A uniform semantics for declarative and interrogative complements*

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Abstract

This paper proposes a semantics for declarative and interrogative complements and for so-called responsive verbs, like know and forget, which embed both kinds of complements. Following Groenendijk and Stokhof (1984), we pursue a uniform account, in the sense that we take both kinds of complements to be of the same semantic type and we assume a single lexical entry for each responsive verb. This approach avoids a number of problems for non-uniform theories, such as the reductive theories of Karttunen (1977), Heim (1994), Lahiri (2002), Spector and Egré (2015), a.o., and the twin relations theory of George (2011).

On the other hand, our account also addresses the main limitation of Groenendijk and Stokhof’ s proposal, which is that it is primarily designed to derive strongly exhaustive readings for interrogative complements. Our account is more flexible in that it straightforwardly derives non-exhaustive and intermediate exhaustive readings as well.

1 Introduction

So-called responsive verbs like know and forget accept both declarative and interrogative complement clauses as their argument:

(1)  
   a. Mary knows/forgot that John left.
   b. Mary knows/forgot who left.

In this paper, we develop a uniform theory of clause embedding, i.e., a theory on which declarative and interrogative complements have the same semantic type. On such an account, every responsive verb can be associated with a single lexical entry, applying to declarative and interrogative complements alike.

By contrast, most existing approaches to clausal complements are non-uniform. At least since Karttunen (1977), it is usually assumed that declarative and interrogative complements differ in semantic type, with declaratives denoting propositions and interrogatives denoting sets of propositions. Under this view, it is prima facie unexpected that there are responsive verbs like know and forget, and non-uniform accounts need to find ways to resolve this tension. The diagram in Figure 1 classifies the most influential works on clausal embedding according to how they do this.1

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1We restrict our attention here to ‘propositional’ theories of interrogatives, leaving out so-called ‘categorial’ theories (e.g., von Stechow, 1991; Krifka, 2001) as well as theories couched in other frameworks, such as situation semantics (e.g., Ginzburg, 1995). Categorial theories are not considered here because their main focus is on root interrogatives rather than embedded ones. Various phenomena involving root interrogatives require a more fine-grained notion of question meaning than the one provided by the propositional framework. These phenomena, however, can also be explained in extensions of the propositional framework that take dynamic aspects of meaning, i.e., the discourse referents that sentences introduce, into consideration (e.g., Aloni et al., 2007; Roelofsen and Farkas, 2015).
So-called reductive approaches take the declarative-embedding use of responsive verbs to be basic, and reduce the interrogative-embedding use to the declarative-embedding one. They assume that responsive verbs want a proposition as their input—not a set of propositions. This means that if the complement of the verb is interrogative, a type mismatch arises. Heim (1994), Dayal (1996), and Beck and Rullmann (1999), among others, propose that this type mismatch is resolved by a type-shifting answer operator, which compresses the set of propositions generated by the interrogative clause into a single proposition and then feeds this proposition to the verb. Lahiri (2002) proposes that the type mismatch is resolved by raising the interrogative clause to a higher position in the syntactic structure, leaving a proposition-type variable in the verb’s argument slot.

A different variant of the reductive approach, briefly suggested by Karttunen (1977) and elaborated in detail by Spector and Egré (2015), is to assume two lexical entries for every responsive verb, one for each kind of complement. For instance, for know we would have the two entries know\textsubscript{d} and know\textsubscript{i}, taking declarative and interrogative complements, respectively. Spector and Egré then formulate a general meaning postulate which, given the declarative entry V\textsubscript{d} of a verb V, determines the corresponding interrogative entry V\textsubscript{i}.

Two other non-uniform approaches are the twin relations approach (George, 2011) and the inverse reductive approach (also briefly considered by George, 2011, but developed in much greater detail by Uegaki, 2015). The twin relations approach derives both the declarative-embedding interpretation and the interrogative-embedding interpretation of responsive verbs from a common lexical core. The inverse reductive approach reduces the declarative-embedding interpretation of responsive verbs to their interrogative-embedding interpretation, rather than the other way around.

Ginzburg (1995) specifically addresses embedded interrogatives but his focus is on phenomena of context-sensitivity which are arguably orthogonal to the issues addressed in this article. To explain these cases, Ginzburg proposes to parametrise the ‘resolvedness’ relation holding between an interrogative and a piece of information with a number of contextual factors including the goals of the conversational participants. Most of the cases of context-sensitivity discussed by Ginzburg can be explained in an extension of the present approach, adopting for example a conceptual cover style of quantification along the lines of Aloni (2001). On this approach, speakers’ goals and other contextual factors, rather than being parameters of the resolvedness relation, would play a role in fixing the quantificational domain of wh-phrases.
The strategy we will pursue in the present paper diverges from all these approaches in that it treats declarative and interrogative complements uniformly. To our knowledge, the only previous account of clause embedding that follows this strategy is the *partition theory* of Groenendijk and Stokhof (1984). On this theory, both declarative and interrogative complements are treated as denoting propositions. A declarative complement denotes the same proposition in every world, while the proposition denoted by an interrogative complement varies across worlds: in any world \( w \), an interrogative complement denotes that proposition which, in \( w \), is the true exhaustive answer to the question that the complement expresses.

For each of the theories mentioned above, certain problems have been identified in the literature. Here, we give a quick overview of the relevant issues, which are also summarized in Table 1.

**Flexibility in exhaustive strength (Sections 3 and 4).** Some existing theories are limited to certain levels of exhaustive strength. In particular, the partition theory is mainly designed to derive so-called *strongly exhaustive* (SE) readings for interrogative complements. It needs to invoke additional machinery to derive non-exhaustive/mention-some (MS) readings and does not derive weakly/intermediate exhaustive (WE/IE) readings at all. Groenendijk and Stokhof argued that this is in fact a desirable feature of their theory, but other authors have disagreed (e.g., Heim, 1994; Beck and Rullmann, 1999; Spector, 2005; Klinedinst and Rothschild, 2011). Recent experimental results seem to show that intermediate exhaustive readings indeed exist (Cremers and Chemla, 2016).

**False answer sensitivity (Section 3).** In the traditional characterization of MS and WE readings, false answers to the embedded question do not play a role. However, Spector (2005), George (2011, 2013), and Klinedinst and Rothschild (2011) point out that this is problematic. In particular, George (2011, 2013) observes that in certain scenarios, whether an individual stands in the knowledge relation to a certain interrogative not only depends on the individual’s true propositional knowledge, but also on whether she believes any *false answers* to that interrogative. This sensitivity to false answers implies that interrogative knowledge cannot generally be reduced to true propositional/declarative knowledge. George (2011, 2013) shows that capturing false answer sensitivity is a problem for all reductive theories as well as for the partition theory, but not for the inverse reductive theory or the twin relations theory.

**Predicates of relevance (Section 5).** Elliott *et al.* (2017) observe that when so-called *predicates of relevance*, such as *care* and *matter*, take a declarative complement, they carry a certain presupposition that is absent when the complement is interrogative. Elliott *et al.*

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Table 1: Pros and cons of existing approaches.

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2The idea to treat declaratives and interrogatives uniformly goes further back, at least to Hamblin (1973, p.48). However, Hamblin was exclusively concerned with *root* clauses; he did not consider declarative and interrogative *complements*, and the repercussions of a uniform treatment for the analysis of verbs that take such complements as their argument, which is our main concern here.

3The partition theory correctly captures false answer sensitivity in the case of strongly exhaustive readings. It does not, however, capture false answer sensitivity in the case of mention-some readings.
argue that this is problematic for reductive theories, and Uegaki (2017) shows that it is also problematic for George’s twin relations theory. On the other hand, it can easily be accounted for on the inverse reductive approach.

**Constraints on verb meanings (Section 6).** Evidently, the interrogative-embedding and the declarative-embedding interpretation of a responsive verb are related, and it is plausible that not all kinds of relationships between them are permissible. For instance, Spector and Egré (2015) propose that a responsive verb is veridical w.r.t. interrogative complements if and only if it is veridical w.r.t. declarative complements. If Spector and Egré’s generalization is correct, this means that, across languages, we will not find any responsive verb that is veridical w.r.t. to one but not the other kind of complement. George (2011) and Spector and Egré (2015) put forward that a comprehensive theory of clause embedding should predict constraints of this kind. Partition theory and the inverse reductive approach fail to do so. Spector and Egré’s reductive theory and George’s twin relations theory, on the other hand, do predict the existence of certain general constraints such as the above veridicality constraint. What we will argue is that predicates of relevance form a counterexample to Spector and Egré’s generalization and that the veridicality constraint should therefore not follow from a theory of clause embedding as a necessary consequence. We will also suggest how a uniform or inverse reductive theory can account for the fact that most verbs do satisfy the constraint.

**Selectional restrictions** Not all embedding verbs are responsive. As illustrated in (2)–(3), there are also verbs that only take interrogative complements, such as wonder, and verbs that only take declarative complements, such as believe.

(2) a. *Bill wonders/investigated that John left.
   b. Bill wonders/investigated who left.

(3) a. Bill believes/hopes that John left.
   b. *Bill believes/hopes who left.

Most existing theories, with the exception of Uegaki (2015), have left these selectional restrictions of non-responsive verbs unexplained. For reasons of space, the present paper will do the same, and focus exclusively on responsive verbs. However, in a companion paper (Theiler et al., 2017) we argue that it is a general advantage of the uniform approach taken here that the selectional restrictions of verbs like wonder and believe may in fact be derived in a rather straightforward way from independently motivated features of their lexical semantics. There, we also compare our account with that of Uegaki (2015), arguing that while his inverse-reductive approach makes it possible to account for the fact that verbs like wonder do not take declarative complements (in a way similar to our uniform approach), it does not make it possible to account in an explanatory way for the fact that verbs like believe do not take interrogative complements (unlike our uniform approach).

The theory developed in the present paper is like partition theory in that it treats declarative and interrogative complements uniformly. However, building on recent work in inquisitive semantics (e.g., Ciardelli et al., 2015, 2017), it also differs from partition theory in crucial respects, overcoming its main limitations. Most fundamentally, declarative and interrogative complements are not treated as denoting propositions, but rather as denoting sets of propositions. In the case of interrogative complements, these propositions do not encode what the true exhaustive

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4 A similar argument was made by Groenendijk and Stokhof (1984, p.94) against the reductive theory of Karttunen (1977). Elliott et al.’s argument, however, is more explicit and targets the reductive approach in general rather than only Karttunen’s specific theory.

5 Observe from Table 1 that those theories which can deal with predicates of relevance are exactly those that don’t derive strict constraints on verb meanings like Spector and Egré’s veridicality constraint.
answer to the interrogative is in any given world \( w \), but rather what its *truthful resolutions* are in \( w \). Such truthful resolutions need not be exhaustive, and need not even be true in \( w \); they just need to be ‘truthful’, which means that they should not imply any false information that is directly relevant w.r.t. the issue expressed by the interrogative. This switch from true exhaustive answers to truthful resolutions will allow us to provide a general account of false answer sensitivity, and to derive not only strongly exhaustive readings but also false-answer sensitive mention-some and intermediate exhaustive readings in a straightforward way. Moreover (independently from the move to truthful resolutions), we will show how the special properties of predicates of relevance can be captured, and we will demonstrate how constraints on the space of possible responsive verb meanings can be implemented within a uniform account.

The paper is structured as follows. Section 2 briefly reviews the main terminology and notational conventions of inquisitive semantics. Section 3 introduces our account of clausal complements, paying special attention to how false answer sensitivity is implemented across the different levels of exhaustivity. Section 4 zooms in on the meaning of *know*, while Section 5 brings in other responsive verbs, including predicates of relevance. Section 6 focuses on capturing constraints on possible responsive verb meanings, and Section 7 concludes.

The paper also has two appendices: Appendix A compares our proposal in some detail to the inverse reductive theory of Uegaki (2015), which, even though it does not assume uniformity, is very close in spirit and empirical reach. Appendix B contains formal proofs for some of the claims made in the paper.

