

# Modals and Conditionals Again

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This is the first chapter of a book manuscript. The book will bring together most of my previous work on the topic of modals and conditionals. The original work has been edited for style, cut or merged to avoid overlap, translated if necessary, and supplemented with additional material for purposes of clarification. In addition, there are a few new chapters connecting the older work to the current state of the art. I will post drafts of the chapters as they become available. Any comments are highly appreciated.

Chapter 1 is a reedited version of “What *must* and *can* must and can mean”.

## Chapter 1

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### What *must* and *can* must and can mean\*

#### 1. *Must* and *can* are relational

Words, phrases, and sentences acquire content when we utter them on particular occasions. What that content is may differ from one context to the next. It is the task of semantics to describe all those features of the meaning of a linguistic expression that stay invariable in whatever context the expression may be used. This invariable element is the meaning proper of an expression. All of this is a simplification, of course, that abstracts away from many complications. Here is one: Nobody would claim that a semantic analysis of the words *must* and *can* should try to capture whatever is common to the meanings of the two respective occurrences of these words in (1):

- (1) You must and you can store must in a can.

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\* The original version of this chapter appeared in 1977 in *Linguistics and Philosophy* 1, 337-355. Copyright © 1977 by D. Reidel Publishing Company, Dordrecht, Holland. The article is reproduced here with many stylistic revisions, clarifications, and occasional glimpses into the more recent literature while leaving the original story line intact. I thank John Bigelow, Max Cresswell, Urs Egli, Irene Heim, David Lewis, and Arnim von Stechow for comments on the original version, and Barry Schein and many generations of students for explicit and implicit hints about how to write a better one.

The two occurrences of *must* in (1) are usually not taken to be occurrences of the same word, but are considered accidental homonyms. The *must* you can store in a can has nothing to do with necessity, and the can you can store your *must* in has nothing to do with possibility.

The word *must* in English has at least two different meanings, then, and the word *can* does, too. So far, we have seen that there is a noun *must* and a modal *must*, and a noun *can* and a modal *can*. I think everyone will accept this. But many scholars have claimed that even if we take just the modals *must* and *can*, they are ambiguous too; there are really many modals *must* and many modals *can*. To justify such claims, sentences like the following four might be offered:

- (2) All Maori children *must* learn the names of their ancestors.
- (3) The ancestors of the Maoris *must* have arrived from Tahiti.
- (4) If you *must* sneeze, at least use your handkerchief.
- (5) When Kahukura-nui died, the people of Kahungunu said: Rakaipaka *must* be our chief.

The *must* in sentence (2) is a deontic *must*: it invokes a duty. The *must* in sentence (3) is an epistemic *must*: it relates to a piece of knowledge or evidence. The kind of *must* in sentence (4) has been called a “dispositional”<sup>1</sup> *must*: it helps us talk about dispositions people have, when they can’t help sneezing or must die, for example. The *must* in (5) is sometimes called a “preferential” or “bouletic” *must*: it relates to preferences or wishes. Maybe the classification

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<sup>1</sup>. See Grabski (1974), for example.

should be refined. Maybe we should consider yet other kinds of *must*. How many? Look at the following four fragments of conversation:

You: The Maori children must learn the names of their ancestors.

I: Do they really? Is there a law in New Zealand which provides that the Maori children learn the names of their ancestors?

You: No, of course there is no such law in New Zealand. At least no official law. But the Maoris have their tribal laws, and it was these laws I had in mind when I said that all Maori children must learn the names of their ancestors.

You: The ancestors of the Maoris must have arrived from Tahiti.

I: No, they could have arrived from somewhere else. We know that their technical means permitted them much longer trips. They could have even arrived from Peru.

You: But we know that they did not arrive from Peru. We know it from their tribal history. We know it from Polynesian mythology. We simply know it, They must have arrived from Tahiti.

You: I must sneeze.

I: Don't be silly. You must not. Everyone knows how to prevent sneezing. You feel that something fuzzy is going on in your nose. You feel it a good time in advance. And you can suppress it. That's all.

You: But once I have missed the right moment, I cannot help sneezing any more. It just comes out. It is too late to suppress it. I simply must sneeze.

You: Rakaipaka must be our chief.

I: No, he must not. The Queen does not like him particularly. She does not dislike him particularly, either. He could be our chief, but there are others who could be just as well.

You: I do not care whether the Queen likes Rakaipaka. I only care about our tribe. I only consider what is good for our tribe. That is why Rakaipaka must be our chief.

How many kinds of *must* do we have to distinguish? How many deontic ones? How many epistemic ones? How many dispositional ones? And how many preferential ones?

Obviously many in each group. We do not just refer to duties. We refer to duties of different kinds, to different duties different persons have towards different persons at different times.

We do not simply refer to a piece of knowledge or information - once and for ever the same.

We refer to different kinds of knowledge or information in different situations.

