Inquisitive semantics

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Many of the papers referred to in the manuscript can be accessed through:
www.illc.uva.nl/inquisitivesemantics/papers
Finally, to become familiar with the framework presented here, one may want to try out the computational tools available at:
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Chapter 1

Introduction

Inquisitive semantics is a new semantic framework mainly intended for the analysis of linguistic information exchange. Information exchange can be seen as a process of raising and resolving issues. Inquisitive semantics provides a new formal notion of issues, which makes it possible to model various concepts that are crucial for the analysis of linguistic information exchange in a more refined and more principled way than has been possible in previous frameworks. In particular:

1. The **semantic content** of both declarative and interrogative sentences can be represented in an integrated way, capturing not only the information that such sentences convey but also the issues that they may raise;

2. Similarly, **conversational contexts** can be modeled as encompassing not just the information that has been established in the conversation so far, but also the issues that have been brought up;

3. And finally, it becomes possible to formally represent a broader range of **propositional attitudes** that are relevant for information exchange: besides the familiar information-directed attitudes like *knowing* and *believing*, issue-directed attitudes like *wondering* can be captured as well.

This book provides a detailed exposition of the most basic features of inquisitive semantics, and demonstrate some of the advantages that the framework has with respect to previously proposed ways of representing semantic content, conversational contexts, and propositional attitudes.

This introductory chapter will proceed to argue in some detail why a framework like inquisitive semantics is needed for a satisfactory analysis of information exchange (Section 1.1), and will end with a global outline of the remaining chapters (Section 1.2).
Chapter 1. Introduction

1.1 Motivation

The most basic question that needs to be addressed in more detail before we introduce the new formal notion of issues that forms the cornerstone of inquisitive semantics is why such a notion is needed at all for the analysis of linguistic information exchange. This will be done in Section 1.1.1.

A second fundamental point that we want to make is that the analysis of linguistic information exchange does not just require a semantic theory of declaratives and another semantic theory interrogatives side by side, but rather an integrated theory of declaratives and interrogatives; neither sentence type can be fully understood in isolation. Reasons for this will be given in Section 1.1.2.

Finally, a third important point is that a semantic theory of declaratives and interrogatives should not employ two different notions of semantic content, one for declaratives and one for interrogatives, but should rather be based on a single notion of semantic content that is general enough to capture both the information that sentences convey and the issues that they may raise. This point will be substantiated in Section 1.1.3.

1.1.1 Why do we need a formal notion of issues?

There are several reasons why a formal notion of issues is needed for the analysis of linguistic information exchange, and each of these is related to one of the three aspects of information exchange listed above: some arise from the need for a suitable notion of semantic content, some from the need for a suitable model of conversational contexts, and yet others from the need for a sufficiently refined representation of the mental states of conversational participants. We will discuss each in turn.

Reason 1: To represent the content of interrogative sentences

The semantic content of a declarative sentence is standardly construed as a set of possible worlds, those worlds that are compatible with the information that the sentence conveys (as per the conventions of the language; additional information may be conveyed pragmatically when the sentence is uttered). This set of worlds is referred to as the proposition that the sentence expresses.

This notion of semantic content works well for declarative sentences, whose main communicative role is indeed to provide information. For instance, the main communicative function of the declarative sentence in (1) below is to convey the information that Bill is coming.

(1) Bill is coming.

But information exchange typically does not just consist in a sequence of declar-
ative sentences. An equally important role is played by interrogative sentences, whose main conversational role is to raise issues.

Can the semantic content of an interrogative sentence be construed as a set of possible worlds as well? Consider the example in (2), a polar interrogative:

(2) Is Bill coming?

Frege (1918) famously proposed that the interrogative in (2) and the declarative in (1) can indeed be taken to have the same semantic content:

“An interrogative sentence and an indicative one contain the same thought; but the indicative contains something else as well, namely, the assertion. The interrogative sentence contains something more too, namely a request. Therefore two things must be distinguished in an indicative sentence: the content, which it has in common with the corresponding sentence-question, and the assertion.”

(Frege, 1918, p.294)

So the idea is that declaratives and interrogatives have the same semantic content—a proposition—but come with a different force—either assertion or request. This idea has been quite prominent in the literature, especially in speech act theory (Searle, 1969; Vanderveken, 1990). However, as noted by Frege himself, it is limited in scope. It may work for plain polar interrogatives, but not for many other kinds of interrogatives, like (3)-(5):

(3) Is Bill coming, or Sue?
(4) Is Bill coming, or not?
(5) Who is coming?

Moreover, as has been argued extensively in the more recent literature (see especially Groenendijk and Stokhof, 1997), even the idea that a plain polar interrogative has the same content as the corresponding declarative is problematic. In particular, when applied to embedded cases it is not compatible with the principle of compositionality, which requires that the semantic content of a compound expression be determined by the semantic content of its constituent parts, and the way in which these parts are combined. To see this, compare the following two examples, which contain embedded variants of the declarative in (1) and the polar interrogative in (2), respectively:

(6) John knows that Bill is coming.

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1 The page reference is to the translated version, Frege (1956).
2 See also recent work on questions in dynamic epistemic logic (van Benthem and Minică, 2012).
(7) John knows whether Bill is coming.

If the embedded clauses had the same content, then by the principle of compositionality the two sentences as a whole should also have the same content. But this is clearly not the case. So the embedded clauses must differ in content.

Thus, the standard notion of semantic content is not suitable for interrogative sentences. Rather, what we need for the semantic analysis of interrogatives is a notion of content that directly captures the issues that they raise.\(^3\)

**Reason 2: To model conversational contexts**

It has been argued extensively in the literature that conversational contexts have to be modeled in a way that does not only take account of the information that has been established in the conversation so far, but also of the issues that have been brought up, often referred to as the *questions under discussion* (Carlson, 1983; Groenendijk and Stokhof, 1984; van Kuppevelt, 1995; Ginzburg, 1996; Roberts, 1996; Büring, 2003; Beaver and Clark, 2008; Tonhauser *et al.*, 2013, among others). We will briefly discuss two reasons why this is important.

First, it is needed to develop a formal theory of pragmatic reasoning and the conversational implicatures that result from such reasoning. And second, it is needed for a theory of information structural phenomena like topic and focus marking. Let us first consider pragmatic reasoning.

A key notion in pragmatic reasoning is the notion of *relevance*. When is a contribution to a conversation relevant for the purposes at hand? One natural answer is that a contribution is relevant just in case it addresses one of the issues under consideration. Even if the issues under consideration only partially characterize what is 'relevant' in a broader sense, this partial characterization is crucial for a formal theory of conversational implicatures. For, the issues under consideration influence which conversational implicatures arise. To see this, consider the following example:

(8) A: What did you do this morning?
   B: I read the newspaper. \(\leadsto\) B did not do the laundry

(9) A: What did you read this morning?
   B: I read the newspaper. \(\not\leadsto\) B did not do the laundry

B’s utterance is exactly the same in both cases, but the issue that it addresses is different. As a result, in (8), where the question under discussion is what B *did*
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this morning, there is a conversational implicature that B did not do anything besides reading the newspaper, i.e., that he did not do the laundry for instance. On the other hand, in (9), where the question under discussion is what B read this morning, there is a weaker conversational implicature, to the effect that B did not read anything besides the newspaper. This does not imply that he did not do other things, such as the laundry. Thus, we see that pragmatic reasoning is sensitive to the issues that are at play in the context of utterance.

Now let us illustrate the importance of contextual issues for information structural phenomena like focus and topic marking. We will concentrate on focus marking. Languages generally have grammatical ways to signal which part of a sentence is in focus and which part is backgrounded. In English, the focus/background distinction is marked intonationally: focused constituents receive prominent pitch accents, while backgrounded constituents do not. In other languages, focus is sometimes marked by means of special particles or by means of word order.

Which constituents should be marked as being in focus and which should be marked as being backgrounded is determined, at least partly, by the issue that is being addressed. To see this, consider the following examples, where capitalization is used to indicate focus marking by means of prominent pitch accents.

(10) A: Who did Alf rescue?
    B: Alf rescued BEA. / #ALF rescued Bea.

(11) A: Who rescued Bea?
    B: ALF rescued Bea. / #Alf rescued BEA.

If the question is who Alf rescued, as in (10), then the response that Alf rescued Bea must be pronounced with a prominent pitch accent on Bea. Placing a pitch accent on Alf instead results in infelicity. On the other hand, if the question is who rescued Bea, as in (11), then the same response, i.e., that Alf rescued Bea, must be pronounced with a prominent pitch accent on Alf rather than Bea. Thus, we see that focus marking, just like pragmatic reasoning, is sensitive to the issue under discussion.4

4Besides pragmatic reasoning and information structural phenomena like topic and focus marking, it has been argued that a model of conversational contexts that comprises the issues that have been raised is also needed for a suitable analysis of discourse particles (see, e.g. Rojas-Esponda, 2013) and presupposition projection (e.g., Tonhauser et al., 2013).
Reason 3: To model issue-directed propositional attitudes and capture the meaning of verbs that report such attitudes

In order to understand linguistic information exchange, it is important to have a way of representing the information that is available to the agents participating in the exchange, as well as the information that they would like to obtain. In other words, we need to be able to model what the agents know or believe at any given time, and also what they would like to know, i.e., what they wonder about. Knowledge and belief are information-directed propositional attitudes; wondering is an issue-directed propositional attitude. The simplest and most common way to model the knowledge and beliefs of an agent is as a set of possible worlds, namely those worlds that are compatible with what the agent knows or believes. Such a set of worlds is thought of as representing the agent’s information state. Similarly, in order to capture what an agent wonders about, we need a representation of her inquisitive state. For such a representation, we again need a formal notion of issues.

Moreover, turning back to language, just like there are verbs like know and believe that describe the information state of an agent, as in (12) below, there are also verbs like wonder and be curious that describe the inquisitive state of an agent, as in (13).

(12) John knows that Bill is coming.
(13) John wonders who is coming.

Clearly, in order to analyze the meaning of verbs like wonder we do not only need a suitable representation of the content of the interrogative clause that the verb takes as its complement (here, who is coming), but also a suitable representation of the inquisitive state of the subject of the verb (here, John).

1.1.2 Declaratives and interrogatives can’t be treated separately

The analysis of linguistic information exchange requires a semantic theory of declaratives and one of interrogatives. A question that naturally arises, then, is whether the two sentence types could be analyzed separately, or whether a more integrated approach is called for. Below we give two reasons why neither declaratives nor interrogatives can be fully understood in isolation, making an integrated approach necessary.

Reason 1: Mutual embedding

Declarative and interrogative sentences can be embedded into one another, as exemplified in (14)-(16).

(14) Bill asked me who won. embedded interrogative
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(15) Who told you that Jane won?  embedded declarative
(16) Bill asked me who told you that Jane won.  two-level embedding

So the meaning of a declarative sentence is sometimes partly determined by the meaning of an embedded interrogative sentence, and vice versa. Clearly, then, a complete semantic account of declaratives cannot be achieved without getting a handle on interrogatives, and the other way around, a complete semantic account of interrogatives is impossible without a treatment of declaratives. Thus, the two have to be analyzed hand in hand; considering them in isolation is bound to lead to incomplete theories.

Reason 2: Interpretational dependencies

As illustrated in (17) and (18), the interpretation of a declarative sentence sometimes partly depends on the issue raised by a preceding interrogative.\(^5\)

(17) A: What did you do this morning?
B: I only read the newspaper.  \(\sim B\) did not do the laundry
(18) A: What did you read this morning?
B: I only read the newspaper.  \(\not\sim B\) did not do the laundry

If the question is what you did this morning, as in (17), then the declarative statement that you only read the newspaper implies that you did not do other things, like the laundry. On the other hand, if the question is what you read this morning, as in (18), then the same declarative statement just implies that you did not read anything else, which leaves open whether you did anything besides reading, such as the laundry. Thus, not just the pragmatic implicatures that a declarative statement may induce, but even its truth-conditional content can depend on the issue that is addressed, which again means that analyzing declaratives in isolation, without taking interrogatives into account as well, is bound to lead to an incomplete theory.

1.1.3 Why do we need an integrated notion of semantic content?

We have seen that the notion of semantic content that is most commonly assumed for declarative sentences is not suitable for interrogative sentences. In principle, this does not mean that there is anything wrong with this standard notion. We could attempt to construe a suitable notion of content for interrogatives, and maintain the existing notion for declaratives. This, indeed, is the

\(^5\)The difference between examples (17)-(18) and examples (8)-(9) discussed above is that B's response in (17)-(18) contains the particle only. As a consequence, examples (17)-(18) show that not only the pragmatic inferences induced by a declarative statement, but also its semantic content, may depend on the issue that it addresses.
approach that has been taken in previous work (see Groenendijk and Stokhof, 1997, for an overview). We will argue, however, that such a divided strategy is ultimately not adequate. Rather, we need a single, integrated notion of semantic content that is general enough to deal with both declaratives and interrogatives at once.

**Reason 1: Common building blocks**

Declaratives and interrogatives are to a large extent built up from the same lexical, morphological, and intonational elements. Clearly, we would like to have a uniform semantic account of these elements, i.e., an account that captures their semantic contribution in full generality, rather than two separate accounts, one capturing their semantic contribution when they are part of declarative sentences and the other when they are part of interrogative sentences.

To make this concrete, consider the following two examples, a declarative and an interrogative which are built up from exactly the same lexical items and also exhibit the same intonation pattern (we use ↑ and ↓ to indicate rising and falling intonation, respectively).

(19) Luca is from Italy↑ or from Spain↓.
(20) Is Luca from Italy↑ or from Spain↓?

In uttering the declarative in (19), a speaker provides the information that Luca is from Italy or from Spain, and she does not request any further information from other conversational participants. On the other hand, in uttering the interrogative in (20), she takes the information that Luca is from Italy or Spain as given, and requests other participants to provide further information determining exactly which of the two countries he is from.

Both sentences contain the disjunction word *or*. In declaratives, *or* is generally taken to yield the *union* of the semantic values of the two disjuncts. In (19), each disjunct expresses a property, which is standardly represented as a set of individuals—those individuals that satisfy the property. Thus the semantic value of the first disjunct is the set of all individuals from Italy, and the semantic value of the second disjunct is the set of all individuals from Spain. The union of these two sets is the set of all individuals that are either from Italy or from Spain. Sentence (19) says that Luca is one of the individuals in this set.

This seems a reasonable account of *or* in declaratives. But what is the role of *or* in interrogatives? Ultimately, we would like to have an account of *or* that is general enough to capture its semantic contribution in both declaratives and interrogatives in a uniform way. And similarly for other lexical and morphological elements, as well as intonational features that play a role in both types of sentences.
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This can only be achieved if we operate with a notion of semantic content, both at the sentential level and at the level of sub-sentential constituents, that encompasses both informative and inquisitive content. The semantic content of a complete sentence should capture both the information that the sentence conveys and the issue that it raises (where of course, one of these, or both, may be trivial), and the semantic content of any sub-sentential constituent should capture the contribution that this constituent makes both to the information conveyed and to the issue raised by the sentence that it is part of.

Reason 2: Entailment

Entailment is normally thought of as a logical relation between declarative sentences. One sentence is taken to entail another if the first conveys at least as much information as the second. This logical relation plays a central role in the standard logical framework for natural language semantics. For one thing, predictions about entailment constitute one of the primary criteria for empirical success of a semantic theory. That is, a theory is assessed by testing its predictions about entailment. But besides this, entailment is important in various other respects as well. For instance, it plays a crucial role in the derivation of quantity implicatures, which involves comparing the sentence that a speaker actually uttered with other sentences that the speaker could have uttered instead. This comparison is done in terms of informative strength, which is captured by entailment (see Grice, 1975, and much subsequent work). Similarly, entailment is needed to formulate interpretive principles like the Strongest Meaning Hypothesis, which has been argued to play a crucial role in the resolution of semantic underspecification, for instance in the interpretation of plural predication (Dalrymple et al., 1998; Winter, 2001). And as a final example, entailment has been used to characterize the distribution of positive and negative polarity items in terms of upward and downward entailment environments (e.g., Ladusaw, 1980; Kadmon and Landman, 1993).

Clearly, we would like our theories of quantity implicatures, plural predication, polarity items, etcetera, to apply in a uniform way to declarative and interrogative constructions. However, since the standard notion of entailment compares two sentences in terms of their informative, truth-conditional content (and sub-sentential expressions in terms of their contribution to the informative content of the sentences that they are part of), it does not suitably apply to interrogatives. For this reason, the scope of entailment-based theories such as the ones just mentioned is currently restricted to declaratives.

What we need, then, is a notion of entailment that is general enough to apply to both declaratives and interrogatives in a uniform way. Such a notion must be sensitive to both informative and inquisitive strength. An obvious prerequisite for this is that we operate with a notion of semantic content that encompasses
both informative and inquisitive content.

**Reason 3: Logical operations**

Two declarative sentences can be combined by means of conjunction and disjunction.

(21) Peter rented a car and Mary booked a hotel.
(22) Peter rented a car or he borrowed one.

This does not only hold for root declaratives, but also for embedded ones.

(23) Simon believes that Peter rented a car and that Mary booked a hotel.
(24) Simon believes that Peter rented a car or that he borrowed one.

This is also true for interrogatives, both embedded and unembedded ones.

(25) Where can we rent a car, and which hotel should we take?
(26) Where can we rent a car, or who might have one that we could borrow?
(27) Peter is investigating where can we rent a car and which hotel we should take.
(28) Peter is investigating where can we rent a car or who might have one that we could borrow.

These parallels between declaratives and interrogatives do not only exist in English, but in many other languages as well: words that are used to conjoin declaratives are also used to conjoin interrogatives, and words that are used to disjoin declaratives are also used to disjoin interrogatives.

What we would like to have, then, is an account of conjunction and disjunction that does not just apply to declaratives, but that is general enough to apply to both declaratives and interrogatives in a uniform way. Such an account again requires an integrated treatment of declaratives and interrogatives, employing a notion of semantic content that encompasses both informative and inquisitive content.

Besides conjunction and disjunction, another logical operation that can be performed both on declaratives and on interrogatives is *conditionalization*, as exemplified in (29) and (30).

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6While the possibility of conjoining interrogative sentences is uncontroversial, the possibility of disjoining interrogatives has been disputed by Szabolcsi (1997, 2015a) and Krifka (2001b). In Section 7.2 we will examine Szabolcsi’s argument in some detail. On the basis of examples such as (26) and (28), we will argue that disjoining interrogatives is in principle possible, and that the meaning of the resulting disjunction is correctly derived by applying inquisitive disjunction to the meanings of the two interrogative disjuncts.
(29) If Bill asks Mary out, she will accept.
(30) If Bill asks Mary out, will she accept?

This calls for a uniform account of conditionals, one that applies uniformly regardless of whether the consequent is a declarative or an interrogative sentence. Again, such an account can only be provided within a semantic framework which encompasses both informative and inquisitive content.

1.2 Outline

The remaining chapters of this book broadly fall into two parts. The first part, spanning Chapters 2-4, provides a detailed exposition of the basic inquisitive semantics framework. The second part, consisting of Chapters 5-8, discusses several applications of the framework in the analysis of linguistic information exchange, and compares the approach to previous work.

In a bit more detail: Chapter 2 introduces the new notions of issues, propositions, and conversational contexts that form the heart of inquisitive semantics; Chapter 3 identifies the basic operations that can be performed on inquisitive propositions; and Chapter 4 presents an inquisitive semantics for the language of first-order logic.

Then, turning to the second part, Chapter 5 provides a concrete illustration of the benefits of treating declaratives and interrogatives in an integrated way; Chapter 6 argues that the richer notion of semantic content that inquisitive semantics provides is beneficial even if one is just concerned with declaratives, because the truth-conditions of certain declarative sentences—in particular, conditionals—depend on the inquisitive content of their constituents; Chapter 7 discusses the advantages of inquisitive semantics as a framework for the semantic analysis of interrogatives in comparison with previous work on interrogatives; and Chapter 8 discusses the representation of information-directed and issue-directed propositional attitudes, as well as the semantics of verbs like know and wonder which are used to report such attitudes.

Finally, Chapter 9 concludes with a brief summary, Appendix A provides a list of publications, manuscripts, and teaching materials that have served as the main sources for this book, and Appendix B provides some pointers to work that further extends or applies the framework presented here.

1.3 Exercises

EXERCISE 1.1. In Section 1.1.1 we discussed several reasons why a formal notion of issues is needed for the analysis of linguistic information exchange. One other area where such a notion is arguably needed is in logical frameworks that
are designed to reason about dependencies (cf., Väänänen, 2007; Yang, 2014; Ciardelli, 2016a). Why are dependencies and issues intrinsically connected? Can you think of other areas where a formal notion of issues is required?

EXERCISE 1.2. In Section 1.1.2 we argued that declaratives and interrogatives cannot be fully understood in isolation. Rather, they need to be analyzed in an integrated way, operating with a notion of semantic content that can deal with both declaratives and interrogatives at once. One domain not discussed above where such an integrated notion of semantic content seems needed is that of discourse particles. Consider the following examples from Iatridou and Tatevosov (2015) involving the particle *even* in interrogative sentences:

(31) A: Let’s go to Oleana’s for dinner.  
    B: Where is that even?

(32) A: I want to study the Penutian language Tunica.  
    B: Where is that even spoken?

The particle *even* can also be used in declaratives, as in (33).

(33) Even Lev came to the party.

We would of course like to have a uniform account of *even* that applies both to occurrences in declaratives and to occurrences in interrogatives. Can you think of one? Can you perhaps think of yet other reasons why it is necessary to pursue an integrated account of declaratives and interrogatives?
In this chapter we introduce four notions that form the cornerstone of inquisitive semantics—*information states, issues, propositions, and conversational contexts*—as well as a number of fundamental relations that may hold between them. In particular, as depicted in Figure 2.1, we will specify what it means for an information state to *resolve* an issue or to *support* a proposition, what it means for a context to be *updated* with a proposition, when one context is an *extension* of another, when one proposition *entails* another, when one information state is an *enhancement* of another, and when one issue is a *refinement* of another.

Before turning to the inquisitive setting, however, we first briefly review how these notions—with the exception of issues—are standardly defined.

### 2.1 The standard picture

The simplest way to construe information states, propositions, and conversational contexts is as *sets of possible worlds* (see, e.g., Hintikka, 1962; Stalnaker, 1978). A set of possible worlds can be thought of as representing a certain *body of information*, namely the information that the actual world corresponds to one of the worlds in the set. Such a body of information may be seen as the information available to a certain conversational participant; in that case it can be taken to represent the information state of that participant. On the other hand, a body of information may also be seen as the information conveyed by a certain sentence; in that case it can be taken to constitute the semantic content of that sentence, the proposition that it expresses. And finally, a body of information could be seen as the information that has so far been commonly established by all the participants in a conversation; in that case it embodies the *common ground* of the conversation, which constitutes a minimal representation
Chapter 2. Basic notions

Figure 2.1: Basic notions in inquisitive semantics.

of the conversational context.\textsuperscript{1} Thus, depending on the perspective one takes, one and the same type of formal object—a set of possible worlds—can be used to model all three basic notions.

Let us now turn to the notions of enhancement (between information states), entailment (between propositions), and extension (between contexts). One information state \(s\) is an enhancement of another information state \(s'\) just in case all the information available in \(s'\) is also available in \(s\), i.e., if every candidate for the actual world that is ruled out by \(s'\) is also ruled out by \(s\). This holds just in case \(s \subseteq s'\). Similarly, one proposition \(p\) entails another proposition \(p'\) if and only if \(p\) contains at least as much information as \(p'\) does, i.e., if \(p \subseteq p'\), and one context \(c\) is an extension of another context \(c'\) if and only if all the information that is commonly established in \(c'\) is also commonly established in \(c\), i.e., if \(c \subseteq c'\). Thus, enhancement, entailment, and extension again formally all amount to the same relation, i.e., \textit{set inclusion}, though in each case we take a

\textsuperscript{1}Sometimes a distinction is made between the \textit{common ground} of a conversation and the \textit{context set} (Stalnaker, 1978). The common ground is then construed as the set of pieces of information that are publicly shared among the conversational participants, and the context set as the set of possible worlds that are compatible with all these pieces of information. For our purposes, it will not be necessary to make this distinction, so we simply construe the common ground as the set of possible worlds that are compatible with the commonly established body of information.
somewhat different perspective on what this formal relation encodes, mirroring
the different perspectives on sets of possible worlds when viewed as information
states, propositions, and conversational contexts.

Now let us turn to the notion of support, which relates information states
to propositions. An information state \( s \) is standardly taken to support a propo-
sition \( p \) just in case the information embodied by \( p \) is already available in \( s \),
i.e., if every candidate for the actual world that is ruled out by \( p \) is ruled out
by \( s \) as well. This holds just in case \( s \subseteq p \). So support, just like entailment,
enhancement, and extension, formally amounts to set inclusion.

Finally, let us consider the notion of update. The result of updating a con-
text \( c \) with a proposition \( p \) is a new context \( c[p] \) which, besides the information
already present in \( c \), also contains the information embodied by \( p \). That is, a
candidate for the actual world is ruled out by \( c[p] \) if it was already ruled out by
the information established in the old context \( c \), or if it is ruled out by the new
information embodied by \( p \). Formally, this means that update amounts to set
intersection: \( c[p] = c \cap p \).

What we have just reviewed is the simplest possible way to define information
states, propositions, conversational contexts, and the relations that may hold
between them in possible world semantics. Various more fine-grained versions of
these basic notions have been proposed in the literature. Our goal here, however,
is to construct the direct counterparts of these basic notions, together with a
new notion of issues, in the inquisitive setting. Once these elementary notions
are in place, one could set out to adapt the various refinements that have been
proposed in the standard setting to the inquisitive setting as well. This will not
be our direct concern in this book, but we will point to other work where such
refinements have been pursued.

We are now ready to start building up the inquisitive semantics framework,
starting with the notion of information states.

2.2 Information states

Information states are modeled in inquisitive semantics just as they are in the
standard setting, namely as sets of possible worlds—those worlds that are com-
patible with the information available in the state. There is no need to change
this formal notion of information states since—unlike in the case of propositions
and conversational contexts, as we will see below—the pre-theoretic notion of
an information state really just concerns the information available to a certain
agent, not the issues that the agent entertains or anything else issue-related.

Even though we straightforwardly adopt the standard notion of information
states, we will define, discuss, and exemplify the notion somewhat more explic-
We use $W$ to denote the entire logical space, i.e., the set of all possible worlds.

\textbf{Definition 2.1. [Information states]}
An information state $s$ is a set of possible worlds, i.e., $s \subseteq W$.

We will often refer to information states simply as states. Figure 2.2 depicts some examples of information states in a logical space consisting of just four possible worlds: $w_1, w_2, w_3, w_4$. Intuitively, an information state can be thought of as locating the actual world within a certain region of the logical space. For instance, the state in Figure 2.2(d) contains the information that the actual world is located in the upper left corner of the logical space, while the state in Figure 2.2(c) contains the information that the actual world is located in the upper half of the logical space.

If $s$ and $t$ are two information states and $t \subseteq s$, then $t$ contains at least as much information as $s$; it locates the actual world with at least as much precision. In this case, we call $t$ an \textit{enhancement} of $s$.

\textbf{Definition 2.2. [Enhancements]}
A state $t$ is called an enhancement of $s$ just in case $t \subseteq s$.

Note that we do not require that $t$ is strictly contained in $s$, i.e., that it contains strictly more information than $s$. If $t = s$, then we call $t$ a \textit{trivial} enhancement of $s$. If $t \subset s$, then we say that $t$ is a \textit{proper} enhancement of $s$.

The four information states depicted in Figure 2.2 are arranged from left to right according to the enhancement order. The state in Figure 2.2(b) is an enhancement of the state in Figure 2.2(a), and so on. The state consisting of all possible worlds, $W$, depicted in Figure 2.2(a), is the least informed of all information states: any possible world is still taken to be a candidate for the actual world, which means that we have no clue at all what the actual world is like. This state is therefore referred to as the \textit{ignorant state}. Every other state is an enhancement of it.
2.3 Issues

At the other far end of the enhancement order is the empty state, ∅. This is an enhancement of any other state. It is a state in which all possible worlds have been discarded as candidates for the actual world, i.e., the available information has become inconsistent. It is therefore referred to as the inconsistent state.

2.3 Issues

We now turn to the notion of issues, in a sense the most central notion in inquisitive semantics. How should issues be represented formally? Our proposal is to characterise issues in terms of what information it takes to resolve them. That is, an issue is identified with a set of information states: those information states that contain enough information to resolve the issue.

We assume that every issue can be resolved in at least one way, which means that issues are identified with \textit{non-empty} sets of information states. Moreover, if a certain state \( s \) contains enough information to resolve an issue \( I \), then this must also hold for every enhancement \( t \subseteq s \). This means that issues are always \textit{downward closed}: if \( I \) contains a state \( s \), then it contains every \( t \subseteq s \) as well. Thus, issues are defined as non-empty, downward closed sets of information states.\(^2\)

\textbf{Definition 2.3.} [Issues]
An issue is a non-empty, downward closed set of information states.

\textbf{Definition 2.4.} [Resolving an issue]
We say that an information state \( s \) resolves an issue \( I \) just in case \( s \in I \). If \( s \) resolves \( I \), we will sometimes also say that \( I \) is \textit{settled} in \( s \).

If \( I \) is an issue and \( s \) an information state such that every world in \( s \) is included in a state that resolves \( I \), i.e., \( s \subseteq \bigcup I \), and vice versa, every state that resolves \( I \) is an enhancement of \( s \), i.e., \( \bigcup I \subseteq s \), then we say that \( I \) is an issue \textit{over} \( s \).

\textbf{Definition 2.5.} [An issue over a state]
Let \( I \) be an issue and \( s \) an information state. Then we say that \( I \) is an issue over \( s \) if and only if \( \bigcup I = s \).

Notice that an issue \( I \) over a state \( s \) may contain \( s \) itself. In this case resolving \( I \) does not require any information beyond the information that is already available in \( s \). If so, we call \( I \) a \textit{trivial} issue over \( s \). Downward closure implies that for

\(^2\)Notice that this means that the inconsistent information state, ∅, is an element of every issue. Thus, it is assumed that every issue is resolved in the inconsistent information state. This limit case may be regarded as a generalization of the usual \textit{ex falso quodlibet} principle to issues.
Any state \(s\) there is precisely one trivial issue over \(s\), namely the issue consisting of all enhancements of \(s\), i.e., the powerset of \(s\), which we denote as \(\wp(s)\). On the other hand, if \(s \not\in I\), then in order to settle \(I\) further information is required, that is, a proper enhancement of \(s\) must be established. In this case we call \(I\) a proper issue over \(s\).

Two issues over a state \(s\) can be compared in terms of what it takes for them to be settled: one issue \(I\) is at least as inquisitive as another issue \(J\) just in case any state that settles \(I\) also settles \(J\). In this case we also say that \(I\) is a refinement of \(J\). Since an issue is identified with the set of states that settle it, the refinement order on issues just amounts to inclusion.

**Definition 2.6. [Issue refinement]**

Let \(I, J\) be two issues over a state \(s\). Then \(I\) is at least as inquisitive as \(J\) if and only if \(I \subseteq J\). In this case, we say that \(I\) is a refinement of \(J\).

Among the issues over a state \(s\) there is always a least and a most inquisitive one. The least inquisitive issue over \(s\) is the trivial issue \(\wp(s)\) whose resolution, as we saw, requires no information beyond the information already available in \(s\). The most inquisitive issue over \(s\) is \(\{\{w\} \mid w \in s\} \cup \{\emptyset\}\), which can only be settled consistently by providing a complete description of what the actual world is like.

Suppose that a given information state \(s\) resolves an issue \(I\), and there is no weaker information state \(t \supset s\) that also resolves \(I\). Then \(s\) contains just enough information to resolve \(I\), it does not contain any superfluous information. Technically, these states are the maximal elements of \(I\), since information states consisting of more worlds contain less information. We will refer to these maximal elements as the alternatives in \(I\).

**Definition 2.7. [Alternatives in an issue]**

The maximal elements of an issue \(I\) are called the alternatives in \(I\).

If an issue \(I\) over a state \(s\) is trivial, i.e., if \(s \in I\), then \(s\) is the unique maximal element of \(I\), i.e., the unique alternative in \(I\). On the other hand, if \(I\) contains two or more alternatives then it must be non-trivial. If an issue contains only finitely many information states, which will be the case in all the examples that we will consider here, then the connection between containing multiple alternatives and being a non-trivial issue also holds in the other direction. That

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3Our use of the term *alternatives* here is closely related to its use in the framework of *alternative semantics* (cf., Hamblin, 1973; Kratzer and Shimoyama, 2002; Simons, 2005; Alonso-Ovalle, 2006; Aloni, 2007, among others). We will return to the connection between inquisitive semantics and alternative semantics in Section 4.7 and in Section 7.1.
2.4 Propositions

Traditionally, the semantic content of a sentence, the proposition that it expresses, is intended to capture the information that a speaker conveys in asserting the sentence (as per the conventions of the language; additional information

Figure 2.3: Some issues over the state \( \{w_1, w_2, w_3, w_4\} \).

is, an issue with finitely many elements is non-trivial if and only if it contains at least two alternatives.\(^4\)

**Fact 2.8.** [Multiple alternatives and proper issues]
An issue containing finitely many elements is non-trivial if and only if it contains at least two alternatives.

Figure 2.3 displays some issues over the information state \( s = \{w_1, w_2, w_3, w_4\} \).
In order to keep the figures neat, only the alternatives in these issues are depicted. Since issues are downward closed, we know that all enhancements of these alternatives are also included in the issues at hand. The issue depicted in subfigure (a) is the most inquisitive issue over \( s \), which can only be settled consistently by specifying precisely which world is the actual one. The issue depicted in subfigure (b) can be settled either by establishing that the actual world is an element of the set \( \{w_1, w_2\} \), or by establishing that it is an element of \( \{w_3, w_4\} \). The issue depicted in subfigure (c) can be settled either by establishing that the actual world is an element of \( \{w_1, w_3, w_4\} \), or by establishing that it is an element of \( \{w_2, w_3, w_4\} \). Finally, subfigure (d) displays the trivial issue over \( s \), which is already settled in \( s \) itself. Both (b) and (c) are less inquisitive than (a) and more inquisitive than (d), while neither of them is more inquisitive than the other.

\(^4\)To see that this does not generally hold for issues containing infinitely many states, consider an issue \( I \) that consists of an infinite chain of states, \( s_1 \subset s_2 \subset s_3 \subset \ldots \), without any maximal element. Such an issue is non-trivial, since \( \bigcup I \not= I \), but it does not contain any alternatives. So, if an issue contains at least two alternatives, then it is always non-trivial, but the reverse implication only holds if we restrict ourselves to finite cases (Ciardelli, 2009).
Chapter 2. Basic notions

may be conveyed through pragmatic implicatures). In inquisitive semantics, propositions are not just intended to capture the information that is conveyed in uttering a sentence, but also the issue that may be raised in doing so. In short, propositions are intended to embody both informative and inquisitive content.

How should such more versatile propositions be modeled formally? The most straightforward option would be to construe a proposition \( P \) as a pair \( \langle \text{info}_P, \text{issue}_P \rangle \), where \( \text{info}_P \) is a classical proposition, i.e., a set of possible worlds, embodying the informative content of \( P \), and \( \text{issue}_P \) an issue, embodying the inquisitive content of \( P \). We can then think of a speaker who utters a sentence expressing the proposition \( P \) as (i) providing the information represented by \( \text{info}_P \), and (ii) raising the issue represented by \( \text{issue}_P \). By the latter we mean that the speaker proposes to enhance the current common ground of the conversation in such a way that it comes to settle \( \text{issue}_P \); for short, we will say that the speaker ‘steers the common ground towards a state in \( \text{issue}_P \).

This notion of propositions is a natural starting point, but note that it does not impose any constraints on how the two components of a proposition should be related to each other. There are two constraints that we think should be enforced. First, all states that resolve \( \text{issue}_P \) should be enhancements of \( \text{info}_P \). It would not make sense for a speaker to steer the common ground towards a state where the information embodied by \( \text{info}_P \) itself is not commonly established. Formally, this means we should have that \( \bigcup \text{issue}_P \subseteq \text{info}_P \).

Second, the information that a speaker conveys should ensure that the issue she raises can be resolved truthfully. This means that we should have that \( \text{info}_P \subseteq \bigcup \text{issue}_P \). To see this, suppose that \( \text{info}_P \nsubseteq \bigcup \text{issue}_P \). Then there is a world \( w \in \text{info}_P \) which is not contained in any state that resolves \( \text{issue}_P \). According to \( \text{info}_P \), \( w \) may well be the actual world. Now, suppose it is the actual world. Then there is no way of resolving \( \text{issue}_P \) without discarding the actual world. So, in this case \( \text{info}_P \) does not ensure that \( \text{issue}_P \) can be resolved truthfully.

Putting the two constraints together, we get that \( \text{issue}_P \) should be an issue over \( \text{info}_P \): \( \bigcup \text{issue}_P = \text{info}_P \). Given this, our formal notion of propositions can be simplified considerably. After all, since \( \text{info}_P \) can always be retrieved from \( \text{issue}_P \), it can just as well be left out of the representation of \( P \). Thus, a proposition \( P \) can simply be represented as a non-empty, downward closed set of information states. The informative content of \( P \) is then represented by the union of all these states, \( \bigcup P \), while the issue embodied by \( P \) is the one which is resolved in a state \( s \) just in case \( s \in P \).

**Definition 2.9. [Propositions]**

- A proposition \( P \) is a non-empty, downward closed set of information states.
- The set of all propositions will be denoted by \( \mathcal{P} \).
2.4. Propositions

**Definition 2.10.** [Informative content]
For any proposition $P$: $\text{info}(P) := \bigcup P$

**Definition 2.11.** [The issue embodied by a proposition]
The issue embodied by a proposition $P$ is the one that is resolved in a state $s$ just in case $s \in P$.

**2.4.1 Truth and support**

We say that a proposition $P$ is *true* in a world $w$ just in case $w$ is compatible with the informative content of $P$, i.e., $w \in \text{info}(P)$.

**Definition 2.12.** [Truth]
A proposition $P$ is true in a world $w$ just in case $w \in \text{info}(P)$.

We say that an information state $s$ *supports* a proposition $P$ just in case it implies the informative content of $P$, i.e., $s \subseteq \text{info}(P)$, and it resolves the issue embodied by $P$, i.e., $s \in P$. But note that if $s \in P$ then it must also be the case that $s \subseteq \text{info}(P)$. So supports just amounts to membership.

**Definition 2.13.** [Support]
An information state $s$ supports a proposition $P$ if and only if $s \in P$.

From the fact that propositions are downward closed it follows that truth and support are closely connected: a proposition $P$ is true in a world $w$ just in case it is supported by the singleton state \{w\}.

