

Building DRT Lexical Entries for Superlatives and Ordinal Numbers

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Abstract. This paper is concerned with the construction of semantic representations of utterances involving superlatives and ordinal number phrases, using the framework of Discourse Representation Theory (DRT) (Kamp & Reyle 1993). I will describe the constructions of semantic representations, in the form of Discourse Representation Structures, for sentences involving superlatives and ordinals. An important part of this task is the specification of the relevant lexical entries for the superlative operator ‘-est’, and the ordinal expressions ‘first’, ‘second’, ‘third’, etc, as well as the contribution these entries make to the overall superlative and ordinal phrases (“the highest mountain”, “the first/second/third time”, “the 26th highest mountain”). A crucial aspect of both superlatives and ordinals is the presuppositions they trigger. Representations of these presuppositions will be built as part of the compositional process of DRS construction. As in van der Sandt (1992), presuppositions are resolved in context only after the preliminary representation in which they are explicitly represented has been constructed. However, I will modify van der Sandt’s resolution procedure by extending his notion of ‘contexts’. Essentially, the contexts we will work with is to include also information that has not been expressed in the antecedent discourse. Such contexts can be seen as a way of articulating the concept of common ground (Stalnaker 1974).

Keywords: presupposition, DRT, superlative, ordinal, lexical entry, context.

1. Preliminaries

1.1 DRT Treatment of Presuppositions as Anaphoric Expressions

Central to our discussion is the treatment of presupposition. The treatment I adopt takes as its point of departure the account developed by van der Sandt, which is best known through its presentation in van der Sandt (1992). Van der Sandt proposed that presuppositions be seen as closely analogous to anaphoric pronouns as treated in DRT: both anaphoric pronouns and presuppositions can be seen as conditions imposed on the context; If the context does not satisfy the condition and cannot be accommodated so that it comes to satisfy it, then the utterance which triggers the condition cannot be given an interpretation. On this view anaphoric pronouns and the presuppositions familiar from the presupposition literature (Karttunen 1979, Heim 1990) become sub-cases of a single phenomenon. These considerations led van der Sandt and Geurts to a general revision of the architecture of DRT, in which interpretation of a sentence S first yields a *Preliminary DRS*, where all presuppositional conditions imposed by S are explicitly represented. In a second step, these presuppositional conditions are then verified in the context and the context is accommodated to the extent this is wanted and within the scope of possibilities.

This revised architecture differs crucially from that which had been widely assumed as standard until that time (Kamp & Reyle, 1993). It is adopted in Kamp, Reyle and van Genabith (2008), and it is this version which I will take as point of departure.

According to this new architecture, the processing of multi-sentence text proceeds as follows. Processing of the first sentence S_1 from a text $\langle S_1, S_2, \dots, S_n \rangle$ yields a Preliminary DRS K'_1 . The presuppositional conditions represented in K'_1 are checked against the initial discourse

context K_0 , and if this is successful (either through binding with an antecedent or accommodation in K_0), the remaining, non-presuppositional part of K'_1 is merged with K_0 , leading to a new discourse context K_1 , which captures the contributions made by S_1 while also serving as the discourse context to the next sentence S_2 .

The first of the two stages, presupposition computation, is part of the syntax-semantic interface; while the second, presupposition resolution, is part of the semantic-pragmatic interface. There have been some attempts to account for the manifestations of these two interfaces (Kamp 2001), yet given the complexity of the task and the many different data that are to be accounted for, there are still many aspects of the theory needing articulation.

1.2 Box + λ Framework

Compositional semantics must rely on some syntactic theory for input. In this regard DRT assumes a simplified version of Montague's PTQ model, for a fragment of English sentences (Montague 1973). The basic syntactic rules adopted in PTQ generate the syntactic structures I will be using. These syntactic rules have much in common with CFG and the simple phrase structure rules of LFG¹.

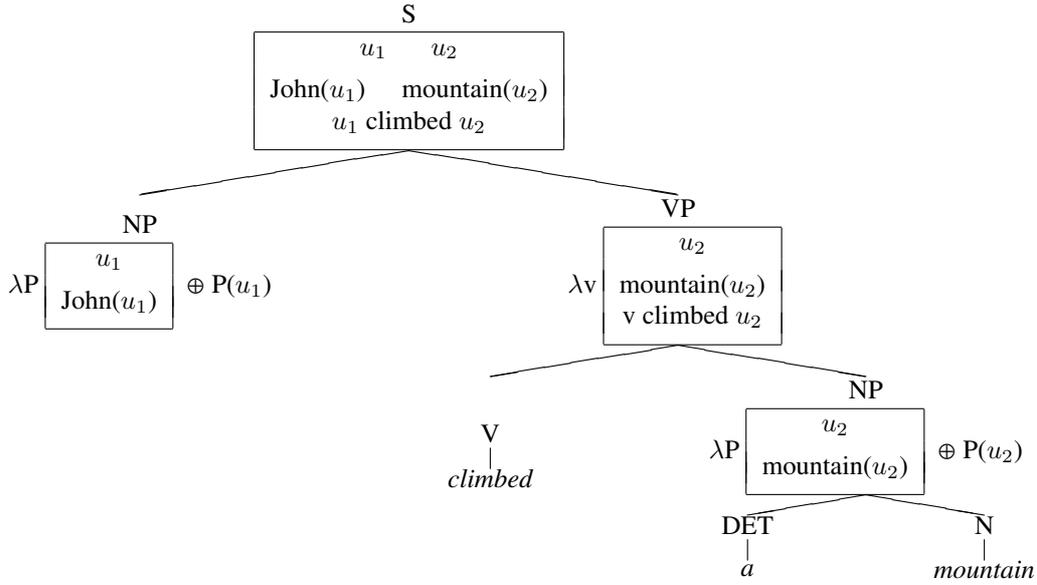
The semantic representations used in this paper are written in the notation of an extended version of DRT that incorporates presuppositions (Kamp/Reyle/van Genabith 2008). The semantic representations, DRSs, are constructed in a similar fashion as in Blackburn and Bos' 'Box + λ ' framework, which is based on Compositional DRT (Muskens 1996). This framework was successfully implemented by Blackburn and Bos in *Verbmobil* (Bos 1994). In this framework, each node on the syntactic structure is annotated with a semantic representation, a DRS. The leaf nodes are lexical entries, each represented by a lambda DRS, and all nodes except the root are intermediate stages of the semantic construction. The result of this construction is a Preliminary DRS. The semantic representation for the entire sentence is represented in the root node, and it is arrived at by unifying the lambda DRS's of lexical entries, and then through further unification of Preliminary DRSs of neighboring intermediate nodes on the given syntactic structure in a 'bottom-up' fashion. Some of the key attributes of this particular framework, which make it ideal for the current pursuit are: it is type-logical, transparent, bottom-up, compositional, and it allows for specification of lexical entries. I will refer to this framework in later sections as 'Box + λ '. The quickest way to illustrate how semantic representations are built from a syntactic structure in the Box + λ framework is probably with an example, take "John climbed a mountain"²:

¹ Lexical entries operate under a syntax-semantics interface. This paper will not be focusing on any particular syntactic theory, but rather try to include some of the necessary elements that belong to any plausible syntax-semantic interface. The syntactic analysis I assume will simply incorporate sets of syntactic properties of analyzed sentences which any viable theory of syntax should make available in some form.

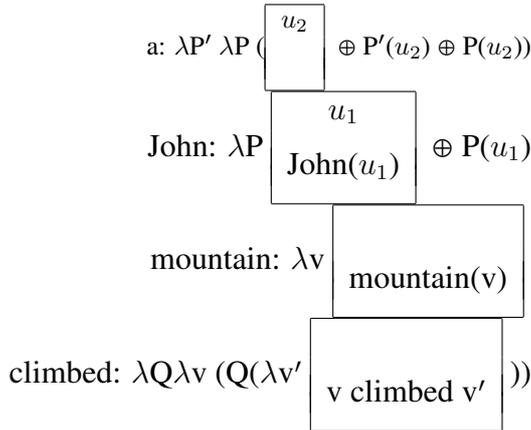
² \oplus is a merge function that merges two DRSs into one. Its definition is as follows:

Let K_1 and K_2 be DRSs, where $K_1 = \begin{array}{|c|} \hline x_1, \dots, x_n \\ \hline \gamma_1, \dots, \gamma_m \\ \hline \end{array}$ and $K_2 = \begin{array}{|c|} \hline y_1, \dots, y_k \\ \hline \mu_1, \dots, \mu_q \\ \hline \end{array}$, and if y_1, \dots, y_k do not occur in any $\gamma_1, \dots, \gamma_m$, then $K_1 \oplus K_2 = \begin{array}{|c|} \hline x_1, \dots, x_n, y_1, \dots, y_k \\ \hline \gamma_1, \dots, \gamma_m, \mu_1, \dots, \mu_q \\ \hline \end{array}$

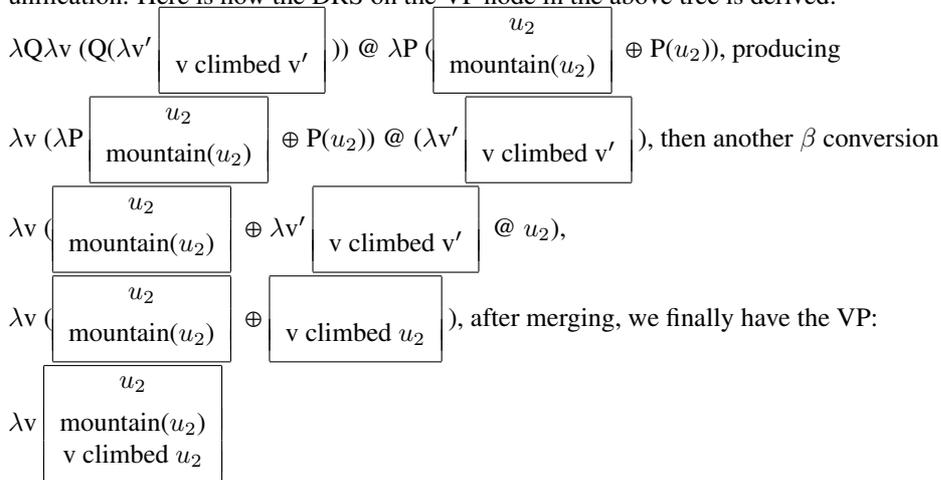
The application of \oplus can be found in Bos (1994) and Blackburn/Bos (1999), and so on. In Muskens (1996) it is written as ' \cdot ', the sequencing operator.



Given the following lexical entries³:



³ Let 'a@b' stand for 'a applied to b' (β conversion). Each time when two nodes on the tree unify, that is a case of β conversion. This is then followed by a merge \oplus . More than one β conversion may take place under a single unification. Here is how the DRS on the VP node in the above tree is derived:



2. The Semantic Representation of Superlatives

2.1 the Meaning of Superlatives and Ordinals

Each lexical entry should make the correct contribution to the DRS of an utterance where the subsequent word appears. This includes presuppositions that the word triggers and also the ways in which these presuppositions may be resolved. I will be looking at the interaction between ordinal and superlative words, as in the ‘ n_{th} highest mountain’ in (1):

- (1) a. Standing at 6962m, Aconcagua is the highest mountain in the Americas.
 b. (However,) it is (only) the 26 $_{th}$ highest mountain in the world.

