* sentence, we have tried to argue that the sentence has the required form underlyingly, and therefore that reconstruction can give us the required LF.

With a wide *or* sentence, no further complications arise. But with the narrow *or* sentences, problems arise as we look at sentences in which the *or* coordination is more deeply embedded. Consider the following:

- (33) Jane may visit Henry or Matilda.
- (34) Jane might renovate her mother's kitchen or dining room.
- (35) Jane might want to arrange to visit Henry or Matilda while she is here.

Each of these sentences has a NS reading.¹² And it is indeed possible to tell a story whereby these sentences have underlying forms of the required form. The story would probably involve a combination of ATB movement and deletion ellipsis.¹³ So, for example, we might argue for a derivation along the following lines for sentence (33): Assume the underlying form:

- (36) [XP [MP may [VP [VP Jane visit Henry] or [VP Jane visit Matilda]]]]

Assume further that at surface form:

- a. The subjects of the VPs ATB raise to matrix subject position, as allowed under identity and
- b. Either
 - i. the verbs ATB raise to the head of some phrase intermediate between XP and MP
 - or
 - ii. the second occurrence of *visit* is elided (deleted), as allowable under identity.

Assume further that everything reconstructs at LF, giving us back (36).

12. Sentence (35) has multiple NS readings, as expected given the Rooth and Partee/Larson data mentioned above (see page 7). These sorts of 'scopal ambiguities' must be left for another time.

13. A pure ellipsis story is also possible, and indeed that is the approach taken in Schwarz (1999) for cases where *either* is displaced from the surface coordination.

If a derivation along these lines were correct, then it would be an illusion that the string *Henry or Matilda* is an *or* coordination. Thus, in particular, we would not expect this string to behave like a constituent. How, it seems to do so.¹⁴ Consider the following:

- (37) Henry or Matilda, Jane might visit.
 (38) It's Henry or Matilda that Jane might visit.
 (39) Q: Who might Jane visit?
 A: Henry or Matilda.

If the surface *or* coordination really is a syntactic constituent, then we need a different story to tell about the derivation of NS readings. The alternative to a syntactic account is to allow the process of semantic composition to produce the propositions which appear in the truth conditions of our sentences. I turn to this semantic proposal in the next section.

4. More steps towards compositionality: Semantic composition of *or*-coordinations

4.1. The semantic value of an *or* coordination

Let's consider for a moment a wide *or* sentence such as (40), whose NS reading we now take to be derived from an LF with the form in (41). The proposed truth conditions for this reading are given again in (42).

- (40) Jane may sing or she may dance.
 (41) MAY [[Jane sing] or [Jane dance]]
 (42) $\exists S. S \subseteq \text{ACC} \ \& \ \{[Jane \text{ sing}], [Jane \text{ dance}]\}$ is a supercover of S

The question now is how the truth conditions are derived from this LF. The proposal is as follows. An *or* coordination is taken to denote a set whose members are the ordinary denotations of the disjuncts. This means that *or* itself is a set formation operator of some kind. For now, I will not try to formulate an explicit translation or denotation for *or* itself, but only for the *or* coordination.¹⁵

- (43) **Translation rule for *or* coordinations**
 $[\alpha_1 \text{ or } \dots \text{ or } \alpha_n]$ translates as the unordered sequence $\langle \alpha_1', \dots, \alpha_n' \rangle$, where α_1', α_n' are the standard translations of $\alpha_1 \dots \alpha_n$.

- (44) **Interpretation rule for unordered sequence**
 For any unordered sequence $\langle \alpha_1', \dots, \alpha_n' \rangle$, $[[\langle \alpha_1', \dots, \alpha_n' \rangle]] = \{[\alpha_1'], \dots, [\alpha_n']\}$

14. Thanks to Sally McConnell-Ginet for this observation. Bernhard Schwarz (p.c.) points out that it is not universally agreed that short answers like *Henry or Matilda* must be constituents, but might be remnants of the same kind of ellipsis which produces the apparent surface coordination.

15. We might posit that at LF, there is a single *or* which somehow takes scope over all of its disjuncts. *Or* can then be treated as an operator which combines with all of these arguments simultaneously, constructing a set from them. One drawback of this approach is that the possibility of constructing *or* coordinations across sentences, and even across speakers, remains highly mysterious.

The supercover condition might then be supposed to be introduced as a consequence of the combination of this set with the modal of which it is an argument. In the usual case, a modal takes a proposition-denoting expression as its argument. In the *or* case, the modal takes as argument an expression denoting a *set* of propositions. It therefore cannot compose with this argument in the usual way. I propose, then, that the supercover condition is the default relation established between an operator *O* of type $\langle a, b \rangle$ and an *or* coordination whose disjuncts are of type *a*.¹⁶ As we have already discussed, it is the standard semantics of the operator which provides the set which must be supercovered by the denotation of the *or* coordination.

The proposal made here is very similar to a proposal in Winter (1995) for the treatment of *and* coordination. Winter argues that *and* is semantically null, and that consequently an *and* coordination introduces a sequence of unrelated constituents. These, he suggests, are interpreted as forming a tuple. He proposes further that Generalized Conjunction – set intersection – may be freely applied to any such tuple at any point in a derivation.

Seen in light of Winter's work, the proposal made here raises the possibility of a unified account of coordination as set-formation.¹⁷ However, if both *and* and *or* coordinations denote sets (or tuples), we could treat neither Generalized Conjunction nor the supercover condition as composition options triggered simply by the occurrence of a set denoting expression. Rather, we must allow that *and* and *or* themselves restrict the composition options. If so, then Winter's proposal that *and* is semantically null must be revised. However, this is another topic which must be left for another time.

4.2. Independent composition

The idea pursued in section 3.2., that phrasal *or* coordinations are reductions of clausal coordinations, is not a new one (see e.g. Stockwell et. al. 1973). One observation which makes this idea attractive is that where no operators are involved, sentences with phrasal *or* coordinations have the same truth conditions as the parallel clausal *or* coordinations. Thus, (45)-(47) are truth conditionally equivalent.

(45) Jane saw Henry or Matilda.

16. Unfortunately, things don't work out quite neatly enough. In some cases, it seems that we need to impose the stronger cover condition in order to get the correct results. Consider first the sentence *Every guest ate or drank*. This can be given supercover truth conditions as follows:

(i) $\exists S. S \in [\text{every guest}] \ \& \ \{[\text{sing}], [\text{dance}]\}$ is a supercover of *S*.

From (i) it follows that $[\text{guest}]$ is a subset of $\cup \{[\text{sing}], [\text{dance}]\}$, which gives us correct results. (There are some nuances which I set aside.) But now consider the case of a sentence with a downward entailing quantifier subject, such as *No guest ate or drank*. A condition parallel to (i) gives incorrect results, failing to guarantee that $[\text{guest}]$ has a non-empty intersection with $\cup \{[\text{sing}], [\text{dance}]\}$. What we need instead is (ii):

(ii) $\exists S. S \in [\text{no guest}] \ \& \ \{[\text{sing}], [\text{dance}]\}$ is a cover of *S*.

This obviously requires further investigation, but I will not undertake it here.

17. Winter proposes that the analyses of *or* and *and* coordination should not be unified, arguing that while *and* is semantically null, *or* just is the Boolean join operator. This allows him to account for certain asymmetries between *or* and *and* coordinations. But as we have seen, the Boolean analysis of *or* fails to account for the interaction of *or* with modals. However, in Winter (1998), the set-formation analysis of coordination indeed is extended to certain cases of *or* coordination. Combined with the possibility of pointwise composition of the set members with an argument (see below), this allows Winter to account for certain scopal peculiarities of *or* coordinations.