2 Semantic framework

Our account will be couched in *inquisitive semantics* (Ciardelli, Groenendijk, and Roelofsen, 2015). More specifically, we will adopt the type-theoretic inquisitive semantics framework developed in Ciardelli, Roelofsen, and Theiler (2017). This framework is particularly suitable for our purposes here, because it offers a natural way of treating declarative and interrogative sentences uniformly (cf., Farkas and Roelofsen, 2017). In this section, we briefly review the basic features of the framework and introduce some notational conventions that will be useful in later sections.

2.1 Sentence meanings in inquisitive semantics

Traditionally, the meaning of a sentence is construed as a *proposition*, i.e., a set of possible worlds. Intuitively, a proposition carves out a region in the space of all possible worlds \( W \), and in asserting a sentence, a speaker is taken to provide the information that the actual world is located within this region. In this way, the proposition expressed by a sentence captures the *informative content* of the sentence.

Inquisitive semantics generalizes this notion of meaning to capture not just informative, but also *inquisitive content*, i.e., the issue raised in uttering a sentence. To achieve this, the meaning of a sentence is construed as a *set of propositions*, namely the set of all those propositions that resolve the issue raised by the sentence. When a speaker utters a sentence with meaning \( P \), she is taken to raise an issue whose resolution requires establishing one of the propositions in \( P \), while at the same time providing the information that at least one of these propositions must be true, i.e., that the actual world is contained in \( \bigcup P \).

It is assumed that if a certain proposition \( p \) resolves a given issue, then any stronger proposition \( q \subset p \) will also resolve that issue. This means that sentence meanings are *downward closed*: if \( p \in P \) and \( q \subset p \), then \( q \in P \) as well. Finally, it is assumed that the *inconsistent* proposition, \( \emptyset \), resolves any issue. This means that any sentence meaning has \( \emptyset \) as an element and is therefore non-empty. These considerations lead to the following characterization of sentence meanings:
Definition 1 (Sentence meanings in inquisitive semantics).
A sentence meaning in inquisitive semantics is a non-empty, downward closed set of propositions. The maximal elements of $P$ are referred to as the alternatives in $P$. We will write $\text{alt}(P)$ for the set of alternatives in $P$. In depicting the meaning of a sentence, we will generally only depict the alternatives that it contains. Finally, $\bigcup P$ is referred to as the informative content of $P$, denoted as $\text{info}(P)$, and a sentence with meaning $P$ is said to be true in a world $w$ just in case $w \in \text{info}(P)$.

Definition 2 (Alternatives, informative content, and truth).
For any sentence meaning $P$ and any world $w$:

- $\text{alt}(P) := \{ p \in P \mid \text{there is no } q \in P \text{ such that } p \subset q \}$
- $\text{info}(P) := \bigcup P$
- A sentence with meaning $P$ is true in $w$ just in case $w \in \text{info}(P)$.

To illustrate these notions, consider the following two sentences.

(4) a. Did Amy leave?
   b. Amy left.

The polar interrogative in (4a) is taken to have the meaning in Figure 2(a), where $w_1$ and $w_2$ are worlds where Amy left, and $w_3$ and $w_4$ are worlds where she didn’t leave. The shaded rectangles are the alternatives contained in the given meanings. By downward closure, all propositions contained in one of these alternatives are also included in the meanings of the sentences. The meaning assigned to (4a) captures the fact that, in uttering this sentence, a speaker (i) provides the trivial information that the actual world must be $w_1$, $w_2$, $w_3$, or $w_4$ (all options are open) and (ii) raises an issue whose resolution requires establishing either that Amy left, or that she didn’t leave. Since $\text{info}([\text{Did Amy leave?}]) = \{w_1, w_2, w_3, w_4\}$, this sentence is true in all of $w_1$, $w_2$, $w_3$, and $w_4$. More generally, since the informative content of a non-presuppositional interrogative sentence always covers the entire logical space, such a sentence is always true in all worlds.

The declarative in (4b) is assigned the meaning in Figure 2(b), which captures the fact that this sentence (i) provides the information that the actual world must be either $w_1$ or $w_2$, i.e., one where Amy left, and (ii) raises an issue whose resolution requires establishing that Amy left. In this case, the information provided by the speaker is already sufficient to resolve the issue that is raised: no further information is needed from other conversational participants. Furthermore, as expected, Amy left is true in worlds $w_1$ and $w_2$.

2.2 Informative and inquisitive sentences

In the case of the interrogative Did Amy leave? the information that is provided is trivial in the sense that it does not exclude any candidate for the actual world. Such sentences are called

Figure 2: The meaning a polar interrogative and a declarative sentence in inquisitive semantics.
non-informative. Conversely, a sentence with meaning $P$ is called informative just in case it does exclude at least one candidate for the actual world, i.e., iff $\text{info}(P) \neq W$.

On the other hand, in the case of the declarative Amy left, the inquisitive content of the sentence is trivial, in the sense that the issue that is raised in uttering the sentence is already resolved by the information provided; no further information is required. Such sentences are called non-inquisitive. Conversely, a sentence with meaning $P$ is called inquisitive just in case resolving the issue that it expresses requires more than the information that it provides, i.e., iff $\text{info}(P) \notin P$.

Given a picture of the meaning of a sentence, it is easy to see whether the sentence is inquisitive or not. This is because a sentence is inquisitive just in case its meaning contains at least two alternatives. For instance, the meaning in Figure 2(a) contains two alternatives, which means that the polar interrogative Did Amy leave? is inquisitive, while the meaning in Figure 2(b) contains only one alternative, which means that the declarative Amy left is not inquisitive. Following Ciardelli et al. (2015), Farkas and Roelofsen (2017) and much other work, we will assume that declarative sentences are never inquisitive, i.e., that their meaning always contains a single alternative.

### 2.3 Composing meanings

We adopt a standard two-step approach for composing the meaning of a sentence, summarized in Figure 3. In the first step, we translate a natural language expression into a type-theoretic language, by translating every lexical item into a certain type-theoretic expression and deriving the translation of complex constituents by means of function application and abstraction. We write $(\cdot)'$ for the translation of a natural language expression $\alpha$. In the second step, type-theoretic expressions are interpreted relative to a model $M$ and an assignment $g$.

The type theory we assume is two-sorted, with basic types $e$, $s$, and $t$, for individuals, worlds, and truth values, respectively. Since sentence meanings are construed as sets of propositions, sentences are taken to be of type $\langle \langle s, t \rangle, t \rangle$, which we abbreviate as $T$. From this, one can reverse engineer the types that should be assigned to various kinds of sub-sentential expressions:

\[
\begin{align*}
\text{John} & : e \\
\text{walks} & : \langle e, T \rangle \\
\text{likes} & : \langle e, \langle e, T \rangle \rangle \\
\text{and} & : \langle T, \langle T, T \rangle \rangle \\
\text{not} & : \langle T, T \rangle \\
\text{somebody} & : \langle \langle e, T \rangle, T \rangle
\end{align*}
\]

For instance, we take the meaning of a sentence like John walks to be the set of propositions $p$ such that John walks in every world $w \in p$:

\[
(\text{John walks})' = \lambda p. \forall w \in p : W(j)(w)
\]

This set of propositions is downward closed since, if $p$ is a proposition such that John walks in every world $w \in p$, then the same goes for any $q \subseteq p$. To obtain the above sentence

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6Strictly speaking, this generalization only holds if any sentence meaning is guaranteed to contain alternatives—which again only holds under the assumption that there are finitely many possible worlds. However, this is a safe assumption to make for all the examples to be considered in this paper.

7There is also work in inquisitive semantics that does not make this assumption (e.g., Groenendijk, 2009; AnderBois, 2012). This requires a view under which uttering an inquisitive sentence does not necessarily involve issuing a request for information. We refer to Ciardelli et al. (2012, p.41-43) for further discussion of this point.
meaning, the verb \textit{walks} should express a function that takes an individual \(x\) and yields the set of propositions \(p\) such that \(x\) walks in every \(w \in p\):

\[
\text{walks}' = \lambda x. \lambda p. \forall w : W(x)(w)
\]

This is all we need to know about type-theoretical inquisitive semantics to give a compositional account of the constructions we are interested in here. For a more systematic introduction to this framework, we refer to Ciardelli et al. (2017).

3 False answer sensitivity across levels of exhaustive strength

We now lay out our account of clausal complements. Our initial focus will be on two of the desiderata mentioned in Section 1, namely implementing false answer sensitivity and deriving the different levels of exhaustive strength. Section 3.1 explains these desiderata in more detail, while the rest of the section spells out our positive proposal. We start by specifying our general assumptions about complement constructions (Section 3.2), then provide an account of interrogative complements (Section 3.3) and of the embedding verb \textit{know} (Section 3.4). Once this is in place, we show how the proposed account captures false answer sensitivity effects across all levels of exhaustivity (Section 3.5), and demonstrate that it also makes correct predictions for declarative complements (Section 3.6).

3.1 Desiderata

Traditionally, three kinds of readings are distinguished for knowledge ascriptions involving interrogative complements: \textit{strongly exhaustive} (SE) readings, \textit{weakly exhaustive} (WE) readings, and \textit{non-exhaustive} readings. The latter are also often referred to as \textit{mention-some} (MS) readings, and we will follow this custom. To see what these readings amount to, consider the following example:

(8) John knows who called.

Let us assume that John knows what the domain of discourse \(D\) is, and let us refer to \(A = \{d \mid d \in D\}\) as the set of answers to the question \textit{who called}. Then the three readings can be characterized as follows:8

\[
\text{(9) a. Strongly exhaustive reading:}
- \text{for any true answer } a \in A, \text{ John knows that } a \text{ is true, and}
- \text{for any false answer } a \in A, \text{ John knows that } a \text{ is false}
\]

\[
\text{b. Weakly exhaustive reading:}
- \text{for any true answer } a \in A, \text{ John knows that } a \text{ is true}
\]

\[
\text{c. Mention-some reading:}
- \text{for at least one true answer } a \in A, \text{ John knows that } a \text{ is true}
\]

Note that in these traditional characterizations of the three different readings, false answers only play a role for SE readings. John’s beliefs about false answers do not matter for WE and MS readings. In the recent literature, however, it has been argued that false answers are relevant for these weaker readings as well. In particular, Spector (2005) and Klinedinst and Rothschild (2011) point out their relevance for WE readings, based on sentences like (10).

\[\text{The characterization of strongly exhaustive readings does not correspond completely to that given in Groenendijk and Stokhof (1984). Under the latter, John would be required to know what the extension of the predicate call is. The two notions do coincide, however, under our current assumption that John knows what the domain of discourse is.}\]
John told Mary who passed the exam.

Suppose that only Ann and Bill passed the exam. Then, under what seems to be the most salient reading of (10), the sentence is judged true if John told Mary that Ann and Bill passed the exam and he didn’t tell her anything else. On the other hand, it is judged false if John additionally told Mary, erroneously, that Chris and Daniel passed the exam as well.

George (2011) argues that false answers are relevant for MS readings as well, based on the following scenario. Suppose that there are three stores, Newstopia, Paperworld, and Celluloid City, of which only two, namely Newstopia and Paperworld, sell Italian newspapers. Janna knows, true to fact, that Newstopia sells Italian newspapers and does not have any beliefs concerning the availability of such newspapers elsewhere. Rupert, on the other hand, while also knowing that one can buy an Italian newspaper at Newstopia, falsely believes that Celluloid City sells such newspapers as well. George (2011) observes that there is a salient reading under which sentence (11a) is judged true in this scenario, while (11b) is judged false.

\[(11)\]
\[
a. \text{Janna knows where one can buy an Italian newspaper.} \\
b. \text{Rupert knows where one can buy an Italian newspaper.}
\]

In order to capture this contrast, the characterization of MS readings should be made sensitive to false answers: it should not just require that the subject knows of at least one true answer to the embedded question that it is true, but also that she does not wrongly believe of any false answer that it is true.

