# The interpretation of superlative modifiers and deontic modals: An experimental investigation

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**Abstract.** There has been much debate recently about the meaning of superlative modifiers like *at least* and *at most*. The main challenge analyses of superlative modifiers face is accounting for the ignorance implication they give rise to, whereby the speaker holds higher (in *at least*) or lower (in *at most*) numbers as possible. In this study, we present results from two experiments that test the interpretation of superlative modifiers when occurring in sentences with deontic modals. We show that the results of the experiments are only partially predicted by the various competing and incompatible analyses in the literature, and thus argue that an alternative, uniform analysis is required.

**Keywords:** superlative modifiers, ignorance inferences, experimental semantics, experimental pragmatics.

## 1. Introduction

There has been much debate recently about the meaning of superlative modifiers like *at least* and *at most* (Geurts and Nouwen, 2007; Büring, 2008; Cummins and Katsos, 2010; Schwarz, 2011; Cohen and Krifka, 2011; Coppock and Brochhagen, 2013). As Geurts and Nouwen (2007) observe, the superlative modifiers *at least* and *at most* give rise to ignorance inferences: By using a superlative modifier, a speaker generally conveys that she is unsure about the precise value, e.g. *at least 15 pages* in (1a) implies that the speaker considers both exactly 15 and higher values possible and *at most 15 pages* in (1b) implies that the speaker considers 15 and lower values possible, but not values higher than 15.

(1)  a. The paper is *at least* 15 pages long.
     b. The paper is *at most* 15 pages long.

Geurts and Nouwen (2007) also observe that when *at least* is embedded under a necessity modal like *have to* in (2a) and when *at most* is embedded under a possibility modal like *can* in (2b), ignorance inferences can be suppressed. In these cases, an authoritative reading emerges: Under this reading (2a) specifies 15 pages as the minimally required paper length and (2b) specifies 15 pages as the maximally allowed paper length.

(2)  a. Your term paper **has to** be **at least** 15 pages long.
     b. Your term paper **can** be **at most** 15 pages long.

According to Geurts and Nouwen (2007), in the other two combinations, namely when *at least* is embedded under a possibility modal like *can* in (3a) and when *at most* is embedded under a necessity modal like *have to* in (3b), there is no suppression of ignorance inference and the authoritative reading does not seem to be available.[1]

(3)    a.  Your term paper **can** be **at least** 15 pages long.
         b.  Your term paper **has to** be **at most** 15 pages long.

The main challenge analyses of superlative modifiers face is accounting for the ignorance implication they give rise to and the mechanisms that lead to the suppression of these inferences as well as to the availability of the authoritative reading in certain combinations with modals. We show in Section 2 that the various analyses that have been proposed for superlative modifiers differ greatly regarding the predictions they make about the derivation and suppression of ignorance inferences under modals.

Following the discussion of previous analyses, we present results from two experiments that test the interpretation of superlative modifiers when occurring in sentences with deontic modals. In Experiment 1, we tested which combinations of modals and superlative modifiers can obviate speaker ignorance and in Experiment 2, we investigated which readings are available for each one of the superlative modifier-modal combinations.

## 2. Previous analyses of speaker ignorance in superlative modifiers

2.1. Geurts and Nouwen (2007)

Geurts and Nouwen (2007) account for ignorance inferences by incorporating epistemic modality into the lexical entries of superlative modifiers. According to their analysis, *at least* $n$ A *are* B means that the speaker is certain that there is a set of $n$ As that are B and considers it possible that there is a larger set of As that are B. *At most* $n$ A *are* B means that the speaker considers it possible that there is a set of $n$ As that are B and is certain that there is no larger set of As that are B.

Regarding the interaction with modals, Geurts & Nouwen assume that superlative modifiers generally take scope over the modal, which they motivate by the general preference of epistemic operators to outscope deontic ones. They also assume a rule of modal concord, which strips off the layer of epistemic modality in case the primary epistemic operator in the lexical entry of the superlative modifier (epistemic necessity for *at least*, epistemic possibility for *at most*) matches the

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[1]Results from Experiment 2 will call into question the observation that the authoritative reading is not available in these superlative modifier-modal combinations. Also, many readers may find these superlative modifier-modal combinations even ill-formed. Results from Experiment 1 suggest that participants do not always find the constructions in (3) significantly much worse that structures in (2).

modal force of the modal. This accounts for the authoritative reading of sentences like (2): When *at least* $n$ is combined with a deontic necessity modal, the reading resulting from modal concord says that $n$ is deontically necessary and higher numbers are deontically possible, which under a one-sided semantics of numerals (or a monotone semantics of gradable adjectives) is equivalent to saying that $n$ is the minimally required number or the lower bound of the deontic range.

The truth conditions for this reading are visually illustrated in (4a), where $\square$ and $\Diamond$ symbolize deontic necessity and possibility, respectively, and where the straight line signifies permissible paper lengths (which we will also call the deontic range). As modal concord is assumed to be optional, a compositional reading is also predicted to be possible in these cases. Under this compositional construal, *at least* $n$ combined with a necessity modal conveys that the speaker is unsure about the minimally-required number and thinks that the lower bound of the deontic range might be $n$ or more. The truth conditions for this reading are shown in (4b), where $\boxdot$ and $\varodot$ symbolize epistemic necessity and possibility, respectively, and the shaded area (marked with forward slashes) signifies the epistemic range.