We do not simply consider dispositions. Dispositions change. My dispositions now are not the same as my dispositions two minutes ago. We do not always refer to the same wishes or preferences when we use a bouletic *must*. Sometimes it is the wish of the Queen, sometimes it is the wish of our tribe, sometimes we consider even our own wish. This leaves us with many different *musts* and *cans*. What produces this variety?

If we look at the four different occurrences of the word *must* in sentences (2) to (5), we see that there is something in their meaning that stays invariable. There is a connection between those four occurrences that is much stronger than the connection between any of those occurrences and the word *must* that stands for the must we can store in a can. The connection between the occurrences of *must* in (2) to (5) can be brought out

more clearly when we try to paraphrase what might be conveyed by possible utterances of those sentences. Consider the paraphrases (2') to (5'), for example:

- (2') In view of what their tribal duties are, The Maori children must learn the names of their ancestors.
- (3') In view of what is known, the ancestors of the Maoris must have arrived from Tahiti.
- (4') If - in view of what your dispositions are - you must sneeze, at least use your handkerchief.
- (5') When Kahukura-nui died, the people of Kahungunu said: in view of what is good for us, Rakaipaka must be our chief.

What happened to the four occurrences of *must* in those paraphrases? In each case a substantial part of the meaning the modal had in the original sentence has been transferred to an *in view of*- phrase. The four occurrences of *must* in (2') to (5') now all have the same meaning. That meaning seems to be the common core we perceive in each occurrence of *must* in (2) to (5). It is that common core that stays the same whenever *must* is used. It is therefore that core that a semantic analysis of *must* should capture. On such an account, there is only one modal *must*. If we insisted on keeping the many different *musts* that are traditionally distinguished we would be forced to accept yet another *must*: the neutral *must* of (2') to (5'). (2') to (5') are English sentences, too, and any adequate account of *must* must therefore recognize a neutral *must*.

Let us now take a closer look at the semantic core of modals like *must*. That core seems to be inherently relational. What has emerged in (2') to (5') is not an absolute *must* but

a relative *must in view of* that has two arguments: a free relative like *what is known* or *what is good for us* etc. and a sentence. Figure 1 is a rough representation of the three crucial components that enter into the composition of the meaning of (3').

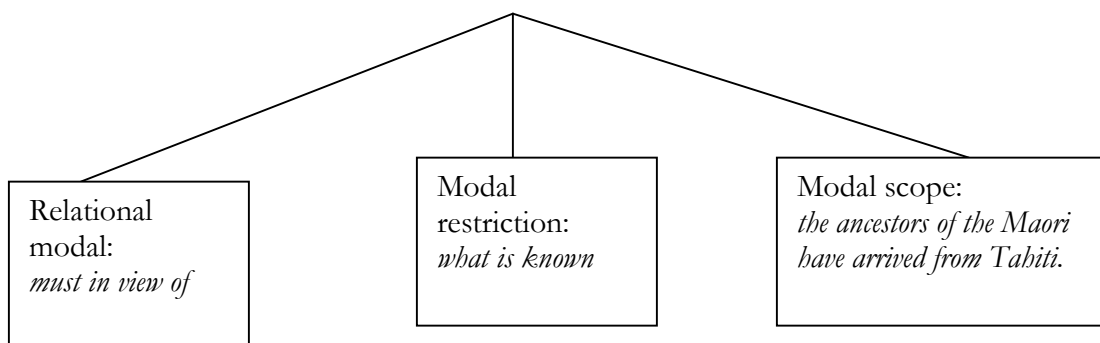


Figure 1

The neutral *must* in (2') to (5') requires two arguments, a modal restriction and a modal scope. The modal restriction can be provided by a free relative clause like *what is known*. The modal scope can come from a sentence like *the ancestors of the Maoris have arrived from Tabiti*. If the neutral *must* in (2') to (5') requires two arguments of a certain kind, the common semantic core of the four occurrences of *must* in (2) to (5) should require two arguments of the very same kind. Sentences (2) to (5), only deliver one such argument explicitly, however. Only the modal scope is overtly represented. The modal restriction is missing and whatever entity it could have contributed to semantic composition seems to have been provided by the context of utterance. The impression that the occurrences of *must* in (2) to (5) were deontic, epistemic, dispositional, and bouletic respectively seems to have been due to the fact that when I uttered those sentences, a contextually provided modal restriction merged with the common semantic core whose presence we feel in all occurrences of *must*. In other

words, a particular contextually provided modal restriction combined with the meaning proper of the modal *must*. It was a fusion of meanings that created the impression that different kinds of *must* were present.

The discussion so far led to the following conclusion: relative modal phrases like *must in view of* and *can in view of* should be considered as representing the semantic core of the modals *must* and *can* respectively. Modals are inherently relational. To be semantically complete, a modal requires two arguments, a restriction and a scope. The restriction may be represented overtly or may be provided by the context of utterance.