- (46) Jane saw Henry or saw Matilda.
 (47) Jane saw Henry or she saw Matilda.

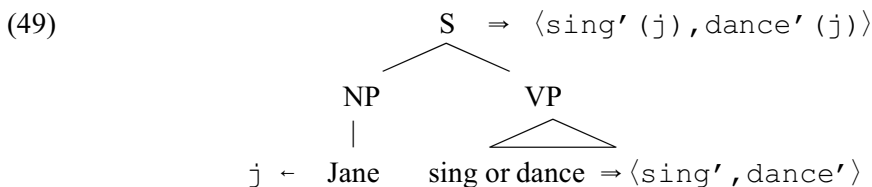
Partee and Rooth (1983) propose a semantic method for deriving this result, which eliminates the need to assume syntactic reduction or ellipsis. They offer a cross-categorial semantics for *or* (and *and*), in which *or* is translated by the Boolean operator “ \sqcup ”, which is equivalent to set union. “ \sqcup ” is a cross-categorial operator i.e. can conjoin expressions of any type (except type *e*). When it conjoins expressions of type *t*, “ \sqcup ” is equivalent to “ \vee ” i.e. the inclusive disjunction operator. When it conjoins expressions of other types, the result denotes a function of the same type as the disjuncts, which applies its input to each of the disjuncts independently. At the level of the translation language, the result is that the final translation of sentence (45) is identical to the translation of sentence (47).

On the account proposed here, an *or* coordination is taken to denote the set of the denotations of its disjuncts. This immediately raises the question of how semantic composition proceeds. The answer which I propose is similar in spirit to Partee and Rooth’s: given a subtree α where one daughter denotes a set of semantic objects each of which can compose independently with the second daughter, the denotation of α is the set of semantic objects which results from these two independent compositions.

- (48) **Rule of independent composition for unordered sequences (optional)**¹⁸
 Let α be a node whose daughters are β and γ . Let the translation of β be $\beta'_{\langle \gamma, \alpha \rangle}$ and let γ be an *or* coordination translating as the sequence $\langle \gamma'_1, \dots, \gamma'_n \rangle$. Then the translation of α is the unordered sequence $\langle \beta'(\gamma'_1), \dots, \beta'(\gamma'_n) \rangle$.

Similarly: Let the translation of β be β'_β , and let γ be an *or* coordination translating as the sequence $\langle \gamma'_{1\langle \beta, \alpha \rangle}, \dots, \gamma'_{n\langle \beta, \alpha \rangle} \rangle$. Then the translation of α is the unordered sequence $\langle \gamma'_1(\beta'), \dots, \gamma'_n(\beta') \rangle$.

We thus have the following (where the arrows mean “translates as”):



We have yet to see how to interpret an unembedded S which denotes a set of propositions i.e. how to assign truth conditions to such an S. We will return to this question when we turn to the derivation of disjunctive readings. For now, we are still concerned with the cases in which such expressions occur embedded under a modal.

Now, the modal/*or* sentences we are considering here clearly show that we cannot pursue this strategy of independent composition ‘all the way up’.¹⁹ Another case where independent composition

18. This rule is optional because, as we have already seen, in some cases a head (such as a modal) will combine directly with the set denotation of an *or* coordination.

19. Rooth and Partee (1982) recognize the limitations of their original account, and offer an alternative semantics for *or*, at least for the cases in which *or* occurs in the scope of an intensional verb such as *seek*.

must be halted somewhere on the way up is where a phrasal *or* coordination occurs under a quantificational DP subject, as in:

(50) Every guest got drunk or overate.

The usual interpretation of this sentence is that represented by the first order formula in (51), not the one in (52):

(51) $\forall x. \text{guest}(x) \Rightarrow (\text{g.d.}(x) \vee \text{o.a.}(x))$

(52) $\forall x. \text{guest}(x) \Rightarrow \text{g.d.}(x) \vee \forall x. \text{guest}(x) \rightarrow \text{o.a.}(x)$ ²⁰

But note that the following are equivalent:

(53) Every guest drank wine or beer.

(54) Every guest drank wine or drank beer.

The examples suggest that the presence of an operator – perhaps simply of a quantifier – can, and in some cases perhaps must, put a halt to the independent composition.

Winter (1995), in his account of *and* coordination, also suggests that independent composition (which he calls *pointwise composition*) is an available option for the composition of a head with a set (or tuple) argument. As noted in footnote 16, this allows him to account for the scope effects observed in sentences such as (22) above. Such an account is totally in keeping with the proposal made here.

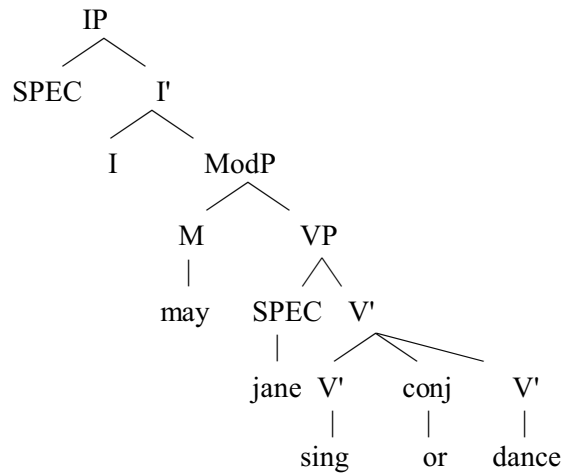
Let's return now to the current account, and to the NS readings of narrow *or* sentences like (3):

(3) Jane may sing or dance.

We still assume that underlyingly and/or at LF, the modal verb *may* sits in a position which has scope over the residue of the sentence. We now assume, though, that *or* coordinates only the V-bars *sing* and *dance*, i.e. we have:

20. This disjunctive interpretation may be available if intonation and context are right. But it seems highly dispreferred. The observation that there is no general equivalence between sentences with QDP subjects such as *Every guest ate or drank* and the parallel clausal disjunction *Every guest ate or every guest drank* provided some of the initial evidence against the early syntactic reduction accounts of phrasal *or* coordination.

(55)



We assume that the complex V' denotes the set of the denotations of its V' daughters, and that these compose independently with the subject. For reasons yet to be clarified, the presence of the modal halts this independent composition, and the modal takes the set as its argument, giving us the truth conditions we have been assuming.

The advantage of this proposal is that it doesn't matter how deeply embedded the phrasal *or* coordination is. Consider sentence (33) from above:

(33) Jane may visit Henry or Matilda.

What looks like an NP (DP) coordination can be treated as such. It is the process of semantic composition which gives us the clausal 'disjuncts' i.e. the pair of propositions which, by the proposed truth conditions, are required to be a supercover of a subset of ACC_d . The only syntactic assumption now required is that the rather standard one, that the modal dominates the residue at LF.

Note that this proposal also allows us to deal with the case in which the *or* coordination occurs in the subject, as in:

(56) Jane or Henry may visit Matilda.

We can assume that this has the underlying form:

(57) $[_{XP} [_{MP} \text{ may } [_{VP} [_{DP} \text{ Jane or Henry }] \text{ visit Matilda }]]]$

The verb and its object compose as usual. Each member of the denotation of the subject DP then composes independently with the predicate, giving us the equivalent of $\{[Jane \text{ visit Matilda}], [Henry \text{ visit Matilda}]\}$ as the argument of the modal.