(4)    $\square$ + *at least*: The paper has to be at least 15 pages long.

    a.  Modal concord reading: $\square$ LENGTH(p) $\geq$ 15pp $\wedge$ $\Diamond$ LENGTH(p) $>$ 15pp

$$\vdash\!\!\!-\!\!\!-\cdots$$
$$13 \qquad 15 \qquad 17$$

    b.  Compositional reading: $\boxdot\square$ LENGTH(p) $\geq$ 15pp $\wedge$ $\varodot\square$ LENGTH(p) $>$ 15pp

$$[\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!-\!\!-\cdots$$
$$13 \qquad 15 \qquad 17$$

The combination of *at most* $n$ with a possibility modal under the modal concord reading says that $n$ is deontically possible whereas numbers higher than $n$ are deontically impossible; that is, $n$ is the maximally allowed number or the upper bound of the deontic range, cf. (5a). *At most* $n$ plus possibility modal under the compositional reading says that the speaker is unsure about the maximally-allowed number and is only certain that the upper bound of the deontic range is not more than $n$. For all she knows, it might be $n$, cf. (5b).

(5)    $\Diamond$ + *at most*: The paper can be at most 15 pages long.

    a.  Modal concord reading: $\Diamond$ LENGTH(p) $\geq$ 15pp $\wedge$ $\neg\Diamond$ LENGTH(p) $>$ 15pp

$$\cdots\!-\!\!\!-\!\!\!-\dashv$$
$$13 \qquad 15 \qquad 17$$

    b.  Compositional reading: $\varodot\Diamond$ LENGTH(p) $\geq$ 15pp $\wedge$ $\neg\varodot\Diamond$ LENGTH(p) $>$ 15pp

$$\cdots\!-\!\!\!-\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!]$$
$$13 \qquad 15 \qquad 17$$

In the other two combinations, given that the epistemic modal in the superlative modifier and the deontic modal do not correspond in their modal force, modal concord is not possible and thus only a compositional reading conveying speaker ignorance is available. *At least* $n$ combined with a possibility modal is predicted to mean that the speaker is unsure about the maximally allowed number and thinks that the upper bound of the deontic range might be $n$ or more, cf. (6a). *At most* $n$ plus necessity modal says that the speaker is unsure about the minimally required number. While she is sure that the lower bound of the deontic range is not more than $n$, it might be $n$, cf. (6b).

(6)  a.  $\Diamond$ + *at least*: The paper can be at least 15 pages long.
         Compositional reading: $\Box\Diamond$ LENGTH(p) $\geq$ 15pp $\wedge$ $\Diamond\Diamond$ LENGTH(p) $>$ 15pp

         $\cdots$ ——[//////////////////
         13      15      17

     b.  $\Box$ + *at most*: The paper has to be at most 15 pages long.
         Compositional reading: $\Diamond\Box$ LENGTH(p) $\geq$ 15pp $\wedge$ $\neg\Diamond\Box$ LENGTH(p) $>$ 15pp

         //////////////////]—— $\cdots$
         13      15      17

## 2.2.  Nouwen (2010)

Rather than hard-wiring speaker ignorance into the lexical meaning of superlative modifiers as in Geurts and Nouwen (2007), Nouwen (2010) derives ignorance inferences from a covert epistemic possibility modal embedded under the superlative modifier. He proposes that superlative modifiers are degree quantifiers that indicate minima (for *at least*) or maxima (for *at most*). The proposal is built on two additional assumptions: The first is that for gradable predicates, a functional (in terms of =) as well as a relational (in terms of $\geq$) meaning is generally available. The second assumption is that linguistic expressions compete: If a certain meaning can be expressed by linguistic expressions differing in their complexity, the simpler expression is preferred, and thus more complex expressions are blocked. The components of the analysis, taken together, predict that in many cases superlative modifiers cannot be used because the resulting sentences either express a contradiction or a meaning that is equivalent to the sentence with a bare numeral and thus blocked. If superlative modifiers, however, apply to a degree property denoting a range of values, the result is non-contradictory truth-conditions, which are not equivalent to the ones expressed by the bare numeral. This is the case, in particular, if a possibility modal is in their scope. Nouwen thus argues that a covert epistemic possibility modal can be inserted in the scope of the superlative modifier to rescue a structure that would otherwise be ruled out, giving rise to speaker ignorance. That said, there is no need to have an additional covert epistemic one in cases with an overt possibility modal. Nouwen's account thus predicts that authoritative readings always arise if *at least* and *at most* co-occur with a deontic possibility modal, as shown in (7). (The narrow scope reading is either contradictory or blocked by the bare numeral).

(7)  a.  *at least* $n \succ \Diamond$ :                    b.  *at most* $n \succ \Diamond$ :

$$\vdash\!\!\underset{n}{\rule{2cm}{0.4pt}}\cdots \qquad\qquad \cdots\underset{n}{\rule{2cm}{0.4pt}}\!\!\dashv$$

Regarding the interaction with necessity modals, Nouwen's analysis predicts that neither *at least* nor *at most* expresses sensible truth-conditions. (The narrow as well as the wide scope readings are either contradictory or blocked.) But we can assume that these combinations too can be rescued by inserting a covert epistemic possibility modal in the scope of the superlative modifier and above the deontic necessity modal. *At most* combined with a necessity modal will then convey speaker ignorance regarding the lower bound of the deontic range, as illustrated in (8)

(8)  a.  *at most* $n \succ \Diamond \succ \Box$:

$$\text{//////////////}\!\!\vdash\!\!\underset{n}{\rule{1.5cm}{0.4pt}}\cdots$$

Nouwen (2010) moreover proposes that a necessity modal is interpreted as a possibility modal when minimality is at stake, such that *at least* plus necessity modal comes out equivalent to *at least* plus necessity modal and thus has the authoritative reading shown in (7a).

## 2.3.  Büring (2008) and Schwarz (2011)

Büring (2008) proposes an account in which the ignorance implications of superlative modifiers arise as pragmatic inferences. It starts from the intuitive equivalence of *at least n* with *n or more* and builds on the observation that ignorance inferences also arise from disjunction (9).