In order to determine the truth condition of a simple clause containing a DP of the form ‘the n_{th} N’ (or ‘the n_{th} P_{est} N’), we need a contextually relevant *set of entities* that are extensions of N. This set must have a cardinality of *at least* n . We will also need a partial order \leq between the members of this set, such that the set is well ordered and linear. Under these premises, (1-a), as a case of superlative, is uttered against a background of a linearly ordered set of mountains, all of which are geographically located in the Americas. Here, a binary ordering relation (\leq) can be retrieved from the meaning of the superlative ‘highest’. This ordering by height is applied to the set of mountains in the Americas such that the highest mountain is singled out and identified as Aconcagua. As for (1-b), a case of superlative ordinal, it seems that not only there is a set of linearly ordered mountains being compared with regard to their height, but it is necessary that there are *at least* 26 mountains in the world in order for the utterance ‘the 26 $_{th}$ highest mountain in the world’ to be felicitous. How these 26 mountains are represented in our framework, and whether they are ‘presupposed’ in the traditional sense of e.g. van der Sandt (1992), is discussed in the following sections. For the moment, let us just assume that when a sentence with the clause ‘the n_{th} N’ (or ‘the n_{th} P_{est} N’) is true, it ‘presupposes’ a set of n many N’s, linearly ordered by \leq_P . Let ‘ $b \leq_{high} a$ ’ stand for ‘ a is at least as high as b ’, the semantic structure for (1-b) is illustrated as follows:

$$\dots \leq_{high} \text{mountain}_{27} \leq_{high} \text{mountain}_{26} \leq_{high} \text{mountain}_{25} \leq_{high} \dots$$

“Aconcagua”

2.2 Articulated Context

In this chapter I argue that the linearly ordered set of N’s ‘presupposed’ by the clause ‘the n_{th} N’ is *not* actually presuppositional, and also that an extension of the current theory of DRT is needed in order to account for the proper expression of the semantics of superlatives and ordinals. In addition, even though they are not presuppositions in the traditional sense, the DRT mechanism for treating presuppositions as anaphoric expressions may equally apply to them given an extended notion of context. Consider the following:

- (2) ? The third student’s name is Ralph.

It was said that a phrase of the form ‘the n_{th} N’ presupposes some ordering \leq_P on the extension of N, but what precisely is this ordering between the students in (2)? In addition, what are the contexts in which (2) can be felicitously uttered? There are many such contexts: a context where the interlocutors see a group of students waiting in line, or a context where a number of students arrived in the class at different times, or students each earning a different grade for their

exams, etc. Each of these contexts will result in a different \leq_P as well as a different subset of the extension of students where \leq_P is well founded. One must look for the precise meaning of \leq_P from the relevant context in which (2) occurs. My observation is that not only the discourse context, which represents the content of the preceding text/discourse, provides the contextual information needed to resolve the presuppositions carried by an utterance, but also our general knowledge about the material, social and cultural world, as well as *the particular context of the conversation of which the utterance is part*, all play important roles in presupposition resolution. A theory of presuppositions should take into account *all* contextually available resources, as opposed to previous theories that treat presuppositions as anaphoric expressions, such as van der Sandt (1992) and Kamp (2001), where the discourse context is the only place in which the antecedents for presupposition justification is sought.

As soon as we begin to utilize contextual resources beyond the discourse, many presuppositions previously considered as needing accommodation disappear. To give an example, it was observed in Vieira/Poesio (2000) that roughly 50 percent of definite uses in daily life are for the purpose of introducing new discourse referents, which under van der Sandt's theory calls for accommodation. This overwhelming proportion of discourse new uses of definites is rather unexpected because in his theory, accommodation is meant as a repair strategy that corrects a 'deficient' context- by adding the information that helps resolve the presupposition of the given description. According to this theory, accommodation is a last resort, which suggests that the tendency for anaphoric definite usages should be more prevalent than discourse new usages. An explanation for what seems an excessive proportion of discourse new occurrences in the data is that perhaps not all such definite descriptions should be considered candidates for genuine accommodation. For Heim (1982), the distinction between definites and indefinites is that definites presuppose *familiarity*, while indefinites do not. The familiarity presupposition is the presupposition that the referent is already familiar to the recipient and can be identified by him on the basis of some information he has at the point where he has to interpret the description. In other words, there is already information corresponding to the familiar definite description in the local context of interpretation. What is meant by 'context' here is characterized as a store of information held in common by the interlocutors, a version of Stalnaker's (1974) *common ground*. The notion of common ground includes not only information that is given by the text, but rather, as I have argued, *all* contextually available resources. To utter a familiar definite is to bring to attention something that already exists in the common ground but not in the immediate center of attention. Definite descriptions which are *completely unfamiliar* are the only ones that should rely on accommodation. This conclusion is reinforced by Vieira and Poesio's experimental data. Roughly 31% of the discourse new definite occurrences can be classified as references to entities in the common ground, or entities that can be easily identified with knowledge from the common ground- 7% of these are definite uses that are new in the text but can be deduced from the discourse through identifying head nouns (e.g. 'the cigarette filter', referring back to 'a filter'), or bridging descriptions based on simple reasoning with world knowledge and knowledge about the meaning of words (e.g. 'the morbidity rate' related to a prior 'total of deaths', 'the details of the spinoff' related to a prior 'documents'); About 24% of the cases of discourse new definite usage in the corpus refer directly to something in the common ground (e.g. 'the 1950s', 'the U.S.', 'the pope', what Hawkins (1978) would call 'larger situation' definites). Once we eliminate these 31% of the discourse new definite occurrences, we are left with a much more plausible portion of genuine accommodations. A referent is familiar because it is in the common ground, even though it might not be explicitly uttered in a prior discourse (the

case of *weak familiarity* according to Roberts (2003)). But what exactly constitutes *familiarity*? What does the common ground look like in our theory?

The notion of familiarity is explicitly spelled out in DRT terms by an *articulated context* (Kamp 2003). Articulated context is a quintuple of distinct but interacting components, each represented by a DRS:

$\langle K_{Dis}, K_{Gen}, K_{Env}, K_{Ut}, K_{Env} \rangle$, where

K_{Dis} is the discourse context. This is the traditional DRT notion of ‘context’. Everything that has been said so far in the given discourse gets stored in this context.

K_{Gen} is the general knowledge context. Similar to the TBox in AI, it is a store of general knowledge about the world. One can see it as a store of words and their ontological relations, much like that of WordNet, or more complex knowledge representations like OWL ontologies or FrameNet. The representation of K_{Gen} would be a DRS lacking discourse referents in its main universe, but only certain general knowledge in the form of either conditional or duplex conditions. An example of general knowledge would be “All humans are mortal”.

K_{Enc} is the encyclopedic context. Similar to the ABox in AI, this is a store of information about particular entities of various kinds: names, people, artifacts, places, events, etc. An example of an item in the encyclopedic knowledge would be the representation of a man named Socrates (which verifies the statement: “Socrates is a human”), together with some of his better known properties, such as that he was a citizen of Athens in the 5th century BC, that he was a philosopher, that his most famous student was Plato, etc.

K_{Ut} is the utterance context. It only contains elements that are associated with the given utterance, specifically: the speaker, the utterance time and the addressee (if there is one). Other elements such as the location where the utterance takes place could be included as needed.

K_{Env} is the environment context. This component contains information about the objects and events that are perceptually accessible to the interlocutors.

From the point of view of this paper, the task of constructing the semantic representation of a given discourse is the same as the task of building up the common ground in the form of an articulated context. Information only enters the common ground when all the participants of a discourse know and agree on it. Anything that is privy to one participant is not considered part of the common ground. New information gets added to the common ground through an utterance, either because it is directly asserted or as a presupposition that needs accommodation. My basic premise here is similar to that of Stalnaker (1998). Speakers tend to take lots of familiar facts for granted, and tend to assume that everyone already knows them when engaged in a conversation. For Stalnaker, information that defines the context can be represented as a set of possible worlds, known as the *context set*. My main goal is to spell out how common ground

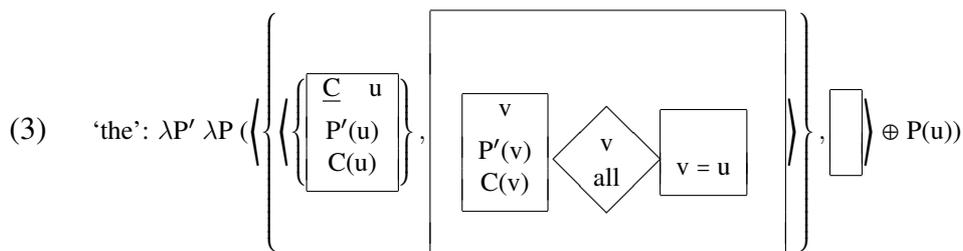
representations are built as a discourse unfolds. My focus will be on the contributions made by particular words and morphemes: the definite determiner, the superlative morpheme ‘-est’, and the ordinal words ‘first’, ‘second’, and so on.

2.3 the Definite Determiner

The ordinal expressions in all of the examples so far have been definite descriptions. Therefore the meaning contributions from the definite determiner ‘the’ cannot be ignored. I will discuss briefly two main features of the definite determiner, namely the existence and the uniqueness presuppositions triggered by definites.

It was observed by Frege that a singular definite description ‘the N’ has two preconditions associated to its referential function: *There is at least one N*- a presupposition of N’s non-emptiness; and *there is at most one N*- a presupposition that N is unique. These two presuppositions must be satisfied in order that the description can refer- otherwise the sentence containing the description will fail to have a proper truth value. According to van der Sandt (1992), ‘the’ is a presupposition trigger that instead of directly introducing the discourse referent, generates a presupposition that makes it behave much the same way as an anaphoric expression. It is obvious but important to note that in general uniqueness must be understood as uniqueness within some contextually restricted domain. Definite descriptions typically involve more descriptive content than pronouns, furthermore, the presuppositions connected with definite descriptions have the potential to accommodate in case a suitable antecedent cannot be found. According to van der Sandt (1992), the interpreter of a definite description should first seek to identify it with an antecedent in the existing context as it is available to him. The antecedent is first sought in the sub-DRS nearest to where the presupposition is triggered, and if a suitable antecedent is not found there, it is sought in the next nearest sub-DRS, and so on. If necessary, this process repeats all the way up to the global DRS until an antecedent is found. When an antecedent is found, the discourse referent of the definite and its antecedent are identified. This process is called *binding*. If there is no suitable antecedent in the context for binding, the presuppositional DRS triggered by the definite is added to the context. This second process is called *presupposition accommodation*. There are three main scope operations in which accommodation can take place- global, intermediate, and local. Global accommodation is the most preferred option- the case where the presuppositional DRS is added to the global DRS (this is also known as *presupposition projection* in earlier literature). If global accommodation would produce an inconsistency, intermediate accommodation is resorted to, and when that also fails, local accommodation remains as the last option. Intermediate accommodation is when the presuppositional DRS is added to the antecedent of a conditional, or the restrictor of a duplex condition, while local accommodation takes place in the consequent, or the nuclear scope.