These assumptions about the derivation require one technical modification to the formulation of truth conditions given so far. We are not, in fact, providing truth conditions for *English* sentences (or LFs of English sentences) of the form $MODAL[\phi \text{ or } \psi]$, but rather for expressions of the translation language of the form $MODAL[\langle \phi, \psi \rangle]$. What we have now seen is that sentences containing phrasal *or* coordinations may have translations of this form.

5. Disjunctive Readings

5.1. Disjunctive truth conditions

Consider a simple clausal *or* coordination such as (58):

(58) John sang or Jane danced.

By the current proposal, this sentence denotes a set of two propositions, i.e. a set of two sets of possible worlds. How do we assign truth conditions to this object?

Standardly, we take a sentence to denote a set of possible worlds, and take the sentence to be true at a world w just in case w is a member of the set. By the standard treatment of *or*, we would take sentence (58) to be true at w just in case w was a member of the union of the two disjoint propositions. What we want is a formulation of truth conditions which is essentially equivalent to this standard treatment but which also utilizes the idea that an *or* coordination serves to divide up a given set.

Recall that the truth conditions for modal/*or* sentences require the existence of a set which has two properties: it is related in a specified way to some other semantic object; and it is supercovered by the denotation of the (embedded) *or* coordination. Let us suppose that sentences containing *or* coordinations always have truth conditions of this form. We can achieve the intuitively correct results for (58) by assigning it the following truth conditions:

(59) $\exists S. w^* \in S \ \& \ \{[\text{John sang}], [\text{Jane danced}]\}$ is a supercover of S

where w^* is the world of evaluation.

According to these truth conditions, w^* is a member of a set S which is supercovered by the disjuncts. As every member of S must be a member of (at least) one member of its supercover, then w^* must be either a world at which John sang or a world at which Jane danced, as required.²¹

Returning now to the disjunctive reading of modal/*or* sentences, one case is now very simple: the disjunctive reading of wide *or* sentences like (1) and (2):

- (1) Jane may sing or she may dance.
 (2) Jane must sing or she must dance.

21. There is one case which is problematic for these truth conditions, which is the case in which one disjunct is necessarily false. (Thanks to Graeme Forbes for this observation.) E.g.

(i) Jane danced or $2+2=5$.

On the assumption that “ $2+2=5$ ” is true at no possible worlds, then $\{[\text{Jane danced}], [2+2=5]\}$ cannot be a supercover of *any* set, as a supercover cannot have the empty set as a member. (See p.6.) Nonetheless, there is an intuition that sentence (i) would be true if it were true that Jane danced. I am not entirely sure what to say about such cases, but it does appear to me that there are two different sub-cases to consider. One is the case in which (i), or something like it, is uttered seriously i.e. the speaker appears to be unaware of the necessary falsity of the second disjunct. The second case is where the speaker *uses* the obvious falsity of the second disjunct to assert the first. Other related cases are ‘monkey’s uncle’ disjunctions, like:

(ii) Either Jane is in love, or I’m a monkey’s uncle.

In the latter case, there is an intentional violation of some kind, which the hearer is intended to recognize and from which she is supposed to draw conclusions as to the communicative intent of the speaker.

Here we have a clausal *or* coordination, so we can simply plug these disjuncts into the truth conditions in (59), which gives us, for (2):

(60) $\exists S. w^* \in S \ \& \ \{[Jane \text{ must sing}], [Jane \text{ must dance}]\}$ is a supercover of S

This is true just in case, at the world of evaluation, Jane has at least one of the obligations specified.

5.2. Disjunctive readings of narrow *or* sentences

In the case of the disjunctive reading of a narrow *or* sentence like (61), we again have a mismatch between surface syntax and the interpretation:

(61) Jane may dance the waltz or the rhumba.

Recall that to account for the non-disjunctive, NS readings of sentences like (61), I proposed that the set which is the semantic value of the *or* coordination propagates itself up the tree. To derive the NS reading, however, we have to halt this process of independent composition when we reach the modal. (Recall the assumption that at LF, the surface subject is under the modal.) If we were to keep going all the way up, we would wind up, at the top level, with a set whose members are the denotations of *Jane may dance the waltz* and *Jane may dance the rhumba*.

Of course, that is exactly what we want in order to derive the disjunctive reading of (61). So, the proposal is that the disjunctive reading of narrow *or* sentences is derived by applying this process of independent composition all the way up. But that means that we need some way to distinguish the two cases: why does the modal sometimes, but not always, put a halt to this process?

A tempting idea is that this has something to do with the position of *either*. As Schwarz (1999) observes, there is a long-standing view that *either*, when it occurs, marks the left edge of a disjunction. Larson (1985) points out that the situation is not straightforward, but also argues that *either* has a crucial role in determining the interpretation of an *or* coordination. Given the account offered here, it seems plausible that the function of *either* is to close off the independent composition process. (In the absence of *either*, we might follow Larson in positing a null operator which serves the same function.) Thus, if *either* occurs above the VP, but below the modal, we get the NS reading; if *either* occurs above the modal, we get the disjunctive reading. But the data do not, to my mind, bear this out convincingly. Consider:

(62) Jane either may dance the waltz or dance the rhumba.

(63) Jane may either dance the waltz or the rhumba.

I find that both sentences have both the NS reading and the disjunctive reading. Both may naturally be followed up by “...*whichever she prefers*” (indicating the NS reading) and by “...*but I don’t know which*” (indicating the disjunctive reading).²²

There is a further issue. *Either* can also occur adjacent to the *or* coordination, as in (64).

(64) Jane may dance either the waltz or the rhumba.

22. This observation does not count against the specific claims made by Larson or Schwarz, but only against the suggestion that *either* closes off independent composition as proposed here.

But it is not clear what it would mean to halt the independent composition at the level of the coordination itself, as it is not clear how else the transitive verb *could* combine with the argument.

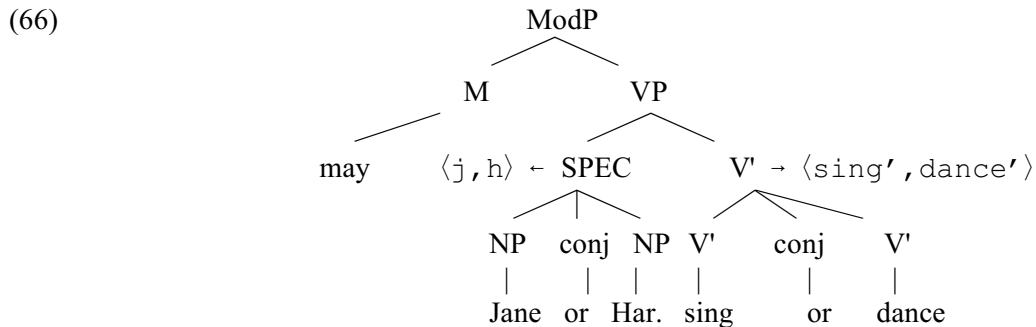
For now, I will leave open the possible role of *either* in determining the interpretation, and tentatively suggest the following alternative: we simply have in these cases a choice. Independent composition can halt at any node where there is an alternative option for composition i.e. whenever composition arrives at a head which can combine with a set argument directly. The preference for NS readings of modal/*or* sentences may be a consequence of the fact that NS readings are derived by halting independent composition at the first opportunity.

5.2.1. Multiple *or* coordinations

A further question which arises is the composition strategy for sentences containing multiple *or* coordinations, such as:

(65) Jane or Harriet may sing or dance.²³

By our current assumptions, this has the underlying form shown in (66), with the lower nodes translated as shown:



We must now provide a rule for the composition of two nodes each of which is translated as a sequence (and denotes a set whose members are the denotations of the elements in the sequence).