(9)   Ernie or Bert called.
      ⇝ The speaker is not certain but considers it possible that Ernie called.
      ⇝ The speaker is not certain but considers it possible that Bert called.

The ignorance inferences arising with disjunction are generally analyzed as quantity implicatures. Büring proposes that the lexical semantics of *at least n* corresponds to *n or more* and thus involves disjunction. He assumes, therefore, that the account of ignorance inferences arising with disjunction carries over to superlative modifiers, but semantic equivalence is in fact not sufficient for the generation of implicatures (see Coppock and Brochhagen, 2013).

Schwarz (2011) generalizes Büring's proposal and shows that the effect of speaker uncertainty can be derived systematically as ignorance inferences under a neo-Gricean approach, similarly to scalar implicatures (Sauerland, 2004). Assuming the Horn set {*at least, exactly, at most*} of scalar modifiers in addition to the Horn set of numerals, the stronger scalar alternatives for *at least n* are

*at least* $n'$ for $n' > n$ and *exactly* $n'$ for $n' \geq n$; for *at most*, $n$ the stronger alternatives are *at most* $n$ for $n' < n$ and *exactly* $n'$ for $n' \leq n$. These scalar alternatives are symmetric and thus block the generation of scalar implicatures while leading to ignorance inferences. But if *at least* and *at most* are interpreted in the scope of a necessity modal, the alternatives are not symmetric, an consequently scalar implicatures rather than ignorance inferences are generated, assuming the speaker is competent. These authoritative readings are illustrated in (10a, b).

(10)  a.  $\Box \succ$ *at least* $n$:

$$\underset{n}{\vdash\!\!-\!\!-\!\!-} \cdots$$

b.  $\Box \succ$ *at most* $n$:

$$\cdots \underset{n}{-\!\!-\!\!-\!\!\dashv}$$

c.  *at least* $n \succ \Box$:

$$\underset{n}{[\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!-\!\!-} \cdots}$$
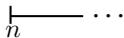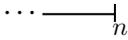
d.  *at most* $n \succ \Box$:

$$\underset{n}{/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!]\!-\!\!-\!\!-} \cdots}$$

This pragmatic account thus predicts that both *at least* and *at most* are able to suppress ignorance inferences and give rise to authoritative readings when they are embedded under a necessity modal. In addition, readings with speaker ignorance are also available if *at least* and *at most* take wide scope over a necessity modal, cf. (10c, d).

With regards to possibility modals, in contrast, the neo-Gricean approach predicts obligatory ignorance inferences for both *at least* and *at most*, since the narrow as well as the wide scope reading leads to symmetric scalar alternatives.[2] The truth-conditions of the readings where superlative modifiers take wide scope over the modal—which are the ones which lead to sensible ignorance implications—are shown in (11).

(11)  a.  *at least* $n \succ \Diamond$:

$$\cdots \underset{n}{-\!\!-\![\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/}}$$

b.  *at most* $n \succ \Diamond$:

$$\cdots\underset{n}{-\!\!-\!\!-\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!/\!]}}$$
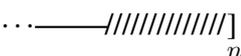
### 2.4. Coppock & Brochhagen (2013)

Coppock and Brochhagen (2013), casting their analysis in Inquisitive Semantics, analyze superlative modifiers as expressions that denote sets of alternatives that are ranked higher (for *at least*) or lower (for *at most*) according to some pragmatic ranking. Speaker ignorance follows from the

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[2]An obvious way to extend the pragmatic account would be to build on the fact that disjunction in combination with possibility modals leads to free choice inferences, which would go beyond the neo-Gricean approach (Fox, 2007, among others). If we assume that the Büring-Schwarz-account can be extended along the line of free choice, we would expect that both *at least* and *at most* give rise to authoritative readings under possibility modals. The predictions of this extended version of the analysis would then be the same as the ones discussed for Coppock & Brochhagen's (2013) account, albeit with a different pragmatic analysis.

MAXIM OF INTERACTIVE SINCERITY, according to which a speaker should only utter a sentence denoting a set of alternatives if her information state is consistent with those alternatives. Regarding the interaction with modals, we again have to consider the wide and narrow scope configurations.

The configurations in which superlative modifiers take wide scope over a modal denote sets of alternatives and thus convey speaker ignorance. The configurations in which superlative modifiers take narrow scope under a modal, no ignorance inferences arise due to Existential Closure, which applies in the scope of modals and whose function is to gather all the alternatives into a single proposition corresponding to the disjunction of all these alternatives. Coppock and Brochhagen (2013) therefore predict that for each superlative modifier-modal combination, both a reading with and without speaker ignorance is possible. In the scope of a necessity modal, *at least* and *at most* specify the lower and upper bound of the deontic range, respectively. For *at most* $n$ in the scope of a possibility modal, Coppock and Brochhagen (2013) argue that the resulting reading, which says that values up to $n$ are permissible, is strengthened by an exhaustivity implicature, according to which values higher than $n$ are not permissible. The same reasoning should apply to *at least* $n$ in the scope of a possibility modal: The truth conditions derived from this structure specify that $n$ and higher numbers are permissible and are subsequently strengthened by an implicature to the effect that lower numbers are not permissible. Note that this strengthening by implicature effectively makes *at least* $+ \Diamond$ equivalent to *at least* $+ \Box$, and *at most* $+ \Diamond$ equivalent to *at most* $+ \Box$, as shown in (12).