A simplified lexical entry for the definite determiner ‘the’ is expedient for our current pursuit:



The above lexical entry consists of a Preliminary DRS with the two presuppositions enclosed within the curly brackets, one nested within the other. The outer curly bracket contains the DRS for the uniqueness presupposition on the right hand side. P' represents the descriptive content of the descriptions and P represents the descriptive content of the predicate to which the description is an argument phrase. Except for the context dependent predicate C , and the presuppositional status of the descriptive content of the description, the structure in (3) is reminiscent of Russell's proposal for the logical forms of sentences with definite descriptions (Russell 1905). For a sentence in which the description is the sentence subject, N is the predicate representing the description's descriptive content, while VP represents the content of the verb phrase, Russell proposes the following logical form:

$$\exists(x)\forall(y)(N(x) \wedge (N(y) \rightarrow x = y) \wedge VP(x))$$

Existence and uniqueness presupposition for singular descriptions must allow for contextual restriction (von Stechow 1994). In (3), this contextual restriction is represented in the form of the predicate C . The underlining of C indicates that C is anaphoric, and that the context must provide a suitable value for it. Like anaphoric pronouns, C is to be identified with some contextually salient entity/set of entities in such a way that the interpreter is able to see the contextualised existence and uniqueness presupposition as fulfilled. Exactly how this is done is discussed in the following section. For now, simply treat C as identified with the discourse universe of the context DRS. In a framework of articulated contexts, one of the context components must be selected before C can be identified with the universe of that particular context component.

Note that the entire presupposition structure in (3) (between the outer curly brackets) represents the existence and uniqueness presuppositions. It does so in a slightly unusual form in that the existence presupposition is 'hidden' in the subordinate presupposition (within the inner curly brackets), which is resolved by determining a value for the contextual restrictor referent C . The constraint on C is that there must be at least one thing falling under C which satisfies the overt descriptive content of the description. In other words, during the presupposition resolution phase there must be a referent in the selected context component DRS such that it satisfies P' . The uniqueness condition is expressed in the main presupposition, i.e. in the DRS on the right of the embedded presupposition concerning C . This guarantees that it will be the only entity satisfying both P' and C .

2.4 Superlatives

A definite NP is not only uniquely identified in one of the components of the articulated context. If and when identification succeeds, it must also be represented in K_{Dis} , for otherwise it could not be explained how pronouns occurring in subsequent utterances can refer back to the description once it has been introduced. This much is true for any singular definite descriptions. But descriptions which contain ordinal/superlative expressions are special in that these expressions contribute some further presuppositions: There is a unique set in the context called the *comparison set* (Bos/Nissim 2006). This comparison set consists of entities in the extensions of the common noun phrase to which the superlative is left-adjoined. Furthermore the comparison set must be ordered (for simplicity at this point, we will assume that the ordering is linear) by some binary relation (e.g. height). This relation- the so called *comparison dimension*- is generally determined by context. However, in cases where the ordinal NP also contains a superlative, as in 'the 26th highest mountain', the order is identified with the comparison dimension of the

adjective, e.g. ‘high’. For ‘the n-th wealthiest person in the world’, the members of the comparison set are all the people in the world, as stated by the utterance, and the comparison dimension is the level of their wealth; ‘the n-th most talented child’ compares a set of children in a salient context with respect to their talent. For (1-a), the set of mountains in the Americas would constitute the comparison set for the second sentence, and the set of mountains around the world would constitute the comparison set for (1-b), while the comparison dimension is height.

There are primarily three types of expressions that concern us- Superlatives, Ordinals (‘the third student’ as in (2)), and Ordinal Superlatives (‘the third highest mountain’). The distinction between pure ordinal expressions and ordinals used in combination with a superlative is that pure ordinal expressions require the interpreter to recover the comparison dimension from context, while ordinal superlatives make this explicit. This difference is important during the presupposition resolution phase. Throughout this article I will refer to these expressions as **S-NP** (for Superlative expressions), **OS-NP** (Ordinal Superlatives), and **O-NP** (Ordinals). These terms are not to be confused with the use I will make of the word *superlative*, which only refers to the superlative adjective, and *ordinal*, which refers to the ordinal adjective.

I argue here that the semantic representations of O-NP and OS-NP are based on the semantics of S-NP, and an investigation of the former should begin with the latter. Intuitively, S-NP’s should have the same semantic content as the very first of the OS-NP. ‘*The first highest mountain’ has the same meaning as ‘the highest mountain’, the ‘*first earliest time’ is the same as ‘the first time’ (as entailed by the meaning of the word ‘time’), and so on. Furthermore, ordinals in general can be used to modify a S-NP, as in (1-b). In (1-b), the superlative merely makes clear what the comparison dimension is, but it is the ordinal that specifies the relative placement of the referred mountain in the list of mountains around the world. Another feature that O-NP/OS-NP and S-NP share is that they are both ambiguous between a comparative reading and an absolute reading (Szabolcsi 1986, Heim 1999). All this suggests that we begin the investigation of the semantics of ordinals by looking at the superlatives.

Following the footsteps of many others (Heim 2004, Cresswell 1976) and extending the previous notion of comparison dimension, I will assume in the case of comparative NP’s/S-NP’s, that their adjective stems express a relation between an object and a degree. So for example: $\text{height}(x, d)$ means object x has a height of degree d . Assuming degree for height is measured in meters, the first sentence of (1-a) provides us with an instantiation: $\text{height}(\text{Aconcagua}, 6962\text{m})$. In addition, if ‘ x is higher than y ’, then $d_x > d_y$. Degrees of unaffixed adjectives are downward monotonic:

A relation R between objects and degrees is downward monotonic iff

$$\forall x \forall d \forall d' (R(x, d) \wedge d > d' \rightarrow R(x, d'))$$

According to the above convention, if Aconcagua is 6962m high, it is also (at least) 6961m and (at least) 6960m high, and so on, but it is not 6963m high. It follows that if another mountain is higher than Aconcagua, then it must also be (at least) 6962m high, in addition to being high to a degree that Aconcagua is not. To be the ‘highest’ then means ‘having a degree of highness such that nobody else in the salient context has it’. This *salient context* is represented by a contextual restrictor C . A superlative *R-est* then, has the following semantics:

$$R\text{-est}(x, R, C): \exists d(R(x, d) \wedge \forall y(y \neq x \wedge y \in C \rightarrow \neg R(y, d)))$$

I begin with a preliminary semantic representation for the word ‘highest’ in singular definite superlative descriptions/SS-NP (e.g. ‘the highest mountain’). A (definite) S-NP inherits the

contextual restrictor of its determiner, so the C below is really a contribution from the definite. It is essential that this contextual restriction becomes part of the lexical entry of a superlative:

$$(4) \quad \text{'(the) highest': } \lambda Q \lambda u \left(\left\{ \left(\begin{array}{c} \underline{C_{Art}} \quad X \\ C_{Art}^*(X) \\ |X| \geq 2 \end{array} \right) \oplus Q^*(X) \right\}, \left(\begin{array}{c} d \\ \text{high}(u,d) \\ u \in X \\ \left(\begin{array}{c} y \\ y \in X \\ y \neq u \end{array} \right) \oplus Q(y) \Rightarrow \neg \left(\begin{array}{c} \text{high}(y,d) \end{array} \right) \end{array} \right) \oplus Q(u) \right)$$

X is a discourse referent for a set. The operator $*$ transforms a predicate P of individuals into one that is true not only of individuals but also of collections consisting exclusively of individuals of type P⁴ (Link 1983). So the set denoted by Q^* can be either a singular element of M or a set of members of M, all of which satisfy the condition Q. Q is obtained through β -conversion in the ‘Box + λ ’ construction, and is identified with the descriptive content of the adjoining noun.

The part within the curly brackets in (4) is the ‘presuppositional’ part of the superlative ‘highest’, it is a representation of the comparison set X, a set of minimally two members required for the superlative to be applicable. The comparison set ‘presupposition’ has in common with traditional presuppositions that there is a requirement to find an antecedent for it. A standard resolution algorithm would try to bind the comparison set presupposition X to some antecedent set, and if this fails, X is accommodated following van der Sandt’s Acceptable and Admissible Resolution Rules. These rules include the Informativity and Consistency Constraints, etc. for details, see van der Sandt (1992). We know at this point that this ‘presuppositional’ DRS is not really presuppositional in the traditional sense of the word (thus the ‘ ’ around ‘presupposition’). This is because the contexts used in familiar studies (mainly in dynamic semantics, such as van der Sandt (1992), Kamp & Reyle & van Genabith (2008)) refer mostly to the Discourse Context K_{Dis} in the articulated context framework, while other context components are rarely considered for presupposition resolution. In these earlier works, when presuppositions bind, they are binding with antecedents in contexts that were established from prior utterances. But since it is very unusual for people to declare explicitly the comparison set prior to using the superlative, accommodation becomes the only prescription, according to existing literature, to deal with the comparison set ‘presupposition’ triggered by a S-NP. This is the wrong approach given that we now have a framework which spells out the common ground in DRT terms.

C_{Art}^* is a *contextual restrictor* for the S-NP: C denotes the kind of general contextual restriction that is conceptually similar to the one that was used for defining the existential presupposition of the definite determiner, and ‘Art’ stands for articulated context. Essentially C_{Art}^* is a discourse referent for a set that determines a subset of the universe of one of the components of the articulated context. The underlines indicate that C_{Art}^* is anaphoric. There are two layers of underlining. They indicate that two separate operations must be carried out to resolve the ‘presuppositions’: one for Art and one for C^{*}. Following DRT conventions, the innermost underline must be resolved first⁵. This first resolution requires Art to find the relevant articulated

⁴ For details, see Chapter 4 of Kamp/Reyle (1993).

⁵ C_{Art}^* is really the shorthand for:

$$\left\{ \left\{ \underline{\text{Art}} \right\}, \left[\begin{array}{c} \underline{C} \\ C^* \subseteq U_{Art} \end{array} \right] \right\}$$

context component where the representation of the comparison set can be located. Generally speaking, $Art \in \{ K_{Dis}, K_{Gen}, K_{Enc}, K_{Utt}, K_{Env} \}$. Once the context component is defined (Art identified with one of the five components), the second resolution will be to find the relevant subset C of entities which satisfies Q that makes up the comparison set in which the S-NP is a member of.