**Fact 2.14.** [Truth and support]
A proposition $P$ is true in a world $w$ if and only if $P$ is supported by \{w\}.

The notion of support will become very useful later on. Notice that the relation between propositions and support is exactly the same as that between issues and resolution: a proposition consists of all states that support it; an issue consists of all states that resolve it. Moreover, the relation between propositions and support in inquisitive semantics is also parallel to the relation between classical propositions and truth: a classical proposition is the set of all worlds in which it is true. In the present setting, truth does not relate directly to propositions in this way, but rather to the informative content of a proposition: the informative content of a proposition is the set of all worlds in which the proposition is true. Evidently, the fact that the connection between truth and propositions is more direct in the classical setting is an immediate consequence of the fact that classical propositions exclusively encode informative content.
2.4.2 Informative and inquisitive propositions

We will say that a proposition $P$ is informative just in case its informative content is non-trivial, i.e., $\text{info}(P) \neq W$. On the other hand, we will say that $P$ is inquisitive just in case establishing its informative content is not sufficient to settle the issue that it raises, i.e., $\text{info}(P) \notin P$.

**Definition 2.15.** [Informative and inquisitive propositions]

- A proposition $P$ is informative iff $\text{info}(P) \neq W$.
- A proposition $P$ is inquisitive iff $\text{info}(P) \notin P$.

Just as we did in the case of issues, we refer to the maximal elements of a proposition as the alternatives in that proposition. These are states that support the proposition and cannot be weakened in any way without losing support. That is, they contain just enough information to support $P$.

**Definition 2.16.** [Alternatives in a proposition]

- The maximal elements of a proposition $P$ are called the alternatives in $P$.
- The set of alternatives in $P$ is denoted as $\text{alt}(P)$.

When discussing issues, we noted that there is a close connection between containing multiple alternatives and being non-trivial. For propositions, there is a parallel connection between containing multiple alternatives and being inquisitive. Namely, if $P$ contains two or more alternatives then it cannot contain $\text{info}(P)$ and therefore must be inquisitive. On the other hand, if a proposition $P$ is non-inquisitive, i.e., if $\text{info}(P) \in P$, then it always contains a unique alternative, namely $\text{info}(P)$. If a proposition contains only finitely many information states, which is the case in all the examples that we will consider, then the connection between multiple alternatives and inquisitiveness is even stronger. Namely, a proposition with finitely many elements is inquisitive if and only if it contains multiple alternatives.\(^5\)

**Fact 2.17.** [Inquisitiveness and alternatives]

A proposition containing finitely many elements is inquisitive if and only if it contains multiple alternatives.

\(^5\)See footnote 4 for an example showing that an inquisitive issue with infinitely many states does not necessarily contain multiple alternatives; it may not contain any alternatives at all. A parallel example can easily be constructed for propositions.
2.4. Propositions

Figure 2.4 depicts a number of propositions. In each case, we only depict the alternatives that the proposition contains. The proposition depicted in Figure 2.4(a) contains just one alternative and is therefore not inquisitive, but it is informative, since its informative content does not cover the entire logical space. The proposition depicted in Figure 2.4(b) contains two alternatives and is therefore inquisitive; on the other hand, it is not informative, because its informative content, i.e., the union of the two alternatives, covers the entire logical space. The proposition depicted in Figure 2.4(c) is both informative and inquisitive, since it contains two alternatives and the union of these two alternatives does not cover the entire logical space. Finally, the proposition depicted in Figure 2.4(d) contains a single alternative, $W$, and is therefore not inquisitive; it is not informative either because its informative content covers the entire logical space.

2.4.3 Statements, questions, hybrids, and tautologies

It will be convenient to distinguish several classes of propositions, based on whether they are informative and/or inquisitive. First, we will refer to non-inquisitive propositions as statements, and to non-informative propositions as questions. The conversational effect of a statement, if any, is just to provide information, while the conversational effect of a question, if any, is just to raise an issue. Second, we will refer to a proposition that is both informative and inquisitive as a hybrid, and to a proposition that is neither informative nor inquisitive as a tautology.

Definition 2.18. We say that a proposition $P$ is:

- a statement iff it is non-inquisitive;
- a question iff it is non-informative;
- a hybrid iff it is both informative and inquisitive;
Questions

Hybrids

Tautologies

Statements

Figure 2.5: Propositions in a two-dimensional space.

- a tautology iff it is neither informative nor inquisitive.

Each of these four classes of propositions is instantiated by one of the propositions depicted in Figure 2.4 above. The proposition in Figure 2.4(a) is not inquisitive, so it is a statement; the proposition in Figure 2.4(b) is not informative, so it is a question; the proposition in Figure 2.4(d) is both informative and inquisitive, so it is a hybrid; and finally, the proposition in Figure 2.4(d) is neither informative nor inquisitive, so it is a tautology.

Propositions can be thought of as inhabiting a two-dimensional space, as depicted in Figure 2.5. The horizontal axis is inhabited by statements, which are non-inquisitive. The vertical axis is inhabited by questions, which are non-informative. The ‘zero-point’ of the space is inhabited by the tautology, which is neither informative nor inquisitive. The rest of the space is inhabited by hybrids, which are both informative and inquisitive.

Spelling out what it means to be non-informative and/or non-inquisitive we obtain the following direct characterization of statements, questions and tautologies.

**Fact 2.19.** [Direct characterization of statements, questions and tautologies]

- $P$ is a statement iff $\text{info}(P) \in P$.
- $P$ is a question iff $\text{info}(P) = W$.
- $P$ is a tautology iff $W \in P$.

It will be insightful (and useful for later) to consider a number of alternative characterizations of statements as well.

**Fact 2.20.** [Alternative characterizations of statements]

The following are equivalent for any proposition $P$:
2.4. Propositions

1. $P$ is a statement;

2. $P = \wp(\text{info}(P))$

3. $P$ is supported by a state $s$ just in case $P$ is true in all worlds in $s$;

4. $P$ has a greatest element.

The equivalence between 1 and 2 immediately follows from the direct characterization of statements in Fact 2.19. After all, if $P = \wp(\text{info}(P))$ then clearly $\text{info}(P) \subseteq P$. Vice versa, if $\text{info}(P) \subseteq P$ then by downward closure every substate of $\text{info}(P)$ must we in $P$ as well, so we have that $\wp(\text{info}(P)) \subseteq P$. But since by definition $\text{info}(P) = \bigcup P$, we also have that $P \subseteq \wp(\text{info}(P))$. Putting the two together, we get that $P = \wp(\text{info}(P))$.

The equivalence between 2 and 3 is also quite immediate. If $P = \wp(\text{info}(P))$ then every state $s$ that supports $P$, i.e., every $s \in P$, must be contained in $\text{info}(P)$. But this means that for every $w \in s$ we have that $w \in \text{info}(P)$, which is just to say that $P$ is true in $w$. Vice versa, if 3 holds then $\text{info}(P)$, which is the set of all worlds where $P$ is true, must support $P$. Thus, $\text{info}(P) \subseteq P$. From this we can derive, as we did above, that $P = \wp(\text{info}(P))$.

Finally, let us establish the equivalence between 4 and the direct characterization of statements in Fact 2.19. Clearly, if $P$ has a greatest element, then this greatest element must be $\bigcup P$, which is $\text{info}(P)$. So we have that $\text{info}(P) \subseteq P$. Vice versa, if $\text{info}(P) \subseteq P$, then $\text{info}(P)$ must be the greatest element of $P$.

The characterization of statements given in 4 makes it particularly easy to say whether a proposition is a statement given a visualization of it—we just have to check whether it has a greatest element. We already established in Fact 2.17 that a proposition containing finitely many elements is inquisitive if and only if it contains at least two alternatives, and therefore a statement if and only if it contains just one alternative, i.e., one maximal element. The present characterization in terms of greatest elements is more general since it applies to infinite propositions as well.

The characterization of statements in 3 is of particular interest as well because it brings out the fact that statements are precisely those propositions which have the classical feature that being supported by an information state amounts to being true in all worlds in that state; in short, support is fully determined by truth. This is not the case for inquisitive propositions. For instance, the proposition depicted in Figure 2.4(c) is true in both in $w_2$ and in $w_3$, but it is not supported by the state consisting of these two worlds.

Finally, one particular consequence of the characterization of statements given in 2 is that there is only one proposition that counts as a tautology, namely $\wp(W)$. After all, tautologies are non-informative statements, i.e., ones whose informative content is trivial. So if $P$ is a tautology, then we must have
that \( \text{info}(P) = W \). But then, according to the characterization in 2, it must be the case that \( P = \varphi(W) \).

2.4.4 Entailment

Propositions can be ordered both in terms of their informative component and in terms of their inquisitive component. First, a proposition \( P \) is at least as informative as another proposition \( Q \) if and only if the informative content of \( P \) determines with at least as much precision what the actual world is like as the informative content of \( Q \), i.e., \( \text{info}(P) \subseteq \text{info}(Q) \).

**Definition 2.21.** [Informative order on propositions]
For any \( P, Q \in \mathcal{P} \):

- \( P \models_{\text{info}} Q \) iff \( \text{info}(P) \subseteq \text{info}(Q) \)

Similarly, we say that \( P \) is at least as inquisitive as \( Q \) just in case any state that settles the issue embodied by \( P \) also settles the issue embodied by \( Q \), i.e., if and only if \( P \subseteq Q \).

**Definition 2.22.** [Inquisitive order on propositions]
For any \( P, Q \in \mathcal{P} \):

- \( P \models_{\text{inq}} Q \) iff \( P \subseteq Q \)

Combining these two orders, we say that \( P \) entails \( Q \) just in case \( P \) is both at least as informative and at least as inquisitive as \( Q \). But note that if \( P \subseteq Q \), then it must also automatically hold that \( \text{info}(P) \subseteq \text{info}(Q) \). So entailment simply amounts to inclusion.

**Definition 2.23.** [Entailment]
For any \( P, Q \in \mathcal{P} \):

- \( P \models Q \) iff \( P \subseteq Q \)

Entailment between two propositions can also be characterized in terms of support, just like classical entailment can be characterized in terms of truth: one proposition entails another just in case any state that supports the former also supports the latter.

**Fact 2.24.** [Entailment in terms of support]
For any \( P, Q \in \mathcal{P} \):

- \( P \models Q \) iff any state that supports \( P \) also supports \( Q \)
Entailment forms a partial order on the set of all propositions, i.e., it is a reflexive, transitive, and anti-symmetric relation. The tautology, \( \varphi(W) \), is entailed by any other proposition, i.e., it is the weakest element of the partial order. On the other hand, the partial order also has a strongest element, namely \( \{\emptyset\} \), which entails all other propositions. We refer to this proposition as the contradictory proposition. We will denote the tautological and the contradictory proposition as \( \top \) and \( \bot \), respectively.

**Definition 2.25.** [Tautology and contradiction]

- \( \top := \varphi(W) \)
- \( \bot := \{\emptyset\} \)

**Fact 2.26.** [Partial order]

- \( \models \) forms a partial order on \( \mathcal{P} \)
- For every \( P \in \mathcal{P} \): \( \bot \models P \) and \( P \models \top \)

### 2.4.5 Some linguistic examples

The notion of propositions as non-empty, downward closed sets of information states allows us to capture the informative and inquisitive content of a wide range of declarative and interrogative sentences in natural languages in a uniform and transparent way. We provide a brief illustration here; a more detailed case-study is presented in Chapter 5. Consider the following sentences in English, where \( \downarrow \) and \( \uparrow \) indicate falling and rising intonation, respectively:

1. Peter will attend the meeting.\( \downarrow \).
2. Peter will attend the meeting?\( \uparrow \).
3. Will Peter attend the meeting?
4. Will Peter\( \uparrow \) attend the meeting, or Maria\( \uparrow \)?
5. If Peter attends the meeting, will Maria attend it too?
6. Who will attend the meeting?

These examples instantiate five different sentence types: (1) a declarative with falling intonation, (2) a declarative with rising intonation, (3) a polar interrogative, (4) an open disjunctive interrogative,\(^6\) (5) a conditional polar interrogative,

\(^6\)Open disjunctive interrogatives, with rising intonation on all disjuncts, are to be distinguished from closed disjunctive interrogatives, often referred to as alternative questions, with falling intonation on the final disjunct (Roelofsen and van Gool, 2010; Pruitt and Roelofsen, 2013). The latter characteristically carry a non-at-issue implication that exactly one of the
and (6) a wh-interrogative. Notice that the classical notion of propositions as sets of possible worlds only allows us to capture the informative content of (1), a declarative with falling intonation; it does not allow us to deal appropriately with the rising declarative in (2) and the various types of interrogatives in (3)-(6).

In the framework that we have laid out so far these sentences can all be treated in a uniform way, expressing the propositions depicted in Figure 2.6. We assume here for simplicity that $W$ consists of just four possible worlds: one world where both Peter and Maria will attend the meeting, one where only Peter will attend, one where only Maria will attend, and one where neither Peter nor Maria will attend—in Figure 2.6, these worlds are labeled 11, 10, 01, and 00, respectively. As before, each proposition is visualized by depicting just the alternatives it contains, i.e., its maximal elements.

The falling declarative in (1) can be taken to express the proposition depicted in Figure 2.6(a). This proposition contains a single alternative, which is the set of all worlds where Peter will attend the meeting. Since this single alternative does not cover the entire logical space, the proposition is informative. Moreover, since there is just one alternative, the proposition is not inquisitive. This captures the fact that in uttering (1), a speaker provides the information that Peter will attend the meeting, without requesting any further information.

The rising declarative in (2) and the polar interrogative in (3) can both be taken to express the proposition depicted in Figure 2.6(b). This proposition consists of two alternatives: the set of all worlds where Peter will attend the meeting, and the set of all worlds where Peter will not attend the meeting. Since the two alternatives together cover the entire logical space, the proposition is not informative. On the other hand, since it contains two alternatives, it is inquisitive. This captures the fact that in uttering either (2) or (3), a speaker

\begin{itemize}
\item disjuncts is supposed to hold (Karttunen and Peters, 1976; Han and Romero, 2004; Beck and Kim, 2006; Pruitt and Roelofsen, 2011; Biezma and Rawlins, 2012; Aloni et al., 2013, among others). This non-at-issue implication cannot be captured in the basic framework presented here, which is only concerned with at-issue content. However, there are natural ways to extend the framework in such a way that non-at-issue content can be captured as well (see, e.g., AnderBois, 2012; Ciardelli et al., 2012, 2015).
\end{itemize}
2.4. Propositions

raises the issue whether Peter will attend the meeting or not.

The open disjunctive interrogative in (4) can be taken to express the proposition depicted in Figure 2.6(c). This proposition consists of three alternatives: the set of all worlds where Peter will attend the meeting, the set of all worlds where Maria will attend, and the set of all worlds where neither Peter nor Maria will attend. Since these alternatives together cover the entire logical space, the proposition is not informative. However, since it contains three alternatives, it is inquisitive. It raises an issue which can be resolved by establishing that Peter will attend the meeting, or by establishing that Maria will attend, or by establishing that neither of the two will attend.

The conditional interrogative in (5) can be taken to express the proposition depicted in Figure 2.6(d). This proposition consists of two alternatives, which correspond to the following answers to (5): 7

(7) a. Yes, if Peter attends the meeting, then Maria will attend as well.
    b. No, if Peter attends the meeting, then Maria won’t attend.

Since the two alternatives cover the entire logical space, the proposition is not informative. However, since it contains two alternatives, it is inquisitive. This captures the fact that in uttering (5), a speaker requests enough information to determine whether Maria will attend the meeting, under the assumption that Peter will.

Finally, consider the wh-interrogative in (6). In uttering this sentence, a speaker may be taken to request a complete, exhaustive specification of all people who will attend the meeting. Alternatively, she may be taken to request enough information to identify at least one person who will attend the meeting, if there is such a person, and otherwise enough information to establish that nobody will attend. 8 These two interpretations of wh-interrogatives are generally

7 Intuitively, there are other natural answers to this conditional interrogative as well:

(i) a. No, it might be that Peter will attend the meeting and that Maria won’t.
    b. Peter won’t attend the meeting.

The first assumes a modal interpretation of (5), the second does not resolve the conditional interrogative as intended but rather denies its antecedent. The basic inquisitive semantics framework that we are presenting here is not fine-grained enough to capture the special nature of these answers. However, the framework may be further extended in such a way that a more refined semantic analysis of conditional interrogatives becomes possible, capturing both modal interpretations and denials of the antecedent (Groenendijk and Roelofsen, 2015).

8 In uttering a wh-interrogative like (6), a speaker is often taken to presuppose that at least one person will attend the meeting. Such a presupposition cannot be captured directly in the basic inquisitive semantics framework that we are presenting here, which is only designed to capture at-issue informative and inquisitive content (see also footnote 6 above). There are natural ways to extend the framework in such a way that presuppositional content can be captured as well (see, e.g., AnderBois, 2012; Ciardelli et al., 2012, 2015).
Chapter 2. Basic notions

referred to as a mention-all and a mention-some interpretation, respectively. Under a mention-all interpretation, (6) expresses the proposition depicted in Figure 2.6(c). Under a mention-some interpretation, it expresses the proposition depicted in Figure 2.6(c). In both cases, the proposition expressed is inquisitive but not informative. Under the mention-all interpretation the proposition expressed is more inquisitive than under the mention-some interpretation.

2.5 Contexts

In Section 1.1.1 we reviewed a number of reasons why conversational contexts should be modeled in a way that does not only take account of the information that has been established in the conversation so far, but also of the issues that have been brought up, often referred to as questions under discussion (see, e.g., Carlson, 1983; Groenendijk and Stokhof, 1984; van Kuppevelt, 1995; Ginzburg, 1996; Roberts, 1996; Groenendijk, 1999; Büring, 2003; Beaver and Clark, 2008; Tonhauser et al., 2013). This can be done using the notion of issues introduced above. The most straightforward way of doing so would be to model a context \( C \) as a pair \( \langle \text{info}_C, \text{issues}_C \rangle \), where \( \text{info}_C \) is an information state representing the information that the conversational participants have commonly established so far, and \( \text{issues}_C \) a set of issues which have been raised in the conversation so far and which the conversational participants would like to see commonly resolved. That is, while \( \text{info}_C \) represents the current common ground, the issues in \( \text{issues}_C \) determine what kind of common ground the conversational participants would like to establish, namely one in which every issue in \( \text{issues}_C \) is settled. The initial context would then be \( \langle W, \emptyset \rangle \), consisting of the trivial information state, which does not rule out any world, and the empty set of issues. As the conversation progresses, worlds would be removed from \( \text{info}_C \) and issues would be added to \( \text{issues}_C \).

This way of modelling conversational contexts is a good starting point, but just as in the case of propositions, it is natural to impose certain constraints on how the informative and the inquisitive component of a context are related to each other. First, every issue \( I \in \text{issues}_C \) should be one that is settled only in information states that enhance the current common ground, \( \text{info}_C \). It would not make much sense if the kind of common ground that the conversational participants would like to establish were not an enhancement of the current common ground. Formally, this means that for every \( I \in \text{issues}_C \) we should have that \( \bigcup I \subseteq \text{info}_C \).

\footnote{It can also be done—and indeed has been done—using different formal notions from the literature on questions (e.g., Hamblin, 1973; Groenendijk and Stokhof, 1984; Groenendijk, 2009; Mascarenhas, 2009). Chapter 7 provides a detailed comparison between the current notion of issues and these previous notions.}
Second, for every issue $I \in \text{issues}_C$, the information available in the current common ground should ensure that $I$ can be resolved truthfully, i.e., without discarding the actual world. This means that for every $I \in \text{issues}_C$ we should have that $\text{info}_C \subseteq \bigcup I$. To see this, we follow the same line of reasoning that we did when considering propositions above. Suppose that $\text{info}_C \not\subseteq \bigcup I$. Then there exists a world $w \in \text{info}_C$ which is not included in any state that resolves $I$. According to the information available in $\text{info}_C$, $w$ may well be the actual world. Suppose it is the actual world. Then any state that resolves $I$ discards the actual world. So, in this case $\text{info}_C$ does not guarantee that $I$ can be resolved truthfully.

Thus, for every $I \in \text{issues}_C$, it should hold on the one hand that $\bigcup I \subseteq \text{info}_C$ and on the other hand that $\text{info}_C \subseteq \bigcup I$. Putting the two together, we get that every $I \in \text{issues}_C$ should be an issue over $\text{info}_C$: $\bigcup I = \text{info}_C$.

If we impose this constraint, the considered notion of conversational contexts is in principle a suitable notion. However, for our current purposes, it can be simplified. We will do this in two steps. First, rather than thinking of a context $C$ as a pair $\langle \text{info}_C, \text{issues}_C \rangle$ where $\text{issues}_C$ is a set of issues over $\text{info}_C$, we may just as well think of it as a pair $\langle \text{info}_C, \text{issue}_C \rangle$ where $\text{issue}_C$ is a single issue over $\text{info}_C$. This simplification is justified by the fact that any set of issues $\Omega$ over a state $s$ can be merged into a single issue over $s$:

$$I_{\Omega} := \{ t \subseteq s \mid t \in J \text{ for every } J \in \Omega \}$$

which is settled precisely by those enhancements $t \subseteq s$ that settle all issues in $\Omega$. Notice that if $\Omega \neq \emptyset$ the issue $I_{\Omega}$ amounts to the intersection $\bigcap \Omega$ of all issues in $\Omega$, whereas if $\Omega = \emptyset$, $I_{\Omega}$ amounts to the trivial issue $\varnothing(s)$ over $s$.\(^{10,11}\)

So we can think of a context $C$ as a pair $\langle \text{info}_C, \text{issue}_C \rangle$, where $\text{info}_C$ is an information state and $\text{issue}_C$ a single issue over $\text{info}_C$. We can then take the initial context to be the pair $(W, \varnothing(W))$, consisting of the trivial information state, which does not rule out any world, and the trivial issue over this state, which is settled even if no information is present yet.

But this representation can be simplified further. After all, since $\text{issue}_C$ is an issue over $\text{info}_C$, we always have that $\text{info}_C = \bigcup \text{issue}_C$. That is, $\text{info}_C$ can

\(^{10}\)Recall from footnote 1 that we are implicitly already assuming a similar simplification concerning the informative component of a context: we do not keep track of all the separate pieces of information that have been established in the conversation so far, but rather of the set of worlds that are compatible with all these pieces of information—formally, this is again the intersection of all the separately established pieces of information. For certain purposes it is necessary to keep track of all the separate pieces of information and/or issues that have been established/raised in a conversation (see, e.g., Roberts, 1996; Farkas and Bruce, 2010; Farkas and Roelofsen, 2016). For our current purposes, however, this would only add unnecessary complexity.

\(^{11}\)Notice that $I_{\Omega}$ is guaranteed to be an issue in the sense of Definition 2.3. In particular, it is guaranteed to be non-empty, since it always contains the inconsistent information state.
always be retrieved from $\text{issue}_C$. But then $\text{info}_C$ can just as well be left out of the representation of $C$. Thus, a context $C$ can simply be represented as an issue, i.e., a non-empty, downward closed set of information states. The information commonly established in $C$ is then embodied by $\bigcup C$.

**Definition 2.27.** [Contexts]

- A context $C$ is a non-empty, downward closed set of information states.
- The set of all contexts will be denoted by $\mathcal{C}$.

**Definition 2.28.** [The information available in a context]

- For any context $C$: $\text{info}(C) := \bigcup C$

We have moved from the commonplace notion of a context as a set of possible worlds—representing the information established so far—to a richer notion of contexts as non-empty, downward closed sets of information states—representing both the information established so far and the issues raised so far. We will now identify some special properties that contexts may have (§2.5.1), some relations that may hold between them (§2.5.2), and some operations that can be performed on them (§2.5.3).

### 2.5.1 Informed and inquisitive contexts

First of all, we say that a context $C$ is *informed* just in case some non-trivial information has been established in it, i.e., $\text{info}(C) \neq W$. Otherwise we say that the context is *ignorant*.

**Definition 2.29.** [Informed and ignorant contexts]

- A context $c$ is informed iff $\text{info}(C) \neq W$.
- A context $c$ is ignorant iff $\text{info}(C) = W$.

Similarly, we say that a context $C$ is *inquisitive* just in case the information that has been established so far does not yet settle the issues that have been raised, i.e., $\text{info}(C) \notin C$. On the other hand, if all issues are settled we say that $C$ is *indifferent*.

**Definition 2.30.** [Inquisitive and indifferent contexts]

- A context $C$ is inquisitive iff $\text{info}(C) \notin C$.
- A context $C$ is indifferent iff $\text{info}(C) \in C$. 
2.5. Contexts

Figure 2.7: Some contexts.

There are two special contexts: the initial and the absurd context. The initial context, $C_\top$, is the only context that is both ignorant and indifferent. The absurd context, $C_\bot$, is one in which the established information is inconsistent and therefore rules out all possible worlds.

**Definition 2.31.** [The initial and the absurd context]

- $C_\top := \wp(W)$
- $C_\bot := \{\emptyset\}$

Some example contexts are depicted in Figure 2.7, where as before it is assumed that $W = \{w_1, w_2, w_3, w_4\}$. Only the maximal states in each context are depicted. Since contexts are downward closed, we know that all enhancements of these maximal states are also part of the context at hand. The context in (a) is the initial context, $\wp(W)$, which is neither informed nor inquisitive. The one in (b) is still not informed, but it is inquisitive. In order to resolve the issue that is present in this context, it either needs to be established that the actual world is one of $\{w_1, w_2\}$ or that it is one of $\{w_3, w_4\}$. The context in (c) is both informed and inquisitive. In this context it is common ground that the actual world is one among $\{w_1, w_2, w_3\}$, i.e., $w_4$ has been ruled out as a candidate for the actual world, but in order to resolve the issue that has been raised, more precise information is needed—namely, it either needs to be established that the actual world is one of $\{w_1, w_2\}$ or that it is one of $\{w_1, w_3\}$. Finally, the context in (d) is informed, but not inquisitive. It is common ground in this context that the actual world is one among $\{w_1, w_2\}$, and no issues have been raised whose resolution would require more precise information.

### 2.5.2 Context extension

Two contexts can be compared in terms of the information that has been established or in terms of the issues that have been raised. One context $C'$ is at least as informed as another context $C$ if and only if $\text{info}(C') \subseteq \text{info}(C)$.
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Definition 2.32. [Informative order on contexts]
For any \( C, C' \in \mathcal{C} \):

- \( C' \geq_{\text{info}} C \) if and only if \( \text{info}(C') \subseteq \text{info}(C) \)

Similarly, we say that \( C' \) is at least as inquisitive as \( C \) if and only if every state that settles all the issues that have been raised in \( C' \) also settles all the issues that have been raised in \( C \), i.e., if and only if \( C' \subseteq C \).

Definition 2.33. [Inquisitive order on contexts]
For any \( C, C' \in \mathcal{C} \):

- \( C' \geq_{\text{inq}} C \) if and only if \( C' \subseteq C \)

Combining these two orders, we say that \( C' \) is an extension of \( C \) just in case \( C' \) is both at least as informed and at least as inquisitive as \( C \). But note that if \( C' \subseteq C \), then it must also be the case that \( \text{info}(C') \subseteq \text{info}(C) \). So context extension simply amounts to inclusion.

Definition 2.34. [Extending contexts]
For any \( C, C' \in \mathcal{C} \):

- \( C' \) is an extension of \( C \), \( C' \geq_{\text{ext}} C \), if and only if \( C' \subseteq C \)

The extension relation forms a partial order on \( \mathcal{C} \), and \( C_{\top} \) and \( C_{\bot} \) constitute the extremal elements of this partial order: \( C_{\bot} \) is an extension of every context, and every context is in turn an extension of \( C_{\top} \).

Fact 2.35. [Partial order]

- \( \geq \) forms a partial order on \( \mathcal{C} \)
- For every \( C \in \mathcal{C} \): \( C_{\bot} \geq C \) and \( C \geq C_{\top} \)

In Figure 2.7, the contexts in (b), (c), and (d) are all extensions of the trivial context in (a). Moreover, (d) is also an extension of (b) and (c), but neither (b) nor (c) is an extension of the other.

2.5.3 Updating contexts

Recall that in the standard setting, where both contexts and propositions are construed as sets of possible worlds, the result of updating a context \( c \) with a proposition \( p \) is a new context \( c[p] \) which, besides the information already present in \( c \), also contains the information embodied by \( p \). That is, a candidate for the actual world is ruled out by \( c[p] \) if it was already ruled out by the
information established in the old context $c$, but also if it is ruled out by the new information embodied by $p$. Thus, formally, update amounts to set intersection in the standard setting: $c[p] = c \cap p$.

In inquisitive semantics, we want the result of updating a context $C$ with a proposition $P$ to be a new context $C[P]$ which does not only incorporate the informative content of $P$, but also the issue that it embodies. Thus, on the one hand, a candidate for the actual world must be ruled out by the information established in $C[P]$ if it was either already ruled out by the information established in the old context $C$, or if it is ruled out by the informative content of $P$. Formally, this means that we must have that $\text{info}(C[P]) = \text{info}(C) \cap \text{info}(P)$.

On the other hand, a state must resolve the issues present in $C[P]$ if and only if it resolves the issues already present in $C$ and also the issue embodied by $P$. Formally, this means that we must have that $C[P] = C \cap P$. Now, note that if the latter condition is satisfied, then the former condition is automatically satisfied as well. This means that, just as in the standard setting, update can simply be defined as set intersection.

**Definition 2.36.** [Updating contexts]

For any $C \in \mathcal{C}$ and any $P \in \mathcal{P}$:

- $C[P] := C \cap P$

Some examples of context update are given in Figure 2.8. In the first case, the initial context is informed but not inquisitive. More specifically, in this context it is commonly established that the actual world is one among $\{w_1, w_2, w_3\}$, and no issues have been raised that require more precise information. This context is updated with a proposition which is informative—embodying the information that the actual world is one among $\{w_1, w_2, w_4\}$—but not inquisitive. The result of the update, obtained by intersection, is a new context in which it is established that the actual world is among $\{w_1, w_2\}$, and where there are still no issues that require more precise information. Note that in this case, where neither the initial context nor the proposition involved in the update are inquisitive, our framework reproduces exactly the same result that is obtained in the standard setting. This holds in full generality.

**Fact 2.37.** [Update without inquisitiveness yields standard results]

For any non-inquisitive context $C$ and any non-inquisitive proposition $P$, $C[P]$ is a non-inquisitive context as well, and its unique maximal element is the intersection of the unique maximal element of $C$ and that of $P$.

The second example in Figure 2.8 is one where the initial context is the same as in the first example, but now the proposition with which it is updated is inquisitive, embodying the issue whether the actual world is among $\{w_1, w_2\}$
Initial context: Proposition: New context:

\[
\begin{array}{c}
\{w_1, w_2\} \cap \{w_3, w_4\} = \{w_3, w_4\} \\
\{w_1, w_2\} \cap \{w_3, w_4\} = \{w_3, w_4\} \\
\{w_1, w_2\} \cap \{w_3, w_4\} = \{w_3, w_4\}
\end{array}
\]

Figure 2.8: Some update examples

or among \(\{w_3, w_4\}\). The context resulting from the update is one in which this issue is present, together with the information that was already available beforehand. That is, after the update it is still established that the actual world is among \(\{w_1, w_2, w_3\}\), as in the initial context, but now there is also an issue as to whether it is \(w_3\) or among \(\{w_1, w_2\}\). Note that in order to obtain this result simply by means of intersection, it is important that both contexts and propositions are downward closed. Made fully explicit, the initial context is represented as the following set of information states:

\[
\{\{w_1, w_2, w_3\}, \{w_1, w_2\}, \{w_1, w_3\}, \{w_2, w_3\}, \{w_1\}, \{w_2\}, \{w_3\}, \emptyset\}
\]

The proposition considered is:

\[
\{\{w_1, w_2\}, \{w_3, w_4\}, \{w_1\}, \{w_2\}, \{w_3\}, \{w_4\}, \emptyset\}
\]

Applying intersection to these two sets yields the new context:

\[
\{\{w_1, w_2\}, \{w_1\}, \{w_2\}, \{w_3\}, \emptyset\}
\]

whose two maximal elements, \(\{w_1, w_2\}\) and \(\{w_3\}\), are the ones that are depicted. This result would not be obtained if we discharged downward closure and identified contexts and propositions exclusively with their maximal elements. In
2.6. Summary and pointers to possible refinements

that case, the initial context would be represented as \{w_1, w_2, w_3\}, the proposition at hand as \{w_1, w_2\}, \{w_3, w_4\}, and applying intersection to these two sets would yield the empty set, clearly not the desired result.

The third example in Figure 2.8 is one where the initial context is already inquisitive. The issue that is present is whether the actual world is among \{w_1, w_3\} or among \{w_2, w_4\}. The proposition with which this context is updated is the same as in the previous example, embodying the issue whether the actual world is among \{w_1, w_2\} or among \{w_3, w_4\}. The update results in a context in which these two issues have been merged. In order to resolve the issue that is present in this new context it is necessary to determine exactly which of \(w_1, w_2, w_3, w_4\) is the actual world. That is, it is necessary to resolve the issue that was already present in the initial context, and also the issue that was embodied by the proposition involved in the update.

Thus, while our update procedure yields standard results in the case of non-inquisitive contexts and propositions, it also smoothly generalizes to cases involving inquisitive contexts and/or propositions.

2.6 Summary and pointers to possible refinements

We have now introduced all the notions that we set out to introduce (recall the diagram in Figure 2.1 at the beginning of the chapter). We adopted the standard notion of information states as sets of possible worlds. In terms of this familiar notion, we defined a new notion of issues. We represent an issue as a non-empty, downward closed set of information states, those information states that contain enough information to resolve the issue. With this crucial notion in place, we turned to propositions and contexts. We moved from the standard notion of a proposition as a set of possible worlds, which just allows us to capture the information that a sentence conveys, to a more fine-grained notion, which also allows us to capture the issue that a sentence raises. Similarly, we replaced the standard minimal notion of contexts, which just captures the information that has been commonly established in the conversation so far, by a richer notion that also allows us to capture the issues that have been brought up. Formally, both propositions and contexts are not modeled as sets of possible worlds in our framework, but rather, just like issues, as non-empty, downward closed sets of information states.

Turning to the relations that may hold between the various kinds of objects, we have seen that entailment between propositions, enhancement of information states, and extension of contexts all amount to set inclusion, just as in the standard setting, and the same is true for the new notion of issue refinement. Support, a relation between information states and propositions, is no longer defined as inclusion, but rather as membership. This is a consequence of the
fact that an information state no longer necessarily supports a proposition if it implies the informative content of that proposition; rather, it should also contain enough information to resolve the issue embodied by the proposition. Finally, context update still amounts to set intersection. However, since the operation no longer applies to sets of worlds but rather to sets of information states, we have seen that it can deal in a uniform way with cases involving purely informative propositions and indifferent contexts, as well as cases involving inquisitive propositions and/or contexts.

We briefly illustrated how the informative and inquisitive content of various types of sentences in English can be captured using the proposed notion of propositions. On the other hand, we also drew attention to several aspects of meaning that are beyond the scope of the basic inquisitive semantics framework that we are presenting here. However, this basic framework is set up in such a way that it allows for several natural refinements. We briefly mention three such refinements, with references to other work for further detail.

First, instead of the static view on meaning that we have assumed here, one may also adopt a dynamic view on meaning (see, e.g., Kamp, 1981; Heim, 1982; Groenendijk and Stokhof, 1991; Veltman, 1996). Under this view, the meaning of a sentence is conceived of as its context change potential, modeled formally as a function $F$ that maps any context $C$ to a new context $F(C)$, which would result from uttering the given sentence in $C$. This new context $F(C)$ need not necessarily be obtained by intersecting $C$ with the proposition $P$ expressed by the sentence. In fact, on a dynamic view, there is no need to associate sentences with propositions at all. This allows for greater flexibility, which has led to important advances in the treatment of various linguistic phenomena, including anaphora and presuppositions. While we have taken a static perspective here, the formal notions that we have introduced, in particular the notion of contexts, also form a suitable starting point for a dynamic inquisitive semantics (see Ciardelli et al., 2012, 2013a, for initial work in this direction, though much remains to be done).

Second, rather than starting out with the commonplace notion of information states as sets of possible worlds, which imposes a very specific (Boolean) algebraic structure on the space of information states, we may also construe information states as primitive objects and assume that the space of information states has a more generic algebraic structure (Punčochář, 2015a).

Third, in order to model more than just informative and inquisitive content we may further enrich our notion of propositions and/or contexts, either by explicitly encoding additional dimensions of meaning (see, e.g., Roelofsen and Farkas, 2015; AnderBois, 2016b), or by weakening the downward closure constraint that we have placed on contexts and propositions here (Ciardelli et al., 2014; Punčochář, 2015b; Groenendijk and Roelofsen, 2015). Such amendments lead to richer notions of meaning, and further broaden the range of linguistic
phenomena that can be captured in the framework. However, it largely remains to be investigated whether these further refinements are as well-behaved as the basic framework presented here.

2.7 Exercises

Exercise 2.1. [Contexts]

1. Give a representation of a context in which (i) it is commonly established that Bill is going to the party, and (ii) there is an issue as to whether Mary is going as well.

2. Give a representation of a context in which (i) it is commonly established that if Bill is going to the party, then Mary will go as well, and (ii) there is an issue as to whether Bill is going.

3. Give a representation of a context in which (i) it is commonly established that either Bill or Mary is going to the party (not both), and (ii) there is an issue as to which of them is going.

4. For each of the above contexts, determine whether it is an extension of the others.

Exercise 2.2. [Propositions]

1. Give a representation of the proposition whose informative content is that Bill is only going to the party if Mary is going, and which embodies the issue whether Mary is going.

2. Give a representation of the proposition whose informative content is trivial and which embodies the issue which of Bill and Mary are going (Bill, Mary, both, or neither).

3. Give a representation of the non-inquisitive proposition whose informative content is that only Mary is going to the party, not Bill.

4. For each of the above propositions, determine whether it entails the others.

Exercise 2.3. [Update]

1. Determine the result of updating each of the contexts in Exercise 2.1 which each of the propositions in Exercise 2.2.

2. Prove Fact 2.37, showing that the notion of update as intersection yields standard results when applied to non-inquisitive contexts and propositions.
Exercise 2.4. [Statements and questions]
Let $P$ and $P'$ be two statements, and $Q$ and $Q'$ two questions.