(12)    a.  $\Box \succ$ *at least* $n$, $\Diamond \succ$ *at least* $n$:            b.  $\Box \succ$ *at most* $n$, $\Diamond \succ$ *at most* $n$:

$$\underset{n}{\vdash\!\!\!-\!\!\!-\!\!\!-} \cdots \qquad\qquad\qquad \cdots \underset{n}{-\!\!\!-\!\!\!-\!\!\!\dashv}$$

## 2.5. Summary of previous analyses

In sum, all the analyses discussed here make clear predictions regarding (i) which combinations of superlative modifiers and modals potentially give rise to the authoritative reading and which only have a reading conveying speaker ignorance, and (ii) what exactly the truth conditions are for the authoritative reading and the speaker ignorance reading. As the discussion in this section made clear, the different analyses vary considerably regarding the predictions. An experimental investigation that examines these questions is thus warranted. In what follows, we present an experimental paradigm that is designed to answer these research questions, report on the results of our investigation, and discuss their implications. In Experiment 1, we test which combinations bring about ignorance inferences. In Experiment 2, we determine which readings are available for each of the superlative modifier-modal combinations vis-à-vis the truth conditions predicted by the various analyses, as summarized in (13).

(13)  a.  $\square$ + *at least* $n$:        $n$

  i.                    ⊢——— ⋯                          G&N, N, B/S, C&B

  ii.              [//////////////⊢——— ⋯                 G&N, B/S

  b.  $\square$ + *at most* $n$:

  i.              ⋯ ——⊣                                  B/S, C&B

  ii.        ////////////////⊣——— ⋯                      G&N, N, B/S

  c.  $\lozenge$ + *at least* $n$:

  i.              ⊢——— ⋯                                 N, C&B

  ii.        ⋯ ——[/////////////////                       G&N, B/S

  d.  $\lozenge$ + *at most* $n$:

  i.              ⋯ ——⊣                                  G&N, N, C&B

  ii.  ⋯ ——⊣////////////// ]                             G&N, B/S


## 3. Experiment 1: Ignorance inferences of superlative modifier-modal combinations

### 3.1. Research question

In Experiment 1, we tested which combinations of superlative modifiers and modals can suppress ignorance inferences. In order to test these questions, we pitted the speaker's epistemic state, i.e. whether a speaker is knowledgeable or ignorant with respect to the number under discussion, against the ignorance inferences conveyed by a superlative modifier. If it is clear from the context that the speaker has the relevant information, then only utterances that can suppress ignorance inferences will be acceptable. If it is clear from the context that the speaker does not have the information, then utterances that do not lead to ignorance inferences will be ruled out. And finally, if it is not clear from the context whether the speaker has the knowledge or not, then either an utterance that leads to ignorance inference or not would be compatible with the speaker's epistemic state.

### 3.2. Methods

In Experiment 1, 40 participants (18 Female; Mean Age: 43.8; Age range: 26-62) were asked to rate the coherence of a speaker uttering a sentence with a modal and a superlative modifier. The task was conducted on Amazon's Mechanical Turk. Participants received $3.24 for answering 80 experimental items at a rate of $0.03 per item.

In this experiment, participants read scenarios like (14), in which Speaker A asks for information and Speaker B provides this information in the form of an utterance with one of the two types of deontic modals and one of the two types of superlative modifiers. Following the presentation of

the context and utterance, participants were asked to evaluate the speaker's coherence on a Likert scale of $-5$ to $+5$, where $-5$ is *definitely* **not** *coherent* and $+5$ is *definitely coherent*. Cummins and Katsos (2010) show that this scale provides a way to differentiate between semantic falsity and pragmatic infelicity, whereby participants give false statements ratings in the $-5$ region, true and pragmatically felicitous statements in the $+5$ region, and statements that are true but pragmatically infelicitous in the 0 region. However, in contrast with Cummins and Katsos (2010), who asked participants to rate the coherence of the utterance, we directed participants' attention to the *speaker's* coherence instead.[3]

(14)  Boris is applying for a graphic designer position at an ad company. He called the secretary of the company asking for the amount of art works in the portfolio. The secretary, who was involved in the selection process, said:

"You are $\left\{ \begin{array}{c} allowed \\ required \end{array} \right\}$ to have $\left\{ \begin{array}{c} at\ least \\ at\ most \end{array} \right\}$ 3 works in the portfolio you send us."

In light of the context given above, how coherent do you think the speaker is on a scale ranging from -5 to +5, where -5 is definitely not coherent and +5 is definitely coherent?

As discussed in Section 3.1, in order to determine which superlative modifier-modal combinations suppress ignorance inferences, we aimed to contrast the potential ignorance inference or authority reading that the utterances may generate with contextually-explicit speaker ignorance. We expected that mismatches in the explicitly-stated speaker epistemic state and the one inferred by the utterance would have an effect on the coherence rates attributed to the speaker.[4] We created three different speaker epistemic state conditions (with illustrations from the example stimulus in (14)):

(15)   a.  $\pm$knowledgeable: the speaker may or may not have the knowledge
        "The secretary said:"
       b.  $+$knowledgeable: context makes it explicit that the speaker has the knowledge
        "The secretary, who is involved in the selection process and knows all the requirements, said:"

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[3]We modified the task from Cummins and Katsos further by changing the conversational and situational context in which the statement was evaluated. While they had participants evaluate the utterance containing the bare or modified numeral in light of a subsequent sentence in which the numeral was modified by *exactly*, we had participants evaluate the utterance in light of what they could infer about the epistemic state of the speaker.