The insight that the core of modality is always relative modality is not new<sup>2</sup>. We find the following thoughts in Peirce's Collected Papers, for example:

“... first let me say that I use the word *information* to mean a state of knowledge, which may range from total ignorance of everything except the meanings of words up to omniscience; and by *informational* I mean relative to such a state of knowledge. Thus by ‘informationally possible’, I mean possible so far as we, or the person considered know. Then the *informationally possible* is that which in a given information is not perfectly known not to be true. The *informationally necessary* is that which is perfectly known to be true ...”<sup>3</sup>

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<sup>2</sup>. Kratzer (1978, part 2, chapter 4) has a detailed discussion of predecessors.

<sup>3</sup>. Peirce (1883; quoted from Peirce 1933), 42, 43.

“The information considered may be our actual information. In that case, we may speak of what is possible, necessary or contingent, *for the present*. Or it may be some hypothetical state of knowledge. Imagining ourselves to be thoroughly acquainted with all the laws of nature and their consequences, but to be ignorant of all particular facts, what we should then not know not to be true is said to be *physically possible*; and the phrase *physically necessary* has an analogous meaning. If we imagine ourselves to know what the resources of men are, but not what their dispositions and desires are, what we do not know will not be done is said to be *practically possible*; and the phrase practically necessary bears an analogous signification. Thus the possible varies its meaning continually.”<sup>4</sup>

To limit the scope of this chapter, I will in what follows only consider examples where the modal restriction is overtly represented. In real life, this is very seldom the case, however, even though being aware of a missing modal restriction might help us avoid or settle misunderstandings. Consider, for example, the following case. Many years ago, I attended a lecture in ethics given by a man called “Professor Schielrecht”. Professor Schielrecht is a third-generation offspring of the Vienna Circle, so his main concern in philosophy is to show that most of what most people say most of the time does not make sense. Suppose a judge asks himself whether a murderer could have acted otherwise than he eventually did. Professor Schielrecht claimed that the judge asks himself a question that does not make sense. Why not? Professor Schielrecht’s answer was: Given the whole situation of the crime, which includes of course all the dispositions of the murderer, this man could not

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<sup>4</sup>. Peirce (1883; quoted from Peirce 1933), 43.

have acted otherwise than he did. If he could have acted otherwise than he eventually did, he would have. So the answer to the question is trivial; there is no need to spend a single second on the problem. There is really no problem. But there IS a problem. The answer to the question of the judge is not trivial. The judge asked himself: Could this murderer have acted otherwise than he eventually did? Professor Schielrecht claimed that the judge asked himself whether - given the whole situation of the crime - the murderer could have acted otherwise than he eventually did. The judge did not make explicit the modal restriction for the modal *could* he used. Professor Schielrecht provided the restriction *given the whole situation*, but that restriction trivialized what the judge said. Rather than ridiculing the judge in this way, Professor Schielrecht should have asked him: In view of WHAT could the murderer have acted otherwise than he did? Maybe the judge would not have been able to answer the question. Maybe what he meant was genuinely underdetermined. This made it possible for Schielrecht to fill in an obviously unintended interpretation and thereby submit the judge to ridicule.

Since I will be explicit about modal restrictions in the remainder of this chapter, I will largely abstract away from context-dependency. The context-dependency of modal expressions and the resulting indeterminacy is central to Kratzer (1978), though, and was the driving force behind my work on conditionals. It will be taken up in the following chapter, which can be seen as an extended postscript to the original (1977) article reproduced here.

## 2. *Must* and *can* in a premise semantics

This section presents a first analysis of relational modals like *must* and *can* within what Lewis (1981) called a “premise semantics”<sup>5</sup>. The meaning of *must* is related to logical consequence: a proposition is necessary with respect to a premise set if it follows from it. The meaning of *can* is related to logical compatibility: a proposition is possible with respect to a premise set if it is compatible with it. All analyses in this book are cast within a possible worlds framework where possible worlds are assumed to be particulars, as advocated in Lewis (1986), rather than maximal consistent sets of sentences, for example. But the guiding ideas of a premise semantics for modals can be implemented in any framework that provides suitable notions of logical consequence and compatibility. In fact, one of the main virtues of a premise semantics for modality is that it links the semantics of modals to general principles of rational inquiry that apply whenever we reason from a set of premises.

In the possible worlds semantics assumed here, propositions are identified with sets of possible worlds. If  $W$  is the set of possible worlds, the set of propositions is  $P(W)$ , the power set of  $W$ . The basic logical notions can now be defined as follows:

DEFINITION 1. A proposition  $p$  is true in a world  $w$  in  $W$  iff  $w \in p$ .