1. Is $P \cap P'$ guaranteed to be a statement? If so, give a proof; if not, give a counterexample.

2. Is $Q \cap Q'$ guaranteed to be a question? If so, give a proof; if not, give a counterexample.

3. Is $P \cap Q$ either guaranteed to be a statement or to be a question? If so, give a proof; if not, give a counterexample.
Chapter 3

Basic operations on propositions

Now that we have introduced a new notion of propositions, a natural question that arises is what the basic operations are that can be performed on such propositions. In the classical setting, where propositions are simple sets of worlds, we can form the intersection or the union of two propositions, or the complement of a single proposition. These operations play a central role in logic and in semantic analyses of natural languages: conjunction and disjunction are standardly taken to express intersection and union, respectively, while negation is standardly taken to express complementation. Do these operations have natural counterparts in the inquisitive setting, where propositions are no longer simple sets of worlds?

We will address this question in Section 3.1, adopting an algebraic perspective. We will find that the basic algebraic operations on classical propositions can indeed be applied to inquisitive propositions as well. This result facilitates a very natural way of dealing with connectives and quantifiers, and will allow us to define an inquisitive semantics for the language of first-order logic which is, from an algebraic perspective, the exact counterpart of classical first-order logic in the inquisitive setting.

In Section 3.2, we will consider two additional operations, which trivialize the informative or the inquisitive content, respectively, of any given proposition. For reasons that will become clear below, we refer to such operators as projection operators. One projection operator turns any proposition into a statement, trivializing its inquisitive content, while the other turns any proposition into a question, trivializing its informative content. Clearly, these operations do not have a counterpart in the classical setting, where propositions capture only informative content to begin with; but in the inquisitive setting they naturally arise, and we will suggest that they also have an important role to play in the semantic analysis of natural languages.
Chapter 3. Basic operations on propositions

3.1 Algebraic operations

In this section we will identify the basic algebraic operations that can be applied to inquisitive propositions. To illustrate our approach, we will first briefly review the algebraic perspective on classical logic.

3.1.1 The algebraic perspective on classical logic

In the classical setting a proposition $P$ is simply a set of possible worlds. Let us denote the set of all classical propositions as $\mathcal{P}_C$. The proposition expressed by a sentence can be thought of as carving out a certain region in the logical space—the set of all possible worlds—and in asserting a sentence, a speaker is taken to provide the information that the actual world is located within this region. One proposition $P$ entails another proposition $Q$, $P \models Q$, just in case $P \subseteq Q$, which means that $P$ carves out a smaller region in the logical space than $Q$ does, thereby encoding more information as to what the actual world is like. Entailment forms a partial order on the set of all classical propositions, i.e., it is a reflexive, transitive, and anti-symmetric relation.

Now, every partially ordered set has a certain algebraic structure and comes with certain basic algebraic operations. The set of classical propositions ordered by entailment, $\langle \mathcal{P}_C, \models \rangle$, forms a so-called Heyting algebra, which comes with four basic operations: meet, join, relative pseudo-complementation and absolute pseudo-complementation.

The meet of $P$ and $Q$ is the greatest lower bound of $P$ and $Q$ with respect to entailment, i.e., the weakest proposition that entails both $P$ and $Q$. As depicted in Figure 3.1(a), this greatest lower bound amounts to the intersection of the two propositions: $P \cap Q$. More generally, the meet of a (possibly infinite) set of propositions $\Sigma$ amounts to the intersection of all the propositions in that set:

$$\bigcap \Sigma = \{ w \mid w \in P \text{ for all } P \in \Sigma \}$$

If $\Sigma$ is empty, then $\bigcap \Sigma$ is the proposition consisting of all possible worlds, $W$. This is the weakest of all propositions, since it is entailed by all other propositions. It is denoted as $\top$. On the other hand, if $\Sigma$ is the set of all propositions, then $\bigcap \Sigma$ is the empty proposition, $\emptyset$. This is the strongest of all propositions, since it entails all other propositions. It is denoted as $\bot$.

The join of two propositions $P$ and $Q$ is the least upper bound of $P$ and $Q$ with respect to entailment, i.e., the strongest proposition that is entailed by both $P$ and $Q$. As depicted in Figure 3.1(a), this least upper bound amounts to the union of the two propositions: $P \cup Q$. More generally, the join of a (possibly infinite) set of propositions $\Sigma$ amounts to the union of all the propositions in that set:

$$\bigcup \Sigma = \{ w \mid w \in P \text{ for some } P \in \Sigma \}$$
3.1. Algebraic operations

If Σ is empty, then $\bigcup \Sigma$ is the empty proposition, $\bot$. On the other hand, if Σ is the set of all propositions, then $\bigcup \Sigma$ is the proposition consisting of all possible worlds, $\top$.

The existence of meets and joins for arbitrary sets of propositions implies that $\langle P, \models \rangle$ forms a complete lattice, bounded by $\bot$ and $\top$ as its strongest and weakest elements, respectively.

Now let us turn to relative and absolute pseudo-complementation. The pseudo-complement of a proposition $P$ relative to another proposition $Q$, which we will denote as $P \Rightarrow Q$, can be thought of intuitively as the difference between $P$ and $Q$: it is the weakest proposition $R$ such that $P$ and $R$ together contain at least as much information as $Q$. More formally, it is the weakest proposition $R$ such that $P \cap R \models Q$. This is visualized in Figure 3.1(b). The shaded area in the figure is the set of all propositions $R$ which are such that $P \cap R \models Q$. The weakest among these, i.e., the topmost one, is the pseudo-complement of $P$ relative to $Q$. This proposition consists of all possible worlds which, if contained in $P$, are also contained in $Q$:

$$P \Rightarrow Q = \{w \mid \text{if } w \in P \text{ then } w \in Q \text{ as well}\}$$

Absolute pseudo-complementation is a limit case of its relative counterpart. The absolute pseudo-complement of a proposition $P$, which we will denote as $P^*$, is the weakest proposition $R$ such that $P \cap R$ entails any other proposition. Since the only proposition that entails any other proposition is $\bot$, $P^*$ can be characterized as the weakest proposition $R$ such that $P \cap R = \bot$. It consists simply of all worlds that are not in $P$ itself:

$$P^* = \{w \mid w \notin P\}$$
In a Heyting algebra it always holds, by definition of $P^*$, that $P \cap P^* = \bot$. In the specific case of $\langle P_C, \models \rangle$, we also always have that $P \cup P^* = \top$. This means that in this particular setting, $P^*$ is in fact the Boolean complement of $P$, and that $\langle P_C, \models \rangle$ forms a Boolean algebra, a special kind of Heyting algebra.

Thus, classical propositions are amenable to certain basic algebraic operations. Classical first-order logic is obtained by associating these operations with the connectives and the quantifiers. Indeed, the usual definition of truth can be reformulated as a recursive definition of the set $\models \phi$ of models over a domain $D$ in which $\phi$ is true. The inductive clauses then run as follows:

- $|\neg \phi| = |\phi|^*$
- $|\phi \land \psi| = |\phi| \cap |\psi|$
- $|\phi \lor \psi| = |\phi| \cup |\psi|$
- $|\phi \rightarrow \psi| = |\phi| \Rightarrow |\psi|$
- $|\forall x. \phi(x)| = \bigcap_{d \in D} |\phi(d)|$
- $|\exists x. \phi(x)| = \bigcup_{d \in D} |\phi(d)|$

Negation expresses absolute pseudo-complementation, conjunction and disjunction express binary meet and join, respectively, implication expresses relative pseudo-complementation, and quantified formulas, $\forall x. \phi$ and $\exists x. \phi$, express the infinitary meet and join, respectively, of $\{|\phi(d)| \mid d \in D\}$.

Notice that everything started with a notion of propositions and a natural entailment order on these propositions. The entailment order induces certain basic operations on propositions, and classical first-order logic is obtained by associating these basic semantic operations with the connectives and quantifiers.

3.1.2 Algebraic operations on inquisitive propositions

Recall that in inquisitive semantics propositions are not sets of worlds, but rather sets of information states, non-empty and downward closed. In this setting, one proposition $P$ entails another proposition $Q$ just in case $P$ is at least as informative and at least as inquisitive as $Q$. We have seen that this condition is satisfied just in case $P \subseteq Q$. So technically entailment still amounts to inclusion, just like in classical logic, though now it encompasses both informative and inquisitive strength.

Let us consider the algebraic structure of the space of all inquisitive propositions ordered by entailment, $\langle P, \models \rangle$, in order to determine which operations could be associated with the connectives and quantifiers in an inquisitive semantics for the language of first-order logic. What kind of algebraic operations
3.1. Algebraic operations

can be performed on inquisitive propositions? Does every set of propositions still have a unique greatest lower bound \((\textit{meet})\) and a unique least upper bound \((\textit{join})\) w.r.t. entailment? Does every proposition still have a pseudo-complement relative to any other proposition?

It turns out that these questions can be answered in the positive: \(\langle P, \models \rangle\) forms a complete Heyting algebra, just like \(\langle P_C, \models \rangle\). First, any set of propositions \(\Sigma \subseteq P\) still has a meet and a join, which can moreover still be characterized in terms of intersection and union.

**Fact 3.1.** [Meet] Any set of propositions \(\Sigma \subseteq P\) has a meet, which amounts to:

\[
\bigcap \Sigma = \{ s \mid s \in P \text{ for all } P \in \Sigma \}
\]

**Fact 3.2.** [Join] Any set of propositions \(\Sigma \subseteq P\) has a join, which amounts to:

\[
\bigcup \Sigma = \{ s \mid s \in P \text{ for some } P \in \Sigma \}
\]

if \(\Sigma \neq \emptyset\), and to \(\{\emptyset\}\) otherwise.

The existence of meets and joins for arbitrary sets of propositions implies that \(\langle P, \subseteq \rangle\) forms a complete lattice. This lattice has a unique strongest element, \(\bot := \emptyset\), and a unique weakest element, \(\top := \wp(W)\).

Furthermore, just as in the classical setting, for every two propositions \(P\) and \(Q\), there is a unique weakest proposition \(R\) such that \(P \cap R\) entails \(Q\). Recall that this proposition, the pseudo-complement of \(P\) relative to \(Q\), can be thought of intuitively as the difference between \(P\) and \(Q\).

**Fact 3.3.** [Relative pseudo-complement]

For any \(P, Q \in P\), the pseudo-complement of \(P\) relative to \(Q\) amounts to:

\[
P \Rightarrow Q \ := \{ s \mid \text{for every } t \subseteq s, \text{if } t \in P \text{ then } t \in Q \}
\]

The existence of relative pseudo-complements implies that \(\langle P, \subseteq \rangle\) forms a Heyting algebra. Finally, recall that the \(\text{absolute}\) pseudo-complement of a proposition \(P\), denoted \(P^*\), is defined as the pseudo-complement of \(P\) relative to \(\bot\). We saw that in the classical setting, \(P^*\) amounts to the set of worlds that are not in \(P\). In the inquisitive setting, \(P^*\) amounts to the set of states that are incompatible with any state in \(P\).

**Fact 3.4.** [Absolute pseudo-complement] For any proposition \(P \in P\):

\[
P^* = \{ s \mid s \cap t = \emptyset \text{ for all } t \in P \}
\]
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A state $s$ is incompatible with all states in $P$ just in case it is incompatible with $\bigcup P$, which in turn holds just in case $s$ is a subset of $\bigcup P$. Thus, $P^*$ can also be characterized as $\varphi(\bigcup P)$. This means in particular that $P^*$ always contains a single alternative, $\bigcup P$, and is therefore never inquisitive.

The algebraic operations that we have identified are exactly the ones that are present in the classical setting. One notable difference, however, is that the absolute pseudo-complement of an inquisitive proposition is not always its Boolean complement. In fact, most inquisitive propositions do not have a Boolean complement at all. To see this, suppose that $P$ and $Q$ are Boolean complements. This means that:

(i) $P \cap Q = \bot$

(ii) $P \cup Q = \top$

Since $\top = \varphi(W)$, condition (ii) can only be fulfilled if either $P$ or $Q$ contains $W$. Suppose $W \in P$. Then, since $P$ is downward closed, $P = \varphi(W) = \top$. But then, in order to satisfy condition (i), we must have that $Q = \{\emptyset\} = \bot$. So the only two elements of our algebra that have a Boolean complement are $\top$ and $\bot$. Hence, the space $\langle P, \models \rangle$ of inquisitive propositions does not form a Boolean algebra, unlike the space $\langle P_C, \models \rangle$ of classical propositions.

This difference has repercussions for the behavior of the logical system that we will specify, in particular for negation (for instance, the law of double negation will no longer hold). However, the similarity between $\langle P, \models \rangle$ and $\langle P_C, \models \rangle$ that we identified, i.e., the fact that both form a Heyting algebra, is much more important for our current purposes. In particular, the existence of meets, joins, and relative and absolute pseudo-complements in $\langle P, \models \rangle$ will allow us to specify an inquisitive semantics for the language of first-order logic which is, from an algebraic perspective, the exact counterpart of classical first-order logic in the inquisitive setting. We will turn to this in Chapter 4. Before that, however, we will consider two additional operations that are particularly natural to perform on propositions in inquisitive semantics.

### 3.2 Projection operators

We noted in Section 2.4.3 that propositions in inquisitive semantics can be seen as inhabiting a two dimensional space, with statements living on one axis and questions on the other. Given this picture, it is natural to consider whether it is possible to define general projection operators on this space, i.e., operators that turn any given proposition either into a statement or into a question, otherwise preserving as much as possible of the proposition’s characteristics. We will refer to such operators as s-projection operators and q-projection operators, respectively.
3.2. Projection operators

Let us first consider more precisely what would be required for an operator \( \pi \) to qualify as an s-projection operator. On the one hand, when applied to a proposition \( P \), \( \pi \) should trivialize the inquisitive content of \( P \), so that the resulting proposition is a statement. On the other hand, \( \pi \) should preserve the informative content of \( P \), i.e., \( \pi P \) should have exactly the same informative content as \( P \) itself.

**Definition 3.5. [Requirements on s-projection]**
An operator \( \pi \) qualifies as an s-projection operator just in case for any \( P \in \mathcal{P} \):

- \( \pi P \) is a statement
- \( \text{info}(\pi P) = \text{info}(P) \)

Now, in Section 2.4.3 we saw that if \( P \) is a statement, then we always have that \( P = \wp(\text{info}(P)) \). This means that in order to satisfy the above requirements, \( \pi P \) must amount to \( \wp(\text{info}(P)) \) for any proposition \( P \). Thus, the semantic behavior of \( \pi \) is uniquely determined by the given requirements.

**Fact 3.6. [Unique characterization of s-projection]**
An operator \( \pi \) qualifies as an s-projection operator just in case for any \( P \in \mathcal{P} \):

- \( \pi P = \wp(\text{info}(P)) \)

Now let us consider which requirements \( \pi \) should fulfil in order to qualify as a q-projection operator. Obviously, we should require that \( \pi \) trivializes the informative content of the proposition to which it applies, i.e., \( \pi P \) should always be a question. But, given this basic requirement, we cannot further demand that \( \pi \) always preserve the inquisitive content of \( P \). For, if \( P \) and \( \pi P \) do not have the same informative content, then their inquisitive content will differ as well.

Fortunately, there is a natural way to overcome this obstacle. Namely, what we can require is that \( \pi \) preserve the decision set of \( P \), i.e., the set of states that either settle the issue embodied by \( P \), or contradict the informative content of \( P \) and thereby establish that it is impossible to settle the issue altogether.

**Definition 3.7. [Contradicting and deciding on a proposition]**
Let \( s \) be an information state and \( P \) a proposition. Then we say that:

- \( s \) **contradicts** \( P \) just in case \( s \cap \text{info}(P) = \emptyset \);
- \( s \) **decides** on \( P \) just in case \( s \) either supports or contradicts \( P \).

**Definition 3.8. [Decision set]**
The decision set \( D(P) \) of a proposition \( P \) is the set of states that decide on \( P \).
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The decision set of a proposition can be characterized explicitly as follows.

**Fact 3.9.** [Decision set explicated] For any proposition $P$:

- $D(P) = P \cup P^*$

Now, what we require of a q-projection operator $\pi$ is that, besides trivializing the informative content of the proposition it applies to, it preserves the proposition’s decision set. This is a requirement that can in principle be met, since $P$ and $\pi P$ can very well have the same decision set even if they differ in informative content.

**Definition 3.10.** [Requirements on q-projection]

An operator $\pi$ qualifies as a q-projection operator just in case for any $P$:

- $\pi P$ is a question;
- $D(\pi P) = D(P)$.

Now suppose that $\pi$ fulfills these requirements. Then for any $P$, $\pi P$ is a question, which means that the informative content of $\pi P$ is $W$. But then $(\pi P)^* = \{\emptyset\}$, and therefore $D(\pi P) = (\pi P) \cup (\pi P)^* = \pi P$. But since $\pi$ should preserve the decision set of $P$, we also have that $D(\pi P) = D(P) = P \cup P^*$. Putting these facts together, we obtain that $\pi P = P \cup P^*$. Thus, the requirements we placed on $\pi$ again uniquely determine its semantic behavior.

**Fact 3.11.** [Unique characterization of q-projection]

An operator $\pi$ qualifies as a q-projection operator just in case for any $P$:

- $\pi P = P \cup P^*$

Thus, by spelling out the natural requirements on s-projection and q-projection we have arrived at a unique characterization of two projection operators, which we will denote as $!$ and $?$, respectively.

**Definition 3.12.** [Projection operators] For any proposition $P$:

- $! P := \wp(\text{info}(P))$
- $? P := P \cup P^*$

As depicted in Figure 3.2, the projection operators $!$ and $?$ turn any proposition $P$ into a statement $! P$ which has the same informative content as $P$, and a question $? P$ which has the same decision set as $P$. $P$ itself can always be reconstructed as the meet of these two ‘pure components’.
3.3. Linguistic relevance

Questions

\[ P = !P \cap ?P \]

\[ P = !P \cap ?P \]

Statements

Fact 3.13. [Division] For any proposition \( P \):

- \( P = !P \cap ?P \)

Finally, let us consider how \( ? \) and \( ! \) are related to the algebraic operations identified in Section 3.1. Notice that \( ?P \) is already explicitly characterized in terms of the algebraic operations: it amounts to the join of \( P \) and its absolute pseudo-complement \( P^* \). It turns out that \( !P \) can also be characterized in terms of pseudo-complementation. Namely, for any proposition \( P \), \( !P \) amounts to \( P^{**} \), i.e., to the proposition that results from two successive applications of the absolute pseudo-complementation operator to \( P \).

Fact 3.14. [Projection operators and algebraic operators]

For any proposition \( P \):

- \( !P = P^{**} \)
- \( ?P = P \cup P^* \)

This concludes our discussion of the basic semantic operations that can be performed on propositions in inquisitive semantics. We end this chapter with a brief remark on the linguistic relevance of these operations, which will be further substantiated in later chapters.

3.3 Linguistic relevance

Even though virtually any introduction to natural language semantics starts out with a discussion of classical logic, the linguistic relevance of the classical treatment of the connectives and quantifiers is not often explicitly argued for.
What makes this particular logical system so special? Why is it even called classical logic? Is this terminology just a historical coincidence, or does it carry some real substance?

The algebraic perspective that we adopted here makes it possible to answer these questions. Namely, what is special about classical logic is that it takes the connectives and quantifiers to express the most basic algebraic operations on propositions. This also explains why classical logic is of particular interest from a linguistic point of view. After all, since the algebraic operations on propositions that are associated with the connectives and quantifiers in classical logic are so elementary, it is to be expected that natural languages will generally have ways to express them as well; just like one would expect, for instance, that the basic algebraic operations in arithmetic—addition, subtraction, multiplication, and division—are generally expressible in natural languages. And indeed, in language after language words have been found that may be taken to fulfill exactly this purpose (see, e.g., Haspelmath, 2007; Gil, 2013; Szabolcsi, 2015b). The algebraic perspective provides a simple explanation for the cross-linguistic ubiquity of such words. This makes the treatment of the connectives and quantifiers in classical logic attractive from a linguistic point of view. This is one important reason for us to try to establish the exact counterpart of classical logic in the inquisitive setting, which is what we will do in Chapter 4.

Similar considerations apply to the projection operators. Again, since these operators are so elementary, it is to be expected that they too are expressible in many natural languages. More specifically, it seems plausible to hypothesize that they are expressed in English and many other languages by declarative and interrogative clause type markers. Such markers may include word order, intonation, as well as specific particles. For instance, on a first approximation, we may hypothesize that declarative word order in English invokes the ! operator, and interrogative word order the ? operator. A more detailed account of clause type marking in English in terms of the projection operators will be presented in Chapter 5.

3.4 Exercises

Exercise 3.1. [Working through some examples]
Consider the four propositions depicted in Figure 2.4 on page 27.

1. Determine the absolute pseudo-complement of each of these propositions.
2. Determine the meet and the join of any pair among these propositions.
3. Determine the outcome of applying the projection operators to each of these propositions.
3.4. Exercises

Exercise 3.2. [Meets and joins]
Show that every set of propositions in inquisitive semantics has a meet (Fact 3.1) and a join (Fact 3.2) with respect to entailment.

Exercise 3.3. [Absolute pseudo-complementation]
Show that every proposition $P$ in inquisitive semantics has an absolute pseudo-complement. That is, show that for any proposition $P$ there is a weakest proposition $R$ such that $P \cap R = \bot$ (Fact 3.4).

Exercise 3.4. [Projection operators]
Suppose we apply both projection operators to a given sentence, one after the other. Does it matter in which order we do this? That is, does the following hold for every proposition $P$:

$$?!P = !?!P$$
In this chapter we define an inquisitive semantics for the language of first-order logic, making use of the operations on propositions identified in the previous chapter. We will highlight some of the main features of the system, and illustrate it with a range of examples.

### 4.1 Logical language and models

We will consider a standard first-order language $\mathcal{L}$, based on a signature that consists of a set of function symbols $\mathcal{F}_\mathcal{L}$ and a set of relation symbols $\mathcal{R}_\mathcal{L}$, each with an associated arity $n \geq 0$. As usual, 0-place function symbols will be referred to as individual constants. We assume that the language has $\neg$, $\lor$, $\land$, $\rightarrow$, $\exists$, and $\forall$ as its basic logical constants.

We will interpret $\mathcal{L}$ with respect to \textit{first-order information models}. Such models consist of a set of possible worlds $W$, each associated with a standard first-order model. A standard first-order model, in turn, consists of a domain of individuals $D$ and an interpretation function $I$ which maps any function symbol in $\mathcal{F}_\mathcal{L}$ to a function over $D$ and every relation symbol in $\mathcal{R}_\mathcal{L}$ to a relation over $D$.

In order to avoid certain issues arising from quantification across different possible worlds, we will restrict our attention to \textit{rigid} first-order information models, in which the domain of individuals as well as the interpretation of function symbols is fixed across worlds. The only thing that may differ from world to world is the interpretation of relation symbols.

**Definition 4.1.** [Rigid first-order information models]

A rigid first-order information model for $\mathcal{L}$ is a triple $\langle W, D, I \rangle$, where:

- $W$ is a set, whose elements are referred to as possible worlds;
- $D$ is a non-empty set, whose elements are referred to as individuals;
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• \( I \) is a map that associates every \( w \in W \) with a first-order model \( I_w \) such that:
  
  – For every \( w \in W \), the domain of \( I_w \) is \( D \);
  
  – For every \( n \)-ary function symbol \( f \in \mathcal{F}_L \), \( I_w(f) : D^n \rightarrow D \), with the condition that for every \( w, v \in W \), \( I_w(f) = I_v(f) \);
  
  – For every \( n \)-ary relation symbol \( R \in \mathcal{R}_L \), \( I_w(R) \subseteq D^n \).

Unless specified otherwise, we will assume a fixed model throughout our discussion and we will often omit explicit reference to it. So, while in the previous chapters, where we were not yet considering a concrete logical language, we simply assumed a set of possible worlds \( W \) as our logical space, we now consider a triple \( (W, D, I) \), where \( W \) is still a set of possible worlds, and the other elements specify the interpretation of the function symbols and relation symbols in our language w.r.t. these possible worlds.

In order not to have assignments in the way all the time, we will assume that for any \( d \in D \), our language \( \mathcal{L} \) contains an individual constant \( d' \) such that \( I_w(d') = d \) for all \( w \in W \): if this is not the case, we simply expand the language by adding new constants, and we expand each \( I_w \) accordingly. In this way we can define our semantics just for formulas without free variables, and we can do without assignments altogether. This move is not essential, but it simplifies notation and terminology somewhat.

Finally, it will be convenient to have a notation for the set of worlds in our model in which a given sentence \( \varphi \) is classically true. We will denote this set as \( |\varphi| \) and refer to it as the classical truth-set of \( \varphi \).

**Definition 4.2.** [Classical truth-set]

For any \( \varphi \in \mathcal{L} \), the set of worlds where \( \varphi \) is classically true is denoted as \( |\varphi| \).

**4.2 Semantics**

We are now ready to recursively associate each sentence of our first-order language with an inquisitive proposition. The proposition expressed by an atomic sentence \( R(t_1, \ldots, t_n) \) is defined as the set of all information states that consist exclusively of worlds where \( R(t_1, \ldots, t_n) \) is true, i.e., the set of all subsets of \( |R(t_1, \ldots, t_n)| \). The connectives and quantifiers are taken to express the basic algebraic operations that we identified in Section 3.1.

**Definition 4.3.** [First-order inquisitive semantics]

1. \( [R(t_1, \ldots, t_n)] := \varphi(|R(t_1, \ldots, t_n)|) \)
2. \( [\neg \varphi] := [\varphi]^* \)
4.2. **Semantics**

3. \[ \varphi \land \psi \] := \[ \varphi \] \cap \[ \psi \]
4. \[ \varphi \lor \psi \] := \[ \varphi \] \cup \[ \psi \]
5. \[ \varphi \rightarrow \psi \] := \[ \varphi \] \Rightarrow \[ \psi \]
6. \[ \forall x. \varphi(x) \] := \bigcap_{d \in D} \[ \varphi(d') \]
7. \[ \exists x. \varphi(x) \] := \bigcup_{d \in D} \[ \varphi(d') \]

We refer to this first-order system as \( \text{InqB} \), where \( \text{B} \) stands for *basic*. We refer to \[ \varphi \] as the proposition expressed by \( \varphi \). The clauses of \( \text{InqB} \) constitute a proper inquisitive semantics in the sense that they indeed associate every sentence \( \varphi \in \mathcal{L} \) with an inquisitive proposition, i.e., a non-empty downward closed set of information states.

**Fact 4.4.** [Suitability of the semantics] For any \( \varphi \in \mathcal{L} \), \[ \varphi \] \in P.

Many notions that were introduced in Chapter 2 with reference to propositions can now be formulated with reference to the sentences in our logical language. For instance, we define the *informative content* of a sentence \( \varphi \), \( \text{info}(\varphi) \), as the informative content of the proposition it expresses, \( \text{info}(\[ \varphi \]) \). Similarly, the set of *alternatives* induced by \( \varphi \), \( \text{alt}(\varphi) \), is the set of alternatives in \( \text{alt}(\[ \varphi \]) \); \( \varphi \) is true in a world \( w \) just in case the proposition it expresses is true in \( w \), i.e., \( w \in \text{info}(\varphi) \); and the issue raised by \( \varphi \) is the issue embodied by \[ \varphi \], which is resolved by an information state \( s \) just in case \( s \in \[ \varphi \] \).

**Definition 4.5.** [Informative content, alternatives, and issues]

For any \( \varphi \in \mathcal{L} \):

- \( \text{info}(\varphi) := \bigcup_{\varphi} \)
- \( \text{alt}(\varphi) := \text{alt}(\[ \varphi \]) \)
- \( \varphi \) is true in \( w \) if and only if \( w \in \text{info}(\varphi) \)
- The issue raised by \( \varphi \) is one that is resolved by a state \( s \) just in case \( s \in \[ \varphi \] \).

We say that one sentence \( \varphi \) *entails* another sentence \( \psi \), \( \varphi \models \psi \), just in case the proposition expressed by \( \varphi \) entails the proposition expressed by \( \psi \), and we say that \( \varphi \) and \( \psi \) are *equivalent*, \( \varphi \equiv \psi \), just in case they express exactly the same proposition.

**Definition 4.6.** [Entailment and equivalence]

For any \( \varphi, \psi \in \mathcal{L} \):

- \( \varphi \models \psi \) just in case \( \[ \varphi \] \subseteq \[ \psi \] \)
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- \( \varphi \equiv \psi \) just in case \([\varphi] = [\psi]\)

Finally, we say that an information state \( s \) supports a sentence \( \varphi \), notation \( s \models \varphi \), just in case it supports the proposition expressed by \( \varphi \), which holds precisely if \( s \) implies \( \text{info}(\varphi) \) and resolves the issue that \( \varphi \) raises.

**Definition 4.7. [Support]**

An information state \( s \) supports a sentence \( \varphi \), notation \( s \models \varphi \), if and only if:

- \( s \subseteq \text{info}(\varphi) \), and
- \( s \) resolves the issue raised by \( \varphi \).

It is easy to see that these two conditions are satisfied just in case \( s \in [\varphi] \). So, just like the proposition expressed by \( \varphi \) in classical logic is the set of worlds where \( \varphi \) is true, the proposition expressed by \( \varphi \) in \( \text{InqB} \) is the set of states where \( \varphi \) is supported.

**Fact 4.8. [Support and propositions]**

For any information state \( s \) and any sentence \( \varphi \):

- \( s \models \varphi \iff s \in [\varphi] \)

As a consequence of this fact, \( \text{InqB} \) can be characterized by a recursive definition of the support conditions for the sentences in the language. These support conditions are as follows.

**Fact 4.9. [Support conditions]**

1. \( s \models R(t_1, \ldots, t_n) \iff s \subseteq [R(t_1, \ldots, t_n)] \)
2. \( s \models \neg \varphi \iff \forall t \subseteq s : t \neq \emptyset \text{ then } t \not\models \varphi \)
3. \( s \models \varphi \land \psi \iff s \models \varphi \text{ and } s \models \psi \)
4. \( s \models \varphi \lor \psi \iff s \models \varphi \text{ or } s \models \psi \)
5. \( s \models \varphi \rightarrow \psi \iff \forall t \subseteq s : t \models \varphi \text{ then } t \models \psi \)
6. \( s \models \forall x \varphi(x) \iff s \models \varphi(d') \text{ for all } d \in D \)
7. \( s \models \exists x \varphi(x) \iff s \models \varphi(d') \text{ for some } d \in D \)
In much early work on inquisitive semantics (Ciardelli, 2009; Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2011) as well as in some more recent work (e.g., Ciardelli et al., 2015; Ciardelli, 2016c,d), $\text{InqB}$ is in fact characterized directly in terms of support conditions. The proposition expressed by a sentence is then defined in terms of these support conditions, i.e., as the set of all states that support the sentence. An advantage of this approach is that it parallels the usual presentation of classical logic, with truth conditions as the basic notion. Another advantage, at least for certain purposes, is that it allows for a very efficient presentation of the system, bypassing all the more abstract notions that we introduced here before even starting to consider a concrete logical language.

There are two main reasons why we have chosen a less direct route here, following Ciardelli et al. (2013a) and Roelofsen (2013a). First, the current presentation of the new inquisitive notion of propositions (Chapter 2) brings out very explicitly how the standard information-centred notion of semantic content is enriched, why the new notion is shaped exactly the way it is, and that it naturally allows for various further extensions and refinements (see the references on page 43 as well as Appendix B). Second, the algebraic perspective adopted here (Chapter 3) makes it possible to motivate the treatment of the connectives and quantifiers in $\text{InqB}$ in a solid way, relying only on the structure of our new space of propositions. Moreover, it shows that $\text{InqB}$ is, in a very precise sense, the exact counterpart of classical logic in the inquisitive setting. Thus, unlike a support-based exposition, this mode of presentation flows directly from the abstract motivations and conceptual underpinnings of the system to its concrete implementation. On the other hand, for many particular applications of inquisitive semantics, it is more practical to present $\text{InqB}$ directly in terms of support conditions. The support based perspective will also be employed in Chapter 8 of the present book.

### 4.3 Semantic categories and projection operators

We say that a sentence is informative, inquisitive, a statement, a question, a hybrid, or a tautology just in case the proposition that it expresses is. This amounts to the following.

**Definition 4.10.** [Informativeness and inquisitiveness] For any $\varphi \in \mathcal{L}$:

- $\varphi$ is informative just in case $\text{info}(\varphi) \neq W$.
- $\varphi$ is inquisitive just in case $\text{info}(\varphi) \notin [\varphi]$.

**Definition 4.11.** [Semantics categories] We say that a sentence $\varphi \in \mathcal{L}$ is:

- a statement iff it is non-inquisitive;
• a question iff it is non-informative;
• a hybrid iff it is both informative and inquisitive;
• a tautology iff it is neither informative nor inquisitive.

**Fact 4.12.** [Direct characterization of statements, questions and tautologies]
• $\phi$ is a statement $\iff \text{info}(\phi) \in [\phi] \iff [\phi]$ has a greatest element.
• $\phi$ is a question $\iff \text{info}(\phi) = W$.
• $\phi$ is a tautology $\iff W \in [\phi]$.

Just like propositions, then, the sentences in our logical language can be thought of as inhabiting a two-dimensional space (see Figure 2.5 on page 28). The horizontal axis is inhabited by statements, which are non-inquisitive. The vertical axis is inhabited by questions, which are non-informative. The ‘zero-point’ of the space is inhabited by tautologies, which are neither informative nor inquisitive. The rest of the space is inhabited by hybrids, which are both informative and inquisitive.

In Section 3.2 we characterized two projection operators on propositions, which turn any given proposition into a statement or a question. Now that we are considering a concrete logical language, we will introduce two one-place connectives that express these projection operators. We will denote these connectives as $!$ and $?$, just like the operators they express.

**Definition 4.13.** [Projection operators]
For any $\phi \in \mathcal{L}$:
• $![\phi] := ![\phi]$
• $?\phi := ?[\phi]$

Recall from Fact 3.14 on page 53 that the projection operators on propositions can be characterized algebraically:
• $!P = P^{**}$
• $?P = P \cup P^*$

Since negation expresses absolute pseudo-complementation and disjunction expresses the join operation, this means that the connectives $!$ and $?$ can be characterized in terms of negation and disjunction.

**Fact 4.14.** [Projection operators in terms of negation and disjunction]
For any $\phi \in \mathcal{L}$:
4.4. Examples

- \( !\varphi \equiv \neg \neg \varphi \)
- \( ?\varphi \equiv \varphi \lor \neg \varphi \)

This means that ! and ? do not have to be added to our logical language as primitive connectives; \( !\varphi \) and \( ?\varphi \) can simply be regarded as abbreviations of \( \neg \neg \varphi \) and \( \varphi \lor \neg \varphi \), respectively.

Finally, we have that a sentence \( \varphi \) is always equivalent to the conjunction of its two ‘pure components’ \( !\varphi \) and \( ?\varphi \) (the analogue of Fact 3.13 on page 53).

**Fact 4.15.** [Division] For any \( \varphi \):

- \( \varphi \equiv !\varphi \land ?\varphi \)

### 4.4 Examples

Now let us consider some concrete sentences in \( \text{InqB} \) and the propositions that they express. We will assume that our language contains just one unary predicate symbol, \( R \), and two individual constants, \( a \) and \( b \). Accordingly, we will assume that the domain of discourse consists of just two objects, denoted by \( a \) and \( b \), respectively. Our logical space, then, consists of four worlds, one in which both \( Ra \) and \( Rb \) are true, one in which \( Ra \) is true but \( Rb \) is false, one in which \( Rb \) is true but \( Ra \) is false, and one in which neither \( Ra \) nor \( Rb \) is true. These worlds will be labeled 11, 10, 01, and 00, respectively. As usual, in order to keep the pictures orderly we display only the maximal elements of a proposition. For concreteness, we will informally read \( Ra \) as ‘\( a \) is running’, and similarly for \( Rb \).

**Atomic sentences.** Let us first consider the proposition expressed by the atomic sentences \( Ra \) and \( Rb \). According to the clause for atomic sentences, \( [Ra] \) consists of all states \( s \) such that every world in \( s \) makes \( Ra \) true, i.e., the state \{11, 10\} and all substates thereof. Thus, as depicted in Figure 4.1(a), \( [Ra] \) has a unique greatest element, \{11, 10\}. Fact 4.12 therefore ensures that \( Ra \) is a statement. It provides the information that \( a \) is running, and it does not request any further information. So it behaves just as in classical logic. Analogously, \( Rb \) is a statement which provides the information that \( b \) is running, without requesting any further information. The proposition expressed by \( Rb \) is depicted in Figure 4.1(b).

**Disjunction.** Next, consider the disjunction \( Ra \lor Rb \). According to the clause for disjunction, \( [Ra \lor Rb] \) consists of those states that are either in \( [Ra] \) or in \( [Rb] \). These are \{11,10\}, \{11,01\}, and all substates thereof, as depicted in Figure 4.1(c).
Chapter 4. A first-order inquisitive semantics

Figure 4.1: Atomic sentences, disjunction, and negation.

Since \( \text{info}(Ra \lor Rb) = \bigcup [Ra \lor Rb] = \{11, 10, 01\} \neq W \), the disjunction \( Ra \lor Rb \) is informative. It provides the information that at least one of \( a \) and \( b \) is running. However, unlike in the case of atomic sentences, in this case there is no unique greatest element in \([Ra \lor Rb]\) that includes all the others. Instead, there are two maximal elements, \([Ra] = \{11, 10\}\) and \([Rb] = \{11, 01\}\), which together contain all the others. Thus, \( Ra \lor Rb \) is not a statement; besides being informative it is also inquisitive. In order to settle the issue that it raises, one has to establish either that \( a \) is running, or that \( b \) is running.

A note of caution is perhaps in order here: it is important to keep in mind that \( \text{InqB} \) does not embody an analysis of sentences in natural language, it only provides the tools to formulate such analyses. In particular, a disjunctive sentence in \( \text{InqB} \) like \( Ra \lor Rb \) does not necessarily correspond to a disjunctive declarative sentence in English like (1) below, or to a disjunctive interrogative sentence like (2) for that matter.

(1) Ann is running or Bill is running.

(2) Is Ann running or is Bill running?

In Chapter 5 we will present a concrete analysis of sentences like (1) and (2), using \( \text{InqB} \). On that analysis, (1) corresponds to \(!((Ra \lor Rb)\) and (2) either to \(?((Ra \lor Rb)\) or to \(Ra \lor Rb\), depending on intonation.

Negation. Next, we turn to negation. According to the clause for negation, \([\neg Ra]\) consists of all states \( s \) such that \( s \) does not have any world in common with any state in \([Ra]\). Thus, \([\neg Ra]\) consists of all states that do not contain the worlds 11 and 10, which are \([\neg Ra] = \{01, 00\}\) and all substates thereof, as depicted in Figure 4.1(d). Since this set of states has a greatest element, Fact 4.12 ensures that \( \neg Ra \) is a statement. It provides the information that \( a \) is not running, and does not request any further information.