[4]The reader may observe a potential confound in the design, noting that the speaker's epistemic state can be stated explicitly in the context or (automatically, as some may argue) inferred from the authority of the speaker. In this example stimulus as well, the secretary's knowledge regarding the job search information may be inferred by virtue of her being an employee in the company who holds a position that entails being familiar with the intricacies of, and the processes taking place in, the company. We discuss this potential confound in Section 5. We thank Irene Heim and Ede Zimmerman for discussing this issue with us.

c.  −knowledgeable: context makes it explicit that the speaker does not have the knowledge

"The secretary apologized for not knowing the requirements for the application, and said:"

The experimental design was 2 (modals) × 2 (superlative modifiers) × 3 (speaker conditions). In addition to the 40 target items, there were also 20 control items that consisted of 10 contradictions and 10 entailments, which served as false and true (and pragmatically felicitous), respectively, baseline for comparison with the target items, as well as 20 filler items.

## 3.3. Results

Participants rated entailments as coherent (Mean = +4.41, SD = 0.89) and contradictions as incoherent (Mean = −4.70, SD = 0.63), thereby establishing that the negative and positive ends are associated with false and true (and pragmatically felicitous) sentences, respectively.

First looking at the −knowledgeable speaker condition, participants found statements uttered by an unknowledgeable speaker less coherent than statements uttered by a knowledgeable speaker in general. This was shown by the overall lower coherence in the −knowledgeable condition, regardless of the type of modal or superlative modifier, in comparison with the other two speaker conditions, as well as a main effect of speaker condition in an ANOVA ($F(2,1587) = 98.69$, $p<0.01$). We found that □ + *at least* (Median = 2.5, Mean = 1.14, SD = 3.33) was significantly more coherent than ◊ + *at least* (Median = 1, Mean = 0.32, SD = 3.14; $t(392.59) = 2.51$, $p<0.05$) and that ◊ + *at most* (Median = 2, Mean = 1.32, SD = 2.86) was significantly more coherent than ◊ + *at least* (Median = 1, Mean = 0.32, SD = 3.14; $t(387.69) = 3.31$, $p<0.01$), but did not find any other differences between the various superlative modifier-modal combinations.

In the ±knowledgeable speaker condition, we found a main effect of superlative modifier (ANOVA: $F(1,410) = 4.75$, $p<0.05$) and that superlative modifier was a significant predictor ($\beta = 0.25$, $p < 0.01$), and this manifested itself by ◊ + *at most* (Median = 4, Mean = 3.17, SD = 2.47) judged as significantly more coherent than ◊ + *at least* (Median = 3, Mean = 2.21, SD = 2.79; $t(195.21)=2.57$, $p<0.05$) and □ + *at most* (Median = 4, Mean = 3.04, SD = 2.44) judged as significantly more coherent than ◊ + *at least* ($t(197.65) = 2.27$, $p <0.05$).

And finally, in the +knowledgeable speaker condition, participants judged □ + *at least* to be more coherent (Median = 5, Mean = 3.59, SD = 2.50) than ◊ + *at least* (Median = 3, Mean = 2.42, SD: 2.85; $t(194.80) = 3.08$, $p<0.01$). Likewise, ◊ + *at most* was judged as more coherent (Median = 4, Mean = 3.44, SD = 2.16) than □ + *at most* (Median = 4, Mean = 2.60, SD = 2.99; $t(160.26) = 2.19$, $p<0.05$). Note also that the less coherent combinations in this speaker conditions still had ratings higher than contradiction.
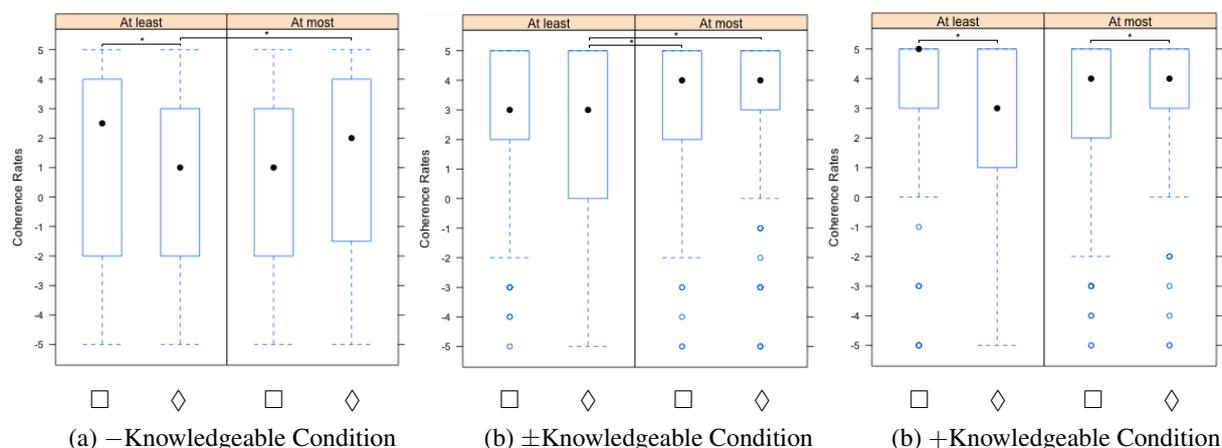
Figure 1: Experiment 1: Boxplots of Coherence Rates in the Three Speaker Conditions

## 3.4. Discussion

In the −knowledgeable and ±knowledgeable conditions, we have found differences between some superlative modifier-modal combinations, but no differences emerged in these speaker conditions that would suggest a grouping predicted by any of the four analyses discussed in the previous section. The results from the +speaker condition, however, show a robust contrast in coherence rates between two groups of superlative modifier-modal combinations. In this speaker condition, the combinations □ + *at least* and ◇ + *at most* are judged as more coherent than the other two combinations, ◇ + *at least* and □ + *at most*. This corresponds to the grouping delineated in Geurts and Nouwen's analysis, but not to the predictions of any of the other analyses discussed in Section 2. Under Geurts and Nouwen's analysis, □ + *at least* and ◇ + *at most* are able to suppress ignorance inferences and allow for an authoritative reading, whereas ◇ + *at least* and □ + *at most* have the speaker insecurity reading only. Under their account, the irrepressible, so to speak, ignorance inferences in the latter two combinations conflict in terms of inferred epistemic certainty with contexts in which the speaker has the knowledge and therefore should be certain about the information conveyed in the utterance. Moreover, since the coherence rates for these combinations in the +knowledgeable condition were low but distinct from those given to contradiction items— that is, since the mismatch between the speaker's certainty and the ignorance inferences generated by these combinations was not judged as a contradiction—we conclude that ignorance inferences are pragmatic rather than semantic (contra Geurts and Nouwen 2007 and in line with, e.g., Büring 2008; Cummins and Katsos 2010; Coppock and Brochhagen 2013).