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<sup>5</sup>. The term “premise semantics” is used in Lewis (1981) to refer to the semantics for conditionals presented in Kratzer (1979) and (1981). Since Kratzer (1979) is just an extension of Kratzer (1977) to conditionals, the term “premise semantics” is applicable to Kratzer (1977) as well. My approach to relative modality via premise sets took its direct inspiration from Rescher (1973) (see also Rescher (1964)). Veltman (1976) independently developed a formally parallel premise semantics for the special case where premise sets are taken to represent beliefs. My own work on modality has always been semantic in nature and has aimed for an account of the full range of modalities expressed in natural languages.

DEFINITION 2. If  $A$  is a set of propositions and  $p$  is a proposition, then  $p$  follows from  $A$  iff  $\bigcap A \subseteq p$ , that is, iff there is no possible world where all members of  $A$  are true but  $p$  is not.

DEFINITION 3. A set of propositions is consistent iff  $\bigcap A \neq \emptyset$ , that is, iff there is a world where all members of  $A$  are true.

DEFINITION 4. A proposition  $p$  is compatible with a set of propositions  $A$  iff  $A \cup \{p\}$  is consistent.

With those set-theoretic tools in hand, we can go back to figure 1 in the previous section, and think about the three crucial pieces that enter into the computation of the meaning of a modalized sentence like (3') above, repeated here as (6).

(6) In view of what is known, the ancestors of the Maoris must have arrived from Tahiti.

Abstracting away from context dependency and shamelessly neglecting all matters of tense, the meaning of the modal scope of (6) is the proposition  $p$  that is true in exactly those possible worlds where the ancestors of the Maoris have arrived from Tahiti. What is the meaning of the free relative *what is known*, then? What is known may change from one world to the next. If Lord Rutherford had not existed, we would not know many things we do in fact know. If Darwin had never traveled with Captain Fitzroy, our close connection to the great apes might not yet be known. We can imagine worlds where people know more than

we do. There are possible worlds where it is known who made the statues on Easter Island, for example. We can conclude, then, that the meaning of a free relative like *what is known* is an individual concept, that is, a function that assigns to every possible world whatever it is that is known in that world. What is it that is known in a world? In our world it is known, for example, that Lord Rutherford was a physicist, that Darwin visited New Zealand, that 1 plus 1 equals 2, and so on. What is known in a possible world is a set of propositions, then, a premise set. Consequently, the meaning of the phrase *what is known* is a function from possible worlds to sets of propositions. To be more specific, it is that function  $f$  from  $W$  to  $P(P(W))$  (the power set of the power set of  $W$ ) that assigns to every possible world  $w$  the set of propositions that are known in  $w$ .

We are now in a position to say what the meaning of the relational modal *must in view of* is. In figure 1, *must in view of* semantically composes with two arguments: the modal scope, which denotes the proposition  $p$ , and the modal restriction, which denotes the individual concept  $f$ . The meaning of *must in view of* must then be a function that maps pairs consisting of a proposition and a function of the same type as  $f$  to another proposition. In the case of (6) that other proposition is the set of possible worlds  $w$  such that  $p$  follows from  $f(w)$ . In other words, the proposition expressed by (6) is true in those worlds  $w$  such that it follows from what is known in  $w$  that the ancestors of the Maoris arrived from Tahiti.

If we replace *must* in (6) by *can*, the proposition expressed by the resulting sentence would be true in a world  $w$  just in case it is compatible with what is known in  $w$  that the ancestors of the Maori arrived from Tahiti. These considerations lead to the following definitions for the meaning of relational *must* and *can*:

DEFINITION 5. The meaning of *must in view of* is that function  $\nu$  that satisfies the following conditions:

- (i) The domain of  $\nu$  is the set of all pairs  $\langle p, f \rangle$  such that  $p \in \mathcal{P}(W)$  and  $f$  is a function from  $W$  to  $\mathcal{P}(\mathcal{P}(W))$ .
- (ii) For any  $p$  and  $f$  such that  $\langle p, f \rangle$  is in the domain of  $\nu$ :  

$$\nu(p, f) = \{w \in W: \cap f(w) \subseteq p\}.$$

DEFINITION 6. The meaning of *can in view of* is that function  $\mu$  that satisfies the following conditions:

- (i) As in definition (5).
- (ii) For any  $p$  and  $f$  such that  $\langle p, f \rangle$  is in the domain of  $\mu$ :  

$$\mu(p, f) = \{w \in W: \cap (f(w) \cup \{p\}) \neq \emptyset\}.$$

The general idea behind these definitions is simple. The semantics of *must in view of* and *can in view of* is given by means of a function  $f$  that assigns sets of propositions to every possible world. A proposition is necessary in a possible world  $w$  in view of  $f$ , if it follows logically from the set of propositions that  $f$  assigns to  $w$ . A proposition is possible in a possible world  $w$  in view of  $f$  if it is logically compatible with the set of propositions that  $f$  assigns to  $w$ . Since the set of propositions a given  $f$  assigns to a world may vary from one world to the next, there could be worlds  $w$  and  $w'$ , such that a proposition  $p$  follows from  $f(w)$ , but not from  $f(w')$ , or is compatible with  $f(w)$ , but not with  $f(w')$ . This feature of the analysis makes it possible for sentences like (2) to (6) to express contingent propositions.