One obvious possibility is to allow all possible pointwise combinations of the elements in the two sequences, and to form a new sequence of the results. This would give for the VP denotation:

(67) $VP \rightarrow \langle \text{sing}'(j), \text{dance}'(j), \text{sing}'(h), \text{dance}'(h) \rangle$

Composition can now proceed using either of the two strategies identified: pointwise composition of the modal *may* with each element of the sequence; or application of the sequence as argument to the modal. The first of these strategies would result in a four-way disjunctive reading, paraphrasable as in (68):

(68) Either Jane has permission to sing or Jane has permission to dance or Harriet has permission to sing or Harriet has permission to dance (but I don't know which).

The second strategy would result in a four way NS reading, paraphrasable as in (69):

23. Thanks to Robert May for raising this problem.

- (69) Any of the following (but not necessarily any combinations of these) are permissible: Jane singing, Jane dancing, Harriet singing, Harriet dancing.

Both of these readings indeed seem to be available for sentence (65). But the sentence also seems to have an *additional* two readings. Paraphrases of these additional readings are given in (70) and (71).

- (70) Jane or Harriet (I don't know which) has permission to sing or dance (whichever she chooses.)
 (71) Jane or Harriet (whichever) either has permission to sing or has permission to dance (I don't know which).

Let's call (70) the *subject disjunctive* reading; and (71) the *predicate disjunctive* reading. Obviously, the composition strategies posited so far do not generate either of these readings.

Under current assumptions, the form of the truth conditions for the subject disjunctive reading is as follows:

- (72) $\exists S. w^* \in S \ \& \ \{[MAY \langle \text{sing}'(j), \text{dance}'(j) \rangle], [MAY \langle \text{sing}'(h), \text{dance}'(h) \rangle]\}$ is a supercover of S.

This says that w^* (the world of evaluation) is a member of a set which is supercovered by a set whose members are:

- a. the set of worlds in which Jane has permission to sing and permission to dance and
 - b. the set of worlds in which Harriet has permission to sing and permission to dance.
- i.e. that w^* is a world at which at least one of these states of affairs holds.

But how can we compose the constituents of our sentence in such a way that the modal combines independently with subsets of the possible pointwise combinations of the elements of the two *or* coordinations? I propose as follows. Suppose we allow the elements of the subject sequence $\langle j, h \rangle$ to each 'saturate' the predicate sequence separately, with each pair of combinations forming a separate sequence. That is, the elements of the sequence $\langle \text{sing}', \text{dance}' \rangle$ each compose with, let's say, j . Once the predicate sequence is saturated (i.e. all of its members have combined with an argument) the new sequence thus formed is closed off. However, the subject sequence still has a member which has not entered into the composition. So the same process is repeated, this time with h . The result is a pair of items which are not in any logical relation, and hence must be treated as a sequence: this time, a sequence of sequences, as shown in (73):

- (73) $\langle \langle \text{sing}'(j), \text{dance}'(j) \rangle, \langle \text{sing}'(h), \text{dance}'(h) \rangle \rangle$

Now, we have seen that when the argument of a modal is a sequence of proposition-translations (or proposition denotations, in a system of direct interpretation), the modal may either combine pointwise with these elements, or compose directly with the sequence as a whole. However, in this case, the argument of the modal will be a sequence of *sequences*. Consequently, it is plausible to assume that the only composition option for the modal is pointwise composition. This will give us:

- (74) $\langle MAY \langle \text{sing}'(j), \text{dance}'(j) \rangle, MAY \langle \text{sing}'(h), \text{dance}'(h) \rangle \rangle$

This expression will give us exactly the truth conditions in (72).²⁴

To generate the predicate disjunctive reading, we need to do the same thing in the opposite direction, i.e. to allow the saturation of the subject *or* coordination to trigger closing off of the sequence. A simple way to do this is to lift the type of the translations of the proper names, so that it is the elements of the subject *or* coordination which are in need of arguments. So we begin with the pair of sequences:

- (75) a. $\langle \lambda P.P(j), \lambda P.P(h) \rangle$
 b. $\langle \text{sing}', \text{dance}' \rangle$

We first saturate the subject sequence with, let's say, sing' , and then close off the resulting sequence. Then we generate a new sequence by saturating the subject sequence with dance' . The result is this sequence of sequences:

- (76) $\langle \langle \text{sing}'(j), \text{sing}'(h) \rangle, \langle \text{dance}'(j), \text{dance}'(h) \rangle \rangle$

This sequence will combine with the modal in the manner described above. The result will be an expression which has the truth conditions in (77):

- (77) $\exists S. w^* \in S \ \& \ \{ [\text{MAY} \langle \text{sing}'(j), \text{sing}'(h) \rangle], [\text{MAY} \langle \text{dance}'(j), \text{dance}'(h) \rangle] \}$ is a supercover of S .

Apparently, then, interpretation of sentence (65) involves considerable complications. This is perhaps borne out by the fact that it is surprisingly hard to make sense of this sentence on a first encounter – surprisingly, as its surface form does not suggest much complexity. The readings are not easy to isolate. However, I do sense a distinction between the readings discussed above and the following:

- (78) Either Jane has permission to sing and Harriet has permission to dance or else Harriet has permission to sing and Jane has permission to dance – I don't know which.
 (79) Either Jane has permission to sing or else Jane has permission to dance and Harriet has permission to sing and Harriet has permission to dance.

(78) and (79) are *not* possible readings for sentence (65). These would involve alternative permutations of the propositions generated by pointwise combination of the original sequences. That these readings (and other imaginable ones) are not available provides some support for the composition options proposed. Further investigation of the composition possibilities given multiple disjunctions is needed, but I will not pursue this issue further here.²⁵

24. In (72), we are assuming that the modal *inside* the members of the supercover combines directly with its sequence argument, so that the predicate *or* coordination is read non-disjunctively. If the modal combines pointwise with the elements of the sequence argument, then (72) comes out equivalent to the four-way disjunctive reading.

25. From an initial and superficial look at the data, it appears that the usual strategy for combining multiple disjunctions is the initial one proposed here, i.e. construction of a single sequence produced by carrying out all of the available pointwise combinations. For example, sentence (i) has the interpretation in (ii), and (iii), the

5.3. Why epistemic/*or* sentences lack disjunctive readings

5.3.1. *Genuineness*

We observed in the introductory section that modal/*or* sentences whose modal is interpreted epistemically lack a disjunctive reading. Zimmerman (2000) relates this fact to another observation: that the disjuncts of an ordinary unembedded disjunction are usually understood to be epistemic possibilities for the speaker. Although my account of the meaning of *or* is entirely different from Zimmerman's, I think that he is right that these two properties are related.

For Zimmerman, the requirement that disjuncts be epistemic possibilities for the speaker (what he calls the *Genuineness requirement*) is a semantic one. I consider this requirement to be pragmatic in nature, as it is evidently cancellable under certain circumstances. Grice (1989: 45) offers the following example:

I can say to my children at some stage in a treasure hunt, *The prize is either in the garden or in the attic. I know that because I know where I put it, but I'm not going to tell you.* Or I could just say (in the same situation) *The prize is either in the garden or in the attic*, and the situation would be sufficient to appraise the children of the fact that my reason for accepting the disjunction is that I know a particular disjunct to be true [and therefore the other is not an epistemic possibility].²⁶

I propose that we capture the *Genuineness* requirement as a 'pragmatic add-on' to the truth conditions for an unembedded *or* coordination. The truth conditions we have arrived at for a sentence of form ϕ *or* ψ are these:

(80) $\exists S.w^* \in S$ & $\{[\phi], [\psi]\}$ is a supercover of S .