Xiang (2016a) further observes that the assessment of interrogative knowledge ascriptions is not only sensitive to beliefs concerning completely resolving false answers, but also to false partial answers. To see this, consider the same kind of scenario as above but now suppose that only Newstopia sells Italian newspapers. Suppose Rupert knows that Newstopia sells Italian newspapers but also wrongly believes that Paperworld or Celluloid City sells them, although he isn’t certain which of the two. Xiang (2016a) observes that (11b) is still judged false in this scenario. Thus, Rupert’s belief in the false partial answer ‘that Paperworld or Celluloid City sells Italian newspapers’ is sufficient to block interrogative knowledge.

These observations show that false answer sensitivity (FA sensitivity for short) plays a role at all levels of exhaustive strength. For example (8), this yields the following truth conditions, assuming that \(A^\vee := \{ a_1 \lor \ldots \lor a_n \mid a_i \in A \}\) is the set of partial answers to the question who called:

\[(12)\]
\[
a. \text{Strongly exhaustive reading, as before:} \\
   \quad - \text{for any true answer } a \in A, \text{ John knows that } a \text{ is true, and} \\
   \quad - \text{for any false (partial) answer } a \in A^\vee, \text{ John knows that } a \text{ is false}^9 \\
b. \text{FA sensitive weakly exhaustive reading:} \\
   \quad - \text{for any true answer } a \in A, \text{ John knows that } a \text{ is true} \\
   \quad - \text{for any false (partial) answer } a \in A^\vee, \text{ John does not believe that } a \text{ is true} \\
c. \text{FA sensitive mention-some reading:} \\
   \quad - \text{for at least one true answer } a \in A, \text{ John knows that } a \text{ is true} \\
   \quad - \text{for any false (partial) answer } a \in A^\vee, \text{ John does not believe that } a \text{ is true}
\]

We will follow Klinedinst and Rothschild (2011) in referring to FA sensitive WE readings as intermediate exhaustive (IE) readings.

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^9This characterization of strong exhaustivity, which has been chosen here for expository purposes, does not correspond completely to that given in Groenendijk and Stokhof (1984). Under the latter, John would be required to know what the extension of the predicate call is. The two notions do coincide if we assume that John is fully informed about which individuals constitute the domain of discourse.
When it comes to deriving the different readings, some theories focus exclusively on FA sensitive MS readings (George, 2011) while others focus on IE readings (Spector, 2005; Klinedinst and Rothschild, 2011; Uegaki, 2015; Spector and Egré, 2015; Cremers, 2016). Like Xiang (2016a), we will aim to give a general account of FA sensitivity that applies uniformly across the different levels of exhaustive strength.

As noted by George (2011), FA sensitivity poses a problem for reductive theories of clause embedding. To see this, observe that in George’s scenario above, Janna and Rupert know exactly the same set of relevant propositions. Indeed, the only relevant proposition that they both know is the proposition that Newstopia sells Italian newspapers. Rupert additionally believes the proposition that Celluloid City sells Italian newspapers, but he doesn’t know this proposition, simply because it is in fact false. This is problematic for reductive theories, because it shows that ‘interrogative knowledge’ is not always reducible to ‘declarative knowledge’. It reveals that interrogative knowledge may not only depend on true declarative knowledge but also on beliefs about false answers to the interrogative at hand. We will see that this is not problematic under a uniform approach to clause embedding.

Finally, we should note that not all interrogative-embedding verbs exhibit FA sensitivity effects. An example of such a verb is be certain:

(13) Rupert is certain where one can buy an Italian newspaper.

In contrast to (11b), where the same complement was embedded under know, (13) is saliently judged true in George’s scenario. Our account should explain this lack of FA sensitivity effects for verbs like be certain.10

### 3.2 General assumptions about complement constructions

We now lay out our account of clausal complements, starting with some high-level syntactic and semantic assumptions about complement constructions.

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10In talking about FA sensitivity we will draw a distinction between, on the one hand, FA sensitivity itself, which is a theoretical property that, on our account, all interrogative-embedding verbs will have, and, on the other hand, FA sensitivity effects, which are a possible, but not necessary empirical manifestation of FA sensitivity. What we mean when we say that a verb is FA sensitive is that its semantics is sensitive to answers that are false at some relevant world(s). In the case of know, the relevant world is the world of evaluation \( w_0 \), and, as we just saw, know also exhibits FA sensitivity effects. Later, when considering verbs like be certain and agree, we will see that the relevant worlds can be different from \( w_0 \); in particular, they can be worlds in the epistemic state of the attitude holder (in the case of be certain) or another relevant agent (in the case of agree). In the case of be certain, this will have as a consequence that FA sensitivity effects are absent; in the case of agree, FA sensitivity effects will still arise. See footnote 29 for further discussion. Another verb that does not exhibit FA-sensitivity effects, for different reasons, is surprise; see Section 5.2.3.
Figure 5: A wh-interrogative on its exhaustive and non-exhaustive reading.

We assume that complement constructions have the global structure in Figure 4. For reasons that we will return to in Section 3.5, declarative and interrogative complements both involve an embedding operator $E$, which adjoins to a clause that we will call the nucleus of the complement. The nucleus has the same semantic properties—for our current purposes—as a declarative respectively interrogative root clause. So, in particular, the semantic type of the nucleus of a complement is $T$.

The $E$ operator takes the nucleus as its input and returns a function from worlds to sets of propositions. Thus, it is of type $(T, (s, T))$. As we will see, this function maps every world $w$ to the set of propositions that can be thought of as truthful resolutions, in $w$, of the issue expressed by the nucleus.

Typically a verb and its complement together form a verb phrase that has the same semantic type as an intransitive verb like walk, i.e., $(e, T)$, expressing a function from individuals to sets of propositions. To achieve this, a verb that takes clausal complements has to be of type $((s, T), (e, T))$. It takes as its input a function from worlds to sets of propositions, generated by its complement, and it yields as its output a function from individuals to sets of propositions.

3.3 Interrogative complements

In this subsection, we formulate a semantics for interrogative complements, starting at the level of the nucleus (Section 3.3.1), then moving on to the level of the complement, where the $E$ operator and the notion of truthful resolutions will be introduced (Section 3.3.2).

3.3.1 Interrogative nuclei

We assume that a root wh-interrogative like (14), and thus also the nucleus of the corresponding interrogative complement, has two possible readings, an exhaustive and a non-exhaustive one.

(14) Who left?

On the exhaustive reading, the sentence raises an issue whose resolution requires establishing exactly who left and who didn’t. Let’s assume that there are two individuals in the domain, Amy and Bill. Then, in order to resolve the issue raised by (14), one would have to specify for both Amy and Bill whether they left. On the non-exhaustive reading, on the other hand, the issue can be resolved by establishing either that Amy left, or that Bill left, or that neither of them left. The meaning we take (14) to have on the exhaustive reading is depicted in Figure 5(a) and the one we take it to have on the non-exhaustive reading in Figure 5(b). As before, $w_1$ and $w_2$ in the diagrams are worlds where Amy left, while $w_3$ and $w_4$ are worlds where she didn’t leave. Furthermore, $w_1$ and $w_3$ are worlds where Bill left, and $w_2$ and $w_4$ are worlds where he didn’t leave.

Depending on the precise nature of the nucleus, either the exhaustive or the non-exhaustive interpretation may be blocked. For instance, the Dutch example in (15) only has an exhaustive interpretation, due to the presence of the exhaustivity marker allemaal, while the example
in (16), which contains the non-exhaustivity marker zoal, only has a non-exhaustive interpretation (Beck and Rullmann, 1999).

(15) Welke acteurs zijn er **allemaal** genomineerd voor een Oscar dit jaar?
which actors are there *+EXH* nominated for a Oscar this year
‘Which actors are nominated for an Oscar this year?’ (exhaustive)

(16) Welke acteurs zijn er **zoal** genomineerd voor een Oscar dit jaar?
which actors are there *−EXH* nominated for a Oscar this year
‘Which actors are nominated for an Oscar this year?’ (non-exhaustive)

The existence of such explicit (non-)exhaustivity markers in Dutch and other languages (see Li, 1995, for similar data from Mandarin and German) motivates a particular view on the division of labor between semantics and pragmatics. On our account, generating the different levels of exhaustive strength is a semantic affair. The way we suggest our analysis could be integrated into a broader theory of interpretation is the following. We assume that the semantic component makes a range of possible readings available, and the pragmatic component selects from this range that reading which was most likely intended by the speaker, given the particular context of utterance. One reason for assuming this division of labor is that the range of permissible readings of an interrogative clause can be constrained by a number of conventional means, such as the (non-)exhaustivity markers mentioned above. It is difficult to envision how pragmatics, operating at the matrix clause level, could be sensitive to these subentential, conventional ways of marking exhaustive strength. In light of this, it seems that the range of possible readings should be determined within the semantics.

We will focus here on embedding and will treat the compositional derivation of the nucleus meaning as a blackbox, referring to Champollion et al. (2015) for a concrete compositional semantics that is compatible with the account developed here. Our account of embedding is also compatible with other treatments of interrogative nuclei that derive both exhaustive and non-exhaustive interpretations (e.g., Nicolae, 2013; Theiler, 2014).11

### 3.3.2 The $E$ operator: from resolutions to truthful resolutions

As we saw in Section 2, in order to count as a resolution of some issue, a proposition has to provide enough information to resolve this issue. Naturally, if a proposition $p$ resolves an issue $P$, then any more informative proposition $q \subset p$ will resolve $P$ as well. This is the reason why sentence meanings in inquisitive semantics, which are taken to be sets of resolutions, are downward-closed.

However, unlike the meaning of an interrogative nucleus, the meaning of an interrogative complement is not represented as a plain set of resolutions, but rather as a function from worlds to sets of *truthful* resolutions. Truthful resolutions are still resolutions, but in addition they have to fulfill two requirements: (i) they need to be consistent, and (ii) they must not provide false information w.r.t. the given issue.

In order to make this more precise, we first introduce some auxiliary notation. We will write $\text{alt}_w(P)$ for the set of alternatives in $\text{alt}(P)$ that are true in $w$; and we will write $\text{alt}^*_w(P)$ for the set of all alternatives in $\text{alt}(P)$, as well as unions of such alternatives, that are false in $w$. Intuitively, $\text{alt}^*_w(P)$ can be thought of as the set of (possibly partial) answers to $P$ that are false in $w$.

---

11We will largely leave presuppositional interrogative nuclei out of consideration in this paper. For instance, a singular *which*-question like *Which student left?* is often taken to presuppose that exactly one student left. The issue of how presuppositions like this should be modeled and derived is complicated, and orthogonal to our main concerns here. We believe that our account can be extended in various ways to suitably deal with such cases. Whenever we make general claims about interrogative complements, we will indicate in a footnote whether the claim can be expected to hold of presuppositional cases as well.
Definition 3 (True alternatives and false partial answers).
For any sentence meaning $P$ and any world $w$:

- $\text{alt}_w(P) := \{ p \in \text{alt}(P) \mid w \in p \}$
- $\text{alt}^*_w(P) := \{ \bigcup Q \mid Q \subseteq \text{alt}(P) \text{ and } w \not\in \bigcup Q \}$

Given a sentence meaning $P$, we can then say that a resolution $p \in P$ provides false information w.r.t. $P$ in $w$ iff it entails some proposition in $\text{alt}^*_w(P)$. Conversely, $p$ provides no false information w.r.t. $P$ in $w$ iff it does not entail any proposition in $\text{alt}^*_w(P)$.