### 3. Inconsistent premise sets

I have given an account of the meaning of relational *must* and *can* in terms of logical consequence and compatibility. In so doing I must be prepared to face all the old paradoxes connected to these notions. For example, *ex falso quodlibet* rules that any proposition whatsoever follows from an inconsistent set of propositions. This section argues that we need not, and should not, accept this paradox. We have clear intuitions about what does or does not follow from an inconsistent set of propositions, and we also have the technical tools to model those intuitions in a precise way.<sup>6</sup>

Imagine a country where the only source of law is the judgments that are handed down. There are no hierarchies of judges, and all judgments have equal weight. There are no majorities to be considered. It does not matter whether a judgment has a hundred judgments against it – a judgment does not have less importance for all that. Let New Zealand be such a country. Imagine that there is one judgment in New Zealand legal history that provides that murder is a crime. Never in the whole history of the country has anyone dared to attack that judgment. No judgment in the whole history of New Zealand has ever suggested that murder is not a crime. There are other judgments, however. There were judges who did not agree on certain matters and handed down judgments that were in conflict with each other. Here is an example of such a disagreement. In Wellington a judgment was handed down that

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<sup>6</sup>. The method for making the best out of an inconsistent set advocated here was inspired by the account of relative modalities in Rescher (1973) (see also Rescher (1964)), but is different in a number of respects. On the analysis proposed here, *must* and *can* are duals, sentences with modals are allowed to be contingent, and the definitions do not assume compactness for premise sets. See Kratzer (1978) and chapter 2 for more discussion.

ruled that deer are not personally responsible for damage they inflict on young trees. In Auckland a judgment was handed down that ruled that deer ARE personally responsible for damage they inflict on young trees. As a consequence, the set of propositions picked out by the phrase *what the New Zealand judgments provide* in the actual world is inconsistent.

The situation I have just presented is not unusual. It may happen every day that two judges disagree. But the meaning definitions for relational *must* and *can* that I proposed earlier cannot cope with such a situation. According to those definitions, the propositions expressed by sentences (7) and (8) below should be true on the scenario I designed (I put *must be that* in place of *must* so as to get the scope of the negation right in (8)):

- (7) In view of what the New Zealand judgments provide, murder must be a crime.
- (8) In view of what the New Zealand judgments provide, it must be that murder is not a crime.

That (7) should come out true on our scenario is right. But that both (7) and (8) should wind up true is very wrong. Unfortunately, we are committed to this consequence if we accept the definitions I gave for *must*. Since the set of propositions that correspond to the New Zealand judgments in our world is inconsistent, both the proposition that murder is a crime and the proposition that murder is not a crime follow from that set. Although no New Zealand judgment has ever questioned that murder is a crime, our semantic analysis forces us to accept that murder must be a crime and not a crime in view of New Zealand common law.

The situation is no better when we consider the personal responsibility of deer. Since the set of propositions that correspond to the set of judgments in New Zealand history is inconsistent on our story, no proposition can be compatible with it. As a consequence, the propositions expressed by (9) and (10) are both predicted to be false (I am using *it is possible* to fill in for *can* to get the scope of negation right in (10)).

(9) In view of what the New Zealand judgments provide, it is possible that deer are personally responsible for damage they inflict on young trees.

(10) In view of what the New Zealand judgments provide, it is possible that deer are not personally responsible for damage they inflict on young trees.

We have run into an odd situation. Our semantic analysis forces us to conclude that, given our scenario, murder must be both a crime and not a crime in view of New Zealand common law. And it tells us moreover that both a ruling in favor of deer responsibility and a ruling against it would go against the law. Maybe the oddest aspect of our current analysis is that a single disagreement is enough to get us into such trouble.

To clear our path towards a possible way out of the dilemma we ran into, let us simplify our scenario a bit further. Suppose that the whole content of New Zealand common law, that is the collective content of all the judgments that have been handed down in New Zealand legal history, is the set of propositions expressed by (11) to (13):

(11) Murder is a crime.

- (12) Deer are personally responsible for damage they inflict on young trees.
- (13) Deer are not personally responsible for damage they inflict on young trees.

In a situation like this, an adequate analysis of modals should predict that the proposition expressed by (7) is true, and that expressed by (8) is false. Likewise, the proposition expressed by (14) should come out false:

- (14) In view of what the New Zealand judgments provide, it is possible that murder is not a crime.

As for the personal responsibility of deer, there is a judgment that says that deer are personally responsible for damage they inflict on young trees, and there is another judgment that says they are not. These are two incompatible opinions, and we should be free to go along with either one. That is, we would want the propositions expressed by (9) and (10) to both come out true on our scenario. Since intuitions about the truth of sentences (7) to (10) and (14) on our scenario are completely clear and uncontroversial, we have to aim for an analysis of modals that captures those intuitions. This means that we have to give up our earlier proposal for *must* and *can*.