The coherence rates in the −knowledgeable condition strongly suggest that speakers found statements uttered by a speaker who did not have the information incoherent even though those statements allowed for ignorance inferences and therefore for a match between the contextual epistemic state as well as the one conveyed by the utterance. It seems that the effect of an ignorant

speaker is stronger than the ability of participants to choose the available reading of the superlative modifier-modal combination that would match such ignorance, leading all utterances in this speaker condition to be judged as degraded.

## 4. The truth conditions of superlative modifiers-modal combinations

### 4.1. Research question

In Experiment 2, we tested which interpretations were available for certain combinations of deontic modals and superlative modifiers. As discussed in Section 2, the different analyses on the auction block make different predictions regarding the truth conditions and available inference regarding the permitted values. In what follows, we initially take Geurts and Nouwen (2007) analysis as an example for how our experimental task helps us determine whether the predictions regarding available readings of superlative modifiers under deontic modals are borne out. We then proceed to discuss the other analyses in light of our results.

### 4.2. Methods

In Experiment 2, 40 participants (17 Female; Mean Age: 34.2; Age range: 23-59) read scenarios similar to the ones read in Experiment 1, but in this experiment the contexts were underspecified regarding the knowledge of the speaker, as in (16). The utterance was then followed by a description of an action or a state of affairs concerning A (and directly relevant to B's utterance), in which the stated number was either lower (the UNDER CONDITION) or higher (the OVER CONDITION) than the one used with the superlative modifier in B's utterance. Then, participants were asked to judge whether A acted or was in accordance with what B had said.

(16)   Professor Samsa is teaching an Introduction to Semiotics class. Jeremy, a student in his class, asked him about the length of the paper for the class, and Professor Samsa said:

"Your term paper $\begin{Bmatrix} has\ to \\ can \end{Bmatrix}$ be $\begin{Bmatrix} at\ least \\ at\ most \end{Bmatrix}$ 15 pages long."

Jeremy handed in a $\begin{Bmatrix} 13 \\ 17 \end{Bmatrix}$ page-long paper.

Did the length of Jeremy's term paper comply with Professor Samsa's specifications?

The experimental design was 2 (modal conditions) $\times$ 2 (superlative modifier conditions) $\times$ 2 (under/over conditions). In addition to the 40 target times, there were also 20 control items and 20 filler items. The control items consisted of 10 violation conditions, in which A's action or the description sentence violates B's utterance, and 10 compliance conditions, in which A's action or the

description sentence was in accordance with B's utterance. The task was conducted on Amazon's Mechanical Turk. Participants received \$3.24 for answering 80 experimental items.

Before we discuss the response rates in Experiment 2, we would like to illustrate how the experimental paradigm we utilized is helpful in determining which of the analyses makes the correct predictions about which readings are available for the various superlative modifier-deontic modal combinations. We discuss here the predictions the analysis in Geurts and Nouwen (2007) makes, but in Section 4.4 we return to the other analyses to assess them as well.

As exemplified in (17), under the authoritative reading only higher values than 15 would be allowable when *at least 15* is combined with a necessity modal. Therefore, we expect to get No responses in the Under condition and Yes responses in the Over condition. Under the speaker insecurity reading the speaker is unsure about the minimally-required number and thinks that the lower bound of the deontic range might be 15 or more. Therefore, again, we expect to get No responses in the Under condition. But since the speaker only considers it possible, but is not certain, that 15 or 17 are permissible, both Yes and No responses in the Over condition are compatible with this reading. In sentences like (18), under the authoritative reading only lower values than 15 would be allowable when *at most 15* is embedded under a possibility modal. Therefore we would expect to get Yes responses in the Under condition and No responses in the Over condition. Under the speaker insecurity reading, *at most 15* plus possibility modal conveys that the speaker is unsure about the maximally-allowed number and is only certain that the upper bound of the deontic range is not more than 15; for all she knows, it might be 15. Therefore, again, we would expect to get No responses in the Over condition, while both Yes and No responses in the Under condition are compatible with this reading. It is important to point out that Geurts and Nouwen (2007) do not make any predictions about which one of the readings is preferred, but it is safe to assume that the stronger reading is preferred, especially in an experimental setting that does not allow for hedging or opting out.

(17)  □ + *at least*: The paper has to be at least 15 pages long.

    a.
                                 ⊢────── ⋯
                13      15      17                          authoritative reading

    b.
                                [//////////////────── ⋯
                13      15      17               speaker insecurity reading

(18)  ◇ + *at most*: The paper can be at most 15 pages long.

    a.
                      ⋯ ──────┤
                13      15      17                          authoritative reading

    b.  ⋯──────//////////////]
                13      15      17               speaker insecurity reading

In sentences like (19a), higher values than 15 are consistent with the speaker's knowledge and lower values than 15 are within the deontic range when *at least* is embedded under a possibility

modal. Therefore we would expect Yes responses in the Under condition, while both Yes and No responses in the Over condition are compatible with this reading. In sentences like (19b), lower values than 15 are consistent with the speaker's knowledge and higher values than 15 are within the deontic range when *at most* is embedded under a necessity modal. Therefore we would expect Yes responses in the Over condition, and both Yes and No responses in the Under condition are compatible with this reading.