The pragmatic add-on requires the speaker to limit herself to invoking worlds which are (to begin with) epistemic possibilities for her. This requirement is related to the standard requirement to try to say things which are true, to make assertions which are consistent with the context set, and so on.

interpretation in (iv).

- (i) She sang or danced expertly or confidently.
- (ii) Either she sang expertly or she sang confidently or she danced expertly or she danced confidently.
- (iii) There was a red or blue flag at the first or second window.
- (iv) There was a red flag at the first window or there was a red flag at the second window or there was a blue flag at the first window or there was a blue flag at the second window.

Presumably, it is the presence of the modal operator in sentence (65) which gives rise to the multiplication of possible readings.

26. Zimmerman acknowledges that his semantic treatment of *Genuineness* requires him to "classify these cases as abnormal utterances" (fn.24). He goes on to suggest that examples like these "all involve some form of pretense, which suggests that they may be analyzed as referring to a hypothetical or fictional epistemic background." But I think Grice has been careful, in the construction of his example, to eliminate any appearance of pretense; the whole point is that the children know that he knows where the prize is.

We can formulate this requirement as a constraint that the set S satisfying the truth conditions must be a subset of the speaker's epistemic set. Let's incorporate the pragmatic add-on into the truth conditions, putting it in parentheses to show that it is not strictly related to truth:

$$(81) \quad \exists S.w^* \in S \ \& \ \{[\phi], [\psi]\} \text{ is a supercover of } S \ (\& S \subseteq ACC_e)$$

Adopting Zimmerman's terminology, let's call the pragmatic add-on the *Genuineness constraint*. The effect of the constraint is that a speaker who utters *A or B* can be assumed to take both *A* and *B* to be possibly true.

5.3.2. *Epistemic modals and Genuineness*

Suppose we have a wide *or* sentence of the form:

$$(82) \quad \text{MIGHT}[\phi] \text{ or } \text{MIGHT}[\psi]$$

Consider the case in which there is no ATB raising of the modal, and so the disjunctive interpretation is expected. The truth conditions which are expected to give rise to this reading are these:

$$(83) \quad \exists S.w^* \in S \ \& \ \{\text{MIGHT}[\phi], \text{MIGHT}[\psi]\} \text{ is a supercover of } S$$

But given the discussion above, we must add the Genuineness condition, so the full requirement is this:

$$(84) \quad \exists S.w^* \in S \ \& \ \{\text{MIGHT}[\phi], \text{MIGHT}[\psi]\} \text{ is a supercover of } S \ (\& S \subseteq ACC_e)$$

The truth conditional part of this statement requires that w^* (the world of evaluation) be either a world at which ϕ is an epistemic possibility or a world at which ψ is an epistemic possibility. But the pragmatic add-on has the consequence that ACC_e must contain at least one world of each sort. So we have that:

- a. It is an epistemic possibility for S that ϕ is an epistemic possibility for S and
- b. It is an epistemic possibility for S that ψ is an epistemic possibility for S

We can assume further that what is possibly possible is possible (i.e. that $\diamond \diamond p \Rightarrow \diamond p$). Then from a. and b. above we get that:

- c. ϕ is an epistemic possibility for S and
- d. ψ is an epistemic possibility for S

which we can also express as:

$$(85) \quad \text{MIGHT}[\phi] \text{ and } \text{MIGHT}[\psi]$$

i.e. given the pragmatic add-on, the disjunctive reading of an epistemic modal/ *or* sentence reduces to the NS reading.

We noted above that in a context in which epistemic *might* is interpreted in relation to the epistemic state of an agent A distinct from the speaker, a disjunctive reading is available. This is because in such a case the truth conditions plus the pragmatic add-on give us:

- a. It is an epistemic possibility for S that ϕ is an epistemic possibility for A and
- b. It is an epistemic possibility for S that ψ is an epistemic possibility for A

Nothing here allows us to draw an inference equivalent to the NS reading.

6. Further pragmatic constraints

There is an aspect of the interpretation of *or* which is not captured by the analysis given so far. This is the intuition that the expressions conjoined by *or* must be interpretable as alternatives to one another. When this is not possible, the sentence is judged anomalous in some way. But the nature of the anomaly is not always clear. In some cases, the violation seems clearly pragmatic; in others, there is a sense that the violation leads to falsity. Judgments vary from speaker to speaker, and in some cases (like mine) from one occasion to another. I'll begin by looking at the cases which I am pretty confident involve only a pragmatic, and not a truth conditional, violation. Then I'll turn to the more problematic cases.

6.1. Entailment and Epistemic entailment

a. *Entailment*

One very clear case in which we have a failure of alternativeness is where one disjunct entails another. "Entailment" here includes any case in which the interpretation of one disjunct is a subset of the interpretation of the other. We can have both unembedded and embedded occurrences, as exemplified in:

- (86) ?? Jane sang an aria or she sang.
- (87) ?? Jane owns a red truck or a truck.
- (88) ?? Jane may/must wear a red dress or dress.
- (89) ? Jane owns a red truck or a truck or a station wagon.

Sentences (86)-(88) are strongly anomalous, but none of the people I have asked seem willing to judge them false as long as Jane sang, owns a truck, or is permitted (required) to wear a dress, respectively. Most people, however, don't want to give a truth value judgment at all. These are just downright weird. In sentence (89), I have added a third disjunct which is not logically related to either of the others. This addition seems to improve the acceptability of the sentence for some speakers. Let's call examples like these "entailment plus" examples.

b. "*Epistemic entailment*"

These cases are a variant on the entailment cases. In these, no disjunct logically entails any other. But one disjunct combined with some background assumption does entail another i.e. $\llbracket d_1 \rrbracket \cap ACC_e \subseteq \llbracket d_2 \rrbracket \cap ACC_e$. Again, we can have both unembedded and embedded variants, and we can also have "epistemic entailment plus". So, taking it as given that all graduate students are chronically stressed, consider:

- (90) ? Either Henry is a graduate student or he’s chronically stressed.
- (91) ? Either Henry is a graduate student or he’s chronically stressed or he’s upset that the Steelers lost.
- (92) ? Henry might/must be a graduate student, or be chronically stressed.

These are anomalous too, but not as bad as the logical entailment cases. The less salient the background assumption which results in entailment, the better the cases seem. Moreover, even where the background assumption is salient, there also seems to be a strong tendency to interpret the ‘entailed’ disjunct as excluding the ‘entailer’ i.e. to interpret (90) as “Either Henry is a graduate student or he is chronically stressed for some other reason.” Note that we could do the same thing with the logical entailment cases above (e.g. interpret (86) as “Jane sang an aria or she sang something other than an aria”) but there seems to be less of a predilection to do so.

I take it that none of these examples require us to modify the truth conditions; we have here a pragmatic violation, albeit a strong one. However, our formulation of truth conditions in terms of finding a set of which the *or* coordination is a supercover gives us a natural way of explaining, or at least characterizing, this violation.