Some abbreviations will be useful for the considerations that follow. Let  $A$  be the set of propositions provided by the New Zealand judgments in our world. In our simplified example, this set has just three members:  $p$  (the proposition expressed by (11)),  $q$  (the proposition expressed by (12)), and  $\neg q$  (the proposition expressed by (13)). The reason why Definition (5) did not yield the right result was because it was based on a relation between a

premise set and a proposition that was simply logical consequence. Since  $A$  is inconsistent, the relation specified in Definition (5) holds between  $A$  and any proposition whatsoever. In particular, it holds between  $A$  and  $p$  and  $A$  and  $\neg p$ . What we are looking for is a meaning definition for *must* that is based on a more discriminating relation. That relation should hold between  $A$  and  $p$ , but not between  $A$  and  $\neg p$ . One method of coming to terms with the inconsistency of  $A$  would look at the set of all consistent subsets of  $A$ . Let  $\mathcal{X}$  be that set. We have then:

$$A = \{p, q, \neg q\}$$

$$\mathcal{X} = \{ \emptyset, \{p\}, \{q\}, \{\neg q\}, \{p, q\}, \{p, \neg q\} \}$$

Maybe the relation we are looking for could be one that holds between a proposition and  $A$  just in case the proposition follows from every set in  $\mathcal{X}$ . To be sure,  $\neg p$  does not follow from every set in  $\mathcal{X}$ . It does not follow from the set that contains  $q$  as its only member, for example. But, unfortunately,  $p$  does not follow from every set in  $\mathcal{X}$ , either. This shows that the relation we have just came up with is too strict. Here is another attempt. Suppose the relation we are after holds between  $A$  and a proposition just in case for every set in  $\mathcal{X}$  there is a superset in  $\mathcal{X}$  from which that proposition follows. This relation seems to do the job we want it to do. For every set in  $\mathcal{X}$ , there is a superset in  $\mathcal{X}$  from which  $p$  follows. But there is not for every set in  $\mathcal{X}$  a superset in  $\mathcal{X}$  from which  $\neg p$  follows. To see this, consider  $\{p, q\}$ . Since  $\{p, q\}$  is consistent, it is a member of  $\mathcal{X}$ . But there is no superset of  $\{p, q\}$  in  $\mathcal{X}$  from which  $\neg p$  follows. This is a promising result. We seem to have found a method that allows us to draw conclusions from an inconsistent set of propositions while staying as close as possible to the information it contained. In the hope that the method generalizes to the full

range of more complicated examples, I want to propose the following revised meaning definition for relational *must*:

DEFINITION 7. The meaning of *must in view of* is that function  $\nu$  that satisfies the following conditions:

- (i) As in definition (5).
- (ii) For any  $p$  and  $f$  such that  $\langle p, f \rangle$  is in the domain of  $\nu$ :

$$\nu(p, f) = \{w \in W: \forall A [A \in X_{f(w)} \rightarrow \exists B [B \in X_{f(w)} \& A \subseteq B \& \cap B \subseteq p] ] \},$$

$$\text{where } X_{f(w)} = \{A: A \subseteq f(w) \& \text{consistent}(A)\}.$$

Similar considerations lead to an improvement of Definition (6). Definition (6) was inadequate because it required logical compatibility between a premise set and a proposition. Since  $A$  is inconsistent, the required relation fails to hold between  $A$  and any proposition. What we are looking for is a relation that holds between  $A$  and  $q$  and  $A$  and  $\neg q$ , in our example, but not between  $A$  and  $\neg p$ . If  $X$  is again the set of all consistent subsets of  $A$ , we should say that the relation we are after holds between  $A$  and any proposition just in case the proposition is compatible with some set and all of its supersets in  $X$ . In our case, we can easily find a set in  $X$  such that  $q$  is compatible with all of its supersets in  $X$ . The singleton set  $\{q\}$  is such a set. Since  $q$  is in  $A$  and  $\{q\}$  is consistent,  $\{q\}$  is a member of  $X$ , and obviously,  $q$  is compatible with every superset of  $\{q\}$  in  $X$ . Likewise, there is a set in  $X$  such that  $\neg q$  is compatible with all its supersets in  $X$ . The singleton set  $\{\neg q\}$  is such a set. What about  $\neg p$ ? The proposition  $\neg p$  is consistent, but it is not in  $A$ , and therefore not a member of any set in  $X$ . As a matter of fact,  $\neg p$  is compatible with both  $\{q\}$  and  $\{\neg q\}$ , but each of

those two sets has a superset in  $X$  that implies  $p$ , hence is no longer compatible with  $\neg p$ : the two relevant sets are  $\{q, p\}$  and  $\{\neg q, p\}$ . Hoping again that the idea we developed on the basis of a simple example generalizes to more complicated cases, I would like to propose the revised meaning definition 8.