Finding the most *salient and relevant*⁶ context component to resolve a presupposition is an extremely complicated task. Even though it was hinted by Lewis (1979) that certain rules may assist us in keeping track of the salience of entities being talked about (e.g. in the case of definite descriptions, *Rule of Accommodation for Comparative Salience*), it is entirely unclear how far we can take these rules in an open-ended discourse environment. The attention span of the interlocutors and their psychological processes must be weighed in while these factors are far beyond what any linguistic theory can account for. For this reason I will only discuss this problem in general terms while using some examples. Let us suppose that the (comparison) set of mountains in (1-a) was never mentioned in prior utterances, and so its antecedent cannot be found in the discourse context. We would be inclined to assume that the articulated context component where this information can be found is the interlocutors' general K_{Gen} or encyclopedic knowledge K_{Enc} . An example where another component is salient: suppose the interlocutors are standing in front of a landscape consisting of mountains. With the intention to talk about a specific mountain at the scene, one of them utters "the highest mountain...". In this scenario, it seems clear that the most salient context is the environment context, because the (highest) mountain referred to is amongst the mountains within the direct environment of the utterance. The mountains and the height relations between them can be seen as part of K_{Env} . A further complication arises when all of the interlocutors in front of the landscape realize that they are familiar with these mountains: familiar in the sense that they know their names, and perhaps but not necessarily some other attributes such as height, location, etc. These information belong in the interlocutors' encyclopedic knowledge. Therefore, under this circumstance, the very mountains represented in K_{Env} should be anchored to their counterparts in K_{Enc} . These examples are deictic uses of the definite NP 'the highest mountain', where a member in the immediate environment shared by the interlocutors serves as the antecedent for the existence presupposition of the definite. Deictic use of a definite NP is only interpretable after the interlocutors have agreed to have K_{Env} as the most salient context where the comparison set is to be sought. It may appear easy in the case of deictic expressions to select the most salient component of the articulated context, but in general, as a sub-task of interpreting S-NP's, this is not simple at all.

When C_{Art} is resolved to C_{Gen} (or C_{Env} , or any other context component for that matter), the selected articulated context component lies at the center of the interlocutors' attention, and this component is where they believe the referred entities can be retrieved. The remaining underline indicates that a further restriction on that context component's discourse universe is required before the antecedent for the comparison set can be determined. The reason for this further restriction is that even though we know where in the common ground we are to look for the relevant comparison set, there remains the separate question as to *what exactly that set consists of*. (1-a) explicitly states that the comparison set is the set of mountains in the Americas, while (1-b) claims it is the set of all mountains from around the world. However, S-NP do not always make their comparison set explicit as in (1). Suppose without any background context, we have:

U_{Art} is the discourse universe of the subsequent articulated context component. For an ordered-pair notation like the above, the left inner-most presuppositional DRS must be resolved first.

⁶ I am using the concept of salience and relevance very loosely, along the lines of David Lewis (1979).

- (5) Speaker: The highest mountain is Aconcagua.

The comparison set intended for the above S-NP can be mountains in the Americas, or just as likely, mountains in Latin America, South America, mountains in the Andes, or mountains in Argentina: Aconcagua turns out to be the highest of all of the aforementioned sets. More such sets of mountains may be construed, if perhaps less obvious without a background context. There is nothing in (5) that tells us where to draw the line, and for this reason, C has to be left *underspecified* with certain constraints. For (5), if it is known by the interlocutors that Mt. Everest is the highest mountain in the world, for example, then there must be a constraint such that C does not include Mt. Everest as a member, nor any of its associates (mountains near Mt. Everest are also conceivably higher than Aconcagua). One could perhaps induce from this that mountains of a certain geographic area e.g. the Himalayas, or Asia, are not considered part of C. But what are the rules governing such reasoning? Instead of pursuing this, I will simply say that any mountain that is higher than Aconcagua must be excluded from C if at all possible, so that (5) comes out true. Still, simple avoidance of inconsistency is not sufficient enough for determining the precise constituents of C.

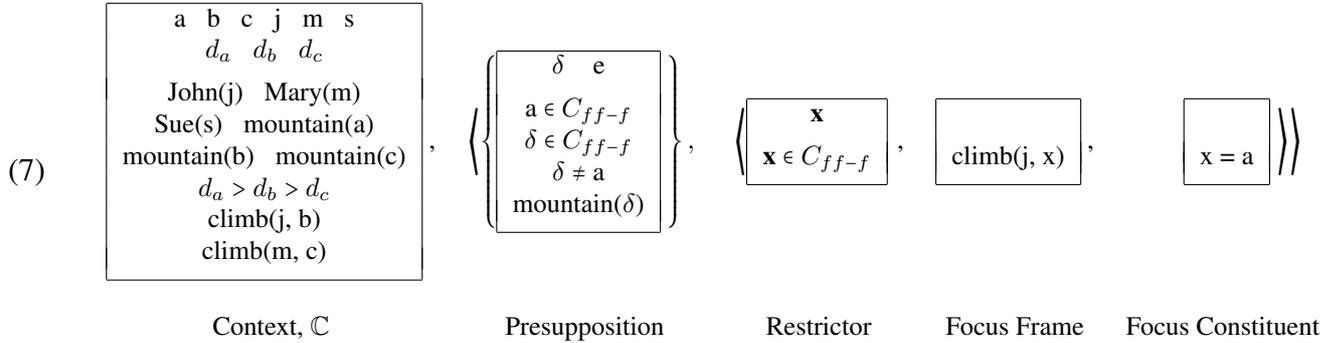
Another issue that made the contextual restrictor an essential component of the semantics of S-NP is the ambiguity between absolute and comparative readings. This ambiguity was first made prominent by Heim (1985) and Szabolcsi (1986), both of which proposed a syntactic solution that was later refuted in Teodorescu (2009). There has been a lot of debate on whether what we are dealing with are truly distinct readings, what precisely is responsible for these different readings, and how they are computed. Unfortunately, very little consensus had been established. One thing is certain however, the cause of these different readings is pragmatic in nature (preconceived context, salience, and information structure). I would like show here that both readings can be generated using the DRT apparatus I am using, and that the difference in these readings is attributed to the difference in the choice of domain restriction for the S-NP. Consider the following sentences with different focus background articulation:

- (6) a. John climbed the highest mountain.
b. John climbed the [*HIGHEST*]_F mountain.
c. [*JOHN*]_F climbed the highest mountain.

(6-a) seems uncontroversial until we consider (6-b) and (6-c). For (6-b), ‘highest’ bears overt focus accent. According to Szabolcsi, it has the absolute reading: John climbed the highest mountain in the world (or rather, as I have been hinting up to this point, the highest mountain in C). Given that the highest mountain in the world is Mt. Everest, (6-b) states that John climbed Mt. Everest. (6-c) on the other hand, has a different meaning. With the focus placed on John, (6-c) means that John climbed a higher mountain than anybody else climbed in some contextually salient domain. Under this reading, (6-c) may well be true when John only climbed a small hill in his neighborhood, as long as nobody else in the relevant context climbed any mountain that is higher. It is well-known that the *wh*-word in interrogative sentences constitutes the focus of that sentence. Gutierrez-Rexach (2005) pointed out that a superlative phrase in an interrogative has the comparative reading when the comparison set is drawn from alternatives in the domain associated with the *wh*-element, (6-c) therefore answers the question “who climbed the highest mountain of all the mountains climbed by anybody?”. I will take it for granted here that *wh*-focus licences the comparative reading of a S-NP. There is nevertheless a *prima facie* case for regarding superlatives as a case of free focus association, because association with focus is not compulsory (Beaver/ Clark 2007). This means for (6-c) the highest mountain

may well be the highest in an independently salient set of mountains C , for example C could be mountains in the Americas, or the Andes.

The determination of C in S-NP utterances in (6) may first be approached by listing all of the alternatives to the focus marked constituent in a focus background articulation. To do this we must first incorporate an account of focus background articulation into DRT. I will only sketch out the framework here and refer to Kamp (2004) for details. (6-b) has the following context representation and focus frame focus division (ff-f division for short):



A ff-f division and its presupposition is separated from the context of the sentence \mathbb{C} ⁷, which is represented on the left end of the representation above. This is because ff-f division deals strictly with focus background articulation, it is a means to represent and derive all the possible alternative semantics attributed to a certain part of a sentence being focus marked. The ff-f division in (7) assumes that presuppositions from other triggers such as proper names (John), the definite determiner ‘the’, are already resolved and accommodated in the context.

The first component of the ff-f division is the DRS on the center right, the one that contains the condition $x \in C_{ff-f}$. This component is called a restrictor. A restrictor consists of the focus variable x , displayed in bold, and a restrictor predicate C_{ff-f} . C_{ff-f} stands for restrictions on the focus variable, such as quantification, abstraction, and so on, that are not explicit in the sentence itself, but must be inferred from context. To provide a simple example, suppose the interlocutors of (6-b) have in their common ground three mountains under their attention: mountain a, b and c, then C_{ff-f} is restricted to these three individuals. In this case, C_{ff-f} is the property of being a member of the set consisting a, b, and c. The restrictor of a ff-f division represents all possible values of the focus variable under the given context. x replaces the focus marked constituent *highest mountain* in the semantic representation of (6-b). This replacement of the focus marked constituent by the focus variable produces the separation of focus constituent from focus frame, which are the other two components of the ff-f division. The focus frame is the outcome of semantic construction from the syntactic tree of the sentence, using the bottom-up Box + λ method. The focus constituent of a ff-f division states that the value of the focus variable is that of mountain a. Each ff-f division comes with a presuppositional constraint on its alternative set (Rooth 1992). This means there must be at least one other element δ in C_{ff-f} besides the focus marked constituent that may serve as its alternative and be identified with the focus variable in the focus constituent. In the case of the above example, mountain b is an alternative to mountain a as a potential candidate for being climbed by John. The presuppositional DRS of the focus is the DRS between the curly brackets located in the

⁷ The context \mathbb{C} is predetermined for this example. For simplicity it encompasses all relevant information from the union of all five components of the articulated context. Notice how according to this context, John in fact did *not* climb the highest mountain. The expression in (6-b) is therefore either *false*, or some form of *accommodation* must take place, such that the information John climbed the highest is recorded in \mathbb{C} .

center. It states that there has to be at least one other mountain (δ) in context \mathbb{C} such that it is not mountain a.