The original intuition was that an *or* coordination functions to divide up some domain. The supercover formulation is intended to capture that. All of these anomalous cases are instances where the supercover condition can be met, but only vacuously i.e. without really dividing up the domain. Consider, for example, the simplest entailment case, sentence (86). Here are its predicted truth conditions:

- (93) $\exists S. w^* \in S \ \& \ \{[jane \ sang \ an \ aria], [jane \ sang]\}$ is a supercover of S

These conditions can easily be satisfied. Figure 1 below illustrates one possibility:

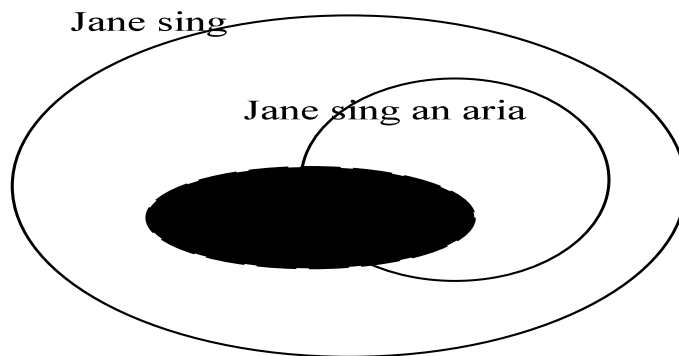


Figure 1

The *or* coordination is indeed a supercover of S: S has a non-empty intersection with each member of the supercover, and the union of the supercover is a superset of S. Nonetheless, the supercover fails to divide up the set in the way that we want it to, as S is included within one member of the supercover. And given the logical relation between the two members of the supercover, this is unavoidable. Any S which satisfies the existential in (93) will do so only in this unsatisfactory way.

What we can do to rule out such cases is to include another pragmatic add-on, this time in the definition of supercover. The add-on will ensure that the supercover really divides up the domain i.e. that the domain is not a subset of any member of the supercover. Let's call this additional constraint the *Non-containment constraint*. The supplemented definition of supercover is as follows:

(94) **Enhanced Supercover: (Non-Containment)**

A set of sets SC is an enhanced supercover of a set S iff:

- (i) Every member of S belongs to some member of SC.
- (ii) Every member of SC contains some member of S.
- (iii) S is not a subset of any member of SC.

Clause (iii) of the new definition places a constraint on the relation between S and SC. But we could also rule out examples like (86) by placing a constraint on the relation between the members of SC. While we want to allow these sets to overlap, we don't want to allow total overlap. Here is the alternative version:

(95) **Enhanced Supercover: (No Total Overlap)**

A set of sets SC is an enhanced supercover of a set S iff:

- (i) Every member of S belongs to some member of SC.
- (ii) Every member of SC contains some member of S.
- (iii) No member of SC is a subset of any other member.

The No Total Overlap constraint captures the same intuition formulated in Stalnaker's (1975) generalization: A disjunctive statement is appropriately made only in a context which allows any disjunct to be true without any other. NTO extends this generalization to cases where the disjuncts are non-clausal and cannot be evaluated independently for truth.²⁷

For the simple case of (86), either NTO or non-containment will do the trick. But for the "entailment plus" cases – where we add a disjunct logically unrelated to the others – we can find an S which satisfies the truth conditions without violating non-containment, as shown in Figure 2. The *or* coordination, however, still violates NTO. So, NTO is needed.

27. NTO also suffers from the same difficulty as Stalnaker's generalization, namely, that it predicts that *or* coordinations of the form *A or B or both* should be unacceptable. Assuming that the last disjunct is equivalent to *A and B*, it will of course be a subset of both of the other disjuncts. Stalnaker suggests that utterances of this form are allowable because, being obvious violations of the requirement, they cannot be misleading; and because the addition of the third disjunct serves to change the appropriateness conditions of the utterance. (See fn. 13). I am unsure what to say about these cases at this point. In earlier work, I resolved the problem of the apparent redundancy of *or both* disjuncts by arguing that all disjuncts in an *or* coordination are interpreted as exhaustive, and thus that *A or B or both* is interpreted as "Only A; or only B; or A and B". (See also Groenendijk and Stokhof 1984.) It may be that this idea of 'exhaustifying' the disjuncts should be incorporated into the current analysis.

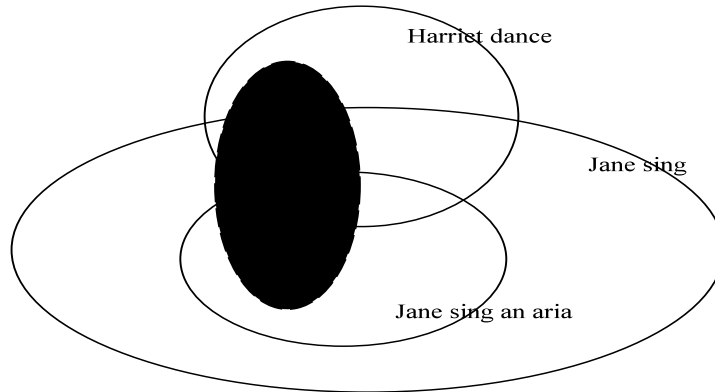


Figure 2

However, we can't simply opt for NTO and throw out non-containment. The cases of *epistemic entailment* satisfy NTO, but violate non-containment. So we must invoke non-containment to rule them out. However, to get any kind of violation at all in these cases, we also have to invoke the Genuineness condition introduced above. So, consider again example (90), repeated below:

(90) Either Henry is a graduate student or he's chronically stressed.

Neither disjunct is a subset of the other, so NTO is not violated. Moreover, we can find a set *S* which is non-vacuously supercovered by the *or* coordination, so Non-containment is also not violated, and the truth conditions can be satisfied without inducing a pragmatic violation. However, once we add the Genuineness constraint, which requires *S* to be a subset of ACC_e , we are forced back into a violation of non-containment. The full set of requirements we are trying to satisfy are these, with the pragmatic conditions given in boldface:

(96) $\exists S. w^* \in S$ & $\{[Henry \text{ is a graduate student}], [Henry \text{ is chronically stressed}]\}$ is a SC of *S*
 & $S \subseteq ACC_e$
 & $\forall X \in \{[Henry \text{ is a graduate student}], [Henry \text{ is chronically stressed}]\}, S \not\subseteq X$

Given the assumption that within ACC_e all worlds in which Henry is a graduate student are worlds in which he is chronically stressed, these conditions are not simultaneously satisfiable.

Two observations are now in order. First, we noted above that the cases of epistemic entailment seem less strongly anomalous than the cases involving logical entailment. Now we see why. The logical entailment cases violate two postulated constraints: NTO and non-containment. Moreover, there is no way to satisfy the truth conditions of these sentences without incurring a violation of these pragmatic constraints. In the cases involving epistemic entailment, only one of these constraints is violated. Moreover, it is not satisfaction of the truth conditions alone which results in this violation. The problem arises only when we also try to satisfy the additional pragmatic constraint of Genuineness. And if the hearer simply assumes that the speaker's epistemic background lacks the relevant assumption, the violation goes away.

So we have reached the following position: the alternativeness requirement is a pragmatic one, which can be captured by enhancing the definition of supercover with two constraints. These are No

Total Overlap (which constrains the relation among the members of SC), and Non-Containment (which constrains the relation between S and SC).

But there is one case that is not yet covered, the one we called “epistemic entailment plus”. Here is the example:

- (91) Either Henry is a graduate student or he’s chronically stressed or he’s upset that the Steelers lost.