DEFINITION 8. The meaning of *can in view of* is that function  $\mu$  that satisfies the following conditions:

- (i) As in definition (5).
- (ii) For any  $p$  and  $f$  such that  $\langle p, f \rangle$  is in the domain of  $\mu$ :

$$\mu(p, f) = \{w \in W: \exists A [A \in X_{f(w)} \& \forall B [ [B \in X_{f(w)} \& A \subseteq B] \rightarrow \text{consistent}(B \cup \{p\}) ] ] \}, \text{ where } X_{f(w)} = \{A: A \subseteq f(w) \& \text{consistent}(A)\}.$$

#### 4. Structuring premise sets

Our hopes that the semantics of relational *must* and *can* proposed in the previous section might be general enough to cope with the problem of making the best out of inconsistent sets of propositions seem to vanish under the impact of the following example<sup>7</sup>.

##### THE STORY OF TE MITI AND TE KINI

The pupils of a Whare Wananga, which was a kind of University in Maori Society, have to be educated according to the recommendations of the former principals of the school. As is to be expected, those principals had different opinions about what is good for a student to

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<sup>7</sup>. I am indebted to Irene Heim (personal communication) for raising a related objection against the analysis of modals presented in section 3.

learn. There was, for example, Te Miti, who recommended that students practice striding and flying. And there was Te Kini's recommendation, which didn't allow students to practice striding under any circumstances. In Te Kini's opinion the practice of striding overstrained the students' legs. He had no objections to the practice of flying.

If the two recommendations mentioned in our story were the only ones those two principals gave during the relevant time period, sentences (15) and (16) can be taken to express the complete content of Te Miti's and Te Kini's respective recommendations.

(15) The pupils practice striding and flying.

(16) The students do not practice striding.

If  $p$  is the proposition that the students practice striding, and  $q$  is the proposition that the students practice flying, the proposition expressed by (15) seems to be  $p \cap q$ , and the proposition expressed by (16) is  $\neg p$ . Assuming furthermore that Te Miti and Te Kini were the only principals our Whare Wananga has ever had, we would now want the propositions expressed by (17) to come out true:

(17) In view of what the former principals of the Whare Wananga recommended, the students must practice flying.

Does our current analysis of *must* match our intuitions about the truth of (17) on our scenario? It seems that it does not. Let  $A$  be the set of propositions which form the content

of the recommendations of the two former principals. On our scenario,  $A$  contains only two members,  $p \cap q$  and  $\neg p$ . The set  $X$  of all consistent subsets of  $A$  is then:

$$X = \{ \emptyset, \{ p \cap q \}, \{ \neg p \} \}$$

Not every set in  $X$  has a superset in  $X$  from which  $q$  follows. The set  $\{ \neg p \}$  does not, and this means that our analysis of *must* seems to predict (17) to be false on the assumed scenario. We have run into a problem. To escape from it we may try to improve our definitions, or else check our intuitions again. The definitions do not match our intuitions. Either the definitions are wrong or our intuitions are misguided. I opt for looking into the second possibility and propose to reexamine the reasoning that led to the impression that our analysis has a problem in the first place.

Te Miti recommended that students practice striding and flying. Te Kini recommended that students do not practice striding under any circumstances. In such a situation, I argued, we certainly wanted to preserve Te Miti's recommendation about the students' flying. This recommendation was obviously not contradicted by what Te Kini recommended. "Certainly," I just said, and "obviously", but I think we cannot be certain about this at all, and our claim is far from being obvious. Look at the following elaboration of our original scenario.

#### TE MITI'S RECOMMENDATION FLESHED OUT

Suppose, as we did before, that Te Miti recommended that students practice both striding and flying. But he really wanted them to do both together. Here are his reasons: striding

stretches the legs and flying stretches the arms. If you do both, that's a good combination. But if you practice striding without flying or flying without striding, the proportions of your body become distorted. Your legs get stretched and your arms remain short, or else your arms get stretched and your legs are left behind. Neither is good. Practicing both sports together leads to a good shape for your body.

If the motivation just mentioned was behind Te Miti's recommendation, Te Kini's view that striding is bad under any circumstances challenges Te Miti's recommendation as a whole, not just the part about striding. If the students do not stride any more, Te Miti would not want them to practice flying either. Representing Te Miti's recommendation as a single proposition  $p \cap q$  correctly captures the intent behind his recommendation. Our analysis handles this case correctly, then: it does not preserve the students' flying. If striding needs to be given up, flying must be, too.