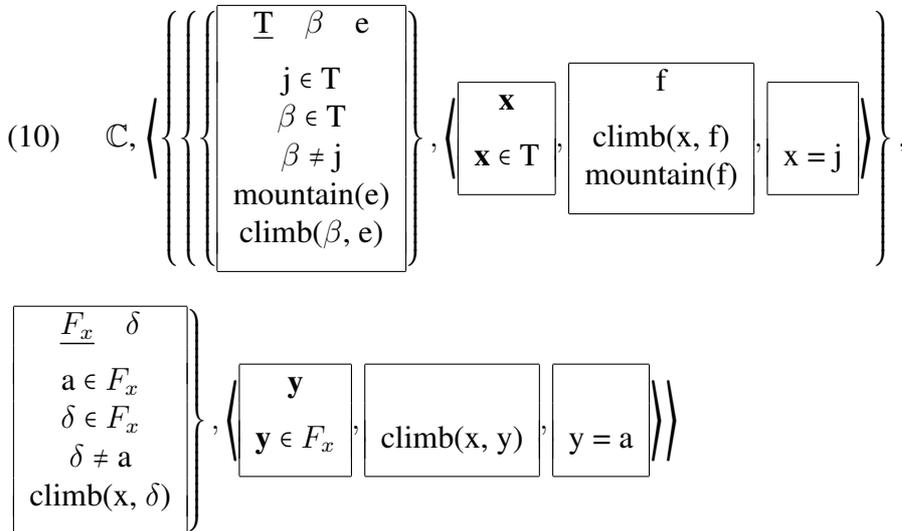
I argue here that in order to get comparative readings for a sentence like (6-c), its information structure should be analyzed as the more complex type of *Topic-Focus* structure that was made prominent in the work of Büring (1997), instead of the simple focus-background type in Rooth (1992). (6-c) should be analyzed as follows:

(8) $[JOHN]_T$ climbed the $[highest]_F$ mountain.

The above annotations come with two presuppositions: The focus marking indicates the presupposition of alternatives to the focus constituent, it is the equivalent of C_{ff-f} that was described earlier. Let us call this the *focus set* F. The topic marking indicates the presupposition of a similar *topic set* T. T is a set that includes the alternatives to John in (8). I use the notation $\llbracket \cdot \rrbracket^0$ to denote semantic values à la Rooth (1992). $\llbracket \alpha \rrbracket^f$ is the focus semantic value for the phrase α , which is a set of alternative propositions when α is a sentence, a set of alternative entities when α is a noun phrase. $\llbracket \alpha \rrbracket^0$ is the ordinary semantic value for α . The ordinary semantic value is always a member of the focus semantic value. $\llbracket \alpha \rrbracket^t$ then, is the topic semantic value of α . According to Büring's original analysis, the topic semantic value of (8), $\llbracket (8) \rrbracket^t$, is a set of sets- for each member d of the topic set T (John and his alternatives in \mathbb{C}) there is a focus set F_d assigned to it (the highest mountain, the second highest mountain, etc.), as in the following:

(9) $\llbracket (8) \rrbracket^t = \{ \{ \text{John climbed the highest mountain, John climbed the second highest mountain, John climbed the third...} \}, \{ \text{Mary climbed the highest mountain, Mary climbed the second...} \}, \{ \text{Sue climbed the highest...} \}, \dots \}$

The preliminary representation for (8) is as follows:



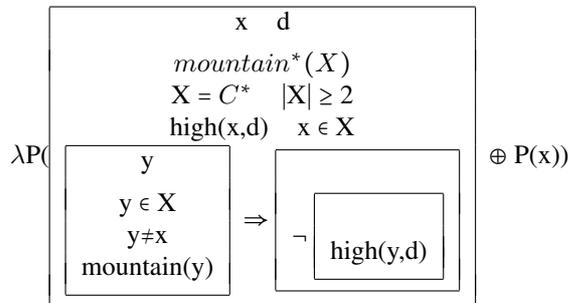
An explanation for (10) is in order, starting from top left: The first DRS is the context \mathbb{C} . Moving to the right of the context we find the first and innermost presupposition. It consists of the topic set T, triggered by the topic marking of (8). It states that John is a member of T, and that there is at least one alternative to John, β , who climbed some mountain. β is the presupposition of the S-topic. Further to the right is the topic frame topic (tf-t) division. Mirroring the ff-f division, the tf-t division has a restrictor, a topic frame and a topic constituent. The tf-t division states that a person from the topic set T climbed a certain mountain, and that person, as specified in the topic constituent, is John. \mathbf{f} in the topic frame is a place holder for the focus variable

y that is yet to come. Moving further down to the second row of the Preliminary DRS is the presupposition triggered by the focus marking, and its ff-f division. The nested presuppositional structure here reflects that the presupposition of the focus is dependent on the presupposition of the topic. The focus presupposition states that there is a focus set F_x and the highest mountain **a** is a member of F_x . There is also another mountain δ in F_d such that the topic variable x climbed δ . We may in fact generate all the semantic values that were listed in (9) using the focus frame of this ff-f division, while the topic marked constituent in combination with the focus marked constituent being John and the highest mountain (a) make up the ordinary semantic value of (8).

I argue here that $F = \cup \{F_d | d \in T\}$ is the set of all possible alternatives to the focus constituent, it denotes in (10) the set of all mountains in context which *could have been climbed* by someone. The contextual restrictor C for the S-NP in comparative readings of focus marked sentences is identified to the F of that sentence. F, however, is *not* the set of mountains that are *actually climbed*. What we are ultimately interested in, the comparison set X, is a subset of F: The set of mountains that are *actually climbed* by somebody from the Topic Set T. These mountains constitute the comparison set X, and we can only obtain them by verifying F with the context to determine which of the mountains are indeed climbed by members of T⁸.

So far we have seen how the contextual restrictor and the comparison set are derived in focus marked S-NP sentences. However, most S-NP utterances have no focus articulation and the derivation method we have been using for topic-focus sentences does not apply. Often one must resort to very complex cognitive processes when interpreting S-NP utterances with an underspecified C. Fortunately, C is sometimes made explicit, as in (1), and we return to it at this point. (1-a) specifies that the comparison set X consists of mountains in the Americas, whilst C without contextual restriction from a prior discourse or other presupposition triggers, is identified to the comparison set. The result of combining the definite determiner (3) with the superlative entry (4) through $\text{Box} + \lambda$, is as follows:

(11) ‘the highest mountain (in the Americas’), K_{Dis} :



$C^* = \{z | z \text{ is a mountain (or some mountains) in the Americas}\}$

3. Ordinals and Ordinals Superlatives

OS-NP have in common with S-NP some key features such as the comparison set, ordered by a comparison dimension. The departure between the two begins as semantic representations of OS-NP’s which require a formal DRS account of what’s involved in counting, i.e. of the natural numbers and of *Well Ordering*. Fortunately, a set of axioms for the natural numbers was

⁸ The distinction between C and X is unfortunately very complex but nevertheless important. For lack of space here I must leave this up to forthcoming works on the same subject.

presented by the 19th century Italian mathematician Giuseppe Peano, and if necessary they can be readily converted into DRT terms. First we define *Well Ordering*:

A set X is Well Ordereded (\mathcal{WE}) by $<$ iff

1. X is linearly ordered by $<$
2. $(\forall Y \subseteq X)(Y \neq \emptyset \rightarrow (\exists y \in Y)(\forall z \in Y)(y < z))$, Every non-empty subset Y of X has a $<$ -first (least) element.

Peano Axioms (\mathcal{P}) for natural numbers are as follows, where S denotes the successor function:

1. $\neg \exists x(S(x) = 0)$
2. $\forall x \exists y(x \neq 0 \rightarrow S(y) = x)$
3. $\forall x \forall y(S(x) = S(y) \rightarrow x = y)$
4. $\forall x(x + 0 = x)$
5. $\forall x \forall y(x + S(y) = S(x + y))$
6. $\forall x(x * 0 = 0)$
7. $\forall x \forall y(x * S(y) = x * y + x)$

The Induction Scheme (\mathcal{IS}) states that if a property is possessed by 0 and also by the successor of every natural number which possesses it, then it is possessed by all natural numbers:

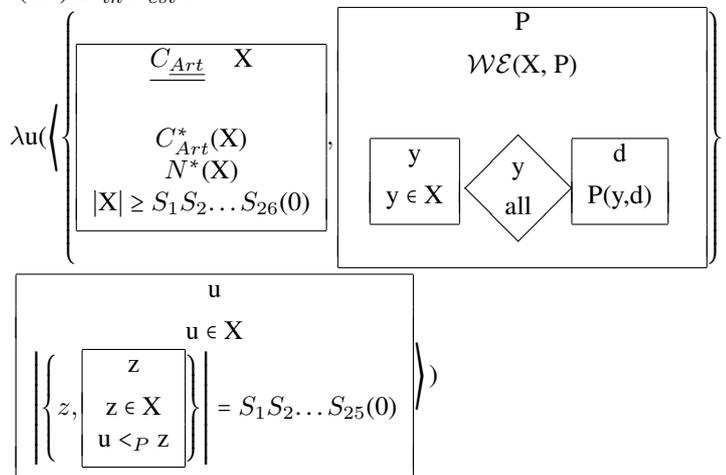
$$\forall v_1 \dots v_n ((F[0, v_1, \dots, v_n] \wedge \forall v_0 (F[v_0, v_1, \dots, v_n] \rightarrow F[Sv_0, v_1, \dots, v_n])) \rightarrow \forall v_0 F[v_0, v_1, \dots, v_n]),$$

Where n is an integer, and $F[v_0, v_1, \dots, v_n]$ is any formula whose only free variables are v_0, v_1, \dots, v_n .

We may incorporate these key definitions and Axioms into our DRT formalism: ‘0’ is the only new discourse referent that needs to be introduced, and it is always anchored to the the number zero. We assume that our models contain the natural numbers as part of their domain of individuals and thus the constants ‘0’, ‘S’, ‘+’ and ‘*’ will always have interpretations that are consistent with Axioms 1-7.

The semantic representation for the OS-NP ‘(the) 26th P_{est} N’ where P is the degree adjective and N is a singular noun, is in (12) below. Note that its contextual restrictor, like its S-NP counterpart, is related to the definite determiner which adjoins the ordinal number:

(12) ‘(the) 26th P_{est} N’:



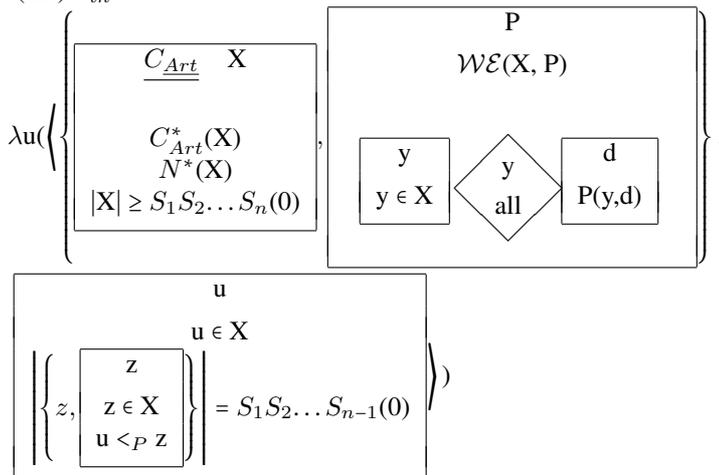
The first presupposition, like S-NP, contains the contextual restrictor C which must first be resolved to an articulated context component, then to a certain subset of the universe of that component depending on the descriptive content of the noun, focus articulation, and any relevant factors from the common ground. The representation of this presupposition uses the successor function S, it states that the comparison set of ‘26_{th}’ must consist of at least 26 members, for otherwise the utterance would be infelicitous. The second presupposition states that the comparison set X is *Well Ordered* with respect to P. P is a relation between members of X and a certain degree d that they possess (in the same sense in which we have been using the term ‘degree’ so far, d should be a natural number that can be represented using the successor function). P is the comparison dimension, it is the predicate obtained from adjectives such as ‘high(er/est)’, ‘big(er/est)’, etc. The comparison dimension is always specified in the superlative for OS-NP’s (‘the 26_{th} highest mountain’), and it is obtained from context for O-NP’s (as in (2)). The second presupposition states that every member of the comparison set must have a P degree, such that X is Well Ordered by P. Finally, the assertional part of (12) states that there are exactly 25 members from within the comparison set besides the asserted OS-NP u that are *higher* than u: $a <_P b$ is defined as follows:

$$a <_P b \text{ iff } (\forall d P(a, d) \rightarrow P(b, d)) \wedge (\exists d' P(b, d') \wedge \neg P(a, d'))$$

It is easy to see that if the set of degrees is linearly ordered, and P is downward monotone in its second argument, then \leq_P is a pre-ordering, and $<_P$ is asymmetric and transitive. So if $a <_P b$, it is entailed by downward monotonicity of degrees that b has a unique degree that a does not have. For ‘the 26_{th} P_{est} N’, there should be exactly 25 N in X, all of which have some P degree d such that u does not have d. When we consider (1-b), it follows that there are exactly 25 mountains in the world which are higher than Aconcagua (because they possess degrees of height that Aconcagua does not possess).