(19)   a.   $\Diamond$ + *at least*: The paper can be at least 15 pages long.

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              · · · ——[/////////////////
          13        15        17                                speaker insecurity reading
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   b.   $\Box$ + *at most*: The paper has to be at most 15 pages long.

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      /////////////////]——— · · ·
          13        15        17                                speaker insecurity reading
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## 4.3. Results

The results, given in Table 1, show that (i) in the $\Box$ + *at least* combination, the number is interpreted as denoting the lower bound, shown by the fact that the vast majority of participants rejected the description of A in the Under Condition; and that (ii) in the $\Diamond$ + *at most* combination, the number is interpreted as the upper bound, shown by the fact that most participants rejected the description of A in the Over Condition. The other two combinations, $\Box$ + *at most* and $\Diamond$ + *at least*, exhibit a less robust contrast but clearly show that participants tended to interpret the number in the $\Box$ + *at most* combination as specifying the upper bound, shown by the lower Yes rates in the Over Condition, and the $\Diamond$ + *at least* combination as specifying the lower bound, shown by the lower Yes rates in the Under Condition.

| Discrepancy Condition | $\Box$ + *at least* | $\Box$ + *at most* | $\Diamond$ + *at least* | $\Diamond$ + *at most* |
|---|---|---|---|---|
| Over | 90% | 16.67% | 84% | 1.05% |
| Under | 4.7% | 77.65% | 16% | 93.91% |

Table 1: Results from Experiment 2: Means of "Yes" Responses

A series of Welch Two-Sample t-tests (after converting a Yes response to 1 and a No response to 0) to compare the eight different combinations shows that the response rates for each one of the superlative modifiers were different when it was embedded under a necessity modal vs. when it was embedded under a possibility modal. The response patterns for $\Box$ + *at least* and $\Diamond$ + *at most*, however, were not significantly different from each other, and neither were $\Diamond$ + *at least* and $\Box$ + *at most*.

| Conditions Compared | Statistics |
|---|---|
| Over: □ + *at least* vs. ◇ + *at least* | t(379.79)=2.25, p<0.05 |
| Under: ◇ + *at most* vs. □ + *at most* | t(318.50)=4.73, p<0.01 |
| Over: ◇ + *at most* vs. □ + *at most* | t(356.99)=2.94, p<0.01 |
| Under: □ + *at least* vs. ◇ + *at least* | t(286.06)=5.27, p<0.01 |
| Over: □ + *at least* vs. Under: ◇ + *at most* | t(391.02)=0.97, p=0.33 |
| Under: ◇ + *at least* vs. Over: □ + *at most* | t(400.57)=1.69, p=0.09 |
| Under: □ + *at least* vs. Over: ◇ + *at most* | t(409.52)=0.86, p=0.39 |
| Over: ◇ + *at least* vs. Under: □ + *at most* | t(401.50)=1.54, p=0.12 |

Table 2: Welch Two-Sample T-Test Results for Experiment 2

Returning to Table 1, notable response patterns are the ones for □ + *at most* and ◇ + *at least*, as they were not as robust as for the other two combinations. A closer look at the response pattern of each participant revealed that participants were not consistent in their response strategies; that is, we did not find that there were participants who consistently responded to the stimuli with, e.g., □ + *at most* in the Over Condition, with No (which happened in 13.37% of the the cases). We likewise did not find specific stimuli that had led to a consistent response pattern that matched those low-percentage response.

### 4.4. Discussion

In order to evaluate the various analyses, we summarize in (20) the predictions the four analyses discussed in Section 2 make regarding the permitted values and compare them with the results of our Experiment 2. Starting with the combinations □ + *at least* and ◇ + *at most*, for which we found a clear response pattern, it turns out that the predictions of all four analyses are compatible with our results. For □ + *at least* (20a), greater values than $n$ were accepted in 90% of the cases, cf. (20a-i), as all four analyses predict. The additional, weaker, reading given in (20a-iii) may be available as well, but we hypothesize that when participants are faced with a choice between a strong and a weak reading, they will tend to choose the stronger one. For ◇ + *at most* in (20d), only smaller values than $n$ were accepted, cf. (20d-i), as is again predicted in all analyses.

The remaining two combinations, □ + *at most* and ◇ + *at least*, are more interesting, as here the different analyses make distinct predictions and the response patterns we found were not as uniform as for the other two combinations. For □ + *at most* (20b), smaller values than $n$ were accepted in 77.65% of the cases but greater values were not always ruled out, cf. (20b-i). This suggests that in most cases, participants got the authoritative reading (20b-ii) predicted by Büring (2008)/Schwarz (2011) and Coppock and Brochhagen (2013), while a minority got the speaker insecurity reading in (20b-iii), which Geurts and Nouwen (2007) and Nouwen (2010) predict to be the only available

reading.[5] For $\Diamond$ + *at least* (20c), greater values than $n$ were accepted in 84% of the cases, but smaller values were not always ruled out (20c-i). This suggests that in most cases, participants got the authoritative reading in (20c-ii), as predicted by Nouwen (2010) and Coppock and Brochhagen (2013), while a minority got the speaker insecurity reading in (20c-iii), which Geurts and Nouwen (2007) and Büring (2008)/Schwarz (2011) predict to be the only available reading.