Now let us consider the negation of an inquisitive disjunction, \( \neg(Ra \lor Rb)\). According to the clause for negation, \([\neg(Ra \lor Rb)]\) consists of all states which do not have a world in common with any state in \([Ra \lor Rb]\). Thus, \([\neg(Ra \lor Rb)]\)
4.4. Examples

Figure 4.2: Projection operators.

consists of all states that do not contain the worlds $11$, $10$, and $01$, which are $\{00\}$ and $\emptyset$, as depicted in Figure 4.1(e). Again, there is a unique maximal element, namely $|\neg(Ra \lor Rb)| = \{00\}$. Thus, $\neg(Ra \lor Rb)$ is a statement, which provides the information that neither $a$ nor $b$ is running, just like in classical logic, and does not request any further information.

These examples of negative sentences exemplify the general observation that we made above concerning pseudo-complementation (just below Fact 3.4): the absolute pseudo-complement of a proposition always contains a unique alternative. This means that a negative sentence $\neg \varphi$ is always a statement, which provides the information that $\varphi$ is false, and does not request any further information.

Projection operators. Next let us consider $!(Ra \lor Rb)$, which abbreviates $\neg\neg(Ra \lor Rb)$. We have just seen that $\neg(Ra \lor Rb)$ expresses the proposition depicted in Figure 4.1(e). Applying negation again, we arrive at the proposition depicted in Figure 4.2(a), which has $|Ra \lor Rb|$ as its unique alternative. Notice that $!(Ra \lor Rb)$ is not equivalent with $Ra \lor Rb$. The two sentences have the same informative content, but the former is a purely informative statement, while the latter is also inquisitive. This exemplifies the general nature of $!$: for any sentence $\varphi$, $!\varphi$ is a statement with the same informative content as $\varphi$. If $\varphi$ itself is already a statement, then $!\varphi$ and $\varphi$ are equivalent; if $\varphi$ is inquisitive, as in the example just considered, the two differ.

Let us now turn to ?. Consider $?Ra$, which is an abbreviation of $Ra \lor \neg Ra$. We have already seen what $[Ra]$ and $[\neg Ra]$ are. According to the clause for disjunction, $?Ra = [Ra \lor \neg Ra]$ consists of all states that are either in $[Ra]$ or in $[\neg Ra]$. These states are $[Ra]$, $[\neg Ra]$, and all substates thereof, as depicted in Figure 4.2(b). Since $\text{info}(?Ra) = W$, $?Ra$ is not informative, which means that it is a question. Moreover, since $?Ra$ contains two alternatives, it is inquisitive. In order to settle the issue that it raises, one has to establish either that $a$ is running, or that $a$ is not running. That is, one has to establish whether $a$ is running. Thus, while $?Ra$ is shorthand for $Ra \lor \neg Ra$, perhaps the most famous classical tautology, it is not a tautology in InqB: instead, it corresponds to the
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(a) \( R_a \land R_b \)
(b) \( ?R_a \land ?R_b \)
(c) \( R_a \rightarrow R_b \)
(d) \( R_a \rightarrow ?R_b \)
(e) \( \forall x. ?R_x \)

Figure 4.3: Conjunction, implication, and universal quantification.

polar question “whether \( R_a \)”\). Analogously, \( ?R_b \), depicted in Figure 4.2(c), corresponds to the polar question “whether \( R_b \)”.

If \( ? \) applies to the disjunction \( R_a \lor R_b \), which is already inquisitive, then it yields the proposition depicted in Figure 4.2(d). \( [R_a \lor R_b] \) already contains two alternatives, \( [R_a] \) and \( [R_b] \); \( ? \) adds a third alternative, which is the set of worlds that are neither in \( [R_a] \) nor in \( [R_b] \). Thus, in order to resolve the issue raised by \( ?(R_a \lor R_b) \), one either has to establish that \( a \) is running, or that \( b \) is, or that neither \( a \) nor \( b \) is.

Finally, let us consider a case where \( ! \) and \( ? \) both apply, one after the other: \( !?(R_a \lor R_b) \). As we already saw above, \( ![R_a \lor R_b] \) contains a single alternative, consisting of all worlds where at least one of \( a \) and \( b \) is running. As depicted in Figure 4.2(e), \( ? \) adds a second alternative, which is the set of worlds where neither \( a \) nor \( b \) is running. Notice that the resulting proposition differs from that expressed by \( ?(R_a \lor R_b) \), which contains three alternatives rather than two. In order to settle the issue expressed by \( !?(R_a \lor R_b) \) it is sufficient to establish that at least one of \( a \) and \( b \) is running. In order to settle the issue expressed by \( ?(R_a \lor R_b) \) this is not sufficient; rather, it needs to be established which of \( a \) and \( b \) is running (or that neither of them is). We will see in Chapter 5 that the ability to capture such subtle differences is crucial to account for various kinds of disjunctive questions in natural languages.

Conjunction. Next, let us consider conjunction. First, let us look at the conjunction of our two atomic, non-inquisitive sentences, \( R_a \) and \( R_b \). According to the clause for conjunction, \( [R_a \land R_b] \) consists of those states that are both in \( [R_a] \) and in \( [R_b] \). These are \( \{11\} \) and \( \emptyset \). Thus, \( [R_a \land R_b] \) has a unique greatest element, namely \( \{11\} \), and accordingly \( R_a \land R_b \) is a statement which, just like in the classical case, provides the information that both \( a \) and \( b \) are running.

Now let us look at the conjunction of two inquisitive sentences, \( ?R_a \) and \( ?R_b \). As depicted in Figure 4.3(b), the proposition \( ?R_a \land ?R_b \) contains four alternatives, \( [R_a \land R_b] \), \( [R_a \land \neg R_b] \), \( [\neg R_a \land R_b] \), and \( [\neg R_a \land \neg R_b] \). Since these alternatives together cover the entire logical space \( ?R_a \land ?R_b \) is a question. Moreover, since there is more than one alternative, this question is inquisitive.
4.4. Examples

In order to settle the issue that it raises, one has to establish one of \( Ra \land Rb \), \( Ra \land \neg Rb \), \( \neg Ra \land Rb \), \( \neg Ra \land \neg Rb \). Thus, our conjunction is a question which requests enough information to settle both the issue whether \( Ra \) is the case, contributed by \(?Ra\), and the issue whether \( Rb \) is the case, contributed by \(?Rb\).

These two examples of conjunctive sentences exemplify a general fact: if \( \varphi \) and \( \psi \) are statements, then the conjunction \( \varphi \land \psi \) is also a statement, which provides both the information provided by \( \varphi \) and the information provided by \( \psi \); and if \( \varphi \) and \( \psi \) are questions, then the conjunction \( \varphi \land \psi \) is also a question, which requests enough information to settle both the issue raised by \( \varphi \) and the issue raised by \( \psi \).

Implication. Next, let us consider implication. Again, we will first consider a simple case, \( Ra \rightarrow Rb \), where both the antecedent and the consequent are atomic, and therefore non-inquisitive. According to the clause for implication, \( [Ra \rightarrow Rb] \) consists of all states \( s \) such that every substate \( t \subseteq s \) that is in \( [Ra] \) is also in \( [Rb] \). These are all and only those states that are contained in \( [Ra \rightarrow Rb] = \{11, 01, 00\} \), as depicted in Figure 4.3(c). So, \( [Ra \rightarrow Rb] \) has a unique greatest element, \( |Ra \rightarrow Rb| = \{11, 01, 00\} \), which means that the implication \( Ra \rightarrow Rb \) is a statement which, just like in the classical setting, provides the information that if \( a \) is running, then so is \( b \).

Now let us consider a more complex case, \( Ra \rightarrow ?Rb \), where the consequent is an inquisitive question. As depicted in Figure 4.3(d), the proposition \( [Ra \rightarrow ?Rb] \) contains two alternatives, \( [Ra \rightarrow Rb] = \{11, 01, 00\} \), and \( [Ra \rightarrow \neg Rb] = \{10, 01, 00\} \). Since these two alternatives together cover the entire logical space, our implication is a question. Moreover, since there is more than one alternative, the implication is inquisitive. In order to settle the issue that it raises, one must either establish \( Ra \rightarrow Rb \), or \( Ra \rightarrow \neg Rb \). In the former case one establishes that if \( a \) is running, then so is \( b \); in the latter case, that if \( a \) is running, then \( b \) isn’t. So \( Ra \rightarrow ?Rb \) is a question which requests enough information to establish whether \( b \) is running under the assumption that \( a \) is.

Again, these two examples of conditional sentences exemplify a general feature of \( \text{InqB} \): if \( \psi \) is a statement, then \( \varphi \rightarrow \psi \) is a statement as well, providing the information that if \( \varphi \) holds, then so does \( \psi \); and if \( \varphi \) a statement and \( \psi \) a question, then \( \varphi \rightarrow \psi \) is a question as well, requesting enough information to settle the issue raised by \( \psi \) assuming the information provided by \( \varphi \).

Quantification. Finally, let us consider existential and universal quantification. As usual, existential quantification behaves essentially like disjunction and universal quantification behaves essentially like conjunction. In fact, since our current domain of discourse consists of only two objects, denoted by \( a \) and \( b \), respectively, \( \exists x.Rx \) expresses exactly the same proposition as \( Ra \lor Rb \), depicted
in Figure 4.1(c), and $\forall x. Rx$ expresses exactly the same proposition as $Ra \land Rb$, depicted in Figure 4.3(a). Finally, consider the proposition expressed by $\forall x. ?Rx$, depicted in Figure 4.3(e). Notice that this proposition induces a partition on the logical space, where each block of the partition consists of worlds that agree on the extension of $P$. Thus, $\forall x. ?Rx$ is a question that asks for an exhaustive specification of the objects that are running.

This concludes our illustration of the behavior or the connectives and quantifiers in $\text{InqB}$.

### 4.5 Informative content, truth, and support

Recall that $\text{info}(\varphi)$ is defined as $\bigcup [\varphi]$, which is a set of worlds. In classical logic, the informative content of a sentence $\varphi$ is also embodied by a set of worlds, namely the set of all worlds where $\varphi$ is true, $|\varphi|$. Thus, the question that naturally arises is how these two notions of informative content relate to each other. It turns out that the two are precisely the same.

**Fact 4.16.** [Informative content and truth] For any sentence $\varphi \in \mathcal{L}$:

- $\text{info}(\varphi) = |\varphi|

This shows that $\text{InqB}$ fully preserves the classical treatment of informative content. The system only differs from classical logic in that, besides informative content, it takes inquisitive content into consideration as well.

Notice that Facts 2.19 and 4.16 together yield the following characterization of questions and statements in terms of classical truth.

**Fact 4.17.** [Questions and statements in terms of classical truth]

- $\varphi$ is a question $\iff |\varphi| = W$
- $\varphi$ is a statement $\iff |\varphi| \in [\varphi] \iff [\varphi] = \wp(|\varphi|)$

Thus, questions in $\text{InqB}$ are precisely those sentences that are classically true at any world. On the other hand, a sentence is a statement in $\text{InqB}$ just in case the proposition it expresses is fully determined by its classical truth-set: $[\varphi] = \varphi(|\varphi|)$. This means that a statement $\varphi$ provides the information that $\varphi$ is true, and does not request any further information. Thus, statements behave exactly as they do in classical logic.

The classical behavior of statements results in a tight connection between their support conditions and their truth conditions. Namely, a statement $\varphi$ is supported by a state $s$ just in case it is true in every world in $s$. This holds only for statements; the moment a sentence becomes inquisitive, the connection between support and truth breaks down.
4.6. Syntactic properties of statements and questions

We have defined statements as non-inquisitive sentences, and questions as non-informative sentences. These characterizations are semantic in nature. Below we provide some syntactic conditions which make it easy to recognize a large class of statements and questions (though not all of them) just based on their form, without inspecting their meaning.

Let us start with statements. The following fact provides some syntactic conditions which guarantee that a sentence is a statement. These conditions generalize some of the more specific observations that were already made in discussing the examples above.

**Fact 4.19.** [Sufficient conditions for statements]

1. !\varphi is always a statement;
2. Atomic sentences are always statements;
3. \neg \varphi is always a statement;
4. If \varphi and \psi are statements, then so is \varphi \wedge \psi;
5. If \psi is a statement, then so is \varphi \rightarrow \psi for any antecedent \varphi;
6. If \varphi(d') is a statement for all \textit{d} \in \textit{D}, then so is \forall x \varphi(x).

Now let us turn to questions. Again we provide some syntactic conditions that guarantee that a given sentence is a question, generalizing some of the more specific observations made in discussing the examples above.

**Fact 4.20.** [Sufficient conditions for questions]

1. ?\varphi is always a question;
2. A classical tautology is always a question;
3. If \varphi and \psi are questions, so is \varphi \wedge \psi;
4. If \psi is a question, then so are \varphi \lor \psi and \varphi \rightarrow \psi, for any \varphi;
5. If \varphi(d') is a question for all \textit{d} \in \textit{D}, then so is \forall x \varphi(x);
6. If \varphi(d') is a question for some \textit{d} \in \textit{D}, then so is \exists x \varphi(x).
4.7 Sources of inquisitiveness

The partial syntactic characterization of statements given in Fact 4.19 implies that disjunction, the existential quantifier, and the ? projection operator are the only sources of inquisitiveness in our logical language.

FACT 4.21. [Sources of inquisitiveness]
Any sentence that does not contain $\lor$, $\exists$, or $\,?$ is a statement.

Note that there is a close connection between disjunction, the existential quantifier, and the $\,?$ operator in $\text{InqB}$. Namely, they all behave as join operators: $[\varphi \lor \psi]$ is the join of $[\varphi]$ and $[\psi]$; $[\exists x. \varphi(x)]$ is the join of $\{[\varphi(d')] \mid d \in D\}$, and $[\,?\varphi]$ is the join of $[\varphi]$ and $[\varphi]^*$. In terms of semantic operators, then, the join operator is the essential source of inquisitiveness: without applying this operator, it is impossible to produce inquisitive propositions from non-inquisitive ones.

This fact may provide the basis for an explanation of the well-known observation that in many natural languages, question markers are homophonous with words for disjunction and/or existentials (see Jayaseelan, 2001, 2008; Bhat, 2005; Haida, 2007; Cable, 2010; AnderBois, 2011; Slade, 2011, among others). For instance, Malayalam -oo and Japanese ka are used for all three purposes:

<table>
<thead>
<tr>
<th>Malayalam</th>
<th>Japanese</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Existential</td>
<td>aar-oo</td>
<td>dare-ka</td>
</tr>
<tr>
<td>Disjunction</td>
<td>Anna-o Peter-oo</td>
<td>Anna-ka Peter-ka</td>
</tr>
<tr>
<td>Interrogative</td>
<td>Anna wannu-(w)oo</td>
<td>Anna wa kita-ka</td>
</tr>
</tbody>
</table>

Szabolcsi (2015b) proposes an account of this cross-linguistic phenomenon in inquisitive semantics, suggesting that the inquisitive join operation can indeed be seen as the semantic common core of disjunctive, existential, and interrogative constructions in languages like Malayalam and Japanese.

It is also interesting to note that there is a close connection between the treatment of disjunction and existentials in $\text{InqB}$, and their treatment in alternative semantics (Kratzer and Shimoyama, 2002; Simons, 2005; Menéndez-Benito, 2005; Alonso-Ovalle, 2006; Aloni, 2007, among others). In both frameworks, disjunction and existentials introduce sets of alternatives. In the case of alternative semantics, this treatment is motivated by a number of empirical linguistic phenomena, including free choice inferences, exclusivity implicatures, and counterfactual conditionals with disjunctive antecedents. The analysis of disjunction and existentials as introducing sets of alternatives has made it possible to develop new accounts of these phenomena which improve considerably on previous accounts. However, while work on alternative semantics has provided ample empirical motivation for its treatment of disjunction and existentials, it has not provided any motivation for this treatment that is independent of the
linguistic phenomena that it has aimed to capture. Therefore, though successful in terms of empirical coverage, it has limited explanatory power. Moreover, the empirical phenomena that have motivated work on disjunction and existentials in alternative semantics have been taken to require a radical departure from the classical algebraic treatment of disjunction and existentials. For instance, Alonso-Ovalle (2006) writes in the conclusion section of his dissertation:

“This dissertation has investigated the interpretation of counterfactuals with disjunctive antecedents, unembedded disjunctions, and disjunctions under the scope of modals. We have seen that capturing the natural interpretation of these constructions proves to be challenging if the standard analysis of disjunction, under which or is the Boolean join, is assumed.”

Similarly, Simons (2005) starts her paper as follows:

“In this paper, the meanings of sentences containing the word or and a modal verb are used to arrive at a novel account of the meaning of or coordinations. It has long been known that such sentences […] pose a problem for the standard treatment of or as a Boolean connective equivalent to set union.”

As discussed in detail in Roelofsen (2015b), the approach we have taken here shows that, once we take both informative and inquisitive content into account, general algebraic considerations lead essentially to the treatment of disjunction that was proposed in alternative semantics, thus providing exactly the independent motivation that has so far been missing. \(^1\) Moreover, it shows that the treatment of disjunction as generating sets of alternatives can actually be seen as a natural generalization of the classical treatment, rather than a radical departure from it: as soon as we adopt a notion of meaning that encompasses both informative and inquisitive content, treating disjunction as a join operator automatically gives it the potential to generate multiple alternatives. Thus, we can have our cake and eat it: we can treat disjunction as a join operator and as introducing sets of alternatives at the same time. In inquisitive semantics, the two go hand in hand.

4.8 Exercises

Exercise 4.1. [De Morgan’s laws]
Below are two well-known equivalences from classical logic, known as De Mor-

\(^1\)It should be noted that, while both in alternative semantics and in inquisitive semantics disjunction generates alternatives, there is also a subtle but important difference. Namely in inquisitive semantics one alternative can never be contained in another. This has certain advantages, which are discussed in Section 7.1.
gan’s laws:
\[ \neg(\varphi \lor \psi) \equiv \neg\varphi \land \neg\psi \]
\[ \neg(\varphi \land \psi) \equiv \neg\varphi \lor \neg\psi \]

Do these equivalences also hold in inquisitive semantics? If yes, give a proof. If no, provide a counterexample.

**Exercise 4.2.** [The law of double negation]
Recall that in classical logic, \( \neg\neg\varphi \rightarrow \varphi \) is a tautology for any \( \varphi \). Show that in \( \text{InqB} \):

1. \( \neg\neg\varphi \rightarrow \varphi \) is a tautology whenever \( \varphi \) is a statement;
2. It is not the case that \( \neg\neg\varphi \rightarrow \varphi \) is a tautology for any \( \varphi \).

Explain why this difference between classical logic and \( \text{InqB} \) arises, even though \( \neg \) and \( \rightarrow \) express exactly the same algebraic operations in both frameworks (absolute and relative pseudo-complementation, respectively).
Chapter 5

Disjunctive lists

In Chapters 2-4 we laid out the basic architecture of inquisitive semantics. In the present chapter and the ones to follow, we will discuss some of its applications and its relation to other frameworks. We will start by formulating an account of certain types of declarative and interrogative sentences in English, highlighting a particular advantage that inquisitive semantics has w.r.t. other frameworks when it comes to formulating a uniform account of lexical and prosodic elements that play a role in both declarative and interrogative constructions.

We will consider sentences consisting of one or more declarative or interrogative clauses separated by disjunction, with different intonation patterns. Some representative examples are given in (1)-(5) below (we use ↑ and ↓ to indicate rising and falling pitch contours, respectively).

(1) Igor speaks English↓.
(2) Igor speaks English↑?
(3) Does Igor speak English↑?
(4) Does Igor speak English↑ or does he speak French↓?
(5) Does Igor speak English↑ or does he speak French↑?

Drawing inspiration from Zimmermann (2000) we will view such declarative and interrogative sentences as lists. Lists either consist of a single clause, as in (1)-(3), or of multiple clauses separated by disjunction, as in (4)-(5). Moreover, we think of lists as being either open (signaled by a final rise), as in (2), (3) and (5), or closed (signaled by a final fall), as in (1) and (4).

We will present an account of such lists in InqB. While our focus will be on English, we expect that the basic semantic operations that our account associates with the relevant lexical, morphological, and prosodic features may play a central role in the interpretation of similar constructions in other languages as well. The division of labor between the various elements is bound to vary from language to language, but we expect that the basic repertoire of semantic
operations that our account draws on will be relatively stable across languages.

A general point that we want to make in this chapter, independently of the
details of the specific account that we will present, is that any account which
aims to treat disjunction and the relevant prosodic features uniformly across
declarative and interrogative constructions, has to be couched within a semantic
framework which treats informative and inquisitive content in an integrated
way. For instance, if we want to give a uniform characterization of the role of
disjunction in declarative and interrogative sentences in English, we have to be
able to capture how it affects both informative and inquisitive content, independ-
ently of the kind of construction that it happens to be part of. And similarly
for the relevant prosodic features. Simply put, the fact that declarative and
interrogative lists are largely built up from the same parts constitutes an im-
portant piece of motivation for a semantic framework like inquisitive semantics,
which treats informative and inquisitive content in an integrated way, as op-
posed to approaches in which the standard truth-conditional notion of meaning
is maintained for declaratives and a separate notion of meaning is invoked for
interrogatives (e.g., Karttunen, 1977; Groenendijk and Stokhof, 1984). Such ap-
proaches do not provide a uniform account of disjunction across declaratives and
interrogatives—the semantic contribution of disjunction in interrogatives like (4)
is taken to be different from its truth-conditional contribution in declaratives.1

We will present a simplified version here of the account of declarative and
interrogative lists outlined in Roelofsen (2013c) and developed in more detail in
Roelofsen (2015a). The simplified account to be presented here has also been
presented in Roelofsen and Farkas (2015), where it serves as the basis for a
theory of answer particles like yes and no. The main reason we present only
a simplified version of the account here is that the full account does not only
aim to capture the informative and inquisitive content of the various types of
lists but also their presuppositional content, which requires an extension of the
InqB system. While such an extension increases the empirical coverage of the
account, a simplified non-presuppositional version should suffice to substantiate
our general point, i.e., to demonstrate the advantages of inquisitive semantics
in formulating a uniform account of lexical and prosodic elements that play a
role in both declarative and interrogative constructions.2

1The uniform treatment of disjunction across declaratives and interrogatives to be pre-
sented here is closely related to the treatment of disjunction in alternative semantics (Simons,
2005; Alonso-Ovalle, 2006; Aloni, 2007). See Section 4.7 for some discussion of how the latter
treatment of disjunction is related to the inquisitive one. For more elaborate comparison, see
Ciardelli and Roelofsen (2016); Ciardelli et al. (2016a); Roelofsen (2015b).

2Besides leaving presuppositions out of consideration, the account of Roelofsen (2013c,
2015a) is simplified here in another respect as well. Namely, in order to derive the fact that
closed lists with multiple disjuncts generally imply that exactly one of the disjuncts holds, the
full account assumes that such lists involve an exclusive strengthening operator. For simplicity,
this operator is not taken into account here. Again, this does not affect our general point.
We will proceed as follows. Section 5.1 provides a more systematic, though still informal characterization of the different types of lists that we are concerned with, Section 5.2 specifies what we take the logical forms of these lists to be, and Section 5.3 specifies how these logical forms can be interpreted in InqB.

5.1 Declarative and interrogative lists

We take lists to differ along three basic parameters: they can be declarative or interrogative, they can be open or closed, and they can consist of a single clause or of multiple clauses separated by disjunction. We will refer to these clauses as ‘list items’. In total, then, we consider $2 \times 2 \times 2 = 8$ basic types of lists, exemplified in (6)-(9).

(6) **Closed declarative lists**
   a. mono-clausal: Igor speaks English↓.
   b. multi-clausal: Igor speaks English↑ or he speaks French↓.

(7) **Open declarative lists**
   a. mono-clausal: Igor speaks English↑?
   b. multi-clausal: Igor speaks English↑ or he speaks French↑?

(8) **Closed interrogative lists**
   a. mono-clausal: Does Igor speak English↓?
   b. multi-clausal: Does Igor speak English↑ or does he speak French↓?

(9) **Open interrogative lists**
   a. mono-clausal: Does Igor speak English↑?
   b. multi-clausal: Does Igor speak English↑ or does he speak French↑?

Closed lists characteristically have falling intonation on the final item, while open lists characteristically have rising intonation on the final item. Non-final list items are canonically pronounced with rising intonation, both in open and in closed lists. Moreover, each item is pronounced in a separate intonational phrase, which means that there is an intonational phrase break after each non-final item, before the disjunction word. In fact, two non-final list items may be separated just by an intonational phrase break, i.e., the disjunction word may be omitted if neither of the items is final.

Disjunction can be used to separate list items, but it may also occur within a list item. Thus, the lists in (10) below all have a single item, containing disjunction, rather than two items separated by disjunction (we use hyphenation to indicate the absence of an intonational phrase break):
Chapter 5. Disjunctive lists

(10) **Lists with a single disjunctive item**

a. *Closed declarative:* Igor speaks English-or-French↓.

b. *Open declarative:* Igor speaks English-or-French↑?

c. *Closed interrogative:* Does Igor speak English-or-French↓?

d. *Open interrogative:* Does Igor speak English-or-French↑?

Note that some types of lists that we consider here are better known under different names. For instance, singleton interrogative lists (either open or closed) are usually referred to as *polar questions* and non-singleton closed interrogative lists are usually referred to as *alternative questions*. We will sometimes use this more familiar terminology alongside our list terminology. The former has the advantage of being easier to recognize; the latter has the advantage of explicating the distinctive features of each type of list and emphasizing that each specific construction is considered here as part of a more general paradigm.

Previous analyses of lists have only been concerned with some types of lists, not with the full range. Zimmermann (2000) focuses on non-singleton declarative lists like (6b) and (7b). On the other hand, Pruitt (2007), Biezma (2009), Biezma and Rawlins (2012), and Aloni *et al.* (2013), who like us also draw inspiration from Zimmermann, focus on singleton open interrogatives (polar questions) like (9a) and (10d), and non-singleton closed interrogatives (alternative questions) like (8b). A uniform analysis of the full range of lists exemplified in (6)-(10) would thus extend the coverage of these previous analyses considerably.³

5.2 Logical forms

Schematically, we assume that a list with n items has the following logical form:

³The idea that disjunction can be used to form lists has also been put forth by Simons (2001, p.616), independently of Zimmermann (2000). In Simons’ work, however, this idea does not form the basis for a particular semantic treatment of disjunctive sentences and their various prosodic features, but is rather part of a pragmatic explanation for the fact that disjunctive declaratives are typically much more natural in response to a given question than truth-conditionally equivalent non-disjunctive sentences. For instance, if the question is why Jane isn’t picking up the phone, then (i) is a much more natural answer than (ii).

(i) Either she isn’t home, or she can’t hear the phone.

(ii) It’s not the case that she is at home and she can hear the phone.

To the extent that Simons’ analysis of this phenomenon is successful, it provides independent motivation for our general outlook on disjunctive sentences as lists. A proper discussion of Simons’ analysis, however, is beyond the scope of this book.
5.2. Logical forms

We will refer to OPEN/CLOSED and DECL/INT as *list classifiers* and to the rest of the structure as the *body* of the list. We assume that each item in the body of the list is a full clause, headed by a declarative or interrogative complementizer, C_{DECL} or C_{INT}, depending on whether the list is classified as DECL or INT, respectively.\(^4\)

To give a concrete example, the open interrogative in (9b), which involves two list items, is taken to have the following structure:

\[
(12)
\]

On the other hand, the open interrogative in (10d), which has a single list item containing a disjunctive phrase, is taken to have the following structure:

\[
\]

\(^4\) We are only considering lists here whose items undeniably amount to entire clauses. That is, we do not consider cases like (i), which may be treated either as a single clause containing a disjunctive phrase, or alternatively as a disjunction of two full clauses, the latter partially elided.

(i) Does Igor speak English\(^\uparrow\) or French\(^\downarrow\)?

Such constructions raise a number of syntactic issues which remain largely unresolved in the literature. See, e.g., Larson (1985); Han and Romero (2004); Beck and Kim (2006); Haida (2010); Pruitt and Roelofsen (2011); Uegaki (2014); Roelofsen (2015a) for discussion.
Chapter 5. Disjunctive lists

5.3 Interpreting logical forms

We will specify a semantic interpretation of these logical forms by translating them into the logical language of \( \text{InqB} \). Thereby we associate each logical form with a proposition, namely the proposition expressed by the formula that serves as its translation.

We will first consider the body of a list, and after that turn to the list classifiers. Recall that the body of a list consists of one or more list items, separated by disjunction. Every list item, in turn, is a full clause headed by a declarative or interrogative complementizer (C\(_{\text{decl/int}}\)). The rest of the clause is a tense phrase (TP), which may itself contain a disjunction.

The translation procedure is very straightforward. Any disjunction is translated as \( \lor \), no matter whether it separates two list items or occurs within one of the list items. Every complementizer, be it declarative or interrogative, is translated as \( ! \). The rationale for this is that every list item is seen, intuitively speaking, as one block, i.e., as contributing a single alternative to the proposition expressed by the list as a whole. This is ensured by applying \( ! \), which turns any proposition \( P \) into a proposition with a single alternative, \( \bigcup P \). Otherwise the procedure is completely standard. Thus, the body of a list is translated according to the rule in (14), where \( \varphi_1, \ldots, \varphi_n \) are standard translations of TP\(_1\), \ldots, TP\(_n\) into the language of propositional logic.

\[
(14) \quad \text{Rule for translating the body of a list: } \\
[C_{\text{decl/int}} \ \text{TP}_1] \lor \ldots \lor [C_{\text{decl/int}} \ \text{TP}_n] \Rightarrow !\varphi_1 \lor \ldots \lor !\varphi_n
\]

Returning to our concrete examples above, if we translate \textit{Igor speaks English} as \( p \) and \textit{Igor speaks French} as \( q \), then we get the following translations for the list bodies of (9b) and (10d), respectively.

\[
(15) \quad [C_{\text{INT}} \ \text{does Igor speak English}] \lor [C_{\text{INT}} \ \text{does he speak French}] \Rightarrow !p \lor !q
\]

\[
(16) \quad [C_{\text{INT}} \ \text{does Igor speak English or French}] \Rightarrow !(p \lor q)
\]

Now let us turn to the list classifiers. To specify their semantic contribution it
is convenient to use some notation and terminology from type theory.\(^5\) Recall
that in inquisitive semantics, propositions are sets of sets of possible worlds,
i.e., objects of type \(\langle\langle s,t\rangle,t\rangle\). Let us abbreviate this type as \(T\). Now, we will
treat \textsc{decl} and \textsc{int} as propositional operators, i.e., as functions that take a
proposition as their input, and deliver another proposition as their output. This
means that \textsc{decl} and \textsc{int} are of type \(\langle T,T \rangle\). On the other hand, we will treat
\textsc{open} and \textsc{closed} as modifiers of propositional operators, i.e., as functions that
take a propositional operator as their input, and deliver a modified propositional
operator as their output. So \textsc{open} and \textsc{closed} are of type \(\langle\langle T,T\rangle,\langle T,T\rangle\rangle\). It
will become clear in a moment why \textsc{open} and \textsc{closed} are treated as having
this somewhat more complex type, rather than simply \(\langle T,T \rangle\), like \textsc{decl} and
\textsc{int}. First, we need to look at each of the classifiers in somewhat more detail.

First consider \textsc{decl}. We will treat \textsc{decl} as making a list purely informative,
i.e., as \textit{eliminating inquisitiveness}. This effect can be achieved straightforwardly
by treating \textsc{decl} as a function that takes the proposition \(P\) expressed by the
body of a list as its input and applies the projection operator \(!\) to it, returning
\(!P\). Using type-theoretic notation, this can be formulated concisely as follows:

\begin{equation}
\textsc{decl} \Rightarrow \lambda P. \! P
\end{equation}

Next, consider \textsc{int}. The proposal in Roelofsen (2013c, 2015a) is to treat interro-
gativity as having two effects. First, whenever possible, it \textit{ensures inquisitiveness}.
This is done by applying a conditional variant of the \(?\) operator, which we will
denote here as \(\langle?\rangle\). If the proposition \(P\) that \(\langle?\rangle\) takes as its input is not yet
inquisitive, then \(?\) is applied to it. On the other hand, if \(P\) is already inquisi-
tive, then it is left untouched. The only case in which this procedure does \textit{not}
yield an inquisitive output is when \(P\) is a tautology or a contradiction. In this
case \(\langle?\rangle P\) is a tautology, which is not inquisitive. In all other cases, \(\langle?\rangle P\) is
inquisitive.

The second effect of interrogativity proposed in Roelofsen (2013c, 2015a)
is that it \textit{ensures non-informativity}, by introducing a presupposition that the
actual world must be contained in \(\bigcup P\). This second aspect of interrogativity
is especially important to account for the presuppositional component of alter-
native questions (see, e.g., Karttunen and Peters, 1976; Biezma and Rawlins,
2012). However, since presuppositions cannot be captured in \textsc{InqB}, we simplify
the analysis here and restrict ourselves to the first aspect of interrogativity de-
scribed above, i.e., to ensure inquisitiveness. Thus, we assume the following
treatment of \textsc{int}:

\(^5\)We will only use some type-theoretical notation here in the meta-language to describe
functions (as in, e.g., Heim and Kratzer, 1998). A more rigorous approach would be to extend
the \textsc{InqB} system to a full-fledged type theoretic framework (as done in Ciardelli, Roelofsen,
and Theiler, 2016a). We leave this step implicit here because it would involve quite some
technicalities which are to a large extent orthogonal to our present concerns.
Chapter 5. Disjunctive lists

Finally, let us consider open and closed. Intuitively speaking, we treat these classifiers as encoding whether the list is left ‘open ended’ or whether it is ‘finished’ and ready to be ‘sealed off’. The role of closed is to mark the list as being finished, and to allow decl or int, whichever is present, to seal off the list. Thus, closed is simply treated as the identity function, leaving the propositional operator $\pi$ expressed by decl or int untouched and letting it apply to the proposition expressed by the body of the list.

(19) $\text{closed} \leadsto \lambda \pi. \pi$

On the other hand, the role of open is to mark the list as being open-ended. It prevents decl/int from sealing off the body of the list, and instead applies the $?$ operator, which adds the set-theoretic complement of $\bigcup P$ as an additional alternative. This captures what we take to be the characteristic semantic property of open lists, which is that they always leave open the possibility that none of the given list items holds. Thus, unlike closed, open prevents the operator $\pi$ expressed by decl or int from becoming operative, and instead applies $?$ to the proposition $P$ expressed by the body of the list.

(20) $\text{open} \leadsto \lambda \pi. \lambda P. ?P$

In total there are four types of lists, each featuring a combination of two classifiers. From the treatment of the individual classifiers given above, it follows that the four types of lists are translated into our logical language as specified in (21) below, where in each case $\varphi$ stands for the translation of the body of the list, obtained according to the rule in (14) above.

(21) **Rules for translating lists:**

- a. $\llbracket \text{closed decl} \mid \text{body} \rrbracket \leadsto !\varphi$
- b. $\llbracket \text{closed int} \mid \text{body} \rrbracket \leadsto (?)\varphi$
- c. $\llbracket \text{open decl} \mid \text{body} \rrbracket \leadsto ?\varphi$
- d. $\llbracket \text{open int} \mid \text{body} \rrbracket \leadsto ?\varphi$

The rules in (14) and (21) together give a complete specification of how to translate declarative and interrogative lists in English into our logical language. In Table 5.1 we provide translations for a number of examples that are representative for all the types of lists that we are concerned with. In the translations of these examples, $p$ stands for *Igor speaks English* and $q$ for *Igor speaks French*. In each case we provide the direct translation and also a simpler formula that is semantically equivalent in $\text{InqB}$ to the direct translation. The propositions expressed by all these simplified translations are depicted in Figure 5.1.

Note that the mapping from logical forms to propositions is not a one-to-
5.3. Interpreting logical forms

—Closed declaratives—

<table>
<thead>
<tr>
<th>Translation</th>
<th>Simplified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Igor speaks English ↓</td>
<td>![p]</td>
</tr>
<tr>
<td>Igor speaks English-or-French ↓</td>
<td>![p ∨ q]</td>
</tr>
<tr>
<td>Igor speaks English ↑, or he speaks French ↓</td>
<td>![p ∨ q]</td>
</tr>
</tbody>
</table>

—Open declaratives—

<table>
<thead>
<tr>
<th>Translation</th>
<th>Simplified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Igor speaks English ↑?</td>
<td>![p]</td>
</tr>
<tr>
<td>Igor speaks English-or-French ↑?</td>
<td>![p ∨ q]</td>
</tr>
<tr>
<td>Igor speaks English ↑, or he speaks French ↑?</td>
<td>![p ∨ q]</td>
</tr>
</tbody>
</table>

—Open interrogatives—

<table>
<thead>
<tr>
<th>Translation</th>
<th>Simplified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does Igor speak English ↑?</td>
<td>![p]</td>
</tr>
<tr>
<td>Does Igor speak English-or-French ↑?</td>
<td>![p ∨ q]</td>
</tr>
<tr>
<td>Does Igor speak English ↑, or does he speak French ↑?</td>
<td>![p ∨ q]</td>
</tr>
</tbody>
</table>

—Closed interrogatives—

<table>
<thead>
<tr>
<th>Translation</th>
<th>Simplified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does Igor speak English ↓?</td>
<td>![p]</td>
</tr>
<tr>
<td>Does Igor speak English-or-French ↓?</td>
<td>![p ∨ q]</td>
</tr>
<tr>
<td>Does Igor speak English ↑, or does he speak French ↓?</td>
<td>![p ∨ q]</td>
</tr>
</tbody>
</table>

Table 5.1: Representative examples of all types of lists considered.

one mapping: various types of lists are given the same translation and are thus associated with the same proposition. For instance, the open polar interrogative in (22a), the closed polar interrogative in (22b), and the open declarative in (22c) are all translated as ![p], which expresses the proposition depicted in Figure 5.1(c).

(22)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>Does Igor speak English ↑?</td>
</tr>
<tr>
<td>b.</td>
<td>Does Igor speak English ↓?</td>
</tr>
<tr>
<td>c.</td>
<td>Igor speaks English ↑?</td>
</tr>
</tbody>
</table>

This is as it should be, because in terms of informative and inquisitive content these sentences are indeed the same: they all raise the issue whether Igor speaks English, and do not provide any information. However, there is of course a contrast between (22a) on the one hand and (22b-c) on the other. Namely, while (22a) is generally seen as the default way of expressing the issue whether Igor speaks English, (22b-c) are perceived as marked ways of doing so, and the range of contexts in which they can be felicitously used is restricted.