For instance, assume that Amy and Bill are the only individuals in the domain and that only Amy left (in the diagrams in Figure 5, this means the actual world is $w_2$). Consider the sentence meaning $P = \{ \text{who } \text{left} \}$, depicted in Figure 5(b), which contains one alternative that is true in $w_2$ (Amy left) and two alternatives that are false in $w_2$ (Bill left; neither Amy nor Bill left). This means that we get:

- $\text{alt}(P) = \{ \square \circ, \square \circ, \circ \square \}$
- $\text{alt}_w(P) = \{ \square \circ \}$
- $\text{alt}^*_w(P) = \{ \square \circ, \circ \square, \Box \circ \}$

Now, let $p$ be the proposition that Amy and Bill left ($\square \circ$). Observe that $p$ entails the alternative $q$ that Bill left ($\Box \circ$). Since $q$ is false in $w_2$ and $q$ is an alternative in $P$, $q \in \text{alt}^*_w(P)$. Hence, $p$ provides false information w.r.t. $P$. As another example, let $p'$ be the proposition that Amy didn’t leave ($\circ \square$). Now consider the alternatives ($\square \circ$) and ($\circ \square$), both of which are false in $w_2$. The union of these two alternatives ($\square \circ$) is an element of $\text{alt}^*_w(P)$. Since this union is furthermore entailed by $p'$, we find that $p'$ provides false information w.r.t. $P$ as well.

We hence arrive at the following definition of truthful resolutions. In what follows, we will occasionally make reference to the crucial third clause in this definition as the no false alternatives (NFA) condition.$^{12,13}$

Definition 4 (Truthful resolution). Let $P$ be a sentence meaning and $w$ a possible world. A proposition $p$ is a truthful resolution of $P$ in $w$ iff:

(i) $p$ is a resolution of $P$ ($p \in P$),
(ii) $p$ is consistent ($p \neq \emptyset$),
(iii) NFA condition: $p$ doesn’t provide information w.r.t. $P$ that is false in $w$ ($\exists q \in \text{alt}^*_w(P) : p \subseteq q$).

We further distinguish between truthful resolutions simpliciter and complete truthful resolutions, which entail all true alternatives.

Definition 5 (Complete truthful resolution). Let $P$ be a sentence meaning and $w$ a possible world. A proposition $p$ is a complete truthful resolution of $P$ in $w$ iff:

(i) $p$ is a truthful resolution of $P$ in $w$,

$^{12}$This NFA condition is stronger than the one we assumed in Theiler et al. (2016), in that it does not only exclude propositions which entail false alternatives, but also those that entail false unions of alternatives. See Section 3.1 for the motivation behind this stronger formulation, due to Xiang (2016a).

$^{13}$Inquisitive semantics is a support-based rather than a truth-based semantic framework. It would therefore actually be more in the spirit of the framework if we didn’t compute truthful resolutions at a specific world of evaluation, but rather relative to a set of worlds, i.e., an information state. For reasons of presentation, we will nonetheless take the former route here.
Formally, we can characterize yields a function mapping every world

We now turn to defining the

diagram). There is no proposition in

First, let

truthful resolutions because they are consistent resolutions and cannot entail any proposition

alt

in

truthful resolution. More interesting, however, is the question which subsets of

truthful resolutions embody a notion of truth radically relativized to a given issue: they must not provide any false information w.r.t. to that issue.

To exemplify the distinction between truthful resolutions simpliciter and complete truthful resolutions, consider a scenario in which there are three people, Amy, Bill and Clara. Assume that in world

Amy and Bill left, but Clara didn’t. Again, let

. Then, the proposition that Amy left, the proposition that Bill left, and the proposition that both of them left are all truthful resolutions of

in

. The proposition that both of them left is additionally a complete truthful resolution of

in

. The proposition that Amy, Bill and Clara left, on the other hand, is not a truthful resolution of

in

. Observe that, once we make the step from resolutions to truthful resolutions, we are not dealing with downward-closed sets anymore: although the proposition that Amy, Bill and Clara left is a subset of the proposition that Amy left, only the latter is a truthful resolution of

in

.

Crucially, although a truthful resolution

has to entail a true alternative, 

itself need not be true. For instance, assume that in the above scenario it is Monday. Consider the proposition

that Amy left and that it is Tuesday. Clearly, 

is false. Nonetheless, it counts as a truthful resolution because it only provides true information w.r.t. the issue of who left; the false information that it provides—namely that it is Tuesday—is not relevant w.r.t. the issue of who left. In this sense we may say that truthful resolutions embody a notion of truth radically relativized to a given issue: they must not provide any false information w.r.t. to that issue.

To get a more visual understanding of this concept, let us return to a scenario in which there are just two people who might have left, Amy and Bill. Let

. This is depicted in Figure 6, where

is the proposition that Amy left and

is the proposition that Bill left. Suppose that the actual world,

, is located in

, but not in

, as depicted in Figure 6. That is, only Amy left in

. This means that

. Let us now reflect on which propositions in the picture count as truthful resolutions in

. Clearly, 

itself is a truthful resolution. More interesting, however, is the question which subsets of

are truthful resolutions and which are not. To begin with, all true propositions entailing

are automatically truthful resolutions because they are consistent resolutions and cannot entail any proposition in

. On the other hand, with false propositions that entail

, we have to distinguish two cases. First, let

be the proposition that both Amy and Bill left (the crossed-out proposition in the diagram). Since 

entails

, it does not count as a truthful resolution. Second, let

be some other consistent proposition such that

, but

, (the one with a tick mark in the diagram). There is no proposition in

that is entailed by

. Hence, although both

and

are false, 

counts as a truthful resolution in

whereas

doesn’t.

We now turn to defining the

operator. When applied to a nucleus meaning

, this operator yields a function mapping every world

to the set of (complete) truthful resolutions of

in

. Formally, we can characterize

, which comes in a complete and a non-complete variant, as follows.\textsuperscript{14}

\textsuperscript{14} Although at first glance the non-complete and complete variant of

might appear to differ only minimally, formally they actually come apart in a fundamental way. The computation carried out by

is an innocent type-shift, in the sense that if we have a function

, we can retrieve the set

from

, since

. In contrast, the computation carried out by

is not an innocent type-shift, in
Definition 6 (The $E$ operator).

\[
E_{[-\text{cmp}]} \equiv \lambda P, \lambda w, \lambda p. \left( p \in P \land p \neq \emptyset \land 
-\exists q \in \text{alt}_w(P) : p \subseteq q \right)
\]

\[
E_{[+\text{cmp}]} \equiv \lambda P, \lambda w, \lambda p. \left( p \in P \land p \neq \emptyset \land 
-\exists q \in \text{alt}_w(P) : p \subseteq q \land 
\forall q \in \text{alt}_w(P) : p \subseteq q \right)
\]

For an illustration of the functions that $E$ yields, consider the examples below, which show the result of applying this operator to typical interrogative nucleus meanings.\(^{15}\)

\begin{align*}
(17) \quad & E_{[-\text{cmp}]}(\begin{array}{c}
\text{w1} \\
\text{w2} \\
\text{w3} \\
\text{w4}
\end{array}) = \left\{ \\
& \begin{array}{c}
\text{w1} \mapsto \{\begin{array}{c}
\bullet \\
\circ
\end{array} \} \\
\text{w2} \mapsto \{\begin{array}{c}
\circ \\
\circ \\
\circ \\
\circ
\end{array} \} \\
\text{w3} \mapsto \{\begin{array}{c}
\bullet \\
\circ
\end{array} \} \\
\text{w4} \mapsto \{\begin{array}{c}
\circ \\
\circ \end{array} \}
\end{array} \right\} \\
\end{align*}

\begin{align*}
(18) \quad & E_{[+\text{cmp}]}(\begin{array}{c}
\text{w1} \\
\text{w2} \\
\text{w3} \\
\text{w4}
\end{array}) = \left\{ \\
& \begin{array}{c}
\text{w1} \mapsto \{\begin{array}{c}
\bullet \\
\circ
\end{array} \} \\
\text{w2} \mapsto \{\begin{array}{c}
\circ \\
\circ \\
\circ \\
\circ
\end{array} \} \\
\text{w3} \mapsto \{\begin{array}{c}
\bullet \\
\circ
\end{array} \} \\
\text{w4} \mapsto \{\begin{array}{c}
\circ \\
\circ \end{array} \}
\end{array} \right\} \\
\end{align*}

\begin{align*}
(19) \quad & E_{[-\text{cmp}]}(\begin{array}{c}
\text{w1} \\
\text{w2} \\
\text{w3} \\
\text{w4}
\end{array}) = \left\{ \\
& \begin{array}{c}
\text{w1} \mapsto \{\begin{array}{c}
\bullet \\
\circ
\end{array} \} \\
\text{w2} \mapsto \{\begin{array}{c}
\circ \\
\circ \\
\circ \\
\circ
\end{array} \} \\
\text{w3} \mapsto \{\begin{array}{c}
\bullet \\
\circ
\end{array} \} \\
\text{w4} \mapsto \{\begin{array}{c}
\circ \\
\circ \end{array} \}
\end{array} \right\} \\
\end{align*}

\begin{align*}
E_{[+\text{cmp}]}(\begin{array}{c}
\text{w1} \\
\text{w2} \\
\text{w3} \\
\text{w4}
\end{array}) = \left\{ \\
& \begin{array}{c}
\text{w1} \mapsto \{\begin{array}{c}
\bullet \\
\circ
\end{array} \} \\
\text{w2} \mapsto \{\begin{array}{c}
\circ \\
\circ \\
\circ \\
\circ
\end{array} \} \\
\text{w3} \mapsto \{\begin{array}{c}
\bullet \\
\circ
\end{array} \} \\
\text{w4} \mapsto \{\begin{array}{c}
\circ \\
\circ \end{array} \}
\end{array} \right\}
\end{align*}

Observe that, as anticipated, sets of truthful resolutions are not always downward-closed. For instance, $E_{[-\text{cmp}]}(\begin{array}{c}
\text{w1} \\
\text{w2} \\
\text{w3} \\
\text{w4}
\end{array})(w_2)$ contains $\begin{array}{c}
\bullet \\
\circ
\end{array}$, but not $\begin{array}{c}
\circ \\
\circ
\end{array}$. Further observe that if $E$ applies to an exhaustive nucleus meaning $P$, as in (19), $E_{[+\text{cmp}]}(P)$ and $E_{[-\text{cmp}]}(P)$ coincide. This is the case because if $P$ is exhaustive, then $\text{alt}_w(P)$ is a singleton set for every $w$, which means that any truthful resolution in $w$ is automatically a complete truthful resolution in $w$.

For easy reference, we will refer below to truthful resolutions that result from applying $E_{[-\text{cmp}]}$ to a non-exhaustive nucleus meaning, as in (17), as mention-some (MS) truthful resolutions; similarly, when $E_{[+\text{cmp}]}$ applies to a non-exhaustive nucleus meaning, as in (18), we will speak of intermediate exhaustive (IE) truthful resolutions, and when $E_{[+\text{cmp}]}$ or $E_{[-\text{cmp}]}$ applies the sense that if $f = E_{[+\text{cmp}]}(P)$, we cannot retrieve $P$ from $f$. To see this, consider the following two sets of resolutions: $P_1 = \{\begin{array}{c}
\bullet \\
\circ
\end{array}, \begin{array}{c}
\circ \\
\circ
\end{array}, \begin{array}{c}
\bullet \\
\circ
\end{array}, \begin{array}{c}
\circ \\
\circ
\end{array}\}$ and $P_2 = \{\begin{array}{c}
\circ \\
\circ
\end{array}, \begin{array}{c}
\circ \\
\circ
\end{array}, \begin{array}{c}
\circ \\
\circ
\end{array}, \begin{array}{c}
\circ \\
\circ
\end{array}\}$, where the downarrow (↓) indicates closure under subsets. Applying $E_{[+\text{cmp}]}$ to either $P_1$ or $P_2$ yields the same function $f = \{w_1 \mapsto \begin{array}{c}
\circ \\
\circ
\end{array}, w_2 \mapsto \begin{array}{c}
\circ \\
\circ
\end{array}, w_3 \mapsto \begin{array}{c}
\circ \\
\circ
\end{array}, w_4 \mapsto \begin{array}{c}
\circ \\
\circ
\end{array}\}$. So, in general it is impossible to retrieve $P$ from $E_{[+\text{cmp}]}(P)$.