Since there are infinitely many integers and there is at least one ordinal number expression for each of them, the entries for the individual ordinals will have to be specified as instances of a general schema. The semantic representation for the O-NP ‘the n_{th} N’ is as follows. The comparison dimension P must be retrieved from context as it is unspecified in the O-NP:

(13) ‘(the) n_{th} N’:



4. Plural Superlatives and Plural Ordinals

Contrary to what was assumed for singular superlative (SS-NP) and ordinal expressions (SO-NP), the comparison set for PS-NP with or without ordinals are not necessarily linearly ordered:

it is possible for two members, a and b, of the comparison set to have the same degrees and be ranked as such: $a \equiv_P b^9$. In addition, a PS-NP/POS-NP may refer to multiple members of the comparison set across adjacent degrees of the comparison dimension:

- (14) a. Mary climbed the highest *mountains* in the Americas.
 b. John climbed the second highest *mountains* in the Americas.

In (14), an unspecified number of mountains with similar (but distinct) heights are considered together as occupying the same rank. In other words, there is an ambiguity with regard to the degrees of height that are being referred to with the same PS-NP/POS-NP, since one may refer to the set {Mt. Everest, K2} as ‘the highest mountains’, just as well as the set {Mt. Everest, K2, Kangchenjunga}, and so on. I will not go into exactly how far we may inflate this set before ‘highest’ loses its conventional meaning, but it is clear that the PS-NP and POS-NP do not necessarily employ the relation $<_P$ in the way that SS-NP’s do. PS-NP presupposes a comparison set with an ‘ordered’ or ‘layered’ partition with a top tier that is the denotation of the highest mountains, a tier immediately below the first tier which can be described as ‘the second highest mountain(s)’, and so on¹⁰. This calls for revision of our earlier analysis of the superlative and the ordinal.

Treatment of the semantics of singular and plural descriptions can generally be unified (Link 1983). Definite descriptions, as we have been dealing with them so far, are decomposed into two components. One that is due to the definite article ‘the’ which is neutral between singular and plural; and there is a further contribution from the singular or plural morphology of the noun. When the definite article is applied to a set denoting expression, it carries the presupposition that there is a set of individuals which satisfies the descriptive content of that expression. Singular and plural morphology are therefore presuppositional. As we have already seen, a singular definite imposes the existence and uniqueness presupposition. A plural definite on the other hand signals that its referent is a set that consists of at least two elements¹¹. The semantic representation of the plural definite description is obtained this way and is given in (15):

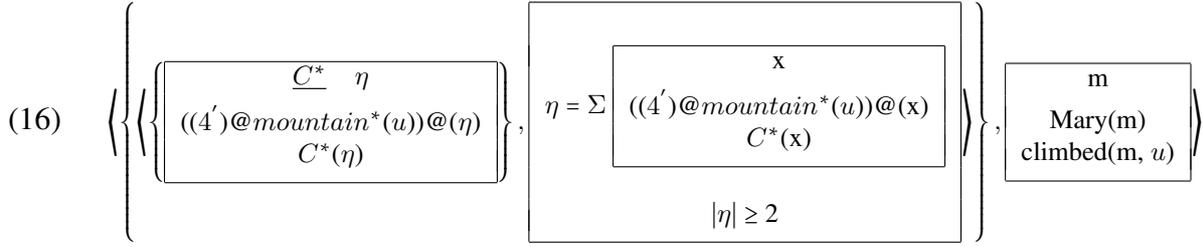
$$(15) \quad \text{‘the’}: \lambda P'^* \lambda P \left(\left\langle \left\langle \left\langle \frac{C^*}{P'^*(\eta)} \right\rangle \right\rangle, \left\langle \left\langle \begin{array}{c} x \\ P'^*(x) \\ C^*(x) \end{array} \right\rangle \right\rangle, \left\langle \left\langle \begin{array}{c} \eta = \Sigma \\ | \eta | \geq 2 \end{array} \right\rangle \right\rangle \right\rangle, \left\langle \left\langle \right\rangle \right\rangle \oplus P(u)$$

Σ is the DRT summation sign (Kamp/Reyle 1993), it states that the discourse referent η is a mereological set consisting of all referents in C that satisfies P'^* , whereas P'^* is filled in by the descriptive content from the adjoining noun. Moving back to superlatives, the Preliminary DRS for (14-a) as a result of combining all the lexical entries involved:

⁹ $a \equiv_P b$ iff $a \leq_P b \wedge b \leq_P a$

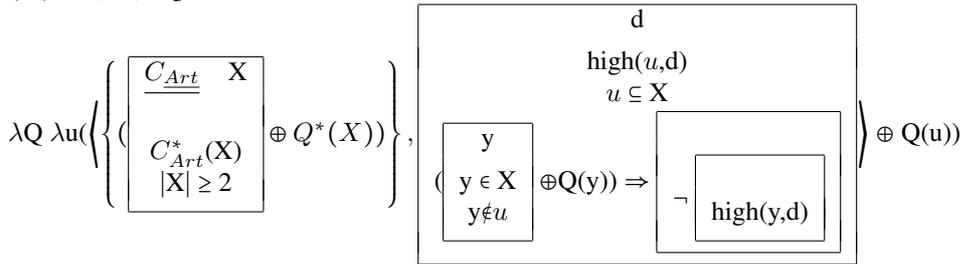
¹⁰ The entries for singular and plural definites can be decomposed into a number neutral semantic for ‘the’- its denotation is the maximum set of N’s that are in C- and the contribution of singular and plural. Singular carries the presupposition that this denotation has cardinality 1. Plural morphology has cardinality > 1

¹¹ Furthermore, this set is uniquely restricted by C within the appropriate articulated context component. For the sake of simplicity, this is not represented in (15).



(4') in this Preliminary DRS refers to the lexical entry of the superlative defined in (4), with a small modification so that u is a discourse referent that can represent either an individual or a set of individuals, and subsequently the ϵ relation for u is replaced by \subseteq for u :

(4') 'the highest':



(16) can be simplified first by applying the predicate 'mountain(u)' to (4'), and then applying η to the result, instantiating the variable u . This gives us the semantics for 'the mountains in the Americas' (I assume here as I did before that 'in the Americas' specifies the comparison set and consequently the contextual restrictor, therefore, $\underline{C_{Art}}$ is resolved the same way as it was in (11)). Finally, 'climbed' and 'John' enter the assertional part of the Preliminary DRS.

What is interesting about (16) is that given (4'), the set of highest mountains referred to collectively become a member of the comparison set partition, whilst these mountains must all possess a degree \mathbf{d} in common, such that no other mountains in the comparison set have \mathbf{d} . \mathbf{d} is taken as the minimal degree of height to qualify a mountain in the comparison set as in the league of 'the highest' - we may regard \mathbf{d} as a *cut-off line* that distinguishes the highest mountains from the second highest and the rest of the mountains.

Things are not quite as simple for OS-NP. The ways in which we use OS-NP to refer to entities is very complex, especially since POS-NP's have unique presupposition triggers that SOS-NP do not. Let M be a context¹², and d_n some degree. There are in M a set of mountains $\{m_1, m_2, m_3, m_4\}$, each and every mountain possesses one or more of the degrees d_1, d_2, d_3 , where $d_1 > d_2 > d_3$:

	M_1	M_2	M_3
d_1	m_1, m_2	m_1	m_1
d_2	m_3	m_2, m_3	m_2
d_3	m_4	m_4	m_3, m_4

Figure 1

Below is a list of all possible POS-NP and the sets of mountains they may refer to in each of the M_1, M_2 , and M_3 :

¹² M denotes a specific context, similar to that of the contextual restrictor C . To make things easier to conceptualize, one may think of M as a Model, while bearing in mind that a Context is really a *set* of Models, all of which are consistent with the contents and restrictions defined the Context.

	M_1	M_2	M_3
‘The highest mountains’	$\{m_1, m_2\}$, $\{m_1, m_2, m_3\}$, $\{m_1, m_2, m_3, m_4\}$	$\{m_1, m_2, m_3\}$, $\{m_1, m_2, m_3, m_4\}$	$\{m_1, m_2\}$, $\{m_1, m_2, m_3, m_4\}$
‘The second highest mountains’	$\{m_3, m_4\}$,	$\{m_2, m_3\}$, $\{m_2, m_3, m_4\}$	$\{m_2, m_3, m_4\}$, $\{m_3, m_4\}$
‘The third highest mountains’	$\{\emptyset\}$	$\{\emptyset\}$	$\{m_3, m_4\}$