The same point applies to the three sentences in (23), which are the disjunc-
(a) \([p]\)

(b) \([!(p \lor q)]\)

(c) \([?p]\)

(d) \([?!(p \lor q)]\)

(e) \([?((p \lor q))]\)

(f) \([p \lor q]\)

Figure 5.1: Propositions expressed by the examples in Table 5.1.

tive counterparts of those in (22):

(23) a. Does Igor speak English-or-French↑? open interrogative
    b. Does Igor speak English-or-French↓? closed interrogative
    c. Igor speaks English-or-French↑? open declarative

These sentences are all translated as ?!(p \lor q) and thus associated with the proposition depicted in Figure 5.1(d). Again, the open interrogative in (23a) is the default way of expressing this proposition, while the closed interrogative in (23b) and the open declarative in (23c) are marked ways of doing so.

Finally, the same point also applies to the sentences in (24), which are both translated as ?!(p \lor q) and thus associated with the proposition in Figure 5.1(e).

(24) a. Does Igor speak English↑ or does he speak French↑? open interrogative
    b. Igor speaks English↑ or he speaks French↑? open declarative

The interrogative is again the default way of expressing the given proposition, while the declarative is marked—indeed, in this case it is difficult to think of any context at all in which it could be used felicitously.

Thus, a general distinction can be made between marked and unmarked sentence types. We will first discuss the unmarked cases in more detail, in Section 5.3.1, and then turn to the marked cases in Section 5.3.2.
5.3 Interpreting logical forms

5.3.1 Unmarked cases

We start with the simplest unmarked case, namely the non-disjunctive closed declarative in (25):

(25) Igor speaks English↓. closed declarative

This sentence is taken to have the following logical form:

(26) [[C\textit{CLOSED DECL} [C\textit{DECL} Igor speaks English]]]

The translation of this logical form is !p, which can be simplified to just p. The proposition expressed by this sentence is depicted in Figure 5.1(a). It is correctly predicted that the sentence provides the information that Igor speaks English, without requesting any additional information.

Next, consider the disjunctive closed declaratives in (27) and (28):

(27) Igor speaks English-or-French↓. closed declarative
(28) Igor speaks English↑ or he speaks French↓. closed declarative

These sentences are taken to have the following logical forms, respectively:

(29) [[C\textit{CLOSED DECL} [C\textit{DECL} Igor speaks English or French]]]
(30) [[C\textit{CLOSED DECL} [[C\textit{DECL} Igor speaks English] or [C\textit{DECL} he speaks French]]]]

These logical forms have the same simplified translation, namely !(p \lor q), which expresses the proposition depicted in Figure 5.1(b). Thus, the sentences are correctly predicted to provide the information that Igor speaks English or French, without requesting any additional information.

Now let us turn to interrogatives. The simplest unmarked case here is the mono-clausal open polar interrogative in (31).

(31) Does Igor speak English↑? open interrogative

This sentence is taken to have the following logical form:

(32) [[C\textit{OPEN INT} [C\textit{INT} does Igor speak English]]]

The simplified translation of this logical form is ?p, which expresses the proposition depicted in Figure 5.1(c). The sentence is correctly predicted to request information as to whether Igor speaks English, and not to provide any information.

Next, consider the disjunctive open interrogative in (33).

(33) Does Igor speak English-or-French↑? open interrogative
This sentence is taken to have the following logical form:

\[(34) \quad \text{[[OPEN INT] } [\text{C}_{\text{INT}} \text{ does Igor speak English or French}]] \]

The simplified translation of this logical form is \(?!(p \lor q)\), which expresses the proposition depicted in Figure 5.1(d). Again, the sentence is predicted to be inquisitive and not informative. In order to resolve the issue that it raises, it either needs to be established that Igor indeed speaks at least one of the two languages, or that he does not speak either.

Next, consider the open interrogative in (35), where the disjunction separates two list items.

\[(35) \quad \text{Does Igor speak English}↑ \text{ or does he speak French}↑? \quad \text{open interrogative} \]

This sentence is taken to have the following logical form:

\[(36) \quad \text{[[OPEN INT] } [[\text{C}_{\text{INT}} \text{ does Igor speak English}] \text{ or } [\text{C}_{\text{INT}} \text{ does he speak French}]]] \]

The simplified translation of this logical form is \(?((p \lor q))\), which expresses the proposition depicted in Figure 5.1(e). As desired, the sentence is predicted to be more inquisitive than (33). Namely, in order to resolve the issue that it raises, it is not sufficient to establish whether or not Igor speaks at least one of the two languages. Rather, it either needs to be established that Igor speaks English, or that he speaks French, or that he speaks neither.

Finally, consider the closed interrogative in (37), again with two list items.

\[(37) \quad \text{Does Igor speak English}↑ \text{ or does he speak French}↓? \quad \text{closed interrogative} \]

This sentence is taken to have the following logical form:

\[(38) \quad \text{[[CLOSED INT] } [[\text{C}_{\text{INT}} \text{ does Igor speak English}] \text{ or } [\text{C}_{\text{INT}} \text{ does he speak French}]]] \]

The translation of this logical form, on the simplified non-presuppositional account presented here, is \(p \lor q\), which expresses the proposition depicted in Figure 5.1(f). Notice that the \(?\) operator is not invoked here because the proposition that \(\text{INT}\) gets as its input is already inquisitive. Since the role of \(\text{INT}\) is not to blindly apply \(?\), but rather just to ensure inquisitiveness, it leaves the input proposition unaltered in this case. The prediction, then, is that the alternative question in (37) provides the information that Igor speaks at least one of the two languages, and raises the issue which of the two he speaks.

This prediction is not entirely satisfactory, because it does not capture the fact that alternative questions presuppose that exactly one of the disjuncts holds (Karttunen and Peters, 1976; Biezma and Rawlins, 2012). As remarked at the outset, however, it is impossible to properly capture this fact in InqB, which concentrates exclusively on informative and inquisitive content and leaves presuppositional aspects of meaning out of consideration. We refer to Ciardelli et al.
5.3. Interpreting logical forms

(2012) and Roelofsen (2015a) for a presuppositional extension of \textsc{lnqB}, and to the latter work for a more sophisticated version of the account of lists presented here, which does make satisfactory predictions about alternative questions.

Aside from this loose end concerning the presuppositional component of alternative questions, we have seen that the basic semantic properties of unmarked declarative and interrogative lists are accounted for in a straightforward and principled way.

5.3.2 Marked cases

We now turn to the marked cases, listed below:

(39) Does Igor speak English↓?

(40) Igor speaks English↑?

(41) Does Igor speak English-or-French↓?

(42) Igor speaks English-or-French↑.

(43) Igor speaks English↑ or he speaks French↑.

Our aim here will just be to account for the marked status of these types of sentences—we will not try to characterize their special discourse effects or the exact range of contexts in which they could be felicitously used. The general idea that we will pursue, familiar from much work in neo-Gricean pragmatics and optimality theory (see, e.g., Horn, 1984; Blutner, 2000), is that an expression is perceived as marked if there is another expression that has the same semantic content and is, other things being equal, better suited to express that content. One reason for this may be that the latter expression is easier to produce; another reason may be that it has a greater chance of being interpreted as intended. This second reason will be most relevant for us.

Notice that every sentence in (39)-(43) either involves the classifier combination [CLOSED INT] or the combination [OPEN DECL]. Vice versa, every type of list with one of these two classifier combinations is represented in (39)-(43), except for closed interrogatives with multiple list items, i.e., alternative questions—we will return to this momentarily. Quite generally, then, there is something marked about closed interrogatives and open declaratives. Why would this be? In Roelofsen (2015a); Farkas and Roelofsen (2016) it is proposed that the source of this markedness lies in the fact that these kinds of lists are generally in competition with open interrogative lists, and that the latter are generally preferred because they maximize the chance of being interpreted as intended. This is because, in many configurations, OPEN and INT have precisely the same seman-

\footnote{See Farkas and Roelofsen (2016) for an analysis of the special discourse effects of open declaratives like (40) that is compatible with the account presented here.}
tic effect, and even more importantly, in these configurations the same overall interpretation would result if either OPEN or INT were to be misinterpreted as CLOSED or DECL, respectively.

Let us look at an example to make this more concrete. Consider the closed interrogative in (39). The simplified translation of this sentence is ?p, which is also the simplified translation of the open interrogative in (44).

(44) Does Igor speak English↑? open interrogative

Now suppose that someone hears (44) in a conversation and has to determine its meaning. If all goes well, the sentence is recognized as an open interrogative—through the interrogative word order and the final rise. However, even if the sentence is mistakenly parsed as an open declarative, or as a closed interrogative, the same interpretation would still be derived. Thus, open interrogatives are very robust: if one piece breaks, the whole construction still functions as intended. This is not the case for the closed interrogative in (39). If this sentence is mistakenly parsed as a closed declarative, the intended interpretation would not be obtained. This explains the marked nature of this sentence type.

Exactly the same reasoning applies to the open declarative in (40). This sentence also has ?p as its simplified translation, so it is also in competition with the open interrogative in (44). And again, it does not have the same robustness as the open interrogative, because if it is mistakenly parsed as a closed declarative, the intended interpretation is not obtained.

The markedness of the closed interrogative in (41) and the open declarative in (42), both involving a single list item containing disjunction, can be explained analogously: these sentences are in competition with the open interrogative in (45), which is favored because of its supreme robustness.

(45) Does Igor speak English-or-French↑? open interrogative

Finally, the markedness of the bi-clausal open declarative in (43) can be explained in a similar way as well, although here the reasoning is somewhat more involved. First note that (43) is predicted to be semantically equivalent with the open interrogative in (46); both are translated as ?(p ∨ q).

(46) Does Igor speak English↑ or does he speak French↑? open interrogative

Now consider which interpretations arise if either (43) or (46) is not parsed as intended. If the open declarative in (43) is mistakenly parsed as an open interrogative it is translated as ?(p ∨ q), which is still its intended interpretation, but if it is mistakenly parsed as a closed declarative it is translated as !(p ∨ q), which is clearly different from ?(p ∨ q).

On the other hand, if the open interrogative in (46) is mistakenly parsed as an open declarative it is translated as ?(p ∨ q), which is its intended interpretation,
but if it is mistakenly parsed as a closed interrogative it is translated as \( p \lor q \), which is different from \(? (p \lor q)\). So if we just count the number of erroneous parses that lead to misinterpretation, there is no reason to prefer the open interrogative over the open declarative in this case. If we take a closer look, however, we find that such reasons do exist.

First note that the difference in word order that encodes the distinction between declarative and interrogative lists is arguably more reliable than the difference in prosody that encodes the distinction between open and closed lists, because the latter may interact with other prosodic factors encoding, e.g., speaker authority, speaker (un)certainty, or politeness. Moreover, the cases under consideration consist of multiple clauses which means that the distinction between declarative and interrogative lists is encoded multiple times, once in each clause, while the distinction between open and closed lists is only encoded once, on the final clause. Thus, it seems reasonable to assume that open interrogatives like (46) are more likely to be interpreted as intended than open declaratives like (43), just like their uni-clausal counterparts.

There is an additional reason to prefer the open interrogative in (46) over the open declarative in (43) as well. Consider the interpretations that arise if the two sentence are misinterpreted. In the case of (46) we obtain \(! (p \lor q)\). Neither of these coincides with the intended interpretation, \(? (p \lor q)\). Arguably, however, the former misinterpretation is less consequential than the latter. To see this, note that \( p \lor q \) entails \(? (p \lor q)\), which means that every resolution of the former is also a resolution of the latter. Thus, even if (46) is misinterpreted as \( p \lor q \), any response that resolves the issue expressed by the sentence under this interpretation also resolves the issue expressed by the sentence under its intended interpretation. On the other hand, \(! (p \lor q)\) does not entail \(? (p \lor q)\). So if (43) is misinterpreted as \(! (p \lor q)\), then it is not the case that any response resolving the issue expressed by the sentence under this erroneous interpretation also resolves the issue expressed by the sentence under its intended interpretation. Thus, the potential misinterpretation of the open interrogative in (46) is less consequential than the potential misinterpretation of the open declarative in (43). This is an additional reason for speakers to prefer (46) over (43) when they want to express the proposition associated with \(? (p \lor q)\), lending further support to a competition-based explanation of the markedness of multi-clausal open declaratives like (43).

Finally, let us return to the case of alternative questions, i.e., multi-clausal closed interrogatives, which are not marked, even though uni-clausal closed interrogatives are, whether they contain a disjunction or not (see examples (39) and (41) above). The reason for this is that multi-clausal closed interrogatives are not generally equivalent with the corresponding open interrogatives. So in this case there is no competition between the two types of lists.
Chapter 5. Disjunctive lists

To make this concrete again, consider the closed interrogative in (47).

(47) Does Igor speak English↑ or does he speak French↓?  

The simplified translation of this sentence is $p \lor q$. Thus, it does not have the same meaning as the corresponding open interrogative in (46), nor is there any other competing list type. This explains its unmarked nature.

This concludes our analysis of declarative and interrogative lists in $\text{InqB}$. Even though there is much more to say about the linguistic properties of such lists, we hope that the bare bones account that we have presented here has succeeded in substantiating the general point that we set out to make, namely that declaratives and interrogatives are to a large extent ‘built up from the same parts’, i.e., the same lexical, morphological, and prosodic elements, and that a uniform account of such elements, which applies across declarative and interrogative constructions, requires a framework like inquisitive semantics, which treats informative and inquisitive content in an integrated way.

5.4 Exercises

Exercise 5.1. Determine the logical form of each of the examples below, and derive, step by step, how these logical forms are translated into $\text{InqB}$ according to the rules in (14) and (21).

(48) a. Martina plays the piano↓.
    b. Martina plays the piano-or-the-cello↓.
    c. Martina plays the piano↑ or she plays the cello↓.
    d. Martina plays the piano↑?
    e. Does Martina play the piano↑?
    f. Does Martina play the piano-or-the-cello↑?
    g. Does Martina play the piano↑ or does she play the cello↑?
    h. Does Martina play the piano↑ or does she play the cello↓?
    i. Does Martina play the piano↓?

Exercise 5.2. Explain how the marked status of (48d) and (48i) is accounted for by the theory presented here.
Chapter 6

Conditionals

In the previous chapter, we have seen that the inquisitive notion of meaning allows us to obtain a uniform semantic analysis of lexical and prosodic elements that occur both in declarative and in interrogative sentences. However, we assumed that the logical form of an (ordinary, falling) declarative sentence is always headed by a projection operator, $!, which makes the sentence non-inquisitive. This may suggest that, as long as we are only concerned with such sentences (and, therefore, not with treating operators like disjunction in a way that works uniformly across declaratives and interrogatives), the standard truth-conditional notion of meaning serves us well enough, and keeping track of inquisitive content is an unnecessary complication.

In this chapter, we will see that this is not the case: even for sentences which are not inquisitive, and whose meaning is therefore completely determined by their truth conditions, these truth conditions may depend crucially on the inquisitive content of some constituent within the sentence. Thus, to derive the right truth conditions for the whole sentence, the inquisitive content of the sentence’s constituents must be taken into account.

We will demonstrate this based on recent experimental work by Ciardelli, Zhang, and Champollion (2016b) on counterfactual conditionals. This work shows that even if two sentential clauses $\varphi$ and $\varphi'$ have exactly the same truth conditions, the counterfactual conditionals $\varphi > \psi$ and $\varphi' > \psi$ may have different truth conditions. This means that it is impossible to give a compositional account of counterfactuals based on a purely truth-conditional notion of meaning.

Ciardelli et al. (2016b) show that the relevant contrast finds a natural explanation once conditionals are analyzed in inquisitive semantics. Moreover, Ciardelli (2016b) argues that an inquisitive analysis of conditionals has other merits as well: on the one hand, it solves a well-known problem that classical analyses of conditionals have with disjunctive antecedents; on the other hand, it does not only allow us to interpret run-of-the-mill conditional statements, but also two other classes of conditional constructions, namely, so-called unconditionals and conditional questions.
In this chapter, we will present the experimental results and theoretical arguments of Ciardelli et al. (2016b) and Ciardelli (2016b) in condensed form. Section 6.1 describes the experiment and explains why the obtained results are problematic for the standard view that equates meaning with truth-conditions. Section 6.2 introduces a recipe for lifting theories of conditionals from truth-conditional semantics to inquisitive semantics, and shows how the experimental results receive a natural explanation once we combine this inquisitive lifting with suitable assumptions about the process of making counterfactual assumptions. Finally, Section 6.3 discusses various further advantages of an inquisitive treatment of conditionals.

6.1 Evidence for truth-conditional effects

6.1.1 The experiment

Imagine a long hallway with a light in the middle and with two switches, one at each end. One switch is called switch A and the other one is called switch B. As the following wiring diagram shows, the light is on whenever both switches are in the same position (both up or both down); otherwise, the light is off. Right now, switch A and switch B are both up, and the light is on. But things could be different...

Which of the following counterfactual sentences are true in this scenario?

(1) a. If switch A was down, the light would be off.
   b. If switch B was down, the light would be off.
   c. If switch A or switch B was down, the light would be off.
   d. If switch A and switch B were not both up, the light would be off.

Figure 6.1: A multiway switch

Which of the following counterfactual sentences are true in this scenario?
6.1. Evidence for truth-conditional effects

<table>
<thead>
<tr>
<th>Sentence</th>
<th>Number</th>
<th>True (%)</th>
<th>False (%)</th>
<th>Indet. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1a)</td>
<td>256</td>
<td>169</td>
<td>6</td>
<td>31.64%</td>
</tr>
<tr>
<td>(1b)</td>
<td>235</td>
<td>153</td>
<td>7</td>
<td>31.91%</td>
</tr>
<tr>
<td>(1c)</td>
<td>362</td>
<td>251</td>
<td>14</td>
<td>26.80%</td>
</tr>
<tr>
<td>(1d)</td>
<td>372</td>
<td>82</td>
<td>136</td>
<td>41.40%</td>
</tr>
<tr>
<td>(1e)</td>
<td>200</td>
<td>43</td>
<td>63</td>
<td>47.00%</td>
</tr>
</tbody>
</table>

Table 6.1: Results of Ciardelli et al.’s (2016b) main experiment

e. If switch A and switch B were not both up, the light would be on.

Ciardelli et al. (2016b) conducted an experiment to test the intuitions of native speakers of English about this question. Participants were recruited online using Amazon’s Mechanical Turk platform; they were first shown the short text above and the diagram in Figure 6.1; they were then presented with one of the sentences in (1) and a filler sentence (one at a time, in random order), and they were asked to judge these sentences as either true, false, or indeterminate. Data from participants who failed to judge the filler correctly, or who otherwise failed to qualify for the task, were rejected. The remaining results are summarized in Table 6.1. For our purposes, the most important result is the contrast between sentences (1c) and (1d): (1c) was judged true by about 70% of participants, while only 22% of participants judged (1d) true.

6.1.2 A problem for the truth-conditional view on meaning

Assuming for the moment that the judgments found in the experiment are due to an actual difference in truth value between (1c) and (1d) in the given context, this is problematic for the standard view that equates meaning with truth-conditions, regardless of the particular account of conditionals one assumes. To see why, consider the clauses (2a) and (2b), corresponding to the two antecedents of (1c) and (1d).

(2) a. Switch A or switch B is down
    b. Switch A and switch B are not both up

Assuming that our switches can only take two positions, up and down, these clauses have the same truth conditions. If switch A or switch B is down, then clearly switch A and switch B are not both up. And conversely, if switch A and switch B are not both up, then either of them must be down. Under the view that the meaning of these clauses can be identified with their truth conditions, this means that (2a) and (2b) have the same meaning.
According to the principle of compositionality, the meaning of a sentence depends only on the meaning of its constituents and the way these constituents are combined. This implies that if, in a sentence \( \varphi \), a constituent \( c \) is replaced by another constituent \( c' \) with the same meaning, the resulting sentence \( \varphi[c'/c] \) must have the same meaning as \( \varphi \).

Now, the counterfactual (1d) can be obtained from (1c) by replacing the sentential constituent corresponding to (2a) with (2b), which has the same meaning. Therefore, (1c) and (1d) must have the same meaning, and thus the same truth conditions. It follows that, in every particular context, these counterfactuals must have the same truth value. But this is not the case: in the context described in the experiment, (1c) is true, but (1d) is not.

This shows that, in combination with the principle of compositionality, the assumption that the meaning of a sentence can be identified with its truth conditions leads to wrong empirical predictions.

### 6.1.3 Ruling out alternative explanations

Ciardelli et al. (2016b) strengthen the argument made in the previous section by ruling out a number of alternative explanations for the data in Table 6.1.

First, one might worry that (1c) and (1d) are judged differently in spite of actually having the same truth value, due to the interference of other factors. The main concern that motivates this worry is that participants may have judged (1d) incorrectly for one of two reasons: they may have misread the phrase ‘not both up’ as ‘both not up’, that is, as ‘both down’; or they may have been confused by the higher processing cost of the antecedent, which involves a negation scoping over a conjunction.

Neither of these hypotheses stands up to further scrutiny. According to the first hypothesis, many participants misread ‘not both up’ as ‘both down’. If so, we would expect many participants to judge the sentence (1e) as true, since the context explicitly specifies that the light is off when both switches are down. This is not what we observe: instead, (1e) is only judged true by about 20% of participants, just like (1d).

According to the second hypothesis, many participants fail to judge (1d) true as a consequence of context-independent feature of this sentence, such as processing cost. This hypothesis predicts that many participants would also not judge this sentence true if the circuit had been wired differently. In a post-hoc experiment, participants were asked to judge the sentences in (1) in a modified scenario, where the light is on only when both switches are up. In this scenario, an overwhelming majority of participants (about 85%) judged (1d) to be true. The contrast between the results in the two scenarios shows that the reason why (1d) was not judged true in the main experiment does not have to do with intrinsic features of the sentence, but rather with the fact that if both switches
were down, the light would not be off. For this is the only difference between
the original scenario and the modified one.

Another way to resist the conclusion drawn in the previous section is to ac-
cept that the difference in truth values between (1c) and (1d) is genuine, but
to deny that the antecedents of these sentences have the same truth conditions.
There are two natural ways to do this: one may point out that down is not logi-
cally equivalent to not up, or hypothesize that the disjunction in the antecedent
of (1c) is interpreted exclusively, i.e., as requiring that only one (and not both) of
the disjuncts is true, for instance as a result of some exhaustification operation
of the kind discussed by Chierchia et al. (2012).

Again, further control experiments render these explanations implausible.
According to the first explanation, the contrast should vanish if the word down
was replaced by the expression not up throughout the sentences in (1). A post-
hoc experiment revealed that this prediction is not borne out: replacing down by not up does not modify the pattern exhibited by the results in Table 6.1.

According to the second explanation, the disjunction in the antecedent of
(1c) is interpreted exclusively, possibly as a result of an exhaustification op-
erator. If so, we would naturally expect the main disjunction in (2a) to be
interpreted exclusively as well, and thus to be judged as false or indeterminate
in a scenario in which both switches are down. In a pre-test, participants were
presented with a picture which displays the circuit with both switches down,
and they were asked to judge the sentences (2a) and (2b) as true, false, or inde-
terminate. Both sentences were judged true by over 80% of participants. This
shows that an exclusive reading of disjunction in (2a) is at best marginal, which
makes it unlikely that it is responsible for the observed contrast.

6.2 Conditionals in inquisitive semantics

In this section, we show that the findings discussed in the previous section have
a natural explanation once we move from a truth-conditional semantic setting
to inquisitive semantics. We start in Section 6.2.1 by showing how inquisi-
tive semantics assigns the same truth-conditions but different meanings to the
antecedents of (1c) and (1d), thus allowing for a compositional account that
assigns different truth conditions to these counterfactuals. In Section 6.2.2, we
introduce the inquisitive lifting operation developed in Ciardelli (2016b), and
explain how a difference in inquisitive content between two antecedents can re-
sult in different truth conditions for the corresponding conditionals. Finally, in
Section 6.2.3 we present the background theory of counterfactuals developed by
Ciardelli et al. (2016b), and show that the inquisitive lifting of this theory yields
the right predictions for the sentences in (1).
Chapter 6. Conditionals

6.2.1 Breaking de Morgan’s law in inquisitive semantics

To see how inquisitive semantics allows us to explain the data in Table 6.1, let us first formalize our sentences in the system $\text{InqB}$ equipped with an additional counterfactual connective $>$.\(^1\) We will assume a predicate $Ux$ for “$x$ is up”, an atomic sentence $O$ for “the light is off”, and two constants $a, b$ which refer to the two switches. We will then analyze the sentences in (1) as follows:\(^2\)

\[
\begin{align*}
(1a) & \quad \neg Ua > O \\
(1b) & \quad \neg Ub > O \\
(1c) & \quad \neg Ua \lor \neg Ub > O \\
(1d) & \quad \neg (Ua \land Ub) > O \\
(1e) & \quad \neg (Ua \land Ub) > \neg O
\end{align*}
\]

Let us consider the antecedents of these counterfactuals. We will assume that our model contains four possible worlds, corresponding to the four possible configurations of the switches. The propositions expressed by the different antecedents in this model are depicted in Figure 6.2.

Crucially, in inquisitive semantics, the antecedent of (1c), $\neg Ua \lor \neg Ub$, and the antecedent of (1d), $\neg (Ua \land Ub)$, are not semantically equivalent: while the two are assigned the same truth-conditions, the former is inquisitive, while the latter is not. $\neg Ua \lor \neg Ub$ generates two alternatives, namely, the set of worlds where A is down, and the set of worlds where B is down. By contrast, $\neg (Ua \land Ub)$ generates a single alternative, namely, the set of worlds where the switches are not both up. This means that the problem we pointed out for truth-conditional semantics no longer arises in inquisitive semantics: the antecedents of (1c) and (1d) have different meanings, and can therefore make different semantic contributions.

\(^1\)Clearly, the implication connective $\rightarrow$ provided by $\text{InqB}$ is not suitable as a translation of counterfactuals. When applied to statements, this operator yields the same truth conditions as the standard material implication connective of classical logic. Since the antecedents of our counterfactual sentences are all false, this would immediately render all these sentences true.

\(^2\)Of course, we do not mean here that the logical form of a sentence like (1a) has to contain a negation operator. We could introduce another predicates $Dx$ for “$x$ is down”, and another atomic sentence $On$ for “the light is on”. However, we would then have to introduce some meaning postulates to enforce that $Dx$ is true exactly when $Ux$ is false, and $On$ is true exactly when $O$ is false. This would then lead to the same results that our analysis gives.
6.2. Conditionals in inquisitive semantics

6.2.2 Lifting conditionals to inquisitive semantics

We now want to explain how the difference in inquisitive content between
\neg Ua \lor \neg Ub and \neg (Ua \land Ub) can lead to a difference in truth-conditions for the
counterfactuals in which these two clauses are embedded as antecedents. For
this, we adopt an idea of Alonso-Ovalle (2006, 2009) (see also van Rooij, 2006).
We assume that an antecedent need not always specify a single counterfactual
assumption; rather, when we have multiple alternatives for an antecedent, each
of them is treated by the semantics as a distinct counterfactual assumption. In
order for the counterfactual to be true, the consequent must follow on each of
these assumptions.

To implement this idea in the inquisitive setting, Ciardelli (2016b) describes
a general procedure for lifting accounts of conditionals to inquisitive semantics.
This lifting procedure takes as its input any truth-conditional account of (indicative
or counterfactual) conditionals, given in the form of a binary operation
\Rightarrow which maps any two classical propositions \alpha and \gamma (expressed by the antecedent
and the consequent of a conditional, respectively) to a third classical proposition
\alpha \Rightarrow \gamma. All the standard theories of counterfactual conditionals, such as selection
function semantics (Stalnaker, 1968), ordering semantics (Lewis, 1973), and
premise semantics (Kratzer, 1981) yield such an operation \Rightarrow.\footnote{In each of these theories, the definition of \alpha \Rightarrow \gamma makes use of some additional piece of
structure: a selection function in Stalnaker (1968), a similarity ordering in Lewis (1973), an
ordering source in Kratzer (1981). However, our lifting recipe only needs access to the resulting
operation on propositions—not to this underlying structure.}

The output of the lifting procedure is an ‘inquisitivized’ version of this truth-
conditional account, which interprets a conditional \varphi > \psi by means of the
following support clause.\footnote{Note that this clause is formulated in terms of \text{alt}(\varphi) and \text{alt}(\psi). It assumes that \text{alt}(\varphi)
and \text{alt}(\psi) completely determine the meaning of \varphi and \psi, respectively, i.e., that \{\varphi\} = \{s \mid s \subseteq \alpha for some \alpha \in \text{alt}(\varphi)\} and similarly for \psi. This will indeed be the case for all the examples
that we will consider in this chapter, but it does not always hold in InqB (see footnote 4
on page 23). For the general case, the clause can be formulated as follows, without making
reference to alternatives:

\begin{align*}
s \models \varphi > \psi \iff \forall \alpha \in [\varphi] \exists \alpha' \in [\varphi] \exists \gamma \in [\psi] : \alpha' \supseteq \alpha \text{ and } s \subseteq (\alpha' \Rightarrow \gamma)
\end{align*}

Provided that the map \Rightarrow is upward monotonic in its second argument, which is true of all
truth-conditional theories of conditionals we are aware of, this clause boils down to the clause
of Definition 6.1 whenever \text{alt}(\varphi) and \text{alt}(\psi) completely determine the meaning of \varphi and \psi.}

\begin{definition}[Inquisitive lifting]
\text{s |\models \varphi > \psi \iff } \forall \alpha \in \text{alt}(\varphi) \exists \gamma \in \text{alt}(\psi) \text{ such that } s \subseteq (\alpha \Rightarrow \gamma)
\end{definition}

When \varphi and \psi are non-inquisitive, we have \text{alt}(\varphi) = \{\varphi\} and \text{alt}(\psi) = \{\psi\},
and the clause therefore boils down to:

\[ s \models \varphi > \psi \iff \forall \alpha \in \{|\varphi|\} \exists \gamma \in \{|\psi|\} \text{ such that } s \subseteq (\alpha \Rightarrow \gamma) \]

Thus, the conditional \( \varphi > \psi \) is predicted to be a statement whose unique alternative is the classical proposition \( |\varphi| \Rightarrow |\psi| \) delivered by the given base account. Except for (1c), all of our counterfactuals have non-inquisitive antecedents and consequents, so they will be interpreted just as they are interpreted by the given base account. As for (1c), translated as \( \neg Ua \lor \neg Ub > O \), the clause yields the following:

\[ s \models \neg Ua \lor \neg Ub > O \iff \forall \alpha \in \{|\neg Ua|, |\neg Ub|\} \exists \gamma \in \{|O|\} \text{ s.t. } s \subseteq (\neg Ua \Rightarrow |O|) \land s \subseteq (\neg Ub \Rightarrow |O|) \]

As in the other cases, the conditional as a whole is a statement. However, the unique alternative for it, the set \( (|\neg Ua| \Rightarrow |O|) \cap (|\neg Ub| \Rightarrow |O|) \), is not the same as the set \( |\neg Ua \lor \neg Ub| \Rightarrow |O| \) that would be delivered by applying the base account directly, without lifting it to inquisitive semantics. Rather, the lifting procedure ensures that the base account is applied twice, once for each disjunct in the antecedent, and the results are then intersected. Thus, disjunctive antecedents are interpreted as providing multiple assumptions, and \( \neg Ua \lor \neg Ub > O \) is predicted to be true just in case both \( \neg Ua > O \) and \( \neg Ub > O \) are true. This explains the strong similarity between the response pattern of (1c) and those of (1a) and (1b).

Now the majority judgments in Table 6.1 could be predicted if we could find a truth-conditional account of counterfactuals according to which (1a) and (1b) are true, but (1d) and (1e) are not. The inquisitive lift of this account would make the same predictions about these sentences, and it would also predict (1c) to be true—something that no purely truth-conditional account could do.

### 6.2.3 The foreground/background theory of counterfactuals

Now that the problem of disentangling (1c) and (1d) is solved, one might expect that we can just take a standard account of counterfactuals, such as the ordering semantics of Lewis (1973), and lift it to inquisitive semantics to obtain an account of our data. However, as Ciardelli et al. (2016b) discuss, this is not the case. The problem is that all standard accounts of counterfactuals validate the following entailment:

\[ \neg Ua > O, \neg Ub > O \models \neg (Ua \land Ub) > O \]
Thus, regardless of how the parameters needed to interpret counterfactuals are set in these theories, they can never predict that (1a) and (1b) are true but (1d) is not. Conceptually, the problem is that all the standard theories are based on the idea that, when making a counterfactual assumption, one is required to minimize the amount of change with respect to the actual world. This means that, when counterfactually assuming that A and B are not both up, one is required to retain the fact that at least one of them is up. This does not seem right: when asked to consider what would happen if the switches were not both up, we are naturally lead to consider the case that just one switch was down, as well as the case that both switches were down, which explains the observed judgments for (1d) and (1e).

To solve this problem, Ciardelli et al. (2016b) adopt a different perspective: they propose to replace the minimal change requirement by a qualitative distinction between aspects of the world that are in the foreground when making a counterfactual assumption, and aspects that are in the background. The latter are held fixed in the counterfactual scenario, while the former are allowed to change, and their change is not subject to any minimality requirement. We will refer to this as the foreground/background theory of counterfactuals.

For a simple example, consider the sentences in (3):

(3) a. If I wore my hair longer, nobody would notice the difference.
   b. If I wore my hair much longer, people would notice the difference.

In both cases, when assuming that the antecedent was true, the length of the speaker’s hair is in the foreground, and we feel no pressure to imagine it to be as close as possible to the actual length; this explains why in normal circumstances we are not inclined to judge (3a) as true. On the other hand, in both cases the fact that people are able to pick up remarkable differences in hair length is in the background, and thus it is held fixed when making the assumption; this explains why in normal circumstances we judge (3b) as true.

Now consider again (1a), (1b), and (1d). When we make the assumption that switch A is down, the position of switch B is naturally regarded as background, and therefore held fixed. This leads us to consider a counterfactual scenario in which A is down, but B is still up. Reasoning by the laws of the circuit, we therefore conclude that the light is off, and judge (1a) as true. Of course, the prediction is analogous for (1b). Now consider the assumption that the switches were not both up: in this case, the positions of both switches are at stake, and thus foregrounded. Therefore, we have no pressure to hold either of them fixed.

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5 This may seem like an over-simplification since, e.g., in ordering semantics, one may well stipulate that toggling two switches is not to be counted as a bigger change than toggling just one. However, this stipulation would make it impossible to account for the truth of $¬Ua > O$ and $¬Ub > O$. A similar argument applies to the other standard accounts.
in the counterfactual scenario. This leads us to consider counterfactual scenarios where just one switch is down as well as scenarios where both switches are down: since these two kinds of scenarios do not agree on the state of the light, neither (1d) nor (1e) are judged true.

Let us now see how an account of the kind just sketched can be formalized, and verify that the predictions we just described are indeed derived. For conciseness, we present a simplified version here of the foreground/background theory of Ciardelli et al. (2016b). This simplified version preserves the essence of the account of our sentences, although it is limited in scope.\footnote{In particular, unlike the original theory, the simplified version it is not equipped to deal with cases in which some causal laws must be broken in order to make a counterfactual assumption.}

The theory employs a formal notion of causal structures inspired by the literature on causal reasoning (Pearl, 2009).\footnote{For related theories of counterfactuals using causal structures, see Schulz (2011), Kaufmann (2013), and Santoro (2016).} For our purposes, such structures can be defined as follows.

**Definition 6.2.** [Causal structures]
A causal structure is a triple $S = \langle V, E, L \rangle$ where:

- $V$ is a set of atomic polar questions, the causal variables of the structure. The settings of a variable $?\varphi$ are the sentences $\varphi$ and $\neg \varphi$.
- $\langle V, E \rangle$ is a directed acyclic graph, whose edges encode causal influence.
- $L$ is a set of statements, the causal laws of the structure; each causal law has the form $\varphi_1 \land \cdots \land \varphi_n \rightarrow \psi$, where $\psi$ is a setting of a variable $v \in V$ and $\varphi_1, \ldots, \varphi_n$ are settings of the parents of $v$ in the graph $\langle V, E \rangle$.

The electric circuit described in Section 6.1 can be modeled naturally as a causal structure as follows. The causal variables are $?Ua$, $?Ub$, and $?O$, corresponding to the states of the switches and the light. The variables $?Ua$ and $?Ub$ have causal influence on $?O$, but not on each other. Thus, the graph $\langle V, E \rangle$ looks as follows:

$$?
\begin{align*}
Ua & \rightarrow ?O \\
Ub & \leftarrow ?O
\end{align*}$$

The causal laws are the following conditionals, encoding the behavior of the circuit:

\begin{equation}
\begin{align*}
(4) & \quad \text{a. } Ua \land Ub \rightarrow \neg O \\
& \quad \text{b. } Ua \land \neg Ub \rightarrow O \\
& \quad \text{c. } \neg Ua \land Ub \rightarrow O \\
& \quad \text{d. } \neg Ua \land \neg Ub \rightarrow \neg O
\end{align*}
\end{equation}
6.2. Conditionals in inquisitive semantics

Within the context of a causal structure, we can associate a possible world with a set of facts—i.e., basic propositions that characterize the world. Moreover, we can equip the set of facts with some structure that reflects the causal relations between them.

**Definition 6.3. [Facts]**

A fact at a world \( w \) is a true setting of a causal variable. The set of facts at \( w \) is denoted \( \mathcal{F}_w \). We say that a fact \( f \) is causally dependent on another fact \( f' \) if \( f' \) is an ancestor of \( f \) in the causal graph.

In our context, the facts are: that switch A is up; that switch B is up; and that the light is on. That is, \( \mathcal{F}_w = \{ U_a, U_b, \neg O \} \). The fact that the light is on is dependent on the other facts, and no other dependencies hold.

We now want to specify, given a certain counterfactual assumption \( \alpha \), which of the facts in \( \mathcal{F}_w \) are called into question by the assumption, and should therefore be considered as potentially different in the counterfactual scenario, and which facts can be regarded as background, and therefore can be held fixed.

The basic idea is simple: an assumption \( \alpha \) calls into question those facts that are logically responsible for its falsity, as well as those facts that are causally dependent on them. Other facts can be regarded as background, although other factors in the context might lead to them being foregrounded as well, and thus varied in the counterfactual scenario.

The idea of a fact being responsible for the falsity of \( \alpha \) can be formalized as follows.

**Definition 6.4. [Facts that contribute to the falsity of a classical proposition]**

Let \( w \in W \), \( \alpha \subseteq W \) a classical proposition, and \( f \in \mathcal{F}_w \) a fact in \( w \). Then, if there exists some set \( F \subseteq \mathcal{F}_w \) such that \( F \) is consistent with \( \alpha \), but \( F \cup \{ f \} \) is inconsistent with \( \alpha \), we say that \( f \) contributes to the falsity of \( \alpha \) in \( w \).