(20)  a.  $\Box$ + *at least* $n$:         $n$

      i.          4.7%    <   90%

      ii.       ⊢——— ⋯                             G&N, N, B/S, C&B

      iii.    [/////////////// ——— ⋯                     G&N, B/S

   b.  $\Box$ + *at most* $n$:

      i.            77.65% <   16.67%

      ii.       ⋯ ———⊣                             B/S, C&B

      iii.   ///////////////⊣——— ⋯                     G&N, N, B/S

   c.  $\Diamond$ + *at least* $n$:

      i.           16%    <   84%

      ii.       ⊢——— ⋯                              N, C&B

      iii.    ⋯ ———[///////////////                     G&N, B/S

   d.  $\Diamond$ + *at most* $n$:

      i.            93.91% <   1.05%

      ii.       ⋯ ———⊣                           G&N, N, C&B

      iii.  ⋯———///////////////⊣                    G&N, B/S

## 5. General discussion

Let us summarize what the results of our experimental study can say about the predictions of different analyses of superlative modifiers. First, both the account of Nouwen (2010) and an approach in line with Büring (2008)/Schwarz (2011) seems untenable in light of the results from our Experiments 1 and 2. Each analysis makes some predictions regarding the suppression of ignorance inferences as well as the available readings that are not borne out. Regarding the predictions about which combinations of superlative modifier and deontic modal can suppress ignorance inferences and for which combinations ignorance inferences are obligatory, the analysis in Geurts and Nouwen (2007) fares best. Only their account predicts that $\Box$ + *at least* and $\Diamond$ + *at most* can suppress ignorance inferences, while $\Box$ + *at most* and $\Diamond$ + *at least* cannot. But they also assume that ignorance inferences are semantic, whereas our results suggests that they are pragmatic, which is in line with Büring (2008)/Schwarz (2011) and Coppock and Brochhagen (2013).

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[5]Note that none of the readings predicted by the discussed analyses is compatible with the fact that participants in our experiment rejected higher values in about 22% of the cases. We thus focus on the 16% acceptance rate of higher values.

Regarding available readings in terms of lower and upper bound of the deontic range, only Geurts and Nouwen's (2007) predictions for □ + *at least* and ◇ + *at most* are fully borne out by our Experiment 2. For the other two combinations, □ + *at most* and ◇ + *at least*, they only predict a reading which we found to be only marginally available. The readings we found to be preferred for these combinations are correctly predicted by Coppock and Brochhagen (2013), as are the readings for the other two combinations. But their account does not explain the difference between these two groups of combinations, i.e. why only one reading is attested for □ + *at least* and ◇ + *at most*, whereas for □ + *at most* and ◇ + *at least* different readings seem to be available simultaneously. Thus, each of the existing analyses only partially predicts the results of this study.

In order to set the stage for a possible explanation for the attested response patterns in Experiment 2, we would like to discuss a potential confound in our experimental set-up. In most of our stimuli the speaker in the contexts could be claimed to be an authority on the topic the utterance evolves around. Recall, for example, our stimulus in which the secretary provides the number of works the applicant must send. The secretary's knowledge regarding the job search information may be inferred by virtue of her being an employee in the company who holds a position that entails being familiar with the intricacies of, and the processes taking place in, the company. This is related to the concept of EPISTEMIC AUTHORITY in psychology and sociology, whereby individuals attribute high confidence to information provided by a source they identify as an epistemic authority, consequently often assimilating it to the common ground as uncontested truth (Kruglanski, 1989). What is relevant to this study is that even in contexts in which the knowledge of the speaker was underdetermined, it could be that participants inferred that the speaker did in fact have sufficient information and thus participants would favour the authoritative reading. Note, however, that in Experiment 1 the coherence rates in the ±knowledgeable condition were not the same as in the +knowledgeable, in which the speaker clearly had the relevant information. This may be so because, lacking sufficient information in the ±knowledgeable condition to determine without doubt that the speaker was certain about the value in question, participants differed in how they evaluated the speaker's epistemic state, perhaps even changing this evaluation from stimulus to stimulus. Moreover, this inferred epistemic authority may have led to the overall low coherence ratings in the −knowledgeable condition. If participants did in fact infer that the speaker should be certain about the relevant information given her authority, then utterances that potentially led to ignorance inference would be judged as incoherent due to the mismatch between the contextually-inferred certainty and the semantically/pragmatically-derived ignorance. In sum, even with the potential confound of inferred epistemic authority, the results from Experiment 1 shed light on the interaction between linguistically- vs. contextually-derived inferences about speaker's certainty.

Having noted that participants in our experiment might have been biased towards an authoritative reading, we return to the question why in the majority of the cases participants in Experiment 2 interpreted the numeral as specifying the lower bound when it was modified by *at least*, independently of whether it co-occurred with a necessity or a possibility modal, and why they interpreted the numeral as specifying the upper bound when modified by *at most*, whether co-occurring with a possibility or a necessity modal. If we assume the analysis in Geurts and Nouwen (2007), we could

hypothesize that participants resorted to a reanalysis of the modal to arrive at a clearer reading that better matched the task, an interpretive strategy we dub MODAL REANALYSIS. In the case of $\lozenge$ + *at least* and $\square$ + *at most*, the only possible reading is one in which all values are potentially allowed, some within the deontic range and some within the epistemic range (that is, the speaker cannot rule out any values, modulo pragmatic restrictions involving relevance). This unrestricted reading might be felt to be at odds with the speaker's utterance, which includes two expressions that normally communicate restriction, namely deontic modals and superlative modifiers. Participants might therefore have decided that the reading conveying speaker ignorance was not felicitous and opted to reanalyze the modal to arrive at an authoritative reading. This would also explain why we got mixed results for $\square$ + *at most* and $\lozenge$ + *at least*: It seems that in the majority of cases, participants opted for the strong reading that necessitated modal reanalysis, but in the minority of cases, participants nevertheless opted for the weak reading conveying speaker ignorance.

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