Figure 2

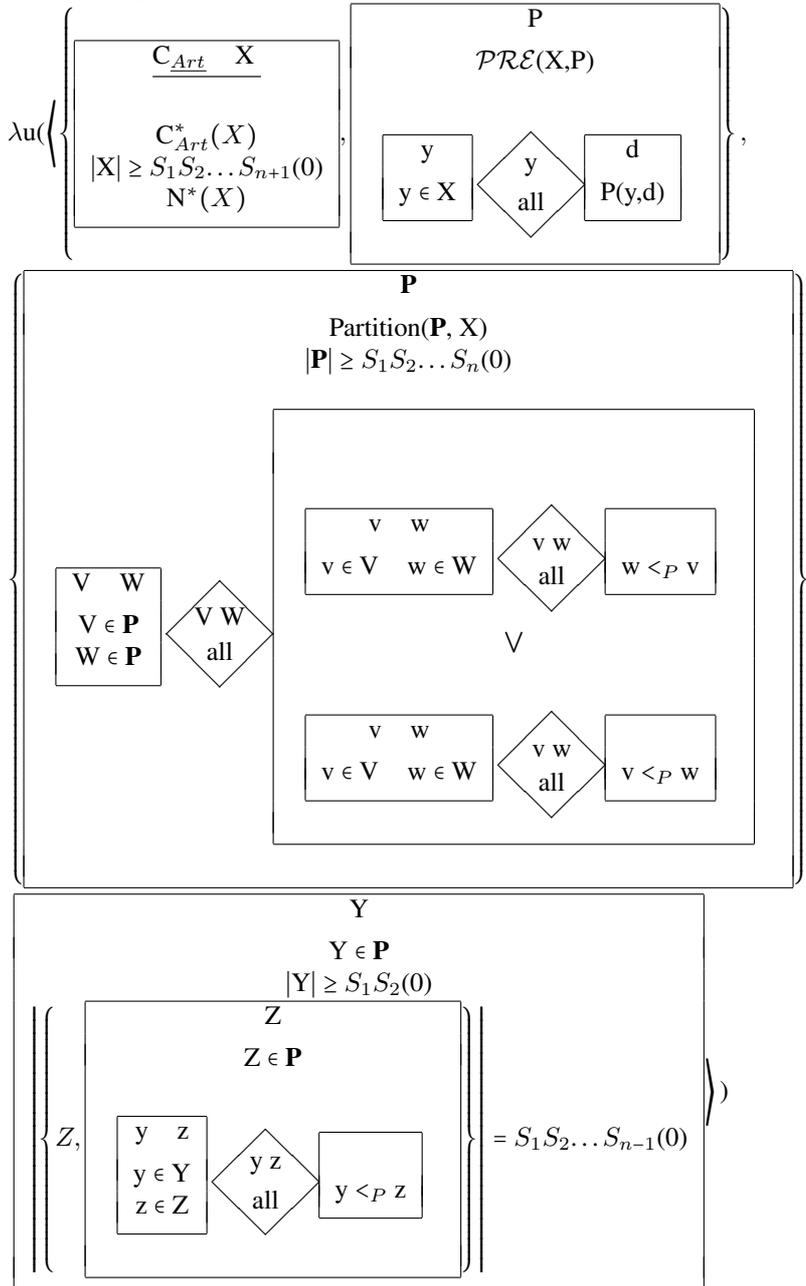
Several important constraints observed from the above jointly determine the set of mountains that a given POS-NP can be used to refer to: First, a POS-NP must refer to at least two mountains. For this reason it is necessary when uttering ‘the highest mountains’ in contexts such as M_2 or M_3 that mountains besides m_1 must be included, even though m_1 is strictly speaking the tallest, i.e. m_1 has a degree of height that none of the other mountains has. However, not all members in the comparison set of a POS-NP need to follow this constraint, as it only applies to the *asserted* POS-NP. This means when referring to m_2 and m_3 under M_2 as ‘the second highest mountains’, m_1 remains ‘the highest’, and m_4 the ‘third highest’. The second constraint on a POS-NP is that it may not refer to one member of a degree while excluding another under the same degree. For example, ‘the second highest mountains’ may not refer to the set $\{m_2, m_3\}$ in M_3 precisely because it separates m_3 from m_4 , which is comparable since they are high to the same degrees. Finally, a POS-NP is ‘inclusive’ in the sense that ‘the n_{th} mountains’ can refer to all the mountains in d_n and below (i.e. d_{n+1} , d_{n+2} , and so on). For instance, in all models, it is felicitous (although a little bit pointless) to refer to all mountains as ‘the highest mountains’, and in M_2 and M_3 , for m_4 to be one of the three ‘second highest mountains’ (the other being necessarily m_2 and m_3).

Due to the ‘inclusive’ nature of POS-NP’s, the choice of the members for the set classified as ‘the n_{th} highest mountains’ makes certain presuppositions about other members of the comparison set, other SOS-NP and POS-NP appearing later in the same discourse must conform to that presupposition. To give an example, if we refer to m_3 and m_4 in M_3 as ‘the second highest mountains’, we presuppose that m_1 and m_2 are considered together as ‘the highest mountains’, any other configuration would be infelicitous. Another example, in M_1 , when one utters ‘the highest mountains’ with the intention to refer to m_1 , m_2 , and m_3 , then it is necessary that m_4 be referred at a latter discourse as ‘the second highest mountain’. From this we may conclude that POS-NP do not refer to entities classified by the different degrees of their comparison dimension, but entities classified by *clusters* of degrees. A cluster of degrees is a group of degrees that are adjacent to one another, e.g. in M_1 and M_3 , d_1 and d_2 can make a cluster, but not d_1 and d_3 . The number of different degrees to be included in a cluster can be arbitrary and underspecified, but the choice of degrees to be included under the same cluster always triggers a presupposition about how the other degrees outside of that cluster ought to be grouped. Because of this added presupposition dimension, POS-NP’s require an analysis of their own.

A POS-NP imposes a *partition* on the comparison set X. A partition is a division of X into non-overlapping and non-empty ‘parts’ (which is synonymous to sets) such that the union of all the parts is X. Each part consists of members of X under a cluster of adjacent degrees. An expression of the form ‘the n_{th} P_{est} N’s’ presupposes at least n many parts, and by virtue of its plural morphology, the n_{th} part has to be a set of *at least two* members, all of which satisfy the descriptive content of N (This entails that X must have at least n+1 members). The parts are

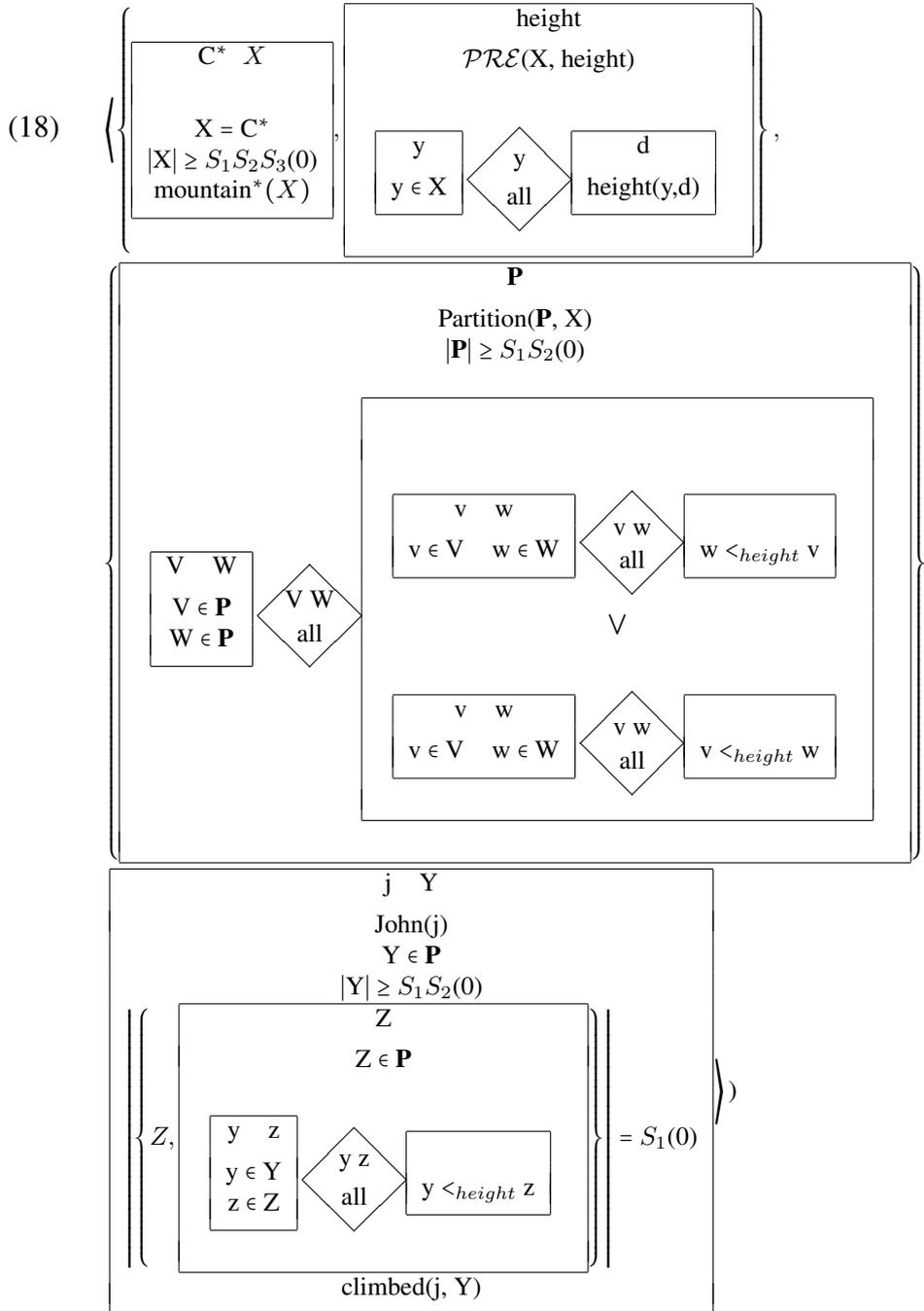
ordered by the comparison dimension P. For every pair of parts V and W, either all members of the part W are $<_P$ than those of the part V, or vice versa. So for example all of ‘the second highest mountains’ are higher than all of the ‘the third highest mountains’, and ‘the third highest mountains’ are higher than ‘the fourth highest’, and so on. A DRS representation for the POS-NP ‘the n_{th} P_{est} N’s’ is as follows:

(17) ‘(the) n_{th} P_{est} N’s’:



The first presupposition of (17) is similar to the SO-NP in (13) with a slight modification to the Well Ordering of X (\mathcal{WE}) earlier: the comparison set X is Pre-ordered by the comparison dimension P. Pre-orders are ordering relations which allows for ties. The added presupposition in the mid-section of (17) introduces the partition. X is partitioned into n many sets. The partition \mathbf{P} is the collection of those sets. \mathbf{P} is a layered partition of X with respect to $<_P$ in the following sense: if V, W are disjoint members of P, either: 1. for all $v \in V, w \in W, v <_P w$ or 2.

for all $v \in V, w \in W, w <_P v$. To repeat, if \mathbf{P} is the partition presupposed by ‘the n_{th} P_{est} N’s’, then \mathbf{P} has at least n members, and its n_{th} member Y (in terms of the comparison dimension relation between members of \mathbf{P}) will have to have a cardinality ≥ 2 . The other members of the partition, however, can be singletons. The following is a Preliminary DRS for (14-b):



$$C^* = \{z \mid z \text{ is a mountain (or some mountains) in the Americas}\}$$

The first presupposition in (18) states that there are at least three mountains in the context where (14-b) is uttered, and that this context is said to be the set of all mountains in the Americas. The second presupposition simply pre-orders the comparison set of the ordinal according to the comparison dimension ‘height’. The third presupposition in the middle row says that \mathbf{P} has

at least two members, V and W , where all members of V are higher than those members of W . Finally, the assertional part of the Preliminary DRS states that there is exactly one part Z whose members are higher than those of the asserted, second highest set of mountains Y . During presupposition resolution, Y is identified to W and Z to V . As (14-b) states, John climbed the set of mountain Y , it is a set of at least two mountains, all of which are lower than another set of mountains Z (the highest mountains), in the comparison set known as the mountains in the Americas.