---

8Ciardelli et al. (2016b) discuss in detail the fact that seeing the filler sentence can affect the way the target sentences are judged. Among the participants who saw the target sentence first, less than 20% judged (1a) and (1b) as indeterminate. By contrast, among the participants who saw the filler sentence before the target sentence, about 45% judged (1a) and (1b) as indeterminate. The background theory offers a natural explanation for this finding. The filler sentence used in the experiment was (i), which calls into question the positions of both switches.

(i) If switch A and switch B were both down, the light would be off.

It is natural to assume that after seeing the filler, many participants kept thinking of the position of switch B as foregrounded even when making the assumption that A was down. Thus, they ended up considering the possibility that both switches are down, leading to the high proportion of ‘indeterminate’ judgments for (1a) (and similarly for (1b)). For a more systematic discussion of order effects, we refer the reader to Ciardelli et al. (2016b).

9We say that a set of facts \( F \) is consistent with \( \alpha \) if there is some world \( w \in W \) where \( \alpha \) is true (i.e., \( w \in \alpha \)) and all facts in \( F \) are true. Notice that if some fact ‘contributes to the
When making a counterfactual assumption $\alpha$, we can no longer take for granted those facts that contribute to the falsity of $\alpha$, nor anything that is causally dependent on such facts. We say that these facts are called into question by $\alpha$.

**Definition 6.5.** [Calling a fact into question]

A classical proposition $\alpha$ calls into question $f \in \mathcal{F}_w$ if either (i) $f$ is a fact that contributes to the falsity of $\alpha$, or (ii) $f$ is causally dependent on some such fact.

In our concrete setting, consider the classical proposition that switch A is down, $|\neg U_a|$. It is easy to see that the only fact $f \in \mathcal{F}_w$ that contributes to the falsity of this proposition is $U_a$. Thus, the counterfactual assumption $|\neg U_a|$ calls into question the fact that A is up, as well as the causally dependent fact that the light is on, but it does not call into question the fact that switch B is up. Similarly, the assumption that switch B is down calls into question the fact that B is up and the fact that the light is on, but not the fact that A is up.

Now consider the classical proposition that the switches are not both up, $|\neg (U_a \land U_b)|$. It is easy to see that the fact that A is up and the fact that B is up both contribute to the falsity of this proposition. Thus, these facts are called into question, and so is the dependent fact that the light is on. Therefore, in this case the assumption calls into question all of the facts in our scenario.

The next step is to use these notions to determine which facts can be regarded as background for a given counterfactual assumption, and thus held fixed in making the assumption and assessing its consequences. We assume that only facts that are not called into question by the assumption can be backgrounded. Furthermore, we assume a requirement to avoid gratuitous changes, and thus to avoid foregrounding anything without a reason. In the absence of contextual cues providing a reason to foreground other facts, the background will consist of all and only the facts that are not called into question. We call this the **maximal background** for the given assumption.\(^{10}\)

**Definition 6.6.** [Maximal background for an assumption]

The maximal background for a classical proposition $\alpha$ at world $w$, denoted $B_w(\alpha)$, is the set of all facts which are not called into question by $\alpha$.

Any fact that is part of the background for a given counterfactual assumption is held fixed in the counterfactual scenario. In other words, in making the assumption $\alpha$, we imagine that $\alpha$ is true in addition to all the background facts.

\(^{10}\)Since background facts are held fixed when making a counterfactual assumption, assuming a preference for maximizing the background may be viewed as an analogue of the minimal change requirement implemented by standard theories of counterfactuals. However, in the background theory there is no requirement to minimize departure from actuality when it comes to those aspects of a state of affairs that are foregrounded.
In our scenario, we have $B_w(\neg U_a) = \{ U_b \}$. This explains why, when we suppose that switch A was down in our scenario, we envisage a situation where switch A is down but switch B is up. Similarly, $B_w(\neg \neg U_b) = \{ U_a \}$: when we suppose that switch B was down, we envisage a situation where switch B is down but switch A is still up. On the other hand, since the assumption $|\neg(U_a \land U_b)|$ calls all facts in $F_w$ into question, we have $B_w(|\neg(U_a \land U_b)|) = \emptyset$, which means that when we suppose that the switches were not both up, no fact carries over from the actual state of affairs to the counterfactual scenario.

We can now define the information state that results from making a counterfactual assumption $\alpha$ in a certain world $w$. This is the information determined by the assumption itself, the background facts, and the underlying causal laws. Under a maximal background interpretation, this amounts to the following.

**Definition 6.7.** [Information state resulting from an assumption]
The information state that results from making an assumption $\alpha$ at world $w$, denoted $S_w(\alpha)$, is the set of worlds in which the following are true: (i) the classical proposition $\alpha$; (ii) all facts in $B_w(\alpha)$; and (iii) all laws in $L$.

The last step is to specify at which worlds the classical proposition $\alpha \Rightarrow \gamma$ is true: this holds if the state that results from making the assumption, $S_w(\alpha)$, supports the conclusion $\gamma$, that is, if $S_w(\alpha) \subseteq \gamma$.

**Definition 6.8.** [Truth-conditional recipe for counterfactuals]
Given two classical propositions $\alpha$ and $\gamma$, the counterfactual proposition $\alpha \Rightarrow \gamma$ is true at a world $w$ in case $S_w(\alpha) \subseteq \gamma$.

This completes the description of the truth-conditional map $\Rightarrow$ that we are going to use as the basis for our inquisitive account. Let us now check that the inquisitive account that results from lifting this map correctly predicts which of the counterfactuals in (1) are true in our scenario.

First consider the counterfactual assumption that switch A was down, $|\neg U_a|$. We saw that $B_w(|\neg U_a|) = \{ U_b \}$. Take any world $v \in S_w(|\neg U_a|)$: at world $v$, (i) our assumption $|\neg U_a|$ is true, that is, switch A is down; (ii) the background facts are true, that is, switch B is up; (iii) all causal laws are true, in particular the law $\neg U_a \land U_b \rightarrow O$. Clearly, $|O|$ must then be true in $v$. This shows that $S_w(|\neg U_a|) \subseteq |O|$, which means that $|\neg U_a| \Rightarrow |O|$ is true at $w$. Since $|\neg U_a| \Rightarrow |O|$ is the unique alternative that our inquisitive account assigns to $\neg U_a > O$, this counterfactual is correctly predicted to be true. Of course, the truth of the counterfactual $\neg U_a > O$ is predicted in an analogous way.

Now consider the counterfactual $\neg U_a \lor \neg U_b > O$. We saw in Section 6.2.2 that our inquisitive lifting account assigns a unique alternative to this sentence, namely, the intersection $(|\neg U_a| \Rightarrow |O|) \cap (|\neg U_b| \Rightarrow |O|)$. Since we have just seen...
that \( w \) belongs to both sets \( \neg Ua \Rightarrow |O| \) and \( \neg Ub \Rightarrow |O| \), \( w \) also belongs to their intersection. Thus, \( \neg Ua \lor \neg Ub > O \) is predicted to be true.

Finally, consider the counterfactuals \( \neg(Ua \land Ub) > O \) and \( \neg(Ua \land Ub) > \neg O \). We saw that \( B_w(|\neg(Ua \land Ub)|) = \emptyset \). Now, consider the state \( S_w(|\neg(Ua \land Ub)|) \): this state consists of those worlds where the switches are not both up, and the causal laws hold; thus, this state contains worlds where only one switch is down and the light is off, as well as worlds where both switches are down and the light is on. Therefore, \( S_w(|\neg(Ua \land Ub)|) \nsubseteq |O| \) and \( S_w(|\neg(Ua \land Ub)|) \nsubseteq |\neg O| \), which means that neither \( |\neg(Ua \land Ub)| \Rightarrow |O| \) nor \( |\neg(Ua \land Ub)| \Rightarrow |\neg O| \) are true at \( w \). Since these are, respectively, the unique alternative for \( \neg(Ua \land Ub) > O \) and the unique alternative for \( \neg(Ua \land Ub) > \neg O \), we predict that neither of these counterfactuals is true in our scenario.

Summing up, combining the foreground/background theory of counterfactuals described in this section with the inquisitive lifting procedure described in Section 6.2.2 we obtain an account that accurately predicts which of the counterfactuals in (1) are true in the given scenario. The crucial ingredients of this account are (i) the fine-grained notion of meaning given by inquisitive semantics, (ii) an account of conditionals which is sensitive to inquisitive content, and (iii) a procedure for making counterfactual assumptions which is not constrained by the requirement to minimize the difference with respect to the actual world.\textsuperscript{11}

### 6.3 Further benefits

In the previous section, we have seen how any truth-conditional account of conditionals, whether indicative or counterfactual, can be lifted to inquisitive semantics. Moreover, we have applied this lifting procedure to a particular truth-conditional account of counterfactuals in order to explain the experimental findings of Ciardelli \textit{et al.} (2016b). In this section, we will demonstrate some further general benefits of the lifting procedure. We will see that no matter what truth-conditional account of (indicative or counterfactual) conditionals we take as our starting point, the lifted inquisitive account will improve on it in three ways: first, it will give a more satisfactory account of conditionals with disjunctive antecedents, avoiding a shortcoming which affects all truth-conditional accounts; second, it will allow us to interpret not only standard

\textsuperscript{11}The theory described here is only concerned with predicting when a sentence is \textit{true}. For a complete account of the data in Table 6.1, one would have to complement it with a component that explains when non-true sentences are judged as indeterminate, as opposed to simply false. It is natural to suppose that ‘indeterminate’ judgments result from the failure of a homogeneity presupposition to the effect that making a counterfactual assumption should lead to a state which settles whether the consequent is true (von Fintel, 1997). However, the issue of how failures of semantic presuppositions are reflected in truth value intuitions is a notoriously tricky one (see von Fintel, 2004).
6.3. Further benefits

if-then conditionals, but also so-called unconditionals; and finally, it will allow us to interpret not only conditional statements, but also conditional questions. We will consider each of these topics in a separate sub-section.

6.3.1 Simplification of disjunctive antecedents

Consider the sentences in (5). One seems justified in inferring (5b) from (5a), but certainly not in inferring (5c) from (5b).

\( \text{(5)} \)
\[ \begin{align*}
\text{a. If Alice or Bea invited Charlie, he would go.} \\
\text{b. If Alice invited Charlie, he would go.} \\
\text{c. If Alice invited Charlie and then canceled, he would go.}
\end{align*} \]

The inference from (5a) to (5b) is an instance of a principle called simplification of disjunctive antecedents (SDA); the inference from (5b) to (5c) is an instance of a principle called strengthening of the antecedent (SA).

\[
\begin{align*}
& \frac{A \lor B > C}{A > C} \quad \text{(SDA)} \\
& \frac{A > C}{A \land B > C} \quad \text{(SA)}
\end{align*}
\]

Intuitively, SDA is valid. Indeed, a conditional like (5a) seems to mean exactly the same as the conjunction in (6).

\( \text{(6)} \)
\[ \text{If Alice invites Charlie, he will go, and if Bea invites him, he will go.} \]

However, classical theories of counterfactuals, such as Stalnaker (1968); Lewis (1973) and Kratzer (1981), fail to validate this principle. This has been widely regarded as a problem for these theories (see, e.g., Fine, 1975; Nute, 1975; Ellis et al., 1977; Alonso-Ovalle, 2009; Fine, 2012) and it has also been clear since Fine (1975) that this problem is more than an accidental shortcoming. Indeed, based on a truth-conditional view on meaning and the classical treatment of connectives, a compositional account that validates SDA is bound to validate SA as well, and this is undesirable in view of the inference from (5b) to (5c).\(^\text{12}\)

In recent years, this problem has motivated approaches to counterfactuals which rely on a more fine-grained semantic representation of antecedents than the truth-conditional one. Perhaps the most prominent account of this kind is

\(^\text{12}\) In fact, the problem is not limited to counterfactuals, but concerns conditionals more generally. The intuitions about the indicative conditionals in (i) are exactly the same as for the corresponding counterfactuals in (5).

\( \text{(i)} \)
\[ \begin{align*}
\text{a. If Alice or Bea invites Charlie, he will go.} \\
\text{b. If Alice invites Charlie, he will go.} \\
\text{c. If Alice invites Charlie and then cancels, he will go.}
\end{align*} \]

The same proof given by Fine (1975) for counterfactuals shows that a compositional account of indicative conditionals based on truth-conditions is bound to make SDA and SA inter-derivable.
due to Alonso-Ovalle (2006, 2009), which we already mentioned above as an inspiration for the inquisitive lifting procedure (for accounts in the same spirit, see also van Rooij, 2006; Fine, 2012; Willer, 2015). The fundamental idea of this account is that disjunctive sentences denote sets of classical propositions, rather than single propositions, and that each proposition in the set serves as a separate counterfactual assumption. Clearly, this approach validates SDA: evaluating a counterfactual with a disjunctive antecedent, \( A \lor B > C \), effectively amounts to evaluating the conjunction \( (A > C) \land (B > C) \). On the other hand, SA is invalid: for non-disjunctive antecedents, Alonso-Ovalle’s account coincides with the ordering semantics of Lewis (1973), which notoriously invalidates SA.

The inquisitive lifting recipe achieves essentially the same: when an antecedent is associated with multiple alternatives, the lifted account leads us to run the base account separately for each of these alternatives. This holds in particular for disjunctive antecedents, which normally present one alternative for each disjunct. Thus, no matter what account of conditionals we take as our starting point, the lifting of this account will interpret \( A \lor B > C \) as equivalent with \( (A > C) \land (B > C) \), validating SDA.\(^\text{13}\) On the other hand, if the base account does not validate SA, neither will its lifting, since the two will coincide in the absence of inquisitiveness. Thus, inquisitive semantics provides a way to disentangle SDA from SA and to avoid one of the central problems faced by standard theories of conditionals.

With respect to Alonso-Ovalle’s own account, the inquisitive treatment of conditionals can be seen as a generalization in three different ways. First, on the inquisitive approach, disjunction is not treated as a special, non-standard connective; instead, all connectives are taken to operate on inquisitive propositions, rather than on classical propositions. As we saw in Chapter 4, this allows us to retain a principled and well-behaved theory of propositional connectives, which preserves the attractive features of the classical theory.

\(^\text{13}\)Disjunctive antecedents where one of the disjuncts entails the other, either logically or contextually, form an exception to this claim. If \( A, B \) are atomic sentences with \( |A| \subseteq |B| \), then \( A \lor B \equiv B \) in inquisitive semantics, and as a consequence, \( A \lor B > C \equiv B > C \). We take this to be a welcome result. For consider a conditional of this special form, such as (i):

(i) If we hire an American or a Californian, we should arrange for a visa.

This sentence is odd if uttered by someone who is aware of the fact that Californians are Americans. This is commonly explained in terms of a ban against logical forms that contain structural redundancy (Katzir and Singh, 2013; Meyer, 2014). In Alonso-Ovalle’s account, this explanation is no longer available: even when \( |A| \subseteq |B| \), we have \( A \lor B > C \neq B > C \), so a sentence like (i) does not involve any structural redundancy. By contrast, on our account we have that \( A \lor B > C \equiv B > C \), which allows us to preserve the standard explanation for the oddity of (i). This observation is not specific to conditionals, but it points to an underlying difference between inquisitive semantics and alternative semantics, the framework in which Alonso-Ovalle’s account is cast. We will come back to this point in Section 7.1, where we compare inquisitive and alternative semantics in detail.
Second, Alonso-Ovalle’s account is based on a specific account of conditionals, namely, the ordering semantics of Lewis (1973). By contrast, the inquisitive lifting recipe can be applied to any base account of conditionals, provided that it is compositional and operates in a truth-conditional setting. In the previous section, we have already made use of this degree of freedom. As we saw, independently of the issue of disjunctive antecedents, minimal change theories could not possibly predict the majority judgments in Table 6.1, as these run against the very logic of these theories. The modularity of the inquisitive lifting strategy allowed us to disentangle the problem of dealing with disjunctive antecedents from the problem of determining the right procedure for making counterfactual assumptions.

Finally, inquisitive lifting is not specifically designed to deal with disjunctive antecedents; rather, it provides a general treatment of the interaction between conditionals and inquisitiveness—an interaction which is manifested not just in conditionals with disjunctive antecedents, but in other classes of conditional sentences as well, as we will discuss in Section 6.3.2 and 6.3.3.

Before turning to the next topic, let us spend a few words on some examples that seem to show that SDA is not in fact generally valid. These examples have a special form, with the consequent coinciding with one of the disjuncts in the antecedent. The most famous such example is (7), due to McKay and Van Inwagen (1977):

(7) If Spain had fought with the Axis or the Allies in WWII, she would have fought with the Axis.

This sentence seems true, even though it is certainly not the case that if Spain had fought with the Allies she would have fought with the Axis. On a standard theory like the one of Lewis (1973), the truth of this sentence would be explained by saying that some worlds where Spain fought with the Axis are more similar to the actual world than any world where Spain fought with the Allies. However, this diagnosis leads us to expect that (8a) is true, since it effectively boils down to (8b).

(8) a. If Spain had fought with the Axis or the Allies in WWII, Germany would have been pleased.

b. If Spain had fought with the Axis, Germany would have been pleased.

As Nute (1980) notes, this is wrong: (8a) is naturally interpreted as implying that Germany would have been pleased if Spain fought with the Allies, in accordance with SDA. This problem, together with the fact that these counterexamples have a very special form, suggests that these cases involve some kind of anomaly.
In our inquisitive account, these counterexamples could be accounted for by stipulating that it is in principle possible to insert a projection operator ! in the antecedent. Thus, (7) would be translated as !(Ax ∨ Al) > Ax, and analyzed as a basic conditional with non-inquisitive antecedent, which would block SDA in this case. However, the possibility to insert ! should be restricted, in order to account for the apparent lack of ambiguity of ordinary conditionals such as (5a) and (8a). One way of explaining why ! is inserted in (7) is based on the observation that a logical form such as Ax ∨ Al > Ax is equivalent with the simpler form Al > Ax.\footnote{We are assuming here that the underlying account of conditionals makes any proposition $\alpha \Rightarrow \alpha$ tautological. This is true for all the obvious candidates for $\Rightarrow$, for both indicative and counterfactual conditionals.} Assuming a general ban against structural redundancy, of the kind proposed by Meyer (2014), this would make the logical form Ax ∨ Al > Ax unavailable for a conditional such as (7), justifying the insertion of ! as a repair strategy. This explanation would account for why SDA only seems to fail in sentences where the consequent coincides with one of the disjuncts in the antecedent.

6.3.2 Unconditionals

In the previous section, we mentioned that our account derives the behavior of disjunctive antecedents as a particular case of a more general pattern of interaction between conditionals and inquisitiveness. Another class of sentences in which this interaction is manifested are unconditionals. These are sentences such as the following:

(9) a. Whether they play Bach or not, Alice will go.
   b. Whether they play Bach or Handel, Alice will go.
   c. Whatever they play, Alice will go.

Following Rawlins (2008), we will analyze unconditionals as conditional constructions where the “antecedent” is an interrogative clause. According to the compositional account given in Chapter 5, we translate the polar interrogative whether they play Bach or not as $?Pb$ and the disjunctive interrogative whether they play Bach or Handel as $Pb \lor Ph$. Moreover, we assume that the antecedent of (9c) corresponds to the interrogative what they play, and we translate this as $\exists x Px$.\footnote{Just like in the previous chapter, we disregard at this point the presuppositional component of these sentences. We will return to this later on in this section.} This gives the following translation for our unconditionals.

(10) a. Whether they play Bach or not, Alice will go. $?Pb > G$
    b. Whether they play Bach or Handel, Alice will go. $Pb \lor Ph > G$
    c. Whatever they play, Alice will go. $\exists x Px > G$
Let us now consider the semantics that our inquisitive account of conditionals assigns to these sentences. Let us start with (9a). The antecedent is inquisitive, while the consequent is not: $\text{alt}(?Pb) = \{|Pb|, |\neg Pb|\}$, $\text{alt}(G) = \{|G|\}$. Our support clause gives:

$$s |\models ?Pb > G \iff \forall \alpha \in \{|Pb|, |\neg Pb|\} \exists \gamma \in \{|G|\} \text{ such that } s \subseteq (\alpha \Rightarrow \gamma)$$

$$\iff s \subseteq (|Pb| \Rightarrow |G|) \cap (|\neg Pb| \Rightarrow |G|)$$

$$\iff s |\models (Pb > G) \land (\neg Pb > G)$$

According to this analysis, (9a) is a statement, and it is true in case Alice will go if they play Bach, and she will go if they don’t. This is precisely the analysis we expect for the unconditional (9a). Similarly, for (9b) and (9c) we obtain the following predictions:

$$s |\models Pb \lor Bh > G \iff \forall \alpha \in \{|Pb|, |Ph|\} \exists \gamma \in \{|G|\} \text{ such that } s \subseteq (\alpha \Rightarrow \gamma)$$

$$\iff s \subseteq (|Pb| \Rightarrow |G|) \cap (|Ph| \Rightarrow |G|)$$

$$\iff s |\models (Pb > G) \land (Ph > G)$$

$$s |\models \exists xPx > G \iff \forall \alpha \in \{|Pd|| d \in D\} \exists \gamma \in \{|G|\} \text{ such that } s \subseteq (\alpha \Rightarrow \gamma)$$

$$\iff s \subseteq \bigcap_{d \in D} (|Pd| \Rightarrow |G|)$$

$$\iff s |\models \forall x(Px > G)$$

Thus, both (9b) and (9c) are statements: (9b) is true in case Alice will go both if they play Bach and if they play Handel; (9c) is true if for every $x$ in the domain, Alice will go if they play $x$. Again, these are indeed the natural truth conditions for these sentences.

Thus, our inquisitive account of conditionals extends very naturally to a general analysis of unconditional sentences. The resulting analysis is in line with the one proposed by Rawlins (2008), and shares its core idea. However, a nice feature of the inquisitive approach is that it is modular: it does not commit us to a specific theory of conditionals, but is compatible with a wide range of theories. A second advantage is that nothing special had to be stipulated to analyze unconditionals: the desired analysis follows for free from the semantics of questions and of the conditional operator, once unconditionals are analyzed as conditionals. Thus, the approach is not merely descriptive, but also has some explanatory power. Another way to put this last point is this: we have given a uniform account of disjunctive and interrogative antecedents as introducing multiple assumptions, and provided an explanation for this commonality based on a feature shared by disjunctive and interrogative clauses, namely, inquisitiveness.
One may complain that, in giving this uniform explanation, we have gone too far: at this point, we have given exactly the same translation for the standard if-conditional (11a) and the unconditional (11b).

\[(11) \begin{align*}
    a. & \quad \text{If they play Bach or Handel, Alice will go.} \quad \text{\(Ph \lor Ph > G\)} \\
    b. & \quad \text{Whether they play Bach or Handel, Alice will go.} \quad \text{\(Ph \lor Ph > G\)}
\end{align*}\]

There is a sense in which this is correct. Both (11a) and (11b) are not inquisitive, and they have the same truth conditions: both are true in case Alice will go if they play Bach, and also if they play Handel. Yet, intuitively there is also a difference between these sentences, which is not reflected by our translation.

The idea pursued in Ciardelli (2016b), proposed already by Zaefferer (1991), is that the difference between (11a) and (11b) is one of presupposition: the unconditional in (11b) presupposes that they will play either Bach or Handel, whereas the conditional in (11a) lacks this presupposition. This can be seen with the following pair of examples, the first of which is adapted from Zaefferer:

\[(12) \begin{align*}
    a. & \quad \text{The meeting might be in London; but if it is in Rome or in Paris, Alice will be there.} \\
    b. & \quad \text{??The meeting might be in London; but whether it is in Rome or in Paris, Alice will be there.}
\end{align*}\]

\[(13) \begin{align*}
    a. & \quad \text{Whether the baby is a boy or a girl, they will be a happy family.} \\
    b. & \quad \text{??If the baby is a boy or a girl, they will be a happy family.}
\end{align*}\]

In the case of (12), the first sentence in the discourse indicates that the speaker cannot presuppose that the meeting is in Rome or in Paris. It is then odd for her to continue with an unconditional which carries this semantic presupposition. By contrast, in the case of (13), it would be natural for a speaker to presuppose that the baby will be a boy or a girl. This makes her use of a standard conditional form odd, as a result of the principle maximize presupposition, which requires speakers to prefer equivalent forms with stronger presuppositions whenever these presuppositions are satisfied (see Ciardelli, 2016b, for a more detailed discussion of these data).\(^\text{16}\)

Importantly, in the analysis we described, this semantic difference between standard conditionals and unconditionals does not have to be stipulated, but can be derived from two standard generalizations about the presuppositions of interrogatives, and the way presuppositions project from conditional antecedents.

1. Interrogative clauses presuppose that one of their alternatives is true.

\(^{16}\text{Ciardelli (2016b) also notes that the \textit{maximize presupposition} principle explains the oddness of a conditional like } \text{\textit{If they play Bach or they don’t, Alice will go.} Since one can always presuppose that either they will or they won’t play Bach, the principle requires that, in any context, a speaker should choose the corresponding unconditional form, \textit{Whether they play Bach or not, Alice will go.}}\)
6.3. Further benefits

(see, e.g., Belnap, 1966)\textsuperscript{17}

2. Conditionals inherit the presuppositions of their antecedent.
(see, e.g., Karttunen, 1973, 1974)

Since we view unconditionals as conditionals with an interrogative clause as their antecedent, it follows from (1) and (2) that unconditionals always presuppose that one of the alternatives for their antecedent is true. For a formalization of these ideas in a system that captures presuppositions, see Ciardelli (2016b).

6.3.3 Conditional questions

Another class of sentences which involve the interplay of conditionals and inquisitiveness is given by conditional questions, such as those in (14) and (15).

(14) a. If Alice goes to the concert, will they play Bach?
    b. If Alice goes to the concert, what will they play?

(15) a. If Alice went to the concert, would they play Bach?
    b. If Alice went to the concert, what would they play?

Standard theories of conditionals, being couched in a truth-conditional semantic framework, cannot be directly applied to analyze these sentences. By contrast, the inquisitive lifting of such theories can be applied directly to these questions, yielding natural results. Let us see how. Here, we will not take a stance on what the semantic difference is between indicative and counterfactual conditionals; we will just suppose that we are given two maps \( \Rightarrow \text{i} \) and \( \Rightarrow \text{c} \) which correspond to these two different classes of conditionals, and we will assume two operators \( \succ \text{i} \) and \( \succ \text{c} \) which are interpreted by lifting these maps to inquisitive semantics.\textsuperscript{18}

As above, we translate the clause \textit{whether} they play Bach by \( \text{i} \mathcal{P}b \), and the clause \textit{what} they play by \( \exists xPx \). This gives the following translations for our sentences:

\[
\begin{align*}
(14a) \quad & G \succ \text{i} \mathcal{P}b \\
(14b) \quad & G \succ \text{i} \exists xPx \\
(15a) \quad & G \succ \text{c} \mathcal{P}b \\
(15b) \quad & G \succ \text{c} \exists xPx
\end{align*}
\]

Let us now see what predictions this yields for the conditional questions in (14) and (15). Since the lifting recipe works in the same way for indicative and counterfactual conditionals, we will suppress subscripts in the derivation. Let us start with the conditional polar questions in (14a) and (15a).

\textsuperscript{17}For further discussion of this generalization, see Ciardelli (2016d, p.21-27).

\textsuperscript{18}This assumption does not preclude the possibility of having a uniform semantics for both classes of conditionals: in this case, the maps \( \Rightarrow \text{i} \) and \( \Rightarrow \text{c} \) will be derived from the same underlying account, perhaps by setting some parameters differently in the two cases.
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\[ s \models G > ?Pb \iff \forall \alpha \in \{|G|\} \exists \gamma \in \{|Pb|, |\neg Pb|\} \text{ such that } s \subseteq (\alpha \Rightarrow \gamma) \]
\[ \iff s \subseteq |G| \Rightarrow |Pb| \text{ or } s \subseteq |G| \Rightarrow |\neg Pb| \]
\[ \iff s \models G > Pb \text{ or } s \models G > \neg Pb \]

Thus, (14a) and (15a) are predicted to be inquisitive. A state supports \( G > ?Pb \) iff it supports the statement \( G > Pb \), or it supports the statement \( G > \neg Pb \); this means that in order to resolve (14a), one must establish either that if Alice goes they will play Bach, or that if Alice goes they won’t play Bach. These are precisely the resolution conditions that we expect for (14a). Similarly, (15a) is supported iff either of the statements \( G > c Pb \) and \( G > c \neg Pb \) is supported, which again gives the natural resolution conditions for this question. Now let us consider the conditional wh-questions in (14b) and (15b).

\[ s \models G > \exists xPx \iff \forall \alpha \in \{|G|\} \exists \gamma \in \{|Pd| \mid d \in D\} \text{ such that } s \subseteq (\alpha \Rightarrow \gamma) \]
\[ \iff s \subseteq |G| \Rightarrow |Pd| \text{ for some } d \in D \]
\[ \iff s \models G > Pd \text{ for some } d \in D \]

Thus, (14b) and (15b) are predicted to be inquisitive. An information state supports \( G > ?xPx \) iff it supports the statement \( G > Pd \) for some \( d \in D \); this means that in order to resolve (14b), one must establish for some specific \( d \) that if Alice goes, they will play \( d \). Similarly, (15b) is supported iff the statement \( G > c Pd \) is supported for some \( d \in D \). Again, these are precisely the resolution conditions that we would intuitively assign to these questions.

These examples illustrate how lifting an account of conditionals to inquisitive semantics immediately yields an extension of this account to conditional questions. This approach differs from previous accounts of conditional questions such as Velissaratou (2000) and Isaacs and Rawlins (2008), which focus on indicative conditional questions like those in (14) and cannot be used directly to analyze counterfactual conditional questions like those in (15). As we have seen, inquisitive lifting applies uniformly to indicative and counterfactual questions. Additionally, inquisitive lifting leaves us with a choice as to the underlying theory of conditionals that we use to interpret these questions.

Before concluding this section, an important issue remains to be addressed. At this point, the reader might be worried that the conditional statement (16a) might end up being assigned the same meaning as the conditional question (16b).

(16) a. If Alice goes, they will play Bach or Handel.
   b. If Alice goes, will they play Bach, or Handel?

This problem is avoided because, were we to specify a compositional analysis of conditional sentences, we would stipulate the same general rule that we already
adopted in Chapter 5: the logical form of a declarative main clause is always headed by a projection operator \( ! \), which ensures non-inquisitiveness, and the logical form of an interrogative clause is always headed by an operator \( \langle ? \rangle \), which ensures inquisitiveness. In all the examples of conditional statements that we considered in this chapter, the consequent was already non-inquisitive, so adding \( ! \) would not make a difference; however, to get the correct interpretation for (16a) it is crucial that we translate it as \( G > !(Pb \lor Ph) \), and not simply as \( G > (Pb \lor Ph) \), which would be the intended translation for the question (16b).

Summing up, in this section we saw that lifting an account of conditionals to inquisitive semantics leads to an account which improves on the original one in various ways: first, it gives a more satisfactory treatment of disjunctive antecedents, which are interpreted as providing multiple assumptions; second, it extends the scope of the original account beyond standard conditional statements, allowing us to analyze two other classes of conditional constructions: unconditionals as well as conditional questions.

### 6.4 Summary

Our goal in this chapter was to show that inquisitive content is relevant even for phenomena that have no obvious link to questions, and that the inquisitive content of a constituent can sometimes play a crucial role in determining the truth conditions of a sentence. We have illustrated this point with conditionals, which provide an especially interesting and rich domain of application. In this domain, taking inquisitive content into account provides a natural explanation for some otherwise puzzling data, solves some long-standing logical problems, and allows for a substantial extension of the scope of standard theories.\(^{19}\) We think that conditionals are not an isolated case, but only one of many environments where inquisitive content plays a role. However, more work, both empirical and theoretical, is needed to substantiate this claim.\(^{20}\)

### 6.5 Exercises

**Exercise 6.1. [Lifting material implication]**

Show that inquisitive implication is the lifting of material implication. That is, show that if \( \Rightarrow \) is defined as material implication (i.e., for every two classical

\(^{19}\)The analysis of indicative conditional statements and questions has also given rise to further refinements of the basic inquisitive notion of meaning presented here (Groenendijk and Roelofsen, 2010, 2015; Aher and Groenendijk, 2015). These refinements address empirical issues that are orthogonal to the ones considered here.

\(^{20}\)For some work in this direction, focusing on modals and imperatives, see Aloni (2007); Aloni and Ciardelli (2013); Willer (2015); Fine (2016).
propositions \( p \) and \( q \), \( p \Rightarrow q \) amounts to \( \overline{p} \cup q \), then the support conditions assigned to \( \varphi > \psi \) by the inquisitive lifting recipe coincide with the support conditions of \( \varphi \rightarrow \psi \) in \( \text{InqB} \).

**Exercise 6.2. [The foreground/background theory]**
Consider sentence (17) in the following two scenarios (Tichý, 1976):

- **Context 1:** Jones has the following habits as regards wearing his hat. Bad weather invariably induces him to wear his hat. Fine weather, on the other hand, affects him neither way: on fine days he puts his hat on or leaves it on the peg, completely at random. Suppose moreover that actually the weather is bad, so Jones is wearing his hat.

- **Context 2:** Jones always flips a coin before he opens the curtains to see what the weather is like. Heads means he is going to wear his hat in case the weather is fine, whereas tails means he is not going to wear his hat in that case. Like above, bad weather invariably makes him wear his hat. Today heads came up when he flipped the coin, and it is raining. So Jones is wearing his hat.

(17) If the weather was fine, Jones would be wearing his hat.

Intuitively, the sentence is true in context 2 but not in context 1. Show how this is derived in the foreground/background theory of counterfactuals, modeling the causal structure of each context.

**Exercise 6.3. [Quantification in the antecedent of a counterfactual]**
Consider an electrical circuit with four switches and one light. The light is on if and only if an even number of switches is up. Currently, all switches are up, so the light is on. Now consider the following sentences:

(18) If any of the switches was down, the light would be off.

(19) If the switches weren’t all up, the light would be off.

Intuitively, (18) is true in the given scenario, but (19) is not. Suppose that the sentences are translated as \( (\exists x. \neg Ux) > O \) and \( (\forall x. Ux) > O \), respectively.

1. Show that the given intuitions cannot be captured by any truth-conditional compositional account of counterfactuals.

2. Show that they are captured by the inquisitive account described above.

**Exercise 6.4. [Conditional questions with disjunctive antecedents]**
Consider the following indicative conditional question:
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(20) If Alice goes to London or to Paris, will she take the train?

1. Translate the sentence into a suitable logical language.

2. Assuming a truth-conditional map $\Rightarrow$ for indicative conditionals, derive the support conditions that the inquisitive lifting of $\Rightarrow$ assigns to the question in (20).

3. What does this predict about the circumstances under which the question is resolved?
Chapter 7

Questions

One of the main purposes of inquisitive semantics is to serve as a framework for the semantic analysis of interrogative sentences in natural languages. In this chapter we will compare inquisitive semantics with some other frameworks which have been proposed for this purpose, and which have been used widely in the literature. In doing so, we will restrict our attention to those previous proposals that are most closely related to our own. That is, we will consider the alternative semantics framework proposed by Hamblin (1973) and Karttunen (1977), the partition semantics developed by Groenendijk and Stokhof (1984) and later cast in a dynamic framework by Jäger (1996), Hulstijn (1997), and Groenendijk (1999), and the inquisitive indifference semantics proposed by Groenendijk (2009) and Mascarenhas (2009). We will argue that the framework presented here preserves the essential insights that have emerged from these previous approaches, while overcoming their main shortcomings.\(^1\)

Figure 7.1 provides a global overview of the different approaches. In this figure, the proposed frameworks are ordered from left to right in terms of expressive power, and the development through time is indicated by the bended arrows. Alternative semantics, developed in the 1970’s, is the most liberal framework in terms of expressive power—leading, as we will see below, to problems of overgeneration. Partition semantics on the other hand, originally proposed in the 1980’s and further developed in a dynamic setting in the 1990’s, is the most restricted in terms of expressive power—leading to problems of undergeneration. More recent work has tried to strike an optimal balance between these two extremes, first leading to indifference semantics and then to inquisitive semantics.

A terminological note: to ease the discussion of previous work, we will use the term ‘questions’ here to refer to interrogative sentences in natural languages, rather than to a class of propositions/sentences in InqB, as we did in Chapter 2-4.

\(^1\)One prominent approach that we will not discuss here is the functional approach (sometimes also called the categorial approach or the structured meanings approach). We refer to Krifka (2001a, 2011) and Groenendijk and Stokhof (1997) for overviews.
Figure 7.1: Semantic frameworks for the analysis of questions, ordered chronologically and in terms of expressive power.

7.1 Alternative semantics

Alternative semantics is probably the most widely used framework for the semantic analysis of questions in natural language. It was first proposed by Hamblin (1973), driven by the following idea:

“Questions set up a choice-situation between a set of propositions, namely those propositions that count as answers to it.”

(Hamblin, 1973, p.48)

Thus, Hamblin takes questions to denote sets of classical propositions. These propositions are often referred to as alternatives, hence the name of the framework. Karttunen (1977) independently proposed a very similar view on question meanings: he also took questions to denote sets of classical propositions, though he restricted the denotation of a question in a particular world to propositions that correspond to answers that are true in that world. In both systems, the meaning of a question, i.e., its intension, is a function from worlds to sets of classical propositions. In Hamblin’s system, this function maps every possible world to the same set of propositions, corresponding to the set of all possible answers; in Karttunen’s system, every world is mapped to a subset of all possible answers, namely those that are true in the given world. As noted by Karttunen (1977, p.10), this difference is inessential. In both cases, the meaning of a question is fully determined by—and could be identified with—the set of all classical propositions that correspond to a possible answer.\(^2\)

\(^2\)It should be noted that there are significant differences between Hamblin’s and Karttunen’s approach concerning the compositional derivation of question meanings. In a nutshell, Hamblin
This classical view on question meanings faces some fundamental problems. We will discuss these, and show that they no longer arise in inquisitive semantics.

**Problem 1: Possible answers**

The first problem is that the framework’s core notion—that of a possible answer—is difficult to pin down. Surely, Hamblin and Karttunen provide a compositional semantics for a fragment of English, and thereby specify what they take to be the possible answers to the questions in that fragment. But in order to assess such a compositional theory, or even to properly understand what its predictions amount to, we first need to have a pre-theoretical notion of possible answers, one that the theoretical predictions can be evaluated against. The problem is that such a pre-theoretical notion is difficult, if not impossible to identify. To illustrate this, consider the question in (1) and the classical propositions in (2):

(1) Is Bill coming to the party tonight?