\(^{15}\)In these examples, we assume that the four worlds are labelled as in Figure 5: w1 is the world in the upper left corner, w2 the one in the upper right corner, etcetera.
to an exhaustive nucleus meaning, as in (19), we will speak of *strongly exhaustive* (SE) truthful resolutions. This terminology is summarized in the following table.

<table>
<thead>
<tr>
<th>E</th>
<th>nucleus_{-exh}</th>
<th>nucleus_{+exh}</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_{-cmp}</td>
<td>mention-some</td>
<td>strongly exhaustive</td>
</tr>
<tr>
<td>E_{+cmp}</td>
<td>intermediate exhaustive</td>
<td>strongly exhaustive</td>
</tr>
</tbody>
</table>

### 3.4 A basic treatment of *know*

So far, we laid out an account of interrogative complements. This account, however, only yields concrete predictions when combined with an analysis of the verbs that take such complements as their argument. Instead of diving right into the full range of verbs, though, we will first zoom in on the case of *know*.

We will formally characterize the meaning of *know* in terms of the subject $x$’s *information state* in a world $w$, which we understand to be the set of worlds compatible with what $x$ takes to be the case in $w$. We will write $\text{DOX}_x^w$ for this set. Crucially, an individual’s information state in $w$ does not have to contain $w$ itself, i.e., it does not necessarily hold that $w \in \text{DOX}_x^w$. As is commonplace in doxastic logic, we do assume that $\text{DOX}_x^w$ is always consistent (i.e., non-empty) and that $x$ always knows what her own information state is (i.e., $\text{DOX}_v^w = \text{DOX}_x^w$ for all $v \in \text{DOX}_x^w$).

We assume the following basic entry for *know*.$^{16}$

\[ \text{know}' := \lambda f(s,T), \lambda x. \lambda p. \forall w \in p : \text{DOX}_x^w \in f(w) \]

In words, *know* takes the complement meaning $f$ and the subject $x$ as arguments, and yields a set of propositions. Recall that $f$ is a function mapping each world to the set of truthful resolutions of the complement in that world. Hence, the set that *know* yields contains only propositions $p$ such that for every world $w \in p$ the information state of $x$ in $w$ exactly matches one of the truthful resolutions in $f(w)$.

This entry for *know* differs from classical accounts in two respects. Firstly, on classical accounts, $f(w)$ has a fixed exhaustive strength; i.e., it is the true WE answer in $w$ (Karttunen, 1977) or the true SE answer in $w$ (Groendijk and Stokhof, 1984). In comparison, our account is more flexible. Depending on the interpretation of the nucleus of the complement (exhaustive or non-exhaustive) and the $E$ operator (complete or non-complete), $f(w)$ will consist of MS, IE or SE truthful resolutions.

The second difference concerns the relation between $\text{DOX}_x^w$ and $f(w)$. Standardly, $f(w)$ is a single proposition rather than a set of propositions and it is required that $\text{DOX}_x^w$ is a *subset* of this single proposition. For us, $f(w)$ is a set of propositions and $\text{DOX}_x^w$ has to be an *element* of this set—we will see shortly that this is instrumental in accounting for FA sensitivity.

### 3.5 False answer sensitivity across levels of exhaustivity

On our account, FA sensitivity is captured by the NFA condition in the definition of truthful resolutions (Definition 4), which says that a proposition $p$ is only a truthful resolution of a sentence meaning $P$ in a world $w$ if it does not entail any proposition in $\text{alt}_w(P)$. Let us see what the consequences of this condition are across the different levels of exhaustive strength.

---

$^{16}$This entry is a refinement of the treatment of the knowledge modality in *inquisitive epistemic logic* (IEL) (Ciardelli and Roelofsen, 2015). In IEL, $\text{DOX}_x^w$ is simply required to coincide with a resolution of the complement. Our entry, on the other hand, requires that $\text{DOX}_x^w$ coincides with a *truthful* resolution of the complement in the world of evaluation. As we will see, it is precisely this refinement that allows us to capture FA sensitivity.
We begin with George’s scenario, involving an MS example. Recall that in the actual world \( w_0 \) only Newstopia and Paperworld sell Italian newspapers. Janna and Rupert know that Newstopia sells Italian newspapers. Additionally, Rupert falsely believes that also Celluloid City sells such newspapers. Janna has no beliefs about Celluloid City. Then, under an MS reading, (22) is judged true, while (23) is judged false.

(22) Janna knows where one can buy an Italian newspaper.

(23) Rupert knows where one can buy an Italian newspaper.

This is indeed what we predict. To see why, assume that the above complements each involve \( E_{[-\text{cmp}]} \) and the nucleus receives a \([-\text{exh}]\) interpretation, resulting in MS readings. Let \( P \) be the nucleus meaning. Observe that \( P \) contains two true alternatives, namely the proposition that one can buy an Italian newspaper at Newstopia and the proposition that one can buy an Italian newspaper at Paperworld, and two false alternatives, namely the proposition that one can buy an Italian newspaper at Celluloid City and the proposition that one cannot buy an Italian newspaper at any of the three stores. Janna’s information state \( \text{DOX}_j^{\text{n}} \) is a truthful resolution of the complement since it is consistent, it entails one of the alternatives in \( \text{alt}(P) \) and it does not entail any alternative in \( \text{alt}^*_r(\text{alt}(P)) \), while Rupert’s information state \( \text{DOX}_r^{\text{n}} \) is not a truthful resolution of the complement since it does entail a proposition in \( \text{alt}^*_r(\text{alt}(P)) \), namely the proposition that one can buy an Italian newspaper at Celluloid City. Thus, (22) comes out as true because \( \text{DOX}_j^{\text{n}} \subseteq E_{[-\text{cmp}]}(P)(w_0) \), while (23) comes out as false because \( \text{DOX}_r^{\text{n}} \not\subseteq E_{[-\text{cmp}]}(P)(w_0) \).

In the case of IE readings, FA sensitivity arises from exactly the same mechanism. Consider example (24), which is a variant of (10) with know rather than tell. Pace Groenendijk and Stokhof (1984), we assume here that know licenses IE readings, just like tell—as will be further discussed in Section 4, this assumption is supported by experimental results of Cremers and Chemla (2016).

(24) John knows who passed the exam.

Suppose that in the actual world \( w_0 \) only Anna and Bill, but not Chris and Daniel passed the exam. An IE reading arises on our account if the complement is headed by \( E_{[-\text{cmp}]} \) and the nucleus receives a \([-\text{exh}]\) interpretation. In this case we predict that the following is required for (24) to be true in \( w_0 \). John’s information state in \( w_0 \), \( \text{DOX}_j^{\text{n}} \), has to be an element of \( E_{[-\text{cmp}]}(P)(w_0) \), where \( P \) is the nucleus meaning. This means that (i) \( \text{DOX}_j^{\text{n}} \) has to be consistent, (ii) it has to entail all true alternatives in \( P \), i.e., it has to entail that Anna and Bill passed the exam, and (iii) in view of the NFA condition, it should not entail any proposition

---

\(^{17}\)Xiang (2016a) argues that there are two different kinds of false answers relevant in the context of MS readings, namely over-affirming and over-denying false answers. George (2011) only takes the former into account. To see what the difference is, consider a modified Italian-newspaper scenario. As before, Newstopia sells Italian newspapers, while Paperworld doesn’t. In addition, however, there is a third store, Celluloid City, which also sells Italian newspapers. Suppose that Janna believes one can get an Italian newspaper at Newstopia and Paperworld. Since this is a falsely positive belief, Xiang classifies it as over-affirming. Now suppose that Janna correctly believes one can buy an Italian newspaper at Newstopia, that she doesn’t have any beliefs about Paperworld, and she wrongly believes one cannot buy an Italian newspaper at Celluloid City. Since this is a falsely negative belief, Xiang classifies it as over-denying.

According to Xiang’s experimental results, in a scenario like the one above, sentences like (22) are judged false by a significant number of participants if Janna believes an over-denying answer. This could either be accounted for by assuming that the respective participants are accessing a mention-all reading instead of an MS reading or by making it part of the truth-conditions of the MS reading that over-denying beliefs are not permitted. Xiang pursues the latter strategy. Our account, as presented here, takes the former route: while over-affirming propositions are excluded from the set of truthful resolutions by virtue of the NFA condition, over-denying propositions are included. It would be easy, however, to expand the NFA condition in Definition 4 in such a way that it also rules out over-denying propositions; all we would have to demand is that a truthful resolution in \( w \) is consistent with every alternative in \( \text{alt}_w(P) \).
in $\text{alt}_{w_0}^*(P)$, i.e., it should not entail that either Chris or Daniel passed. This precisely amounts to the IE reading.

Finally, an SE reading of (24) arises on our account if the nucleus receives a $[+\text{exh}]$ interpretation. In this case the alternatives in the nucleus meaning $P$ form a partition of the logical space such that all worlds in any given partition cell agree on who passed the exam and who didn’t. Now suppose that the complement is headed by $E_{[-\text{cmp}]}$. In this case we predict that for (24) to be true in $w_0$ it is required that (i) $\text{DOX}_{j}^{w_0}$ is consistent, (ii) $\text{DOX}_{j}^{w_0}$ is a resolution of $P$, which means that it entails one of the alternatives in $P$, and (iii) in view of the NFA condition, it should not entail any proposition in $\text{alt}_{w_0}(P)$. Taken together, requirements (ii) and (iii) imply that $\text{DOX}_{j}^{w_0}$ has to entail a true alternative in $P$. There is only one true alternative in $P$, which is the proposition that Anna and Bill passed the exam and Chris and Daniel didn’t. It is required, then, that John’s information state entails this proposition, which is again precisely what we expect under an SE reading.

If we assume that the complement is headed by $E_{[+\text{cmp}]}$ rather than $E_{[-\text{cmp}]}$ we get an additional completeness requirement, namely, that $\text{DOX}_{j}^{w_0}$ should entail all true alternatives in $P$. However, since $P$ forms a partition here, we know that it contains only one true alternative. So the completeness requirement is vacuous in this case, and the end result is exactly the same as with $E_{[-\text{cmp}]}$.

Let us end this subsection with a comment on the division of labor we assume between the $E$ operator and the embedding verb. On our account, the $[\pm\text{cmp}]$ ambiguity is situated at the level of the $E$ operator. One may wonder whether this ambiguity could be incorporated into the meaning of the embedding verb instead. However, coordination data suggest that this would be problematic. To see this, first consider the sentences in (25) and (26) below. As shown experimentally in Cremers and Chemla (2016), the sentence in (25) most prominently receives an IE reading.\(^{18}\) On the other hand, the sentence in (26) most prominently receives an MS reading.

(25) John knows which Spanish newspapers are sold at the corner store.
(26) John knows where one can get an Italian newspaper.

Now, consider (27), in which the two interrogative complements from (25) and (26) are con-joined.

(27) John knows which Spanish newspapers are sold at the corner store and where one can get an Italian newspaper.

The crucial observation is that the most prominent interpretation of (27) seems to be one under which the first complement receives an IE reading, just as in (25), while the second complement receives an MS reading, just as in (26). To derive this interpretation, the first complement needs to bear a $[+\text{cmp}]$ feature, while the second complement needs to bear a $[-\text{cmp}]$ feature. If completeness were taken to be a feature of the verb, it would be impossible to derive the interpretation, since the complex complement clause could only be interpreted $[+\text{cmp}]$ as a whole or $[-\text{cmp}]$ as a whole.\(^{19}\)

This concludes our treatment of interrogative complements embedded under know. What we have seen in this subsection is that our notion of truthful resolutions, in particular the NFA condition, captures FA sensitivity in a uniform way across the different levels of exhaustivity. We now turn to declarative complements.

\(^{18}\)Cremers and Chemla’s experiment will be discussed in more detail in Section 4 below.