A unified treatment can be given to plural superlatives, PS-NP, using the POS-NP entry (17) as template. This strategy would spare the definite determiner from being modified as was in (15). The only difference is that for PS-NP, there must be at least two members in the comparison set (as opposed to $n+1$, the minimum being three, for POS-NP), and only two parts under the partition \mathbf{P} are necessary: The highest set of mountains, and the rest of the mountains in the comparison set bunched up in the other set. Once we have seen that the representations for POS-NP's and PS-NP's can be constructed uniformly, the question arise as whether we should retroactively apply a similar approach to SOS-NP's and SS-NP's, making use of the ordered partition presupposition as well? After all, representations for the SS-NP and SOS-NP can be constructed by restricting the cardinality of each part of the partition \mathbf{P} in the PS-NP/POS-NP representation to one. A natural penchant for more uniformed (and simpler) accounts would certainly suggest we should, but interestingly the answer is less straightforward than might have be thought. There is a strong tendency to assume upon hearing a SS-NP in a discourse initial sentence a linear order of the comparison set X , as we assumed in our earlier treatment of (1-a) in (4) (same can be said about (2)). Of course, this assumption can be easily overwritten as in (19):

- (19) John climbed the second highest mountain in the Americas. The two highest *mountains*, however, were climbed by Mary.

In our judgement, a discourse like (19) is all right, although some hearer may get a slight jolt upon hearing the second sentence. In this respect (19) is different from (20), which begins with the SS-NP 'the highest mountain':

- (20) John climbed the highest mountain in the Americas. The two second highest *mountains*, however, were climbed by Mary.

In the light of these observations, perhaps the most realistic proposal is that the ordinal partition presupposition is always triggered by all the SS-NP, SOS-NP, PS-NP and POS-NP's. However, certain occurrences of SS-NP's (and SOS-NP's) can give rise to the further implicature that all members of the partition \mathbf{P} are singletons (and it follows that in these cases X is well ordered). For these reasons, I leave intact the representations of the SS-NP in (4) and SOS-NP in (13) without further ado.

We close this section by putting the POS-NP schema (17) through tests to see if it indeed delivers its promise to represent POS-NP expressions correctly while ensuring its presupposition constraints are maintained throughout the same discourse. Suppose we are given the following discourse involving a PS-NP and a POS-NP:

- (21) John climbed the **highest mountains** _{Z} in the Americas. The **second highest mountains** _{Y} , however, were climbed by Mary.

Let there be four mountains in the common ground: $\{m_1, m_2, m_3, m_4\}$. There are many ways in which (21) can be interpreted. Suppose we take the contexts in Figure 1 as the only ones available for interpretation, the following are snapshots of the highest and the second highest mountains instantiated under each context as the sentences in (21) are processed:

	M_1	M_2	M_3
Z	$\{m_1, m_2\},$ $\{m_1, m_2, m_3\},$ $\{m_1, m_2, m_3, m_4\}$	$\{m_1, m_2, m_3\},$ $\{m_1, m_2, m_3, m_4\}$	$\{m_1, m_2\},$ $\{m_1, m_2, m_3, m_4\}$

Figure 3, Assignment for the first sentence alone

	M_1	M_2	M_3
Z	$\{m_1, m_2\}$	N/A	$\{m_1, m_2\}$
Y	$\{m_3, m_4\}$	N/A	$\{m_3, m_4\}$

Figure 4, Assignment for both first and second sentences

Figure 3 is a snapshot taken after the first sentence of (21) has been interpreted, but before the second sentence enters the picture. As we can see, there are many ways in which ‘the highest mountains’ can be assigned for each of the given contexts at this stage. Figure 4 is a list of compatible assignments for both ‘the highest mountains’ and ‘the second highest mountains’, which is necessary for the interpretation of (21) as a whole. Notice how as soon as the second sentence is processed, the list of potential assignments for ‘the highest mountains’ (from Figure 3) diminishes. This is because the assignments for ‘the second highest mountains’ and those given to ‘the highest’ must be consistent with each other, as the PS-NP and the POS-NP share the same comparison set. The way in which the comparison set is partitioned, according to (17), must be applicable to the PS-NP and all the POS-NP’s occurring in the same discourse. For this reason, there is only one assignment available for M_1 and M_3 , and none for M_2 , as shown in Figure 4.

If we look at M_1 for example, it is impossible to assign $\{m_1, m_2, m_3\}$ and $\{m_1, m_2, m_3, m_4\}$ to ‘the highest mountains’ for (21) as a whole. Because as soon as we attempt to construct for ‘the second highest mountains’, according to (17), we need at least two mountains available to be assigned to the second highest partition. But if we already assigned $\{m_1, m_2, m_3\}$ to ‘the highest’ partition, then there is only one (m_4) remaining for this task; if we assigned $\{m_1, m_2, m_3, m_4\}$ to ‘the highest’, there will be none left for the second highest. No assignment is available for M_2 , because in order to partition the comparison set into two non-atomic parts, we would have to place m_2 and m_3 in two different partitions, but they have the same degree and to do so violates (17) (that for all $v \in V$ and all $w \in W$, $v < w$ or $w > v$. In M_2 , $m_2 \not< m_3$ and $m_3 \not> m_2$). M_3 is similar to M_1 in that if we assign $\{m_1, m_2, m_3, m_4\}$ to ‘the highest mountains’, we again run into trouble trying to resolve for ‘the second highest mountains’, as are no more left in this context to satisfy further POS-NP’s.

5. Conclusion

The primary goal of this paper is to provide a comprehensive account of the semantics of ordinals and superlatives using the DRT framework. As we have seen, this task touches upon many subjects that need to be further elaborated: The notion of articulated contexts as representation of the common ground raises issues such as how information is introduced to the

common ground, and how it can be accessed and transferred between the different components; When dealing with referential presupposition triggers like the definite article, we are faced with the problem of selecting the most salient and relevant context component- but what are the rules that govern and guide our selection? This open-ended question is further complicated by the problem of contextual restriction- how do we decide which are the relevant entities to be considered for a quantificational/focus articulated/contextualized expression? To this, I have demonstrated with the example of topic-focus marked sentences, where upon construction of the focus frame, one may simply identify the focus set to the contextual restrictor, and select the comparison set from within. A generic lexical schema is arrived at for Singular Superlative NP's, Singular Ordinal Superlative NP's, and Singular Ordinal NP's. These lexical entries have been demonstrated to perform in a plausible syntax-semantic interface such that the correct semantic representations may be constructed for sentences involving the superlative and/or the ordinal.

The issue of Plural Superlative NP's calls a revision of the definite determiner, this solution alone, however, is not enough to deal with POS-NP's. A POS-NP further presupposes a partition of the comparison set. While the comparison set of the POS-NP itself is pre-ordered, every part of the partition is linearly ordered with regard to each other.

An interesting feature of the Plural Ordinal Superlative NP (and Plural Ordinal NP) is that it imposes further presuppositions about how other members of the comparison set should be arranged into different parts of the partition. So for example, given a context of five mountains where $\{m_1, m_2\}$ are of the highest degree, $\{m_3, m_4\}$ the second, and m_5 the third, once we refer to $\{m_1, m_2, m_3, m_4\}$ as 'the highest mountains', it is necessary that m_5 be referred later as 'the second highest' instead of, e.g, 'the third highest'.

Two very curious observations arise from our current analysis. First, consider the context of five mountains as before. Would it be possible to utter in the very beginning of a discourse, "the third highest mountain... ", with the intention to refer to m_5 ? The answer is probably no, even though a partition would be consistent with our analysis. The reason here seems to me is that *too many steps of accommodation* are required- recall that accommodation is a context repair strategy that demands a certain amount of psychological effort from its users. Two steps of accommodations are needed in order for 'the third highest mountain' to successfully refer to m_5 in the given situation. One, the singular ordinal utterance presupposes that the comparison set is *linearly ordered* (Well Ordering), this is specified in (13). But if the comparison set is indeed linearly ordered, then it is forbidden for m_1 and m_2 to be under the same degree, and the same can be said about m_3 and m_4 . So the first accommodation would be to abandon (13) and use (17) as an analysis for the singular expression 'the third highest mountain', but this is quite unnatural. Two, a partition is required (for m_1 to m_4) even though no plural ordinal has been uttered. Again (17) which is intended for plurals, must be used to represent a singular expression, while *ad hoc* sets are construed in order to fill in the parts resulting from the partition. The lesson from this first observation is that there is a strong preference to maintain contextual stability when a singular ordinal is uttered in the absence of other presuppositions (constraints related to the comparison set). To cancel the linear order presupposition and to introduce a partition to the comparison set would require an explicit utterance, entailment will not do.

The second observation is probably related to the first: Once again consider the context as before, when we are looking at those five mountains with their varying heights, does it really make any sense to refer to m_5 as 'the third highest mountain'? Wouldn't it be more intuitive to refer to m_5 as 'the fifth highest mountain' instead? Doing so would imply that we forego using the ordinal to refer to the relative high-ness of m_5 in relation to the rest of the mountains

in the comparison set, but rather, we are counting the number of mountains that come before (as arranged by their high-ness) the one that is asserted. In certain contexts, this seems to be the only option: Consider a context where a married couple has three kids. The first two are identical twins, and the third born several years after. It would be required by convention to refer to the third child as ‘the third’ or ‘the third oldest’, even though by degrees of old-ness, she is in fact ‘the second oldest’. On the other hand, either of the twins can be referred to by ‘the oldest’. But then, where did ‘the second oldest’ go? Suppose another scenario where marathon runners compete, two finished at the same time and won the gold medal, and another who came in third received bronze. Do we refer to the third runner as ‘the second fastest’? If the semantic analysis in this paper is strictly followed, this should be the case. However, this is not how we conventionally refer to the bronze winner. Conventionally, we would regard him as ‘the third fastest’ of the marathon, like ‘the third oldest child’ in the example before.

It appears that for singular ordinals, the ordering is sometimes only indirectly tied to the comparison dimension (especially when convention interferes), and has more to do with the simple counting of the numbers of members in the comparison set that exists before the asserted ordinal itself. In other words, ‘the n_{th} P_{est} N’ suggests that N is not really the n_{th} in a hierarchy of N’s pre-ordered according to their P degrees, but rather, it is merely the n_{th} counting from the beginning of the comparison set, a set which is ordered according to the P degrees. For a more comprehensive answer as to when one usage of the singular ordinal is preferred over the other, I will have to leave that to some large scale survey of the native speakers.

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