(2) a. \( \{w \mid \text{Bill is coming in } w\} \)
b. \( \{w \mid \text{Bill is not coming in } w\} \)
c. \( \{w \mid \text{Bill is coming in } w \text{ unless he has homework to do}\} \)
d. \( \{w \mid \text{Bill is coming in } w \text{ but only until midnight}\} \)
e. \( \{w \mid \text{Bill is coming in } w \text{ but he is bringing his homework}\} \)

In principle, all the propositions in (2) could be seen as possible answers to (1). For Hamblin and Karttunen, only (2a) and (2b) count as such. However, it is not clear what the precise criteria are for being considered a possible answer, and on which grounds (2a-b) are to be distinguished from (2c-e).

In inquisitive semantics, question meanings are also sets of classical propositions, just like in alternative semantics. However, in inquisitive semantics these classical propositions are not thought of as the ‘possible answers’ to the question. Rather, they are thought of as the information states—or equivalently, the pieces of information—that resolve the issue that the question expresses. As a consequence, in inquisitive semantics question meanings cannot be defined as arbitrary sets of classical propositions, which is what Hamblin and Karttunen proposes a rather radical departure from the standard Montagovian approach to meaning composition, adapting the semantic type of all lexical items and letting the operation that is standardly used to compose the meanings of two constituents, i.e., function application, operate in a pointwise fashion. Karttunen on the other hand sticks to the standard Montagovian procedure. While Hamblin’s more radical proposal has probably been adopted more widely, its compositional apparatus faces a number of thorny problems (see, e.g., Shan, 2004; Novel and Romero, 2010; Charlow, 2014). In inquisitive semantics these problems can be overcome in a principled way. A detailed discussion of compositionality, however, is beyond the scope of this book; we refer to Ciardelli, Roelofsen, and Theiler (2016a).
take them to be. Rather, they have to be downward closed. After all, if an information state \( s \) resolves the issue expressed by a given question \( Q \), then any stronger information state \( t \subset s \) will also resolve the issue expressed by \( Q \).

In our view, pre-theoretical intuitions about which pieces of information resolve a given issue are much more robust than pre-theoretical intuitions about what the ‘possible answers’ to a given question are. For instance, in the above example it is clear that (2a-b,d-e) resolve the given issue, while (2c) doesn’t. Thus, evaluating theories of questions formulated in inquisitive semantics is more feasible than evaluating theories of questions which, like Hamblin and Karttunen, view question meanings as sets of possible answers.

This said, we should emphasize that even though the elements of the meaning of a question in inquisitive semantics are not taken to correspond directly to the ‘possible answers’ to that question, this is not to say that such question meanings cannot play a role in characterizing sensible notions of answerhood at all. Quite the contrary.

For instance, it would be natural to characterize the basic answers to a question \( Q \) as those pieces of information that:

(i) resolve the issue expressed by \( Q \), and

(ii) do not provide more information than necessary to do so, i.e., are not strictly stronger than any other piece of information that also resolves the issue expressed by \( Q \).

Under this definition, the basic answers to \( Q \) correspond precisely to what we called the alternatives in the proposition expressed by \( Q \).\(^3\)\(^4\) In the above example, (2a-b) would count as basic answers to the question under this definition, while (2c-e) would not. This is the same distinction that Hamblin and Karttunen made. Now, however, it is clear on which grounds the distinction is made.

Besides defining a notion of basic answerhood along these lines, we may also define notions of partial answerhood and subquestionhood (see Groenendijk and Roelofsen, 2009), which are crucial for the analysis of discourse structure and information structure (Roberts, 1996; Büning, 2003). What is crucial is that in

\(^3\)Recall from footnote 4 on page 23 that some propositions in InqB do not contain any alternatives. According to the characterization of basic semantic answers just given, questions expressing such propositions do not have any basic semantic answers. See Ciardelli (2010), Ciardelli et al. (2013b), and Roelofsen (2013a) for further discussion of such cases.

\(^4\)Notice that our definition of basic answers as minimal resolving pieces of information implies that the set of basic answers to a given question is always a set \( \mathcal{A} \) such that for any \( s, t \in \mathcal{A} \), neither \( s \subset t \) nor \( t \subset s \). After all, if \( s \subset t \), then \( s \) cannot be a minimal resolving piece of information, and vice versa if \( t \subset s \). Thus, even if—in the spirit of Hamblin and Karttunen—we were to identify the meaning of a question with the set of its basic answers, not just any set of classical propositions would count as a proper question meaning. We will return to this point below.
7.1. Alternative semantics

Alternative semantics

Answerhood

Resolution

Inquisitive semantics

Figure 7.2: Primitive and derived notions in alternative/inquisitive semantics.

Inquisitive semantics the meaning of a question is not characterized in terms of basic/possible/complete/partial answers. Rather, as depicted in Figure 7.2, it is the other way around: question meanings, i.e., issues, are defined in terms of what it takes to resolve them, and the basic/possible/complete/partial answers to a question are defined in terms of these resolution conditions. As a consequence, whichever notion of answerhood we choose to adopt, there will be no need for such a notion to correspond directly to some pre-theoretical concept. Rather, it will be grounded, in a precisely circumscribed way, in the pre-theoretical notion of what it takes for a given issue to be resolved. For instance, if as suggested above we characterize the basic answers to a question as those pieces of information that resolve the issue that the question expresses and do not provide more information than is necessary to do so, then in order to evaluate a theory that associates every question with a set of basic answers, we can simply rely on judgments concerning resolution, rather than judgments directly concerning ‘basic answers’.

Problem 2: Entailment

A second fundamental problem for alternative semantics, which was pointed out and discussed at length in Groenendijk and Stokhof (1984), is that it is difficult to define a suitable notion of entailment in this framework that determines when one question is more demanding than another. One consequence of this is that it is hard, if not impossible, to give a principled account of the interaction between questions and logical connectives and quantifiers. For instance, it proves problematic to give a satisfactory treatment of the conjunction of two questions. Without a suitable notion of entailment, conjunction can certainly no longer be treated as a meet operator. See Roelofsen (2013a); Ciardelli and Roelofsen (2016); Ciardelli, Roelofsen, and Theiler (2016a) for more elaborate discussion of this point, and a critical assessment of some concrete notions of entailment and conjunction that may be considered in alternative semantics.
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This problem does not arise in inquisitive semantics, which has a well-behaved notion of entailment. As discussed in Chapter 3, the space of propositions in inquisitive semantics, ordered by entailment, has a familiar algebraic structure, and a natural treatment of the logical connectives is obtained by associating them with the basic operations in this algebra. Thus, as we have seen, the classical treatment of conjunction as a meet operation can be preserved in inquisitive semantics to apply to informative and inquisitive sentences in a uniform way, and the same goes for the other operations.

The two problems that we just discussed for alternative semantics are closely related. After all, if it were possible to ground the notion of ‘possible answers’ in some pre-theoretical notion, then it would most likely also become clear how to characterize entailment. That is, if there were clear criteria for what it takes to count as a possible answer, we would also know better on which grounds two sets of possible answers should be compared, and under which conditions one set should be seen as entailment.

Compare the situation with the one we have in classical logic. There, the proposition expressed by a sentence is a set of possible worlds. These worlds are intended to correspond to situations that are compatible with the information that the sentence conveys. In this case, there is a clear pre-theoretical intuition to build on, as to whether a certain situation is or is not compatible with a given piece of information. As a consequence, it is also clear when one sentence should be taken to entail another, namely if it conveys at least as much information, meaning that the proposition it expresses is a subset of the proposition expressed by the other sentence. In alternative semantics, the meaning of a question is a set of classical propositions which are intended to correspond to its possible answers. However, since it is not clear when exactly a proposition should count as a possible answer, it is also difficult to say when one question should entail another.

In inquisitive semantics, the proposition expressed by a question is a set of information states, which are intended to be those information states that resolve the issue that the question expresses. It is also clear, then, when one question is more demanding than another, namely if every information state that resolves the former also resolves the latter. This immediately delivers the desired notion of entailment, as well as the algebraic operations that are characterized in terms of it.

\[6\] It is not the formal notion of meaning as such that stands in the way of a suitable notion of entailment, but really the conception of these meanings in terms of possible answers. For instance, if we construe the meaning of a sentence as a set of classical propositions, as in alternative semantics, but think of these propositions as those that the sentence draws attention to, rather than as possible answers, then it is quite straightforward to define a suitable notion of entailment, which compares two sentences/meanings in terms of their attentional strength (Roelofsen, 2013b).
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Problem 3: Overgeneration

A third problem, which is again connected to the other two, is that there are question meanings in alternative semantics which seem impossible to express in natural languages. These are question meanings containing two alternatives $\alpha$ and $\beta$ such that one is strictly contained in the other, $\alpha \subset \beta$.

One may think that such meanings may be expressed by disjunctive questions, where each disjunct contributes one of the two alternatives. However, in order to get that $\alpha \subset \beta$, we would have to construct the question in such a way that one disjunct classically entails the other. As illustrated in (3) and (4) below, such questions are infelicitous (Ciardelli and Roelofsen, 2016).

(3) #Is John American, or is he Californian?
(4) #Is the value of $x$ different from 6, or is it greater than 6?

It has been well-known since Hurford (1974) that disjunctive declaratives where one disjunct entails the other are generally infelicitous as well.

(5) #John is American or he is Californian.
(6) #The value of $x$ is different from 6 or it is greater than 6.

This phenomenon, known as Hurford’s constraint, has been given an appealing explanation in terms of redundancy. More specifically, Katzir and Singh (2013) propose the following principle (see also Simons, 2001; Meyer, 2014, for closely related proposals):

**Local redundancy:** a sentence is deviant if its logical form contains a binary operator $O$ applying to two arguments $A$ and $B$, and the outcome $O(A, B)$ is semantically equivalent to one of the arguments.\(^7\)

Let us briefly consider how this principle predicts Hurford’s constraint. In classical semantics, the meaning of a sentence $A$ is a classical proposition $|A|$, the set of worlds where the sentence is true. $A$ entails $B$ just in case $|A| \subseteq |B|$. Moreover, sentential disjunction yields the union of two propositions, that is, $|or(A, B)| = |A| \cup |B|$.

Now, suppose that the logical form of a sentence contains a sentential disjunction operator applying to two arguments $A$ and $B$ such that $|A| \subseteq |B|$, as in examples (5) and (6). Then we have that $|or(A, B)| = |A| \cup |B| = |B|$. So, the output is semantically equivalent with one of the inputs. Thus, the given logical form exhibits local redundancy and is therefore predicted to be deviant.

Now, one would of course hope that this explanation of Hurford’s constraint in terms of redundancy would apply not only to declaratives like (5) and (6), but

\(^7\)Katzir and Singh (2013)’s proposal is relativized to a context of utterance $c$. Since context-dependency plays no role in our discussion, we omit reference to contexts for ease of exposition.
also to questions like (3) and (4). But this is not the case in alternative semantics, where the disjuncts express singleton sets, \{A\} and \{B\}, respectively, and disjunction yields the set \{A, B\}. Since the output of the disjunction operator is different from any of its inputs, the local redundancy condition is not violated, and no deviance is therefore predicted.

In inquisitive semantics, the explanation of Hurford’s constraint in terms of redundancy does naturally apply to questions like (3) and (4), assuming that each of the disjuncts expresses a proposition containing all states that consist exclusively of worlds where that disjunct is true (just like atomic sentences in InqB), and English or is treated as a join operator, just like disjunction in InqB. We then have that \[ A = \wp(|A|), \quad B = \wp(|B|), \quad \text{and } [\text{or}(A,B)] = [A] \cup [B] = \wp(|A|) \cup \wp(|B|) = \wp(|B|) = B. \]

Thus, the output of the disjunction operator is identical to one of its inputs, and redundancy is predicted just as for declarative Hurford disjunctions.

Let us try to better understand this contrast between inquisitive semantics and alternative semantics by considering the notion of ‘alternatives’ that plays a role in the two frameworks. We have seen that both frameworks associate questions with sets of alternatives, but that the status of these alternatives crucially differs from one framework to the other.

In inquisitive semantics, the alternatives in the proposition expressed by a question are characterized as those pieces of information that resolve the issue that the question raises in a minimal way. This implies that sets of alternatives have to be of a particular form: two alternatives are always logically independent, that is, one is never contained in the other.

In alternative semantics on the other hand, there is no such constraint on sets of alternatives: any set will do. This is connected, of course, to the fact that the notion of an alternative is a primitive notion in this framework, not defined in terms of resolution conditions or any other more elementary notion.

Let us refer to sets of classical propositions whose elements are pairwise logically independent as non-nested sets. In inquisitive semantics, then, unlike in alternative semantics, only non-nested sets of classical propositions are regarded as proper sets of alternatives. Thus, certain meanings in alternative semantics do not have a counterpart in inquisitive semantics. It is precisely these additional meanings, i.e., nested sets of alternatives, which seem impossible to express in natural languages, at least ones like English. In principle, a Hurford disjunction would be exactly the right kind of construction to express a nested set of alternatives. But we have seen that such disjunctions are infelicitous. This seems to indicate that there is something wrong with nested sets of alternatives

\footnote{It should be noted that there are apparent counterexamples to Hurford’s constraint, which may seem to undermine the argument that we are making here. For instance:}

(i) \hspace{1cm} Bill solved two of the homework problems, or he solved all of them.
as meanings, which is puzzling from the perspective of alternative semantics, since in this framework nested sets of alternatives have exactly the same status as non-nested sets.

In inquisitive semantics, the puzzle does not arise, because nested sets of alternatives simply do not exist. Importantly, such sets are not ruled out by some special purpose constraint: rather, it just follows from the way alternatives are construed that they are never nested. This means that from the perspective of inquisitive semantics, what is special about Hurford disjunctions is not that they express some distinguished class of meanings, but rather that they involve redundant disjuncts, which fail to contribute an alternative to the meaning of the disjunction. As we have seen, this is precisely what makes it possible to explain their infelicity.

### 7.2 Partition semantics

Departing from Hamblin and Karttunen’s work, Groenendijk and Stokhof (1984) propose that a question does not denote a set of classical propositions at each world, but rather a single classical proposition embodying the true exhaustive answer to the question in that world. For instance, if $w$ is a world in which Paul and Nina are coming for dinner, and nobody else is coming, then the denotation of (7) in $w$ is the classical proposition expressed by (8).

(7) Who is coming for dinner tonight?
(8) Only Paul and Nina are coming.

The meaning of a question, i.e., its intension, then amounts to a function from worlds to classical propositions. In Groenendijk and Stokhof’s framework these classical propositions are required to have two special properties: they have to be mutually exclusive (since two different exhaustive answers are always incompat-ible), and together they have to cover of the entire logical space (since every world is taken to be compatible with at least one exhaustive answer). So the meaning of a question is a set of classical propositions that together form a partition of the logical space.

At first blush, it seems that the second disjunct entails the first, and yet the sentence is felicitous. However, as argued in detail by Chierchia et al. (2009), in such cases the weaker disjunct receives a strengthened interpretation—here, that Bill solved only two of the homework problems—which in effect makes it logically independent from the other disjunct. For a more detailed exposition of the argument that we are presenting here, taking cases like (i) into account, we refer to Ciardelli and Roelofsen (2016).
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Problem: Undergeneration

Partitions correspond to a specific kind of propositions in inquisitive semantics. That is, for every partition $\rho$, there is a corresponding proposition $P_\rho$ in inquisitive semantics, consisting of all states that are contained in one of the cells of the partition:

$$P_\rho := \{ s \subseteq b \mid b \in \rho \}$$

On the other hand, not every proposition in inquisitive semantics corresponds to a partition. This holds in particular for all inquisitive propositions containing overlapping alternatives, and ones whose elements do not cover the entire logical space.

Thus, while we saw that the range of question meanings in inquisitive semantics is more confined than in alternative semantics, it is broader than in partition semantics. In terms of expressive power, then, there is a strict linear order between the three frameworks, as was already anticipated in Figure 7.1.

We have already discussed some of the benefits of inquisitive semantics w.r.t. alternative semantics, and these benefits are all connected in one way or another to the difference in expressive power between the two frameworks. With respect to partition semantics, the main benefit of inquisitive semantics is that, because of its more general notion of meaning, it allows us to deal with a broader range of questions in natural language. In particular, without further amendments it is impossible in partition semantics to give a satisfactory account of conditional questions, disjunctive questions, and mention-some wh-questions, exemplified in (9)–(11).

(9) If Peter attends the meeting, will Maria attend it too?
(10) Will Peter↑ attend the meeting, or Maria↑?
(11) Who will attend the meeting? (on a mention-some reading)

As we discussed in Section 2.4.5, these sentences would be treated in inquisitive semantics as expressing the propositions depicted in Figure 7.3. Note that these propositions have overlapping alternatives, which means that they do not con-
7.2. Partition semantics

stitute partitions of the logical space. And yet they correctly capture what is
needed to resolve the issues raised by (9)–(11). For instance, in order to resolve
the issue raised by the conditional question in (9), it is sufficient to establish
that Maria will go if Peter will, or to establish that Maria will not go if Peter
goes. These two options correspond precisely to the two overlapping alterna-
tives in Figure 7.3(a). And similarly for (10)-(11). Thus, these types of questions
are beyond the scope of partition semantics but can be suitably dealt with in
inquisitive semantics.9

A possible concern: disjunctions of questions

Inquisitive semantics provides a notion of question meaning that is richer than
the one assumed in partition semantics, and we have just seen that this is crucial
in order to accommodate several classes of questions which express issues that
do not correspond to partitions of the logical space. However, this increase in
expressive power may also raise a certain concern.

Consider the following sentence from Szabolcsi (1997, p.325), a disjunction
of two wh-questions, which is decidedly odd.

(12) Who did you marry or where do you live?

Szabolcsi (1997, 2015a) has argued that the oddness of this sentence can be
explained in partition semantics. Namely, a partition may be identified with an
equivalence relation on the space of possible worlds, and while the intersection
of two equivalence relations is itself again an equivalence relation, the same is
not true of the union of two equivalence relations. If conjunction and disjunction
are taken to express intersection and union, respectively, it is to be expected
that conjunction, but not disjunction, can apply to two questions to form a new
question in natural languages.

On the other hand, in inquisitive semantics the oddness of (12) cannot be
explained on purely semantic grounds, because if we take disjunction to express
the join operator it delivers a perfectly sensible issue, one that can be resolved
either by establishing whom the addressee married or by establishing where the
addressee lives. This issue does not correspond to a partition, but it is an issue
nonetheless in our framework. Thus, while the inquisitive notion of meaning
has important advantages w.r.t. partition semantics, it may also seem to have a
certain disadvantage.

However, note that the prediction arising from partition semantics is a very
strong one: it implies that questions cannot be directly disjoined in natural
languages at all. Szabolcsi (1997, 2015a) claims that this general prediction is

9For a more detailed discussion of the difference in expressive power between inquisitive
semantics and partition semantics, we refer to Ciardelli et al. (2015, §5).
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indeed borne out, but we are convinced by examples like (13), repeated from Chapter 1, and (14), repeated from Chapter 5, that it is too strong: disjunctions of questions are not always infelicitous.

(13) Where can we rent a car, or who might have one that we could borrow?

(14) Does Igor speak English or does he speak French?

We should note that Szabolcsi remarks that a sentence like (12) may be marginally acceptable if regarded as a case in which the speaker first asks the question who did you marry, but then reconsiders and proposes to replace this first question by the second, where do you live. In such cases, Szabolcsi suggests, disjunction does not play its usual role but is rather used as a corrective device.

Our examples (13) and (14), however, can be uttered by someone without any reconsideration halfway, and they can each be addressed by an addressee as a single question, to which both disjuncts contribute. So they seem to be a genuine disjunctions of questions.

Szabolcsi (1997) does not base her empirical claim merely on cases like (12) but also on a striking pattern that is found in embedded questions in Hungarian. Hungarian complement clauses, whether declarative or interrogative, are always headed by the subordinating complementizer hogy. Szabolcsi argues that this subordinating complementizer expresses a lifting operation that needs to be invoked before two interrogative complement clauses can be disjoined (just like proper names have to be lifted into generalized quantifiers when they are conjoined or disjoined with a quantificational noun phrase). Support for this idea comes from examples like (15) and (16) below, which indicate that (i) conjoined interrogative complement clauses can have either two occurrences of hogy, applying to both individual conjuncts, or a single occurrence of hogy, applying to the conjunction as a whole, but (ii) disjoined interrogative complement clauses must have two occurrences of hogy, each applying to one of the individual disjuncts.

(15) János meg tudta, hogy kit vettél feleséül és (hogy) hol laksz.
     Janos found.out subord whom you.took as.wife and (subord) where you.live

     'Janos found out whom you married and where you live.'

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(15) János meg tudta, hogy kit vettél feleséül és (hogy) hol laksz.
     Janos found.out subord whom you.took as.wife and (subord) where you.live

     'Janos found out whom you married and where you live.'

10Krifka (2001b) endorses Szabolcsi’s claim, though he offers a different explanation, based on the assumption that questions do not express sets of propositions or partitions, but rather speech acts, which Krifka models as operations on commitment states. Speech act disjunction does not exist according to Krifka, because it “would lead to disjunctive sets of commitments, which are difficult to keep track of” (Krifka, 2001b, p.16).

11Haida and Repp (2013) also challenge Szabolcsi’s empirical claim, although they maintain a weaker version of it: questions can only be disjoined in downward entailing or non-veridical contexts. Our examples (13) and (14) present a challenge for this weaker claim as well.
Szabolcsi concludes from this observation that disjunction cannot directly apply to interrogative complement clauses, but always requires intervention of a lifting operation, expressed overtly in Hungarian by *hogy*.

However, there are counterexamples to the generalization. The Hungarian counterpart of our example (13) is a case in point. When embedded, it may come either with one or with two occurrences of *hogy*, no matter whether the embedding verb is extensional (e.g., *find out*) or intensional (e.g., *investigate*).\(^{12}\)

In (18), a single occurrence of *hogy* favors a reading on which disjunction takes narrow scope w.r.t. the verb, while two occurrences of *hogy* favor a reading on which disjunction takes wide scope (Peter is investigating where we can rent a car or he is investigating who has one we could borrow), a pattern that is in line with Szabolcsi’s idea that *hogy* expresses a lifting operation.

It thus seems that, at least in some cases, disjunction can apply directly to questions, both in English and in Hungarian. A question that naturally arises, then, is whether the general disjunction operation that inquisitive semantics makes available allows us to derive the correct meaning for those disjunctions of questions which are felicitous. For disjunctions of non-*wh*-questions, we have already argued this to be the case in Chapter 5. The predictions for disjunctions of *wh*-questions also seem to be correct. For instance, assuming that (13) is an open interrogative list and that the two interrogative clauses each receive a mention-some interpretation, the sentence is predicted to express an issue which can be resolved either by identifying a place where the speaker can rent a car, or

\(^{12}\text{We are grateful to Donka Farkas, Anikó Liptak, and Anna Szabolcsi for discussion of this datapoint.}\)
by identifying a person who might have a car that the speaker can borrow, or by establishing that there is no such place and no such person. These are indeed the resolution conditions we expect for (13).\footnote{There is a subtlety that we gloss over here. While in the case of a non-\emph{wh}-clause, the clause type marker can simply apply $!$ to the proposition that it takes as its input, as discussed in Chapter 5, in the case of a \emph{wh}-clause, we assume that the clause type marker takes an \emph{n}-place property $R^n$ as its input, where \emph{n} is the number of \emph{wh}-elements, and it delivers the proposition $\bigcup \{ !R^n(t) \mid t \text{ a tuple of } \emph{n} \text{ individuals} \}$ as its output. So the $!$ operator applies to all propositions obtained by feeding $R^n$ an appropriately sized tuple of individuals. In case $n = 0$ this amounts precisely to what we assumed in Chapter 5 for non-\emph{wh}-clauses. We refer to Champollion, Ciardelli, and Roelofsen (2015) for further details.} Notice that this prediction is obtained simply by applying inquisitive disjunction to the propositions expressed by the two interrogative clauses—the same disjunction operation that, in the previous chapter, we took to be at work in disjunctive non-\emph{wh}-questions like (14) above, as well as disjunctive declaratives.

Thus, after all disjunctive questions seem to provide a strong argument in favor of inquisitive semantics over partition semantics, where examples such as (13) and (14) can only be handled at the cost of a significant complication of the framework (and one that gives up some of its most attractive features, such as the general account of entailment and coordination among interrogatives; see Groenendijk and Stokhof, 1989).

Of course, an interesting question that remains to be addressed is why our example (13) behaves so differently from Szabolcsi’s example (12), both as a standalone question and when embedded. We think that the difference may be explained pragmatically. A disjunction of two questions expresses an issue that may be resolved equally well by providing information resolving the first disjunct, or by providing information resolving the second disjunct. Now, it is difficult to see what kind of motivation (or what kind of decision problem, to follow van Rooij 2003) a speaker could have that would lead her to raise or even consider the issue expressed by (12). This is very different in the case of (13): in this case, it is immediate to reconstruct the sort of motivation that may lead a speaker to consider the relevant issue. We suggest that the different cognitive plausibility of the two issues at stake underlies the difference in the perceived felicity of the associated questions.

### 7.3 Dynamic partitions and indifference semantics

While Hamblin (1973), Karttunen (1977), and Groenendijk and Stokhof (1984) all operate under a static view on meaning, there are also a number of proposals that aim to capture the meaning of questions in a \emph{dynamic} framework. The first such proposals, developed by Jäger (1996), Hulstijn (1997), and Groenendijk (1999), essentially reformulate the partition theory of questions in the format...
of an update semantics (Veltman, 1996). This means that they construe the meaning of a sentence as its context change potential, i.e., a function that maps every context in which the sentence may be uttered to a new one. Just like we do here, these theories do not model a context simply as a set of worlds—embodying the information established in the conversation so far—but provide a more refined notion of context, one that also embodies the issues that have been raised so far. More specifically, a context $C$ is modeled as an equivalence relation over a set of worlds $s \subseteq W$. Such an equivalence relation, which induces a partition on $s$, can be taken to encode both information and issues. On the one hand, the information established in $C$ is encoded by the set of all worlds that are in the domain of $C$, i.e., all worlds in $s$. On the other hand, the issues present in $C$ are encoded by the partition that $C$ induces: two worlds $w$ and $v$ are connected by $C$ and therefore included in the same partition cell just in case the distinction between $w$ and $v$ is not (yet) at stake in the conversation. In other words, $C$ is conceived of as a relation encoding indifference (Hulstijn, 1997): if $w$ and $v$ are connected by $C$, the discourse participants have not yet expressed an interest in information that would distinguish between $w$ and $v$.

Both declaratives and questions can then be taken to have the potential to change the context in which they are uttered. A declarative restricts the domain $s$ to those worlds in which the sentence is true (strictly speaking, it removes all pairs of worlds $\langle w, v \rangle$ from $C$ such that the sentence is false in at least one of the two worlds). Questions disconnect worlds, i.e., they remove a pair $\langle w, v \rangle$ from $C$ just in case the true exhaustive answer to the question in $w$ differs from the true exhaustive answer to the question in $v$.

Thus, the dynamic systems of Jäger (1996), Hulstijn (1997), and Groenendijk (1999) provide a notion of context and meaning that embodies both information and issues in an integrated way, in terms of an equivalence relation encoding indifference. However, as discussed in detail by Mascarenhas (2009), these proposals still have a number of shortcomings, both empirically and conceptually. Empirically, just as in classical partition semantics, it is still impossible in these dynamic frameworks, at least without further amendments, to deal in a satisfactory way with conditional questions, disjunctive questions, and mention-some wh-questions.

Conceptually, if $C$ is thought of as a relation encoding indifference, then it is not clear why it should be an equivalence relation. In particular, it is not clear why it should be transitive. The discourse participants could very well be interested in information that distinguishes $w$ from $v$, while not being interested in information that distinguishes either $w$ or $v$ from a third world $u$. To model

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14See the book Questions in dynamic semantics (Aloni et al., 2007) for several papers elaborating on these early proposals.

15Although see Isaacs and Rawlins (2008) for an analysis of conditional questions in a dynamic partition semantics that allows for hypothetical updates of the context of evaluation.
such a situation, we would need an indifference relation $C$ such that $\langle w, u \rangle \in C$ and $\langle u, v \rangle \in C$ but $\langle w, v \rangle \notin C$. This is impossible if $C$ has to be transitive.

These concerns led Groenendijk (2009) and Mascarenhas (2009) to develop a framework in which indifference relations are defined as reflexive and symmetric, but not necessarily transitive relations. The general architecture of this framework is still similar to that of Jäger (1996), Hulstijn (1997), and Groenendijk (1999), but as a consequence of dropping the transitivity constraint on indifference relations, it has a more flexible way of encoding issues, allowing for overlapping alternatives.

Groenendijk (2009) and Mascarenhas (2009) coined the term inquisitive semantics to refer to the resulting framework. Besides avoiding the conceptual problem concerning transitive indifference relations, the fact that the framework had a way of encoding issues with overlapping alternatives, unlike partition semantics, allowed it to deal, at least to some extent, with conditional questions, disjunctive questions, and mention-some wh-questions. However, Ciardelli (2008) observed that the obtained gain in expressive power was not yet sufficient. While conditional questions like (9) and open disjunctive questions with two disjuncts like (10) could be dealt with satisfactorily, disjunctive questions with three or more disjuncts remained problematic, and similarly for mention-some wh-questions. Moreover, Ciardelli (2008) argued that it is impossible to overcome these remaining problems without letting go of the framework’s most fundamental notion, i.e., that of issues encoded as indifference relations. This insight led to the current framework of inquisitive semantics, where issues are instead construed as sets of resolving information states.\footnote{16}

To distinguish the two stages in the development of inquisitive semantics, we refer to the framework proposed by Groenendijk (2009) and Mascarenhas (2009) as inquisitive indifference semantics, or simply indifference semantics.\footnote{17} In terms of expressive power, indifference semantics is situated in between partition semantics and the current inquisitive semantics framework, as was already indicated in Figure 7.1. Many important aspects of the general philosophy behind indifference semantics persist in the present framework. However, its key ingredient—the notion of issues—has been replaced by a more general one, and

\footnote{16}{A bit more historical detail: Groenendijk’s 2009 paper was written and started circulating in 2007, but officially appeared in print only in 2009. Mascarenhas’s 2009 master thesis was also largely written in 2007, but only presented in its final form in 2009 (Mascarenhas started a PhD at NYU in 2008). Ciardelli’s 2008 term paper was written in the fall of 2008, for a course taught by Groenendijk. The arguments presented in the term paper were further elaborated in Ciardelli (2009) and Ciardelli and Roelofsen (2011).}

\footnote{17}{In previous work (e.g., Ciardelli, 2009; Ciardelli and Roelofsen, 2011), the framework proposed by Groenendijk (2009) and Mascarenhas (2009) has been referred to as inquisitive pair semantics, or simply pair semantics, since issues encoded as indifference relations technically amount to sets of world-pairs. Here, we instead use the term inquisitive indifference semantics because it refers more transparently to the framework’s central concept.}
this generality is needed to suitably capture the full range of question types in natural languages. Thus, our framework naturally fits within the existing tradition of semantic theories of informative and inquisitive discourse, but it is more general and able to cover more empirical ground than its predecessors.

7.4 Exercises

Exercise 7.1. [Inquisitive semantics versus alternative semantics]
Explain in your own words what the difference is between inquisitive semantics and alternative semantics in terms of expressive power, how this difference arises, and how it pertains to the suitability of the two frameworks for the analysis of questions in natural languages.

Exercise 7.2. [Inquisitive semantics versus partition semantics]
Explain in your own words in what sense the notion of issues in inquisitive semantics is more general than the notion of question meanings in partition semantics, and why this extended generality is needed for the analysis of questions in natural languages.

Exercise 7.3. [Inquisitive semantics versus indifference semantics]
Determine the interpretation of a disjunction with three disjuncts, \( p \lor q \lor r \), in the semantics of Groenendijk (2009) and Mascarenhas (2009). How does this differ from the proposition assigned to \( p \lor q \lor r \) in \textit{InqB}? How does this difference arise? And how does it pertain to the suitability of the two frameworks for the analysis of questions in natural languages?
Chapter 8

Propositional attitudes

In the previous chapters we have seen that inquisitive semantics makes available a new notion of semantic content, which does not just embody informative content but also inquisitive content, as well as a new notion of conversational contexts, which does not only capture the information that has been established in the conversation so far but also the issues that have been raised. In this chapter we will show that the framework also gives rise to a new view on propositional attitudes, especially those that are relevant for information exchange. Namely, besides the familiar information-directed attitudes like knowing and believing it also allows us to model issue-directed attitudes like wondering.

A perspicuous and widely adopted formal treatment of information-directed attitudes is provided by epistemic logic (EL), sometimes also called the logic of knowledge and belief, which has its roots in the work of Hintikka (1962) and has been further developed by many authors in subsequent work (see, e.g., Fagin et al., 1995; van Ditmarsch et al., 2007; van Benthem, 2011). In this framework, the information state of an agent is modeled as a set of possible worlds, namely those worlds that are compatible with the information available to the agent. As we have seen, this notion of information states also plays an important role in inquisitive semantics. However, while the information-directed attitudes of an agent can be suitably captured in terms of her information state, this clearly does not hold for issue-related attitudes. In order to capture what an agent wonders about, we need a representation of the issues that she entertains, i.e., her inquisitive state.

To this end, we will define an inquisitive epistemic logic (IEL), which brings together ideas from standard EL and lnqB. For simplicity, we will restrict ourselves to a propositional language, leaving the universal and existential quantifier of lnqB out of consideration. On the other hand, the modal part of our logical language will be richer than that of basic EL: while the latter only has a modal operator $K$, which is used to talk about knowledge, the language that we will consider also has a modal operator $E$, which is used to talk about the issues that the agents entertain.
One purpose of IEL is to serve as a formal framework to describe and reason about information- and issue-directed attitudes as such. However, this is not the only purpose. Of equal importance, it also provides a basic semantic treatment of verbs in natural languages that are used to report such attitudes. In English, such verbs include know and wonder, and many other languages have verbs that fulfil precisely the same purpose. While the semantics of know and its cross-linguistic kin has been considered extensively, its treatment in IEL differs from most previous accounts in that it deals completely uniformly with cases where know takes a declarative complement and cases where it takes an interrogative complement, exemplified in (1) and (2), respectively.

(1) John knows that Bill is coming.
(2) John knows whether Bill is coming.

As for wonder, IEL does not only capture its interpretation when taking an interrogative complement, as in (3) below, but it also provides a straightforward semantic explanation of the fact that it cannot take a declarative complement, illustrated in (4).

(3) John wonders whether Bill is coming.
(4) *John wonders that Bill is coming.

We will proceed as follows. Section 8.1 provides a brief review of standard EL, Section 8.2 presents the IEL framework, and Section 8.3 sketches how the treatment of the modalities in IEL may be generalized to other modal constructions.

8.1 Epistemic logic

Epistemic logic is a particular kind of modal logic, which serves as a formal framework to describe and reason about the knowledge of a set of agents $\mathcal{A}$, both about the world (factual knowledge) and about one another’s knowledge (higher-order knowledge).

8.1.1 Logical language and models

The language of epistemic logic is a standard propositional language, based on a set of atomic sentences $S$, enriched with a modal operator $K_a$ for each agent $a \in \mathcal{A}$. A sentence of the form $K_a \varphi$ is read as ‘agent $a$ knows $\varphi$.’

Sentences in this logical language are interpreted with respect to epistemic models, which consist of a set of possible worlds $W$, together with (i) a valuation map which determines, for each world $w \in W$, which atomic sentences in the language are true in that world, and (ii) a set of epistemic maps which specify,
for every world \( w \in W \), the information state of each agent \( a \in A \) in that world, where information states are, as before, sets of possible worlds.

**Definition 8.1.** [Epistemic models] An epistemic model for a set of atomic sentences \( S \) and a set of agents \( A \) is a tuple \( M = \langle W, V, \sigma_A \rangle \) where:

- \( W \) is a set, whose elements are called *possible worlds*.
- \( V : W \rightarrow \wp(S) \) is a *valuation map* that specifies for every world \( w \in W \) which atomic sentences are true in \( w \).
- \( \sigma_A = \{ \sigma_a | a \in A \} \) is a set of *epistemic maps* from \( W \) to \( \wp(W) \), each of which assigns to any world \( w \in W \) an information state \( \sigma_a(w) \).

Note the similarity between epistemic models and the first-order information models we considered in Chapter 4. In both cases, a model consists of a set of possible worlds together with certain elements that describe the state of affairs at each possible world. What these elements are depends on the specific formal language that we consider.

The epistemic maps in an epistemic model are typically required to satisfy certain conditions, depending on the precise kind of knowledge or belief they are intended to capture. For instance, the following conditions are often imposed:

- **Factivity**: for any \( w \in W \), \( w \in \sigma_a(w) \)
- **Introspection**: for any \( w, v \in W \), if \( v \in \sigma_a(w) \), then \( \sigma_a(v) = \sigma_a(w) \)

The factivity condition requires that the information available to agents be truthful. The introspection condition requires that agents know what their own knowledge state is, so that if the information state of \( a \) in \( w \) differs from her state in \( v \), then \( a \) can tell the worlds \( w \) and \( v \) apart. Either of these conditions may be dropped or weakened to model scenarios of false information or not fully introspective agents (see, e.g., Fagin *et al.*, 1995).

The epistemic maps \( \sigma_a : W \rightarrow \wp(W) \) can be equivalently regarded as binary relations \( \sim_a \subseteq W \times W \), where for any \( w \) and \( v \): \( w \sim_a v \) iff \( v \in \sigma_a(w) \). The factivity and introspection conditions on \( \sigma_a \) then translate to the requirement that \( \sim_a \) be an equivalence relation. While the presentation of epistemic models that uses equivalence relations rather than functions is more common in the literature, the functional notation has an important advantage for our current purposes: it brings out more clearly that the maps \( \sigma_a \), together with the valuation \( V \), characterize those aspects of a possible world that are deemed relevant. This suggests that, if we wanted to characterize possible worlds in more detail, taking into account more aspects than just the information available to all the agents involved, we could add further elements to our models to describe these additional aspects. This is indeed the approach we will take in Section 8.2.
Just as we did in Chapter 4, in what follows we will assume a fixed epistemic model $M$ as our logical space and omit reference to it whenever possible.

### 8.1.2 Semantics

Within the context of an epistemic model $M$, the language of epistemic logic is interpreted by means of the following truth-conditional clauses.