\(^{19}\)Thanks to Lucas Champollion (p.c.) for pointing this out. The line of reasoning is borrowed from an argument that has been made in connection with distributivity, cf. Dowty (1987).
3.6 Declarative complements

Even though we focused on interrogative complements so far, our account has been set up in such a way that it can directly be applied to declarative complements as well. Here, we will go into two specific predictions: (i) any truthful resolution of a declarative complement is complete, (ii) the set of truthful resolutions of a declarative complement is always fully downward closed.

3.6.1 All truthful resolutions are complete

In order to see what happens when the $E$ operator applies to a declarative nucleus, let us look at a concrete example:

\[(28) \quad E_{-cmp}(\begin{array}{c}
\text{or} \\
\text{or}
\end{array}) = E_{+cmp}(\begin{array}{c}
\text{or} \\
\text{or}
\end{array}) = \begin{cases}
w_1 & \mapsto \{\begin{array}{c}
\text{or} \\
\text{or}
\end{array}, \begin{array}{c}
\text{or} \\
\text{or}
\end{array}
\}
w_2 & \mapsto \{\begin{array}{c}
\text{or} \\
\text{or}
\}
w_3 & \mapsto \emptyset
\end{cases}\]

In (28), $E_{\pm cmp}$ applies to the nucleus meaning $P = \{\begin{array}{c}
\text{or} \\
\text{or}
\end{array}\}$, which, since it is the meaning of a declarative nucleus, only contains a single alternative. Observe that the complete and the non-complete version of $E$ yield the same result here. This is because any truthful resolution of a declarative complement is automatically also a complete truthful resolution. To see why, suppose that $p$ is a truthful resolution. Since a declarative nucleus meaning only contains a single alternative $q$, we know that $p$ entails $q$ and that $q$ is true. But this means, again because $q$ is the only alternative, that $p$ entails every true alternative in the nucleus meaning. Hence, $p$ is also a complete truthful resolution.

As a consequence, while with interrogative complements our account generates multiple readings (MS, IE, and SE), in the case of declarative complements it always generates just one reading.

3.6.2 The set of truthful resolutions is downward closed

As we have seen in Section 3.3.2, when $E$ applies to a non-exhaustive interrogative nucleus, the resulting set of truthful resolutions only exhibits a restricted form of downward closedness. If $E$ applies to a declarative nucleus, however, the resulting set of truthful resolutions is always fully downward closed. To see why, consider an arbitrary declarative nucleus meaning $P$ and let $q$ be the unique alternative in $P$. Then, if $w \in q$, we have that $E(P)(w) = \{q\}$, while if $w \notin q$, we have that $E(P)(w) = \emptyset$. As a consequence, the set of truthful resolutions is always fully downward closed.

To illustrate this, consider the following example:

\[(29) \quad \text{Rupert knows that one can buy an Italian newspaper at Newstopia.}\]

Let $p$ be the proposition that Newstopia sells Italian newspapers, and $r$ the proposition that both Newstopia and Paperworld sell Italian newspapers. Now, since in the case of a declarative complement, the set of truthful resolutions is downward closed, both $p$ and $r$ are truthful resolutions. This is why it is correctly predicted that (29) is true even if Rupert wrongly believes $r$.\(^{20}\)

\(^{20}\)For a visual understanding, compare the declarative complement meaning in (28) with the interrogative complement meaning in (17). Consider the set of truthful resolutions in $w_2$. In (28), this includes the proposition $\begin{array}{c}
\text{or} \\
\text{or}
\end{array}$, but not so in (17). This is because in (17), but not in (28), the proposition in question implies an alternative that is false in $w_2$. 

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To take stock, we now have a uniform account of declarative and interrogative complements embedded under know. The effects of this semantics depend on whether the complement is declarative or interrogative: (i) in the case of declarative complements, the set of truthful resolutions is fully downward closed, while in the case of interrogative complements it may exhibit a restricted form of downward closedness, which results in FA sensitivity effects; and (ii) the MS, IE and SE reading come apart for interrogative complements, but coincide for declarative complements.

4 Do intermediate exhaustive readings for know exist?

In the previous section, we have derived IE readings for interrogative knowledge ascriptions without considering in any detail whether such readings exist. This is, in fact, a controversial issue. In particular, Groenendijk and Stokhof (1982) explicitly argued that they do not exist, while recent experimental work by Cremers and Chemla (2016) suggests that they do. In this section, we suggest a way to reconcile these findings with Groenendijk and Stokhof's argument.

4.1 Knowledge ascription and introspection

Groenendijk and Stokhof (1982, p.180) argued that know does not license IE readings:

"Suppose that John knows of everyone who walks that he/she does; that of no one who doesn’t walk, he believes that he/she does; but that of some individual that actually doesn’t walk, he doubts whether he/she walks or not. In such a situation, John would not say of himself that he knows who walks. We see no reason to override his judgement and to claim that in this situation, John does know who walks. This seems to suggest that for John to know who walks, he should not only know of everyone who walks that he/she does, but also of everyone who doesn’t that he/she doesn’t."

Many authors have found this argument convincing and have therefore assumed, with Groenendijk and Stokhof, that know only allows for SE and MS readings.

However, recent experimental work by Cremers and Chemla (2016) seems to show quite clearly that know does license IE readings. Cremers and Chemla asked the participants in their experiment to consider the following context:

There is a set of cards, each consisting of four squares. Each square can be blue (B), green (G) or red (R). John is playing a game with these cards: he uncovers a card, looks at it briefly and tries to remember which of the squares on the card were blue. In the first round, the card he looked at was the left one in Figure 7. Now, consider two different scenarios: in scenario A, John’s beliefs about the card he looked at are as represented by the second picture in Figure 7; in scenario B, John’s beliefs about the card he looked at are as represented by the third picture in Figure 7.

Now consider the following sentence:

(30) John knew which squares were blue.

Cremers and Chemla found that (30) was saliently judged false in scenario A, while it was saliently judged true in scenario B. This can only be the case if the complement in (30) received an IE reading. Under an SE reading the sentence would have been judged false in both scenarios.\(^{21}\)

\(^{21}\)An MS reading is in general unavailable for plural which-interrogatives with a distributive predicate, such as the one in (30).
How could this experimental result be reconciled with the widely held view, rooted in Groenendijk and Stokhof’s argument, that \textit{know} does not license IE readings? What is crucial, we believe, is to recognize that knowledge ascriptions are multiply ambiguous: besides the different readings of the complement, the verb itself also allows for two different interpretations. Groenendijk and Stokhof only considered one of these interpretations, namely the one that requires a strong form of \textit{introspection} on the part of the individual to whom knowledge is ascribed. For Groenendijk and Stokhof it is unwarranted to claim that John knows who walks in a situation in which John would not say of himself that he knows who walks. Another interpretation, however, seems to be made particularly salient in the experimental setting of Cremers and Chemla. Here, it is not really at stake whether John would say of himself that he knew which squares were blue; rather, what is at stake is whether we, as external observers, find that there is a sufficient match between John’s beliefs (the second/third picture) and actuality (the first picture).

Thus, Groenendijk and Stokhof assumed an \textit{internal} interpretation of knowledge ascriptions, requiring a strong form of introspection, whereas Cremers and Chemla’s experimental setting lends particular salience to what we will call an \textit{external} interpretation of knowledge ascriptions, which does not come with the relevant introspection requirement.

Our aim will be twofold. First, we want to capture the difference between these two interpretations, i.e., the pertinent notion of introspection. And second, we want our theory to derive that the external interpretation is indeed compatible with IE readings, while the internal interpretation (Groenendijk and Stokhof’s) is incompatible with such readings.

### 4.2 Internal and external interpretation of \textit{know}

The lexical entry for \textit{know} given in Section 3.4 is repeated in (31). This entry captures the external interpretation of \textit{know}.

\begin{equation}
\text{know'} := \lambda f_{(s,T)} \cdot \lambda x. \lambda p. \forall w \in p : \text{DOX}_x^w \in f(w)
\end{equation}

We will now strengthen this entry to capture the internal interpretation. This will be done by requiring a certain kind of introspection on the part of the subject, which goes beyond just knowing what her own information state is. There are at least two natural ways to spell out this introspection condition. We will first introduce what we dub \textit{resolution introspection}, then what we call \textit{Heim introspection}, and finally compare the two, concluding in favor of the former.

The idea of \textit{resolution introspection} is very simple: besides requiring that \(\text{DOX}_x^w \in f(w)\), i.e., that \(x\)’s information state in \(w\) matches one of the truthful resolutions of the complement in \(w\), we also require that \(x\) is fully aware of this match, i.e., that every world she considers possible is one where her information state matches one of the truthful resolutions of the complement.

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\footnote{We are much indebted to Jeroen Groenendijk for discussion of this issue.}
in that world. Formally: $\forall v \in \text{DOX}_x^w : \text{DOX}_x^w \in f(v)$. In other words, under the internal interpretation it is not sufficient if $x$’s information state just happens to coincide with a truthful resolution in the world of evaluation; $x$ also has to take herself to know that this is the case. Incorporating this requirement results in the following entry: 23

\begin{equation}
(32) \quad \text{know}'_{\text{int}} = \lambda f(s, T). \lambda x. \lambda p. \forall w \in p : (\text{DOX}_x^w \in f(w) \land \forall v \in \text{DOX}_x^w : \text{DOX}_x^v \in f(v))
\end{equation}

Recall that for all $v \in \text{DOX}_x^w$, $\text{DOX}_x^v = \text{DOX}_x^w$. Thus, $\text{know}'_{\text{int}}$ can also be formulated as follows, without making reference to $\text{DOX}_x^v$.

\begin{equation}
(33) \quad \text{know}'_{\text{int}} = \lambda f(s, T). \lambda x. \lambda p. \forall w \in p : (\text{DOX}_x^w \in f(w) \land \forall v \in \text{DOX}_x^w : \text{DOX}_x^v \in f(v))
\end{equation}

We now turn to another way of spelling out the introspection condition in the entry for $\text{know}'_{\text{int}}$. Namely, instead of requiring, as we just did, that the subject has to take herself to know a truthful resolution, we could also proceed along the lines of Heim (1994) and demand that the subject has to take herself to know what the set of truthful resolutions is in the world of evaluation. 24 We will refer to this requirement as Heim introspection. Put more loosely, the relevant difference is between taking yourself to know that you have a truthful resolution (resolution introspection) and taking yourself to know what the truthful resolutions are (Heim introspection). Given a world of evaluation $w$, Heim introspection amounts to\footnote{One way to understand the role of this introspection requirement is to compare our system to standard doxastic logic. There, the notion of knowledge is limited to declarative knowledge, and the condition that $\text{DOX}_x^v = \text{DOX}_x^w$ for every $v \in \text{DOX}_x^w$—let’s call this condition information state introspection—guarantees the validity of the positive and negative introspection principles: $\varphi \rightarrow KK\varphi$ and $\neg K\varphi \rightarrow K\neg K\varphi$ for all declarative complements $\varphi$. By contrast, our account additionally models interrogative knowledge, and while information state introspection still guarantees the validity of the introspection principles w.r.t. declarative complements, it does not guarantee their validity w.r.t. interrogative complements. Once we add resolution introspection, however, the principles do become generally valid. In other words, $\text{know}'_{\text{int}}$ guarantees full introspection w.r.t. declarative and interrogative complements, whereas $\text{know}_{\text{int}}$ only guarantees introspection w.r.t. declarative complements.}

\begin{equation}
(34) \quad \text{know}_{\text{Heim}} = \lambda f(s, T). \lambda x. \lambda p. \forall w \in p : (\text{DOX}_x^w \in f(w) \land \forall v \in \text{DOX}_x^w : f(v) = f(w))
\end{equation}

In terms of empirical predictions, $\text{know}_{\text{Heim}}$ and $\text{know}'_{\text{int}}$ only come apart when taking an interrogative complement with an MS reading. To see this, consider (35) in George’s scenario, which was discussed in Section 3.1.

\begin{equation}
(35) \quad \text{Janna knows where one can buy an Italian newspaper.}
\end{equation}

It seems to us that Janna, since she takes herself to know that one can buy an Italian newspaper at Newstopia, would say of herself that she knows where one can buy an Italian newspaper—although she is not certain whether other stores sell such newspapers as well. Accordingly, we think (35) should come out true under an internal interpretation of $\text{know}$ and an MS reading of the complement.