There are subtly different scenarios, however, where we would want to say that Te Kini's recommendation does not contradict Te Miti's recommendation as a whole, but only the part about striding. These would be scenarios where what Te Miti recommended is naturally individuated as two recommendations. He recommended that students stride and also recommended that students fly. He thought that each of those activities was also beneficial on its own. In such cases, we may interpret Te Miti as not recommending the single proposition  $p \cap q$ , but the pair of propositions  $\{p, q\}$ . This makes all the difference on our account. If one of Te Miti's recommendations is challenged, we would still want to keep the other one. Our analysis of *must* matches this intuition as well. If the set of propositions that correspond to the content of what the former principals of our Whare

Wananga recommended is  $A = \{p, q, \neg p\}$ , then the set  $X$  of all consistent subsets of  $A$  looks as follows:

$$X = \{ \emptyset, \{p\}, \{q\}, \{\neg p\}, \{p, q\}, \{q, \neg p\} \}$$

For every set in  $X$  there is now a superset in  $X$  from which  $q$  follows. Our analysis of *must* is correct after all, then, once we acknowledge the difference between giving a single conjoined recommendation and giving a pair of recommendations. There is a subtle, but momentous, distinction here that is all too easy to overlook. Sentence (18) is ambiguous, then.

(18) Te Miti recommended that students practice striding and flying.

The phenomenon we discovered is expected to be very general and should be found with any speech act or attitude whose content can plausibly be thought of as a premise set. For example, if Te Miti believes about a particular student that he practiced striding and flying, what he believes about that student can be individuated as one or two (or more) beliefs. The difference matters when Te Miti's beliefs are challenged. Suppose Te Kini pointed out to Te Miti that the student in question did not actually practice striding. If Te Miti had just a single belief about that student's athletic activities, Te Kini's objection would challenge the whole of what Te Miti believed on that matter. On the other hand, if there were two or more relevant beliefs, Te Kini's objection would only affect part of what his colleague believed.

We have just seen an example of two consistent premise sets that are true in the same set of possible worlds, hence have the same “deductive closures”, but behave differently when consistency needs to be restored after one of the premises has been challenged. That is, even though  $\cap\{p, q\} = \cap\{p \cap q\}$  for any propositions  $p$  and  $q$ , the premise sets  $\{p, q\}$  and  $\{p \cap q\}$  might have to be distinguished in the theory of modality. This is an important property of premise sets that has recently been exploited for theories of rational belief change (see e.g. Rott (2001)). To have another, even more striking, illustration of the same phenomenon, imagine a situation where the content of a given belief state might be represented as one of the two premise sets  $\{p, q\}$  or  $\{p, p \leftrightarrow q\}$ <sup>8</sup>. Even though  $\cap\{p, q\} = \cap\{p, p \leftrightarrow q\}$ , the two sets behave differently in situations where  $p$  has to be given up and is replaced by  $\neg p$ . Rational restoration of consistency yields different results for the two sets. In the first case, we give up  $p$ , retain  $q$ , and add  $\neg p$ , hence end up with  $\{\neg p, q\}$ . In the second case, we give up  $p$ , add  $\neg p$ , and retain  $p \leftrightarrow q$ . The result is the set  $\{\neg p, p \leftrightarrow q\}$ , which implies  $\neg q$ . Representing the content of recommendations, claims, beliefs, orders, wishes, etc. as premise sets thus offers the priceless opportunity to represent connections between propositions in a given premise set. The content of such speech acts and attitudes can now be seen to have an inherent structure that encodes which propositions stand and fall together under challenge. This structure is lost if information contents are directly represented as sets of possible worlds, as is common in possible worlds semantics, following the lead of Hintikka (1962).

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<sup>8</sup>. S. O. Hansson (2006) presents such an example. I have been using “ $\neg$ ” to stand for set complementation, and I am using “ $p \leftrightarrow q$ ” as an abbreviation for “ $(\neg p \cup q) \cap (\neg q \cup p)$ ”.

In conclusion, I have argued for a unified analysis of modals like *must* and *can* where the observed variety of uses is due to their relational nature. Modals require two arguments to be complete: a proposition (their scope) and a function from worlds to premise sets (their restriction). That the premise sets provided by a modal restriction depend on worlds makes sure that modal statements can be contingent. A proposition is necessary or possible if it bears a particular relation to the relevant premise set in the world of evaluation. Since premise sets are not always consistent, the relation between a proposition and a premise set cannot simply be logical consequence or compatibility. We had to design a method that allowed us to model robust intuitions about reasoning from inconsistent sets. This method turned out to be sensitive to subtle potential differences between deductively equivalent premise sets. To quote Rescher (1979, 31), who talks about premise sets that are sets of sentences, not propositions, there is a crucial difference between “juxtaposing commas” and “conjoining ampersands”. Much of my work on modals and conditionals since 1977 has exploited that difference. The difference between  $\{p, q\}$  and  $\{p \wedge q\}$  is not just senseless “notional bondage” (Belnap (1979), 23). It makes all the difference in the theory of modality. “If they are separate items juxtaposed by a comma, the fates of  $p$  and  $q$  are independent of one another, unless there is additional information to the contrary ...; if they are conjoined by an ampersand, they stand and fall together” (Rott (2001), 81).

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