**Definition 8.2.** [Semantics of standard epistemic logic]

1. $w |= p \iff p \in V(w)$
2. $w |= \neg \varphi \iff w \not|= \varphi$
3. $w |= \varphi \land \psi \iff w |= \varphi$ and $w |= \psi$
4. $w |= \varphi \lor \psi \iff w \not|= \varphi$ or $w |= \psi$
5. $w |= \varphi \rightarrow \psi \iff w \not|= \varphi$ or $w |= \psi$
6. $w |= K_a \varphi \iff \text{for all } v \in \sigma_a(w) : v |= \varphi$

The only novelty with respect to classical propositional logic is the interpretation of the modality $K_a$, which relies on the epistemic map $\sigma_a$. Notice that, if we denote by $|\varphi|$ the set of worlds $w$ at which $\varphi$ is true according to the above clauses, the truth-conditions for a modal formula $K_a \varphi$ may be written as follows:

6'. $w |= K_a \varphi \iff \sigma_a(w) \subseteq |\varphi|$

Thus, $K_a \varphi$ is true at a world $w$ in case $\varphi$ follows from the information available to $a$ in $w$. This reformulation of the clause brings out the fact that the modal statement $K_a \varphi$ makes a claim about the relation between two sets of worlds: the information state $\sigma_a(w)$ of the agent $a$ at $w$, and the proposition $|\varphi|$ expressed by the argument. This perspective will help us understand how modalities can be generalized to the inquisitive setting, where both the state of an agent and the proposition expressed by a formula are no longer simple sets of worlds, but richer objects encoding both information and issues.

### 8.1.3 Common knowledge

Besides the agents’ individual knowledge, notions of group knowledge also play an important role in the analysis of information exchange. One notion that is of particular importance is that of common knowledge, i.e., the information that is publicly shared among the group. One might think that treating this notion would require enriching our models with a map $\sigma_s$ that specifies, for each world $w$, an information state $\sigma_s(w)$ embodying the information that is publicly
available to all the agents in $w$. We could then expand our language with a corresponding common knowledge modality $K_*$, interpreted as follows:

$$w \models K_* \varphi \iff \sigma_*(w) \subseteq \varphi$$

However, common knowledge is closely tied to the agents’ individual knowledge: in fact, it is determined by it. A sentence $\varphi$ is common knowledge if and only if every agent $a$ knows that $\varphi$, and every agent $a$ knows that every agent $b$ knows that $\varphi$, and every agent $a$ knows that every agent $b$ knows that every agent $c$ knows that $\varphi$, and so on. Thus, the truth-conditions of the formula $K_* \varphi$ should be completely determined by the following condition:

$$w \models K_* \varphi \iff w \models K_{a_1} K_{a_2} \ldots K_{a_n} \varphi \text{ for any } a_1, \ldots, a_n \in A, n \geq 1$$

One can show that, in order to guarantee this equivalence for any particular valuation $V$, the common knowledge map $\sigma_*$ must be defined precisely as follows:

$$\sigma_*(w) = \{ v \mid \text{there exist } u_0, \ldots, u_{n+1} \in W \text{ and } a_0, \ldots, a_n \in A \text{ such that } u_0 = w, u_{n+1} = v, \text{ and for } i \leq n, u_{i+1} \in \sigma_{a_i}(u_i) \}$$

This means that the common knowledge map $\sigma_*$ is uniquely determined by the set of individual epistemic maps $\sigma_A$, and need not be added to our models as an additional component.

### 8.2 Inquisitive epistemic logic

We now turn to inquisitive epistemic logic, IEL. In this framework it is not only possible to model the information available to a set of agents, but also the issues that they entertain.

#### 8.2.1 Inquisitive epistemic models

While in epistemic logic a possible world $w$ was characterized by (i) a valuation for the atomic sentences in the language, and (ii) an information state for each agent, we now also need to specify (iii) an inquisitive state for each agent, encoding the issues that the agent entertains in $w$. This is where the notion of issues that we introduced in Chapter 2 comes in. Recall that issues were construed as non-empty, downward closed sets of information states, namely precisely those information states that resolve the issue. Moreover, recall that it is only possible to truthfully resolve an issue $I$ if the actual world is contained in at least one $s \in I$, i.e., if the actual world is contained in $\bigcup I$. We say that $I$ is an issue over the information state $\bigcup I$. Finally, recall that the set of all issues is denoted by $I$. 
Chapter 8. Propositional attitudes

In standard epistemic logic, every agent \( a \) is assigned an information state \( \sigma_a(w) \) in every world \( w \), determining the range of worlds that she considers possible candidates for the actual one. Now, every agent will also be assigned an inquisitive state \( \Sigma_a(w) \), which will be modeled as an issue over the information state \( \sigma_a(w) \), encoding which further enhancements of her current information state resolve the issues that she entertains.

Since \( \Sigma_a(w) \) will be modeled as an issue over \( \sigma_a(w) \), we will always have that \( \sigma_a(w) = \bigcup \Sigma_a(w) \). This means that from the inquisitive state \( \Sigma_a(w) \) of an agent \( a \) in a world \( w \), we can always derive the information state \( \sigma_a(w) \) of that agent in that world, simply by taking the union of \( \Sigma_a(w) \). Thus, in effect, \( \Sigma_a(w) \) encodes both the information available to \( a \) and the issues entertained by \( a \) in \( w \). This means that the map \( \Sigma_a \) suffices as a specification of the state of the agent at each world, encompassing both information and issues. We do not have to list \( \sigma_a \) explicitly as an independent component in the definition of an inquisitive epistemic model: we can simply derive \( \sigma_a(w) \) as \( \bigcup \Sigma_a(w) \).

**Definition 8.3. [Inquisitive epistemic models]**

An inquisitive epistemic model for a set of atomic sentences \( S \) and a set of agents \( \mathcal{A} \) is a triple \( M = (W, V, \Sigma_\mathcal{A}) \) where:

- \( W \) is a set, whose elements are called possible worlds.
- \( V : W \rightarrow \wp(S) \) is a *valuation map* that specifies for every world \( w \) which atomic sentences are true at \( w \).
- \( \Sigma_\mathcal{A} = \{ \Sigma_a \mid a \in \mathcal{A} \} \) is a set of *state maps* \( \Sigma_a : W \rightarrow \mathcal{I} \), each of which assigns to any world \( w \) an issue \( \Sigma_a(w) \).

This general characterization of inquisitive epistemic models may again be supplemented with certain constraints on the agents’ information states and inquisitive states. For instance, in analogy with the conditions considered above for standard epistemic models, we may require the following:

- **Factivity:** for any \( w \in W \), \( w \in \sigma_a(w) \)
- **Introspection:** for any \( w, v \in W \), if \( v \in \sigma_a(w) \), then \( \Sigma_a(v) = \Sigma_a(w) \)

The factivity condition is just as before, ensuring that the agents’ information states are truthful. The introspection condition now concerns both information and issues: agents must be introspective in that they must know not only what information they have, but also what issues they entertain. That is, if the state of \( a \) in world \( w \) differs from the state of \( a \) in \( v \), either in information or in issues, then \( a \) must be able to tell \( w \) and \( v \) apart. These conditions are intended here just as an illustration: the choice of the conditions to be imposed on the state...
maps $\Sigma_a$ will depend on the particular intended application of the framework, and in any case, it is orthogonal to the main novelties introduced by IEL.

Clearly, there is much similarity between inquisitive epistemic models and standard epistemic models. Both consist of a set of worlds, each equipped with (i) a valuation for atomic sentences and (ii) a state for each agent. The only difference is that while in standard EL the agents’ states describe just their information, in IEL they encompass both their information and their issues.

8.2.2 Logical language and semantics

Let us now turn to the logical language. We will enrich the language of standard EL by adding a new modality $E_a$ for each agent, which we read as ‘a entertains $\varphi$’. As we will see, this modality allows us to describe the issues that an agent is interested in, and in combination with the modality $K_a$, it allows us to define a modality $W_a$ which is a reasonable formalization of the attitude of wondering about an issue.

In IEL, every sentence will be associated with an inquisitive proposition rather than a classical proposition. In this respect, the transition from EL to IEL is just like that from classical first-order logic to InqB. This means that the semantics will be characterized not in terms of the relation of truth with respect to a world, but in terms of support with respect to an information state. The proposition expressed by a formula $\varphi$, $[\varphi]$, will be defined as the set of information states that support $\varphi$. The recursive definition of support for IEL runs as follows.

**Definition 8.4.** [Semantics of inquisitive epistemic logic]

1. $s \models p \iff \text{for all } w \in s, p \in V(w)$
2. $s \models \neg \varphi \iff \text{for all } t \subseteq s \text{ such that } t \neq \emptyset : t \not\models \varphi$
3. $s \models \varphi \land \psi \iff s \models \varphi \text{ and } s \models \psi$
4. $s \models \varphi \lor \psi \iff s \models \varphi \text{ or } s \models \psi$
5. $s \models \varphi \rightarrow \psi \iff \text{for all } t \subseteq s : t \models \varphi \implies t \models \psi$
6. $s \models K_a \varphi \iff \text{for all } w \in s : \sigma_a(w) \models \varphi$
7. $s \models E_a \varphi \iff \text{for all } w \in s, \text{ for all } t \in \Sigma_a(w) : t \models \varphi$

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1 We use 'entertain' here as a technical term, which—unlike the 'wonder' modality that it allows us to define—is not supposed to correspond precisely with its non-technical use.
As far as the non-modal fragment of the language is concerned, the recursive characterization of support runs just as in Chapter 4. That is, the support clauses for the propositional part of the language still associate the connectives with the basic algebraic operations on inquisitive propositions. The novelty introduced by IEL lies in the clauses for the modalities $K_a$ and $E_a$. To understand these clauses, it is useful to look at the truth-conditions to which they give rise. Recall from Fact 2.14 on page 25 that truth with respect to a world $w$ always amounts to support with respect to $\{w\}$. Thus, by considering what is needed for support with respect to singleton states, we obtain the following truth-conditions for the modalities.

**Fact 8.5.** [Truth-conditions for modal formulas]

1. $w \models K_a \phi \iff \sigma_a(w) \models \phi$
2. $w \models E_a \phi \iff \text{for all } t \in \Sigma_a(w) : t \models \phi$

Given these truth conditions, it becomes clear that $K_a \phi$ is supported by a state $s$ just in case it is true at any world in $s$, and analogously for $E_a \phi$:

$$s \models \Box \phi \iff w \models \Box \phi \text{ for all } w \in s \text{ where } \Box \in \{K_a, E_a\}$$

This means, given Fact 2.20 on page 28, that modal formulas are always statements.

**Fact 8.6.** For any $\phi$, $K_a \phi$ and $E_a \phi$ are statements.

Now, since the semantics of modal formulas is completely determined by their truth-conditions with respect to worlds, we can focus on understanding what it takes for modal formulas of the form $K_a \phi$ and $E_a \phi$ to be true at a world. $K_a \phi$ is true at a world $w$ just in case the information available to $a$ settles the proposition expressed by $\phi$, i.e., $\sigma_a(w) \in [\phi]$. As we will discuss in more detail below, this is a natural generalization of the interpretation of $K_a \phi$ in standard epistemic logic, which does not only deal appropriately with cases where $\phi$ is purely informative, but also with cases where $\phi$ is inquisitive.

On the other hand, $E_a \phi$ is true at a world $w$ just in case the inquisitive state of $a$ entails $[\phi]$, i.e., $\Sigma_a(w) \subseteq [\phi]$. This means that all the states in $\Sigma_a(w)$, i.e., the states that $a$ wants to get to, are ones that settle $[\phi]$. In other words, the issue expressed by $\phi$ is one that $a$ would like to see resolved. This is close to saying that $a$ wonders about the issue expressed by $\phi$, with one exception: if $a$ already has enough information to resolve the issue expressed by $\phi$, i.e., if $K_a \phi$ holds, then according to the above characterization, $E_a \phi$ holds as well. But in such a scenario, we would not say that $a$ wonders about the issue expressed by $\phi$. The situation of an agent $a$ wondering about the issue expressed by $\phi$ can
be characterized as one where the agent does not yet have sufficient information to resolve the issue (so that $\neg K_a \varphi$ holds) but the states she wants to get to are states that do contain such information (so that $E_a \varphi$ holds). So, we can define $W_a$ in terms of $K_a$ and $E_a$ as follows:

$$W_a \varphi := \neg K_a \varphi \land E_a \varphi$$

Let us illustrate the workings of our modal operators in some more detail by means of some concrete examples. First consider $K_a p$: this is true at a world $w$ in case the information state of $a$ at $w$ supports $p$. This simply means that $p$ must be true throughout $\sigma_a(w)$. Thus, when applied to an atomic sentence, $K_a$ boils down to the familiar knowledge modality of standard epistemic logic. This holds more generally if $K_a$ applies to statements, atomic or otherwise.

However, now $K_a$ can also be applied to inquisitive sentences. As an example, consider $K_a ?p$. This formula is true at a world $w$ in case the information state of $a$ supports $?p$. This means that $p$ must either be true throughout $\sigma_a(w)$ or false throughout $\sigma_a(w)$. That is, $a$ either has to know that $p$ holds or that $p$ does not hold—in other words, $a$ has to know whether $p$ holds. Thus, in IEL the $K_a$ modality is generalized in such a way that one and the same semantic clause delivers the expected reading for $K_a \varphi$ both when $\varphi$ is a statement like $p$, and when $\varphi$ is a question like $?p$.

Now let us consider $E_a p$. This formula is true in a world $w$ in case any $t \in \Sigma_a(w)$ supports $p$, that is, in case any $t \in \Sigma_a(w)$ is included in $\text{info}(p)$; clearly, this holds if and only if $\sigma_a(w) = \bigcup \Sigma_a(w) \subseteq \text{info}(p)$. But this is precisely what is required for the truth of $K_a p$. In sum, we have $w \models E_a p \iff w \models K_a p$; since all modal formulas are statements, this implies that $E_a p$ and $K_a p$ are equivalent. This example illustrates a more general fact: when $E_a$ is applied to a statement, atomic or otherwise, it simply boils down to the $K_a$ modality, and thus coincides with the familiar $K_a$ modality of standard epistemic logic.

Things become more interesting when $E_a$ applies to an inquisitive sentence. Consider for example $E_a ?p$, which is true at a world $w$ in case $\Sigma_a(w) \subseteq [?p]$. This means that any information state which settles the issues that $a$ entertains at $w$ also resolves the question $?p$. Now, a trivial way in which this may hold is if $a$’s current information state, $\sigma_a(w)$, already resolves $?p$, that is, if we have $w \models K_a ?p$. In this case, any $t \in \Sigma_a(w)$, being an enhancement of $\sigma_a(w)$, must also resolve $?p$. On the other hand, it may also be the case that the agent’s current information does not settle $?p$, but the states that the agent wants to get to are all ones that do. This holds precisely when we have that $W_a ?p$, which we defined as an abbreviation of $\neg K_a ?p \land E_a ?p$. Thus, $W_a ?p$ captures the fact that $a$ wonders whether $p$, in the sense that $a$ does not know whether $p$ is true but wants to find out.

Interestingly, the fact that $E_a$ and $K_a$ coincide when applied to statements implies that, whenever $\varphi$ is a statement, $W_a \varphi$ is contradictory. That is, applying
the wondering modality to a statement is bound to give rise to a contradiction. This may offer an explanation for the fact that, in English and many other languages, the verb wonder (as well as other inquisitive attitude verbs such as investigate and be curious) cannot take declarative complements.

Now let us briefly consider the nature of the modal operators in iEL from a more mathematical perspective. Clearly, our $K_a$ and $E_a$ are not standard Kripke modalities; that is, they cannot be regarded as quantifiers asserting the truth of their argument at some/all accessible worlds. Yet, there is a sense in which these operators work in our system precisely the way Kripke modalities work in standard modal logic.

In Section 8.1, we remarked that in EL, the modality $K_a$ can be regarded as expressing a relation between two semantic objects of the same kind: the state $\sigma_a(w)$ associated with the world of evaluation $w$, and the proposition $[\varphi]$ expressed by the argument. This relation simply amounts to inclusion:

$$w \models K_a \varphi \iff \sigma_a(w) \subseteq [\varphi]$$

All modal operators of standard modal logic can be seen as working in this way: they express a relation between two sets of worlds, a set of worlds associated with the world of evaluation, and the proposition expressed by the sentence that the operator takes as its argument.

Our modal operators $K_a$ and $E_a$ work in a very similar way: they express a relation between two semantic objects of the same kind, the state $\Sigma_a(w)$ associated with the evaluation world, and the proposition $[\varphi]$ expressed by the argument, as the following re-formulation of their truth-conditions shows.

$$w \models K_a \varphi \iff \bigcup \Sigma_a(w) \in [\varphi]$$
$$w \models E_a \varphi \iff \Sigma_a(w) \subseteq [\varphi]$$

The only difference is that, now, both the state $\Sigma_a(w)$ and the proposition $[\varphi]$ are no longer simple sets of worlds: rather, they are downward closed sets of information states, which capture both information and issues. Notice that, since such semantic objects have more structure than simple sets of worlds do, one may naturally consider several other relations, besides the two expressed by $K_a$ and $E_a$. In this way, the inquisitive perspective suggests a natural generalization of the notion of modal operators. We will briefly come back to this point in Section 8.3 and in the exercises appended to this chapter.

### 8.2.3 Common knowledge and public issues

Besides the information and issues that are private to each agent, agents also share certain public information and jointly entertain certain issues. In Section 8.1 we saw how the common knowledge construction in epistemic logic
allows us to derive a public information map $\sigma_*$ representing the information that is publicly available to all the agents, starting from the epistemic maps $\sigma_a$ encoding the information available to each individual agent. The question is whether this construction can be generalized to the present setting. That is, is it possible to derive a public state map $\Sigma_*$, encoding public information and issues, from the maps $\Sigma_a$ describing the information and issues of each individual agent?

One way to go about answering this question is to consider, as we did in the case of common knowledge, the conditions that a public entertain modality $E_*$ associated with the map $\Sigma_*$ would have to satisfy. This will put constraints on the definition of $\Sigma_*$, which may be sufficient to get at a unique characterization. So, let us consider what it would mean for a sentence to be publicly entertained. In standard epistemic logic, $\phi$ is publicly known in case every agent knows that $\phi$, and every agent knows that every agent knows that $\phi$, and so on. Analogously, it seems natural to say that $\phi$ is publicly entertained in case every agent entertains $\phi$, and every agent knows that every agent entertains $\phi$, and every agent knows that every agent knows, etcetera. Thus, the behavior of the public entertain modality $E_*$ would have to be subject to the following condition:

$$w \models E_* \phi \iff w \models K_{a_1} \ldots K_{a_{n-1}} E_{a_n} \phi \text{ for all } a_1 \ldots a_n \in A, \ n \geq 1$$

If one finds the alternation of the modalities puzzling, there is no need to worry: since $K_a$ and $E_a$ are equivalent when applied to statements, and since any sentence that starts with a modality is a statement, we can simply replace all the $K_a$’s with $E_a$ and obtain the equivalent ‘homogeneous’ condition:

$$w \models E_* \phi \iff w \models E_{a_1} \ldots E_{a_{n-1}} E_{a_n} \phi \text{ for all } a_1 \ldots a_n \in A, \ n \geq 1$$

Does this condition constrain the map $\Sigma_*$ sufficiently to characterize it uniquely? The answer is yes. One can verify that the above condition on $E_*$ holds for any particular valuation $V$ if and only if the map $\Sigma_*$ is defined as follows:

$$\Sigma_*(w) = \{ s \mid \text{there exist } v_0, \ldots, v_n \in W \text{ and } a_0, \ldots, a_n \in A$$

such that $v_0 = w, \ v_{i+1} \in \sigma_{a_i}(v_i) \text{ for all } i < n, \text{ and } s \in \Sigma_{a_n}(v_n) \}$$

Importantly, the public information map $\sigma_*$ corresponding to the public state map $\Sigma_*$, defined as $\sigma_*(w) := \bigcup \Sigma_*(w)$, coincides exactly with the map we would obtain by performing the common knowledge construction on the individual information maps $\sigma_a$. Thus, the standard common knowledge construction from epistemic logic generalizes smoothly to a ‘public state’ construction which encompasses both information and issues.

Given this construction, we can add modalities $K_*$ and $E_*$ to our logical language, to be interpreted as follows:

$$s \models K_* \phi \iff \text{for all } w \in s: \sigma_*(w) \models \phi$$

$$s \models E_* \phi \iff \text{for all } w \in s, \text{ for all } t \in \Sigma_*(w) : t \models \phi$$
If \( \phi \) is a statement, then \( K_* \phi \) gets its standard meaning, expressing that \( \phi \) is common knowledge. On the other hand, in our setting \( K_* \) also applies to inquisitive sentences: if \( \phi \) is inquisitive, then \( K_* \phi \) says that the group’s common knowledge settles the issue expressed by \( \phi \)—in short, that \( \phi \) is publicly settled. For instance, the formula \( K_* ?p \) captures the fact that it is common knowledge among the group whether \( p \).

Moreover, by combining the two public modalities we can define a public version of the wonder modality, \( W_* \): a group of agents jointly wonder about \( \phi \) if they publicly entertain \( \phi \) and \( \phi \) is not yet publicly settled.

\[
W_* \phi := \neg K_* \phi \land E_* \phi
\]

Just like \( K_* \), the modality \( W_* \) is very useful in formally describing an information exchange: while \( K_* \) lets us describe which issues the conversational participants have publicly settled, \( W_* \) lets us describe what the open issues are in the exchange, i.e., what the issues are that the group as a whole would like to see resolved and for which no resolution has been publicly established yet.

An interesting feature of the public wondering operator is that \( W_* \phi \) does not entail \( W_a \phi \) for a particular agent \( a \). While this may come as a surprise at first, it is just as it should be: if \( W_* \phi \) holds, then \( \phi \) is publicly entertained but not publicly settled, that is, the common knowledge of the group does not settle \( \phi \). It may well be that there is some agent whose private knowledge does settle \( \phi \). This does not prevent \( \phi \) from being an open issue for the group, so long as this private information is not made publicly available. In fact, \( W_* \phi \) might even be the case while every individual agent can resolve \( \phi \), but the information needed to resolve \( \phi \) has not been made common knowledge: although the issue is settled for each individual agent in this case, it is still open for the group as a whole.

This concludes our brief presentation of IEL. This basic framework is presented in more detail in Ciardelli and Roelofsen (2015), and the logic that it gives rise to is further investigated in Ciardelli (2014, 2016d). Various extensions and refinements of IEL have been explored in the recent literature as well. In Ciardelli and Roelofsen (2015) and van Gessel (2016) the framework is extended with a dynamic modal operator which makes it possible to describe how the agents’ private and public information and issues change when a statement is made or a question is asked, generalizing the analysis of public and private announcements in dynamic epistemic logic (van Ditmarsch et al., 2007). In Ciardelli and Roelofsen (2014) a refinement of IEL is developed that does not only deal with ‘hard knowledge’ but also with beliefs which may be revised or retracted. This inquisitive believe revision framework can not just be used to model linguistic information exchange, but also other information-related processes such as rational inquiry, where the interplay between issues and beliefs has been argued to play a crucial role (see, e.g., Olsson and Westlund, 2006). And finally, in Theiler et al. (2016a,b) the linguistic treatment of \textit{know} suggested
8.3. *Beyond know and wonder*  

in IEL is refined in order to obtain, among other things, a more sophisticated account of the *factive* nature of the verb. For instance, *John knows that it is raining* implies that it is in fact raining, and *John knows where Bill lives* implies that John knows where Bill in fact lives, but neither of these sentences requires that John’s information state is completely factive, i.e., that he has no false beliefs at all.

8.3 Beyond *know and wonder*  

We have focused our attention in this chapter on a small set of modal operators, in a particular logical setting. However, the approach that we have taken may well be applicable beyond this restricted setting as well, giving rise to a richer view on the linguistic notion of modality in general. We end this chapter with some programmatic remarks on the potential benefits of such an enriched perspective.

In linguistics, modal expressions are standardly characterized as sentential operators that relate the proposition expressed by their argument (their *prejacent*) to a proposition encoding a set of relevant background assumptions (the *modal base*). Some modal expressions indicate that the prejacent is *consistent* with the modal base (possibility modals), while others indicate that the prejacent is *entailed* by the modal base (necessity modals). The nature of the modal base depends on the particular flavor of the modal expression. For instance, epistemic modals relate their prejacent to a relevant body of information, while deontic modals relate their prejacent to a modal base determined by a relevant set of rules. Finally, modal expressions differ in their grammatical category. Among the most widely investigated kinds of modal expressions are propositional attitude verbs like *know*, *believe*, *want*, and *hope*, and auxiliary verbs like *might*, *may*, *must*, and *should*.

Sophisticated theories have been developed to capture the core mechanisms that underlie the linguistic behavior of all these different types of modal expressions in a unified way (see in particular Kratzer 2012 for a collection of influential articles, and Kaufmann and Kaufmann 2015 for a recent survey). However, while the domain that is covered by these theories is indeed impressively broad, the approach taken in inquisitive epistemic logic suggests a substantial further generalization, both of the linguistic notion of modal expressions as such, and of the theories that deal with them.

Namely, rather than construing modal expressions as relating two classical propositions, we may construe them as relating two inquisitive propositions, just as we did with the $K_a$ and $W_a$ modalities in IEL. This would broaden our linguistic view on modality in three ways. First, as exemplified in a very concrete way in IEL, the class of modal expressions would become richer, now also
including operators that take inquisitive constructions as their argument. Thus, it would become possible to pursue a unified account of propositional attitude verbs like know, believe, want, and hope on the one hand, and issue-directed attitude verbs like wonder, be curious, and investigate on the other. Second, a more fine-grained notion of modal bases would become available: we could interpret modal expressions not only in the context of a certain body of information, but also in the context of a relevant background issue. And third, while on the standard account there are only two salient relations between the prejacent and the modal base, i.e., inclusion (entailment) and overlap (consistency), inquisitive propositions have much more structure than classical propositions, and can therefore be related in many more ways. This would allow for a refinement of the basic dichotomy between possibility and necessity modals.

While these remarks are admittedly very programmatic and clearly stand in need of concrete substantiation, the research programme that they suggest seems an exciting one to pursue. The treatment of know and wonder developed in $\text{IEL}$ just constitutes the first step in this direction.

8.4 Exercises

**Exercise 8.1.** [Ignorance]
Consider a new modal operator $N_a$ in $\text{IEL}$, where $N_a \varphi$ is informally read as ‘$a$ is completely ignorant with regard to $\varphi$’.

1. Define a suitable semantic interpretation of $N_a \varphi$.
2. Check whether $N_a \varphi$ is equivalent to $\neg K_a \varphi$ whenever $\varphi$ is a statement.

**Exercise 8.2.** [Agnosticism]
Consider a new modal operator $G_a$ in $\text{IEL}$, where $G_a \varphi$ is informally read as ‘$a$ is completely agnostic with regard to $\varphi$’.

1. Define a suitable semantic interpretation of $G_a \varphi$.
2. Check whether $G_a \? \varphi$ is equivalent to $\neg E_a \? \varphi$ whenever $\varphi$ is a statement.
We will end with a summary of the framework we presented, and will briefly consider to what extent the high-level desiderata discussed in Chapter 1 have been met.

9.1 Summary

Let us start by reviewing the main concepts that play a role in InqB, the basic first-order inquisitive semantics presented in Chapter 4. The diagram in Figure 9.1 provides an overview of these concepts and the dependencies between them. Our starting point was a particular language $L$, in this case the language of first-order logic (the upper leftmost item in the diagram). Given this language, we defined the models relative to which the sentences in our language would be interpreted. A model was construed as a set of possible worlds $W$, associated with a domain of discourse and an interpretation function determining the denotation of the basic elements of our language (function symbols and relation symbols) in each possible world. Thus, a model determines a certain logical space, the set of worlds $W$, as well as a particular connection between the worlds in this space and the basic elements of the language under consideration.

We adopted the standard notion of information states as sets of possible worlds, i.e., subsets of $W$. In terms of information states, we defined a new notion of issues, and based on this notion of issues we introduced a notion of propositions encompassing both informative and inquisitive content. We defined a notion of entailment between propositions, and characterized two kinds of semantic operators on propositions: (i) algebraic operators, which for instance yield the meet or the join of two propositions w.r.t. entailment, and (ii) projection operators, which trivialize either the informative or the inquisitive content of a given proposition. Finally, based on these semantic operators, we defined a semantics for the language $L$ that we started out with, coming full circle.

Having laid out this schematic overview of InqB, we would like to emphasize
that all the notions which play a crucial role in this system, except for the logical language and the models with respect to which the sentences in the language are interpreted, were already characterized in Chapters 2-3, \textit{without reference to any particular logical or natural language}. This makes these notions highly general and widely applicable.

As we saw in Chapter 4, what becomes necessary when turning to a particular language is a more specific characterization of the assumed logical space. In Chapters 2-3, we just assumed a generic set of possible worlds $W$ as our logical space, without any further specification. The moment we fix a particular logical language, we have to establish a connection between the worlds in our logical space and the basic elements of our language. Thus, in Chapter 4, we supplemented the set of possible worlds $W$ with a domain of individuals $D$ and a function $I$ determining the denotation of the basic elements of our language (in this case, function symbols and relation symbols) w.r.t. each world $w \in W$. Having fixed this connection between ‘worlds and words’, all the general notions introduced in Chapters 2-3 could be imported straightforwardly.

In Chapter 8 we considered another logical language, namely a propositional language with modal operators to describe the knowledge and issues of a given set of agents. Accordingly, we re-construed our logical space as a set of possible worlds $W$ together with (i) a valuation function, determining the truth value of the atomic sentences in our language at every world $w \in W$, and (ii) a set of \textit{state maps} $\Sigma_A$, determining the information states and inquisitive states of all the agents at every $w \in W$. Having thus established a suitable connection between the worlds in our logical space and the basic elements of our logical language (in this case, atomic sentences and the modal operators), all the general notions

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**Figure 9.1:** Dependency diagram of the main concepts in InqB.
laid out in Chapters 2-3 could once again be imported straightforwardly.

The fact that the framework is built up in this modular way makes it very flexible. There are many ways in which the basic notions introduced here may be further refined, extended, and applied (see Appendix B for some references).

9.2 Mission accomplished?

Let us now return to the high-level desiderata discussed in Chapter 1, and assess to what extent the framework we presented addresses these desiderata.

The first high-level desideratum was a formal notion of issues that allows for a suitable representation of semantic content, conversational contexts, and propositional attitudes. In Chapter 2 we introduced such a notion of issues, and in terms of it we defined new notions of semantic content (propositions) and conversational contexts. In Chapter 7 we argued that the new notion of semantic content is particularly suitable for the analysis of questions, overcoming the main shortcomings of previous frameworks for question semantics (alternative semantics, partition semantics, and indifference semantics). Finally, in Chapter 8 we showed that the new notion of issues facilitates a richer view on propositional attitudes as well, encompassing both information-directed attitudes like know and believe, and issue-directed attitudes like wonder.

The second high-level desideratum was a framework that allows for an integrated treatment of declarative and interrogative sentences, with a single notion of semantic content which is general enough to deal with both sentence types at once, rather than a separate notion of content for each sentence type. One argument we made to justify this desideratum was that declarative and interrogative sentences are to a large extent built up from the same lexical, morphological, and intonational elements. A general characterization of the semantic contribution of each of these elements should capture both their contribution to the informative content and to the inquisitive content of the sentence that they are part of. This requires a framework in which the semantic content of a sentence—the proposition it expresses—encompasses both informative and inquisitive content.

The notion of propositions introduced in Chapter 2 satisfies this requirement, and the merits of this feature of the framework were illustrated in Chapter 5 with an analysis of declarative and interrogative sentences involving disjunction and various intonation patterns. Both disjunction and the relevant intonational elements were given a uniform treatment across the two sentence types. Another important result of the approach was discussed in Chapter 6: while originally intended to broaden the domain of logical semantics from declaratives to interrogatives, we have seen that it also leads to an improved analysis of declaratives as such. We illustrated this point in the domain of conditionals, whose truth-conditions may be affected by the inquisitive content of their antecedent.
Thus, both desiderata have been met and the ensuing benefits have been concretely substantiated. From a narrow perspective, then, our goals have been achieved. From a broader perspective, however, these results just indicate that our general mission is worthwhile pursuing. We do not see the basic framework presented here as a final product but much rather as a point of departure.
Appendix A

Sources

This book brings together a number of ideas and results from previous publications, manuscripts, and teaching materials. Below we list the main sources for each chapter, which in many cases contain more in-depth discussion of the ideas presented here.

- Chapter 1: Roelofsen (2014)
- Chapter 2: Ciardelli, Groenendijk, and Roelofsen (2013a)
- Chapter 3: Roelofsen (2013a, 2015b)
- Chapter 4: Ciardelli (2009); Groenendijk and Roelofsen (2009); Ciardelli and Roelofsen (2011); Roelofsen (2013a)
- Chapter 5: Roelofsen (2013c, 2015a); Roelofsen and Farkas (2015)
- Chapter 6: Ciardelli, Zhang, and Champollion (2016b); Ciardelli (2016b)
- Chapter 7: Ciardelli, Groenendijk, and Roelofsen (2013a, §6) and Ciardelli and Roelofsen (2016)
- Chapter 8: Ciardelli and Roelofsen (2015)
Appendix B

Pointers for further reading

Despite its relatively recent inception, there has already been a lot of work on inquisitive semantics, much more than we have been able to cover in this book. The basic framework presented here has been further extended, refined, and applied in several ways, the logical properties of the framework have been investigated, and some interesting connections with other logical frameworks have emerged, though in all these areas there are still many open issues to be addressed. Below we provide some pointers for further reading.

Extensions of InqB and IEL.

- A type-theoretical extension of InqB, for full compositionality:
  Ciardelli, Roelofsen, and Theiler (2016a)

- A presuppositional extension of InqB:
  Ciardelli, Groenendijk, and Roelofsen (2012, 2015); Roelofsen (2015a)

- An extension of InqB with propositional discourse referents:
  Roelofsen and Farkas (2015)

- Integration of InqB with a commitment-based discourse model:
  Farkas and Roelofsen (2016)

- An extension of IEL with dynamic operators that model the effects of statements and questions that are publicly observable by all conversational participants: Ciardelli and Roelofsen (2015)

- An extension of IEL with dynamic operators that model the effects of statements and questions that may only be partially observable by some conversational participants: van Gessel (2016)

- An extension of IEL with graded beliefs next to hard knowledge:
  Ciardelli and Roelofsen (2014)
Refinements of InqB.

- A refinement of InqB with a weak negation operator, whose treatment requires the existence of propositions that are not downward closed: Punčochář (2015b)

- A refinement of InqB that does not model information states as sets of possible worlds but as primitive objects in an algebra that is more generic than the Boolean algebra of information states as sets of worlds: Punčochář (2015a)

- A refinement of InqB that does not characterize a proposition just in terms of the states that support it, but also in terms of the states that reject it or ‘dismiss a supposition’ of it, referred to as InqS: Groenendijk and Roelofsen (2015)

- An extension of InqS with operators corresponding to epistemic and deontic modal auxiliaries (might, may, must): Aher and Groenendijk (2015)

- A refinement of InqB that is not only concerned with informative and inquisitive content, but also ‘attentive content’, whose treatment again requires propositions that are not downward closed: Ciardelli, Groenendijk, and Roelofsen (2014)

Logical investigations.

- Logical investigation of InqB: Ciardelli (2009); Ciardelli and Roelofsen (2011); Ciardelli (2016d)

- Logical investigation of IEL: Ciardelli (2014, 2016d)

- Logical investigation of various refinements of InqB, listed above: Punčochář (2015a,b)


- On the general role of questions in logic: Ciardelli (2016c,d)

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1There is also work which argues that it is in fact impossible to capture all three types of content—informative, inquisitive, and attentive—at once using a single semantic object, and pursues a two-dimensional approach instead (Roelofsen, 2013b).
Appendix B. Pointers for further reading

Applications in linguistics.

- Root questions:
  AnderBois (2011, 2012); Champollion et al. (2015); Roelofsen and Farkas (2015); Roelofsen (2015a); Farkas and Roelofsen (2016)

- Embedded questions:
  Theiler (2014); Theiler et al. (2016a,b); Herbstritt (2014); Roelofsen et al. (2016); Roelofsen and Uegaki (2016)

- Implicit questions in discourse: Onea (2013)

- Answer particles (yes/no): Roelofsen and Farkas (2015)

- Disjunction: Winans (2012); Roelofsen (2015a,b); Ciardelli and Roelofsen (2016)

- Modal auxiliaries: Aher (2013); Aher and Groenendijk (2015)

- Conditionals:
  Onea and Steinbach (2012); Starr (2014); Groenendijk and Roelofsen (2015); Willer (2015); Champollion, Ciardelli, and Zhang (2016); Ciardelli (2016b)

- Quantifier particles: Szabolcsi (2015b)

- Ellipsis: AnderBois (2014, 2016a)

- Exhaustivity implicatures: Westera (2012, 2013a,b)

- Imperatives: Aloni and Ciardelli (2013)

- Scalar modifiers: Coppock and Brochhagen (2013a,b); Coppock et al. (2016)

- Directional numeral modifiers: Blok (2015)

- Attentive might: Roelofsen (2013b); Ciardelli et al. (2014); Willer (2015)

Applications in cognitive science.

- Reasoning fallacies: Koralus and Mascarenhas (2014); Mascarenhas (2014)

Applications in epistemology.

- The Gettier puzzle: Uegaki (2012)
- Conversational inquiry: Hamami (2014)
- Belief revision: Ciardelli and Roelofsen (2014)

Related frameworks.

- Dependence logic: Väänänen (2007)
  Discussion of connections with inquisitive semantics: Yang (2014); Ciardelli (2016a)
- Truth-maker semantics: Fine (2014); Yablo (2014)
  Discussion of connections with inquisitive semantics: Ciardelli (2013)
  Discussion of connections with inquisitive semantics: Ciardelli (2016d)
- Dynamic epistemic logic with questions: Minică (2011); van Benthem and Minică (2012)
  Discussion of connections with inquisitive semantics: Ciardelli and Roelofsen (2015); Ciardelli (2016d); van Gessel (2016)
- Knowing value logic: Wang and Fan (2013, 2014); Fan et al. (2015)
  Discussion of connections with inquisitive semantics: Ciardelli (2016d)
- Mental models theory: Johnson-Laird (1983); Mascarenhas (2014); Koralus and Mascarenhas (2014)
- Inferential erotetic logic: Wiśniewski (1995)


Herbsttritt, M. (2014). Why can’t we be surprised *whether* it rains in Amsterdam? A semantics for factive verbs and embedded questions. MSc thesis, University of Amsterdam, supervised by Maria Aloni and Floris Roelofsen.


Roelofsen, F. (2014). Motivation for an inquisitive notion of meaning. Handout for a course on inquisitive semantics at the ILLC, University of Amsterdam.