Let us check which predictions the two introspection requirements make. For simplicity, let us assume that the only two relevant stores are Newstopia and Celluloid City (in George’s original scenario there is a third store as well, Paperworld, but this can be left out of consideration for

\footnote{It should be noted that Heim (1994) is not concerned with formulating an introspection condition, in fact, but with deriving SE answers from complete answers. To do so, she defines two different notions of answers. Given a question $Q$ and a world $w$, her $\text{answer}1(Q)(w)$ is the true complete answer of $Q$ in $w$; her $\text{answer}2(Q)(w)$ is the set of all worlds $v$ such that $\text{answer}1(Q)(v)$ is the same as $\text{answer}1(Q)(w)$. Hence, if you take yourself to know $\text{answer}2(Q)(w)$, you take yourself to know what $\text{answer}1(Q)(w)$ is. Translated into our framework, this amounts to taking yourself to know what the set of truthful resolutions in $w$ is.}
our current purposes). Assume that in \( w_1 \), Italian newspapers are sold at both stores, in \( w_2 \) only at Newstopia, in \( w_3 \) only at Celluloid City, and in \( w_4 \) at neither of the two stores. If we assume an MS interpretation of the complement, the complement meaning is \( f = E_{[\text{cmp}]}(w_2) \), where the actual world is \( w_2 \). Janna’s information state is \( \text{dox}^{w_2}_{j} = \text{dox}^{w_2}_{j} \). Furthermore, we get the following sets of truthful resolutions: \( f(w_1) = \{ \text{dox}^{w_1}_{1}, \text{dox}^{w_1}_{2}, \text{dox}^{w_1}_{3}, \text{dox}^{w_1}_{4} \} \) and \( f(w_2) = \{ \text{dox}^{w_2}_{1}, \text{dox}^{w_2}_{2} \} \). Hence, the resolution introspection requirement, \( \forall v \in \text{dox}^{w_2}_{j} : \text{dox}^{w_2}_{j} = f(v) \), is satisfied since \( \text{dox}^{w_1}_{1}, \text{dox}^{w_1}_{2}, \text{dox}^{w_1}_{3}, \text{dox}^{w_1}_{4} \in f(w_1) \) and \( \text{dox}^{w_2}_{1}, \text{dox}^{w_2}_{2} \in f(w_2) \). On the other hand, the Heim introspection requirement, \( \forall v \in \text{dox}^{w_2}_{j} : f(v) = f(w_2) \), is not satisfied since \( f(w_1) \neq f(w_2) \). This means that \( \text{know}'_{\text{Heim}} \) predicts (35) to be false in \( w_2 \), contra our intuitions, while \( \text{know}'_{\text{int}} \) predicts (35) to be true in \( w_2 \), as desired. Thus, we will use resolution introspection rather than Heim introspection in modelling the internal interpretation of \( \text{know} \).

### 4.3 Availability of IE readings for \( \text{know} \)

Whether a sentence like \( \text{John knows who called} \) is true depends on two factors on our account: whether the verb receives an internal or an external interpretation, and whether the complement gets an MS, IE, or SE reading. Interestingly, however, these two factors interact in such a way that only three distinct readings are predicted (rather than six, as one would expect prima facie). More specifically, as depicted in Table 2, we can establish the following two Facts, the proofs of which are given in Appendix B.1.

**Fact 1.** If the complement receives an MS or an SE interpretation, then the external and the internal interpretation of the verb yield exactly the same reading for the sentence as a whole.

**Fact 2.** If the verb receives an internal interpretation, then the IE and the SE interpretation of the complement yield exactly the same reading for the sentence as a whole.

In view of Fact 1, we will from now on always assume our basic entry for \( \text{know} \) when the complement receives an MS or SE interpretation.

Fact 2 says that, under an internal interpretation of the verb, what is required for \( \text{John knows who called} \) to be true on an IE reading is exactly the same as what is required on an SE reading. Namely, of all people who called, John needs to know that they called, and moreover he needs to know that nobody else called. Thus, Groenendijk and Stokhof’s claim that \( \text{know} \) does not allow for an IE reading is salvaged, though only under an internal interpretation of the verb, the interpretation that they seem to have had in mind.

On the other hand, under an external interpretation of the verb, IE readings are predicted to exist independently of SE ones. This accounts for the findings of Cremers and Chemla (2016), whose experimental setting arguably made the external interpretation of the verb particularly salient.

As Table 2 shows, the three readings that we predict for \( \text{John knows who called} \) can all be derived with our basic entry for the verb, \( \text{know}' \), which was intended to capture the external interpretation. Our second entry, \( \text{know}'_{\text{int}} \), does not yield any additional readings, i.e., it does not overgenerate. Still, these two entries, and the underlying distinction between the internal and the external interpretation of \( \text{know} \), make it possible to reconcile Groenendijk and Stokhof’s argument with Cremers and Chemla’s experimental findings.

An additional prediction is that when we consider self-ascriptions of knowledge by speakers who can be assumed to comply with the Gricean maxims of cooperative conversational behavior, then the IE reading will coincide with the SE reading even under an external interpretation of the verb. To see why, consider the following example.

\[
(36) \quad I \text{ know who called.}
\]
<table>
<thead>
<tr>
<th></th>
<th>external</th>
<th>internal</th>
</tr>
</thead>
<tbody>
<tr>
<td>mention some</td>
<td>$r_1$</td>
<td>$r_2$</td>
</tr>
<tr>
<td>intermediate exhaustive</td>
<td>$r_3$</td>
<td>$r_4$</td>
</tr>
<tr>
<td>strongly exhaustive</td>
<td>$r_5$</td>
<td>$r_6$</td>
</tr>
</tbody>
</table>

Table 2: The predicted readings of interrogative knowledge ascriptions.

Assume an external interpretation of the verb and an IE interpretation of the complement. Then the sentence is true in $w$ just in case the speaker’s information state in $w$ coincides with an IE truthful resolution in $w$, i.e., just in case $\text{DOX}_x^w \in f_{\text{IE}}(w)$, where $x$ is the speaker and $f_{\text{IE}}(w)$ the set of IE truthful resolutions of the complement in $w$. Now, we can assume that the speaker is complying with the Gricean maxims, in particular with Quality. This means that she should believe that her information state coincides with an IE truthful resolution of the complement, i.e., every world $v \in \text{DOX}_x^w$ should be such that $\text{DOX}_x^v \in f_{\text{IE}}(v)$. From here we can derive, as is done in the proof of Fact 2 in Appendix B.1, that it must be the case that $\text{DOX}_x^w \in f_{\text{SE}}(w)$, where $f_{\text{SE}}(w)$ is the set of SE truthful resolutions of the complement in $w$, i.e., it must be the case that the sentence is true under an SE reading.

Thus, we have seen that by distinguishing between an internal and an external interpretation of know, we can reconcile the different views on whether interrogative knowledge ascriptions allow for IE readings.

5 Capturing diversity among responsive verbs

So far we have restricted our attention to only one verb, know. Once we turn to a broader range of responsive predicates, we find interesting differences between them. The aim of this section is to demonstrate that our framework is flexible enough to accommodate these differences. We start out in Section 5.1 by defining the notions of veridicality and factivity. On the one hand this will lead to a refinement of our entry for know, on the other hand it and will help to appreciate the differences between the responsive predicates. In Section 5.2, we then discuss the cases of be certain, be right/be wrong, be surprised, and care, providing a lexical entry for each of these verbs. Finally, in Section 5.3, we point out a number of entailment patterns that are predicted to hold between the various predicates.

5.1 Veridicality and factivity

The notion of veridicality comes in two flavors, pertaining to declarative and interrogative complements, respectively.

5.1.1 Veridicality w.r.t. declarative complements

A verb is veridical w.r.t. declarative complements if, when taking a declarative complement, it gives rise to the implication that this complement is true. We will call this a declarative veridicality implication. For instance, know is veridical w.r.t. declarative complements, as illustrated in (37), while be certain is not, as illustrated in (38).
(37) John knows that Mary called.
\[ \because \text{Mary called.} \]

(38) John is certain that Mary called.
\[ \lnot \text{Mary called.} \]

Our account already captures the fact that know is veridical w.r.t. declarative complements. To see why, suppose that in \( w \) Mary didn’t call, and let \( P \) be the meaning of the declarative nucleus in (38), that Mary called. This is the set of propositions which consist exclusively of worlds where Mary called. Thus, \( P \) contains exactly one alternative, namely the set \( q \) of all worlds where Mary called. Since Mary didn’t call in \( w \), we find that \( \text{alt}^\text{cmp}(P)(w) = \{ q \} \). This means, however, that \( E[\text{cmp}](P)(w) \) is empty, since for a proposition \( p \) to be included in \( E[\text{cmp}](P)(w) \), it would have to hold that \( p \in P \), i.e., \( p \subseteq q \), and \( p \not\subseteq q \), which is impossible. Hence, John’s information state cannot be an element of \( E[\text{cmp}](P)(w) \), and John knows that Mary called comes out as false. Conversely, John knows that Mary called can only be true in \( w \) if Mary called is also true in \( w \).

In the case of know, the observed veridicality implication is actually a presupposition. As illustrated in (39), it projects under negation. Such veridicality implications are referred to as factivity presuppositions, and the verbs that trigger them as factive verbs.

(39) John doesn’t know that Mary called.
\[ \because \text{Mary called.} \]

On our account veridicality implications arise from the interaction between the verb and the \( E \) operator. Now, if the implication is presuppositional in nature, as in the case of know, should this presuppositional nature be determined by the \( E \) operator, or rather by the embedding verb? We opt for the latter, for the following reason. If we were to let \( E \) earmark veridicality implications as presuppositions, then we would be predicting, at least in the absence of any further stipulations, that all verbs which are veridical w.r.t. declarative complements are factive. As pointed out by Uegaki (2015) based on Egéré (2008), this prediction is too strong. There are a number of verbs triggering veridicality implications that are not presuppositional. As illustrated by (40), be right is a case in point. Sentence (40a) implies that Mary called, but this implication clearly doesn’t project under the negation in (40b).

(40) a. John is right that Mary called.
\[ \because \text{Mary called.} \]

b. John isn’t right that Mary called.
\[ \lnot \text{Mary called.} \]

We will give a lexical entry for be right in Section 5.2.2. For now, we conclude that it shouldn’t fall to the \( E \) operator to earmark veridicality implications as presuppositions. Instead, the nature of this implication only gets determined at the level of the embedding verb. For know, this can be implemented by means of a definedness restriction in the lexical entry of the verb, as is done in (41).

(41) \[ \text{know}^!' = \lambda f_{(s,T)} \lambda x. \lambda p. \forall w \in p . f(w) \neq \emptyset . \forall w \in p : \text{DOX}^w_p \in f(w) \]

Recall that \( f(w) = E(P)(w) \) is non-empty if and only if \( w \) is contained in at least one alternative in \( \text{alt}(P) \), i.e., if and only if \( w \) is contained in \( \text{info}(P) \). Also recall that a nucleus with meaning \( P \) is true in a world \( w \) if and only if \( w \in \text{info}(P) \). Taken together, this means that a proposition \( p \) satisfies the definedness restriction of \( \text{know}^! \) just in case the complement nucleus is true in every world in \( p \). In the case of a declarative complement, this amounts to a factivity presupposition. On the other hand, in the case of an interrogative complement, the alternatives in \( \text{alt}(P) \) cover the set of all possible worlds, so there will never be a world \( w \) such that \( f(w) = \emptyset \). Hence, in
In this case, the definedness restriction of know' is trivially satisfied.25

5.1.2 Veridicality w.r.t. interrogative complements

The notion of veridicality w.r.t. interrogative complements is not so straightforward. Spector and Egré (2015, footnote 7) provide the following characterization: a responsive verb V is veridical w.r.t. interrogative complements just in case for every interrogative complement Q, every individual x, and every world w, V(Q)(x) is true in w exactly if V(P)(x) is true in w, where P is a declarative complement expressing the true complete answer to Q in w. However, while the intuition behind this characterization seems clear, the exact formulation needs to be made a little more precise. One issue is that whether V(Q)(x) is true in w generally depends on whether Q receives an SE, IE, or MS interpretation. Another issue is that what constitutes a true complete answer to a given interrogative varies between different theories; for instance, for Groenendijk and Stokhof (1984) it is not the same as for Karttunen (1977).