The truth conditions of this sentence can be satisfied without violating any of the proposed pragmatic constraints, including Genuineness. Concomitantly, the anomaly involved here is relatively weak. However, there is still some sense that an alternativeness constraint has been violated. The intuition is that within the set of worlds we are considering (i.e. the epistemically accessible ones in which all graduate students are chronically stressed), the disjuncts do not all constitute true alternatives to each other. Here, then, we are forced to invoke a true alternativeness constraint to add to the definition of supercover:

(97) **Enhanced Supercover: (Alternativeness)**

A set of sets SC is an enhanced supercover of a set S iff:

- (i) Every member of S belongs to some member of SC.
- (ii) Every member of SC contains some member of S.
- (iii) $\forall X \in SC: \exists x \in X \cap S \text{ s.t. } \forall Y \in SC, Y \neq X: x \notin Y$

The new constraint says that for every member X of the supercover (i.e. each disjunct), there is some member of S which is in X but in no other member.

The Alternativeness constraint can, I think, be justified along the same lines as NTO and Non-Containment. It is a constraint which ensures that the supercover does real work in dividing up the chosen domain. Alternativeness entails both NTO and non-containment, so we might simply adopt the stronger constraint and set the others aside. However, I’m inclined to see them all as being in play. Cases which violate NTO are very blatant violations of Alternativeness, and consequently are strongly anomalous. Cases which satisfy NTO but violate Non-containment require a little more work for the failure of Alternativeness to be recognized, hence the weaker sense of anomaly. Cases which violate Alternativeness without violating either of the other two constraints seem harder to come by, and harder to recognize as violations, and hence are only weakly anomalous. While NTO and Non-containment are not independent constraints, they are ways of characterizing egregious violations of the fundamental Alternativeness requirement.

6.2. Modal-internal violations of Alternativeness

The final set of cases involve interaction between Alternativeness and modal truth conditions. What is particularly interesting about these is that for at least some speakers, Alternativeness seems to work its way into the truth conditions. To see this, let’s consider two sentences relative to specific situations.

Case 1

Jane is allowed (but not required) to eat chocolate. But if she eats chocolate, then she must brush her teeth. She isn’t otherwise required to brush her teeth, but may do so.

- (98) Jane may eat chocolate or brush her teeth.

- (99) $\exists S. S \subseteq ACC_d \ \& \ \{[Jane \text{ eat chocolate}], [Jane \text{ brush her teeth}]\}$ is a supercover of S.

Case 2

Jane is required to brush her teeth. In addition, she's permitted to eat chocolate.

(100) Jane must eat chocolate or brush her teeth.

(101) $\exists S. S = \text{ACC}_d \ \& \ \{[Jane \text{ eat chocolate}], [Jane \text{ brush her teeth}]\}$ is a supercover of S.

I am interested in the NS reading of each sentence in the situation described. The truth conditions which give rise to this reading accompany the sentences above.

In each case, the truth conditions are satisfiable in the given situation. But of course neither situation is the prototypical one we associate with the truth of the sentence. Hearing (98), we form the impression that eating chocolate and brushing her teeth are *alternative* allowable courses of action for Jane. Hearing (100), we form the impression that neither eating chocolate nor tooth-brushing are themselves required courses of action for Jane. The question is whether the failure to satisfy these impressions results in falsity or only in a pragmatic unacceptability.

Intuitions seem to vary here. I've had different responses from different informants, and my own intuitions are unstable. But there is certainly more of a tendency to consider these sentences false in the situations given than there is with the violations of alternativeness examined in the previous section.

Let's first see that we do indeed have violations of Alternativeness here. In fact, we have violations of Non-containment. In both of the cases given, all permissible chocolate-eating worlds are also tooth-brushing worlds. So any set which satisfies the truth conditions of either sentence will be a subset of the tooth-brushing worlds i.e. Non-containment is violated. These examples are parallel to the cases of epistemic entailment: while there is no logical entailment between the disjuncts, the domain from which S may be selected is one in which there is, in effect, such an entailment. The crucial difference between the cases lies in the source of this limit on the domain. In the cases of epistemic entailment, the constraint comes from Genuineness, a pragmatic constraint. In the modal cases, the constraint is semantic; it comes from the modal truth conditions.

For those speakers who find the sentences true but misleading ways of characterizing the situations given, we need say nothing new. The truth conditions of the sentences are satisfied, but a pragmatic constraint is violated in a semi-egregious fashion. For those speakers who find the sentences false under the circumstances described, I am inclined to say that they have semanticized Alternativeness. I would like to say further that this has something to do with the fact that Alternativeness applies, in some sense, under the scope of the modal. I do not yet see, however, how to make this idea precise. So I will leave it at that.

7. Conclusion

I have argued here for an analysis of *or* as a set-formation operator. This analysis necessitates a formulation of unfamiliar-looking truth conditions for sentences containing *or*, as such sentences denote, or have constituents which denote, unexpected sorts of semantic objects. The hope is that the solutions which this analysis offers for the problematic modal/*or* sentences are appealing enough to make this treatment of *or* seem worth pursuing. As additional motivation, I have tried to demonstrate that the analysis allows for a rather natural characterization of some observed pragmatic constraints on *or* coordinations.

8. Appendix: Adapting the account to Kratzer's modal semantics

8.1. A brief review of Kratzer's semantics for modals

In the Kratzer semantics (Kratzer 1977, 1981, 1991), a modal sentence is evaluated for truth relative to two functions from worlds to sets of worlds, which Kratzer calls *conversational backgrounds*.

A conversational background is the sort of entity denoted by phrases like *what the law provides*, *what we know* etc. Take the phrase *what the law provides*. What the law provides is different from one possible world to another. And what the law provides in a particular world is a set of propositions... The denotation of *what the law provides* will then be that function which assigns to every possible world the set of propositions p such that the law provides that p in that world. (1991: 641)

Thus, a conversational background is a function which, applied to a world w , will give back some set of propositions. Let us denote this function with f . $f(w)$ denotes the result of applying this function to w , i.e. denotes some set of propositions. Now, we assume that a proposition is – or can be represented by – a set of possible worlds. So a set of propositions is – or can be represented by – a set of sets of possible worlds. The intersection of such a set, i.e. $\cap f(w)$ is once a gain a set of possible worlds: namely, those worlds at which all the propositions in $f(w)$ are true.

A conversational background can thus do exactly the same work as an accessibility relation in a standard possible world modal semantics. We have:

$$\text{For all } w, w' \in W: wR_j w' \text{ iff } w' \in \cap f(w)$$

Following Kratzer, we call the conversational background which serves the same function as the accessibility relation the *modal base function*. For any world w , we call $f(w)$ the modal base, and $\cap f(w)$ the *modal base set* for w .

But Kratzer's semantics makes use of an additional conversational background, which she calls the *ordering source*, and represents with g . Like a modal base function, an ordering source is a function from worlds to sets of propositions. The function of this set of proposition is to order the worlds in the modal base. Following Lewis (1981), Kratzer defines an ordering \leq_A on a set of worlds W given a set of propositions A as follows:

Ordering

A world w is at least as close to the ideal represented by A as a world w' iff all propositions in A which are true in w' are also true in w .

$$\text{i.e. } w \leq_A w' \text{ iff } \{p: p \in A \ \& \ w' \in p\} \subseteq \{p: p \in A \ \& \ w \in p\}$$

It is important to note that this ordering relation is (possibly) partial. That is, given two worlds $w, w' \in W$ and a set of propositions A , it is possible that neither $w \leq_A w'$ nor $w' \leq_A w$.

The role of the ordering source in the Kratzer semantics is to further restrict the set of worlds which is relevant to determining the truth of a modal sentence. Roughly speaking, in Kratzer's semantics a sentence of the form MUST[ϕ] is true at w relative to a modal base function f and an ordering source g just in case ϕ is true at all those worlds in $\cap f(w)$ which are close enough to the ideal determined by $g(w)$. The question, then, is how to characterize what counts as "close enough."

How difficult it is to give this characterization depends upon whether or not one adopts the *Limit Assumption* (Lewis 1973). The Limit Assumption allows us to assume that in any set of worlds $S \subseteq W$

all of whose members are comparable relative to \leq_A , there is some world or worlds in that set which are at least as close to the ideal as any other world i.e. that given an ordering, we always have a (possibly unit) set of ‘closest worlds’.

Given the Limit Assumption, it is quite straightforward both to formulate the doubly relative truth conditions which Kratzer advocates, and to adapt the proposal in this paper to this doubly relative semantics. Without the Limit Assumption, the situation is more complicated. Kratzer, aiming for maximal generality, eschews the Limit Assumption, so in order to show that my proposal is fully compatible with her account, I must give a version of my truth conditions which does so too. I will do this in section 8.3, but I begin in 8.2. by giving a formulation of the truth conditions for the simpler case.

8.2. Given the Limit Assumption

Given the Limit Assumption, we can define what I will call the set of *g-closest worlds* relative to a world w , a modal base function f , and an ordering source g :

Definition: g-closest worlds

For any world w , modal base function f , and ordering source g , the set of $g_{f,w}$ -closest worlds is that set:

$$\{w' : w' \in \cap f(w) \ \& \ [\forall w^* \in \cap f(w). w' \not\prec w^*]\}$$

i.e. the set of worlds w' which dominate no other world in the modal base set. If a world w' does not dominate any other world then for every other world w^* , either $w' \leq_{g(w)} w^*$ or w' and w^* are incomparable.

It is the set of *g-closest worlds* which will be relevant for determining the truth of a modal sentence on this version of the semantics. We can now formulate Kratzer-style truth conditions for modal sentences using this notion.

$$(102) \quad \llbracket \text{MUST}[\phi] \rrbracket^{w,f,g} = 1 \text{ iff } \forall w' \in \text{g-closest worlds}, w' \in \llbracket \phi \rrbracket^{28}$$

$$(103) \quad \llbracket \text{MAY}[\phi] \rrbracket^{w,f,g} = 1 \text{ iff } \exists w' \in \text{g-closest worlds}, w' \in \llbracket \phi \rrbracket$$

These truth conditions differ from the formulation of truth conditions within a standard accessibility semantics for modals only in that we substitute reference to the set of accessible worlds ACC, with reference to the set of *g-closest worlds*. Exactly the same substitution in the truth conditions offered here for modal/*or* sentences will suffice to adapt these truth conditions to the Kratzer framework.

28. Here, take the superscripted denotation brackets $\llbracket \cdot \rrbracket^{w,f,g}$ to represent the function to the value of the argument at w (relative to f and g), and unsuperscripted brackets to represent the function to the proposition expressed by the argument.

8.3. Without the Limit Assumption

The following truth condition for $\text{MUST}[\phi]$ is given in Kratzer (1991):²⁹

$$(104) \quad [\text{MUST}[\phi]]^{w,f,g} = 1 \text{ iff } \forall u \in \cap f(w). \exists v \in \cap f(w). v \leq_{g(w)} u \ \& \ \forall z \in \cap f(w). z \leq_{g(w)} v \rightarrow z \in [\phi]$$

The complexity of the definition is due to the fact that Kratzer allows for the ordering to be both partial and infinite. It is easier to see what the definition says if we think of the partial ordering as a set of total orderings (any of which may be infinite). The ordering, recall, orders worlds in terms of how close they come to the ideal established by $g(w)$. The definition says that $\text{MUST}[\phi]$ is true just in case, in each of those total orderings, there is a point such that every world from that point on is a ϕ world.

Looking at definition (104), it is difficult to see how to formulate a similar truth condition for the case where the argument of the modal is an *or* coordination. To do this, we need to be able to talk about a set which will be supercovered by the denotation of the coordination. In order to do this, I will make use of a reformulation of (104) proposed by my colleague Horacio Arlo-Costa, to whom I am indebted for this proposal.

The reformulation makes crucial use of the idea just introduced, that we can characterize a partial ordering as the union of a set of total orderings. However, it is not sufficient for our purposes to consider any total ordering which is a sub-part of the partial ordering. We want to restrict our attention to total orderings which go ‘as far down’ as the partial ordering we are trying to characterize. In order to restrict ourselves to these orderings, we define the notion of the bottom of the ordering:

Definition: Bottom of the ordering (minimal worlds)

$$M_{\leq} = \{y \in D : \text{for all } x \in D. y \not\prec x\}$$

i.e. the set of all y in the domain of the ordering s.t. y dominates no x in the domain.

Note that if the ordering \leq contains a ‘bottom’ infinite regress (i.e. an infinite regress such that there are no worlds ‘below’ the regress), then M_{\leq} will be empty.

We now introduce the following theorem:³⁰

Ordering Theorem

For any ordering \leq , there is a set of total orderings $\leq_1, \leq_2, \dots, \leq_n$ such that $\leq = \leq_1 \cup \leq_2 \cup \dots \cup \leq_n$ where for all \leq_i :

- (i) If $M_{\leq_i} \neq \emptyset$, then $M_{\leq_i} \subseteq M_{\leq}$
- (ii) If $M_{\leq_i} = \emptyset$, then \leq_i contains a bottom infinite regress in \leq

Finally, let us introduce one more piece of notation:

29. What Kratzer actually gives is a definition for necessity (in varying degrees). I assume here that MUST expresses necessity (in whatever domain).

30. Arlo-Costa points out that a proof for this theorem has not been given, but that it seems reasonable.

Subordinate set

For any ordering \leq_i and world z in the domain of \leq_i , let:

$$z[\leq_i] = \{x: x \leq_i z\}$$

We can now use the idea of a partial ordering as a union of a set of total orderings to give a truth condition for *must* sentences which includes explicit reference to a set.

- (105) For any ordering source g , let $\leq_{g(w)1}, \dots, \leq_{g(w)n}$ be the set of total orderings whose union is equal to $\leq_{g(w)}$. Then for any world w , modal base function f and ordering source g :
- $$[[\text{MUST}[\phi]]]^{w,f,g} = 1 \text{ iff there is a } z_1 \dots, z_n \text{ in the domains of } \leq_{g(w)1}, \dots, \leq_{g(w)n} \text{ s.t.}$$
- $$z_1[\leq_{g(w)1}] \cup \dots \cup z_n[\leq_{g(w)n}] \subseteq [[\phi]].$$

This says exactly the same as Kratzer's original truth condition i.e. that $\text{MUST}[\phi]$ is true just in case, for each totally ordered subset of $\leq_{g(w)}$ which goes 'as far down' as $\leq_{g(w)}$, there is a point such that all worlds below that point are ϕ worlds. The difference in formulations, however, is that (105) includes explicit reference to the set of 'closest worlds'. This is the set which we need to formulate supercover truth conditions for the disjunctive case. We do this as follows:

- (106) For any ordering source g , let $\leq_{g(w)1}, \dots, \leq_{g(w)n}$ be the set of total orderings whose union is equal to $\leq_{g(w)}$. Then for any world w , modal base function f and ordering source g :
- $$[[\text{MUST}[\langle \phi, \psi \rangle]]]^{w,f,g} = 1 \text{ iff there is a } z_1 \dots, z_n \text{ in the domains of } \leq_{g(w)1}, \dots, \leq_{g(w)n} \text{ s.t.}$$
- $$\{[[\phi]], [[\psi]]\} \text{ is a supercover of } z_1[\leq_{g(w)1}] \cup \dots \cup z_n[\leq_{g(w)n}].$$

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