In its existing form, Spector and Egré’s characterization wrongly classifies know as a non-veridical verb. This is because, as we have already seen, (42) below can very well be true (on an MS reading) even if Rupert doesn’t know the true complete answer (either in Karttunen’s sense or in Groenendijk and Stokhof’s sense) to the question where one can buy an Italian newspaper.

(42) Rupert knows where one can buy an Italian newspaper.

Unintended results of this kind can be avoided by making an assumption that already seems implicit in Spector and Egré’s characterization: to test whether a verb is veridical w.r.t. interrogative complements, one only needs to consider interrogative complements whose SE, IE, and MS interpretation coincide. We will call such complements exhaustivity-neutral. There are two kinds of exhaustivity-neutral interrogative complements: polar interrogatives such as whether it is raining, and wh-interrogatives such as who won the Chess World Cup this year, which involves a property that, in any possible world, applies to a unique individual. For any verb V, individual x, exhaustivity-neutral complement Q, and world w, it is unmistakable whether V(x; Q) is true in w—this doesn’t depend on the reading that Q receives. Similarly, if Q is exhaustivity-neutral it is indisputable what the true complete answer is to Q in w—Karttunen’s notion and Groenendijk and Stokhof’s notion coincide in this case. The complete answers to an exhaustivity-neutral complement always form a partition of the set of all possible worlds.

Using the notion of exhaustivity-neutral complements, we propose the following variant of Spector and Egré’s definition of veridicality w.r.t. interrogative complements. We say that V is veridical w.r.t. interrogative complements just in case for every individual x, every world w, every exhaustivity-neutral interrogative complement Q, and every declarative complement P expressing a complete answer to Q, if V(Q)(x) and P are both true in w, then V(P)(x) is true in w as well.

Under this definition, know is classified as veridical w.r.t. interrogative complements, as intended, because inferences like (43) are valid.

(43) Mary knows where John was born.
John was born in Paris.
⇒ Mary knows that John was born in Paris.

On the other hand, be certain is classified as non-veridical w.r.t. interrogative complements, because inferences like (44) are invalid.

(44) John knows which student called.
⇒ John knows which student called is defined.

In the case of a presuppositional interrogative nucleus it would also hold that f(w) is never empty, although there may be worlds where f(w) is undefined. For instance, in the case of which student called, f(w) would only be defined if exactly one student called in w. As a consequence, John knows which student called(p) would only be defined if p consisted exclusively of worlds where exactly one student called. This way the existence and uniqueness presuppositions of the nucleus would be projected to the root level.

26
Just like know worlds in propositions. Different from know propositions as its first argument, an individual (46)
coincide with a truthful resolution of John is certain who called can be true in a world (only if this happens to be a world that the subject considers possible). For instance, the complement in all worlds that the subject considers possible, not necessarily in the world of differences between the verbs. First, we take be certain (which is non-veridical and non-factive), be right and be wrong (which are veridical but not factive), be surprised (which is veridical and factive but does not exhibit FA sensitivity effects), and care (which is veridical w.r.t. declarative complements but not w.r.t. interrogative complements). We will show how these differences can be captured on our account.

5.2.1 be certain

Clearly, be certain is close in meaning to know. However, we propose that there are two differences between the verbs. First, we take be certain to be sensitive to truthful resolutions of the complement in all worlds that the subject considers possible, not necessarily in the world of evaluation (only if this happens to be a world that the subject considers possible). For instance, John is certain who called can be true in a world w even if John’s information state in w does not coincide with a truthful resolution of who called in w; as long as it does coincide with a truthful resolution of who called in some world that John considers possible. This is captured by the preliminary entry for be certain in (45). For comparison, we repeat the non-presuppositional version of our basic (external) entry for know in (46).

\[
\text{(45) be certain'} = \lambda f_\text{s,T}. \lambda x. \lambda p. \forall w \in p : \exists v \in \text{DOX}_w^w : \text{DOX}_w^w \in f(v) \quad \text{(preliminary)}
\]

\[
\text{(46) know'} = \lambda f_\text{s,T}. \lambda x. \lambda p. \forall w \in p : \text{DOX}_w^w \in f(w)
\]

Just like know, our preliminary entry for be certain takes a function f from worlds to sets of propositions as its first argument, an individual x as its second argument, and yields a set of propositions. Different from know, however, there is a layer of existential quantification over worlds in x’s information state DOX_w^w, and f is fed worlds v ∈ DOX_w^w, rather than the world of evaluation w.

Notice the subtle, but crucial change that this world shift brings: in order to determine whether John is certain who called is true in w, we don’t have to compute the set of truthful resolutions of who called in w itself, but rather in worlds v ∈ DOX_w^w. We will see below that, as a consequence be certain is not veridical and doesn’t exhibit FA sensitivity effects.

We now turn to a second difference between know and be certain. Recall that we argued that know has both an internal interpretation, which requires resolution introspection, and an external interpretation, which does not require such introspection. We propose that be certain

\[
\text{(44)} \quad \text{Mary is certain where John was born.}
\]

\[
\text{John was born in Paris.}
\]

\[
\text{Mary is certain that John was born in Paris.}
\]
only has an internal interpretation, requiring resolution introspection. In order for John is certain who called to be true in w, it is not sufficient if John’s information state in w just happens to match a truthful resolution of who called in some world that John considers possible; rather, in any world that is compatible with John’s information state such a match must obtain.26

Our preliminary entry for be certain needs to be strengthened in order to ensure resolution introspection. This can be done in the same way as we did earlier with our basic entry for know.

\[ (47) \quad \text{be certain'} = \lambda f_{(s,T)} \cdot \lambda x. \lambda p. \forall w \in p : (\exists v \in \text{DOX}^w_x : \text{DOX}^w_x \in f(v)) \]

Now, since \( \text{DOX}^w_x \) is assumed to be consistent, i.e., non-empty, the first conjunct is implied by the second. So we can simplify, and our final entry for be certain is the following:

\[ (48) \quad \text{be certain'} = \lambda f_{(s,T)} \cdot \lambda x. \lambda p. \forall w \in p : \forall v \in \text{DOX}^w_x : \text{DOX}^w_x \in f(v) \]

Our entry for be certain is very similar to that proposed by Uegaki (2015) (though the latter is formulated in a different framework). Uegaki makes two observations in support of his proposal. First, it predicts that be certain does not give rise to IE readings, unlike know. Our treatment also makes this prediction. Moreover, going one step beyond Uegaki’s proposal, it also offers an intuitive explanation for what is responsible for this contrast between be certain and know. Namely, be certain lacks an external interpretation: it is only true to say that an individual \( x \) is certain of something if \( x \) would say of herself that she is certain. On an internal interpretation, both be certain and know require resolution introspection, which is incompatible with IE readings. It is only on the external interpretation of know that it does not require resolution introspection and therefore permits IE readings.27

Uegaki’s second observation is that his entry makes desirable predictions about presupposition projection. For instance, John is certain which student left is predicted to presuppose that John believes that exactly one student left. A detailed account of presupposition projection in our framework is beyond the scope of the present paper, but it seems that under reasonable assumptions, Uegaki’s result would carry over to our treatment.28

Turning now to veridicality, our account correctly predicts that be certain is non-veridical, both w.r.t. declarative and w.r.t. interrogative complements. For instance, the argument in (38), repeated here in (49), is predicted to be invalid, because John is certain that it is raining may well be true in w if it is in fact not raining in w. The only requirement is that it is raining in all worlds that make up John’s information state in w, which may not include w itself. Similarly, the argument in (44), repeated here in (50), is also predicted to be invalid, because even if w is a world in which Mary is certain where John was born and John was born in Paris are both true, it may still be a world in which Mary is certain that John was born in Paris fails to hold. After all, if Mary is certain that John was not born in Paris but, say, in London, we predict that Mary is certain where John was born is true, even if in fact John was born in Paris. Again, the only requirement is that John was born in London in all worlds that make up Mary’s information state.

\[ (49) \quad \text{John is certain that Mary called.} \]

\[ \neg \text{Mary called.} \]

26Thus, in Cremer and Chemla’s experimental setting, if John’s beliefs about the card he saw are as depicted in the third picture in Figure 7, we would say that the sentence John is certain which squares were blue is false.
27Another advantage of our treatment of be certain in comparison with Uegaki’s is that, even though it blocks IE readings, it does allow us to derive FA sensitive MS readings as well as SE readings in a uniform way. On Uegaki’s account, SE readings are readily obtained, but deriving MS readings requires additional assumptions. We refer to Appendix A for a more general and more detailed comparison between our account and Uegaki’s.
28The main assumption that we would have to make is that \( E(\text{which student left})(w) \) is undefined whenever there is not a unique student who left in w. See also footnote 11.
Mary is certain where John was born.
John was born in Paris.
\[\not\exists \text{ Mary is certain that John was born in Paris.}\]

Finally, our account predicts that be certain does not exhibit FA sensitivity effects. For instance, (51) is correctly predicted to be true on an MS reading even if Rupert mistakenly believes that one can buy an Italian newspaper at Paperworld.

(51) Rupert is certain where one can buy an Italian newspaper.

To see that this is predicted, suppose that the complement clause in (51) contains an \(E_{[-\text{cmp}]}\) operator and that the nucleus receives a \([-\text{exh}]\) interpretation, which gives us an MS reading. Then (51) comes out as true even if Rupert mistakenly believes that both Newstopia and Paperworld sell Italian newspapers. This is the case because all the worlds in Rupert’s information state are ones where both Newstopia and Paperworld indeed sell Italian newspapers. This means that in all of these worlds, \(\text{DOX}_r^w\) is a truthful resolution of the complement.29

5.2.2 be right and be wrong

As mentioned in Section 5.1, be right is veridical w.r.t. declarative complements. This is illustrated in (52). It is also veridical w.r.t. interrogative complements, as illustrated in (53).

(52) John is right that Mary called.
\[\therefore \text{ Mary called.}\]

(53) John is right about where Mary was born.
Mary was born in Paris.
\[\therefore \text{ John is right that Mary was born in Paris.}\]

We also observed in Section 5.1 that the declarative veridicality implication of be right is not a presupposition, unlike in the case of know.

(54) John isn’t right that Mary called.
\[\not\exists \text{ Mary called.}\]

What is presupposed by both (52) and (54) is that John believes that Mary called.

(55) John is right that Mary called.
\[\therefore \text{ John believes that Mary called.}\]

\[\text{29At first glance, it might seem that this lack of FA sensitivity effects follows from the fact that be certain is non-veridical. However, there are verbs, such as agree, which are clearly non-veridical but do exhibit FA sensitivity effects. To see this, assume that Rupert believes both Newstopia and Paperworld sell Italian newspapers, while Rachel believes that only Newstopia sells such newspapers. In this context, (i) below is false, meaning that whether Rupert agrees with Rachel depends on an answer that is false according to Rachel’s beliefs.}\]

(i) Rupert agrees with Rachel about where one can buy an Italian newspaper.

A rough first approximation of a lexical entry for agree that would account for this is the following (for a more detailed discussion of this verb, see Chemla and George, 2016; Uegaki, 2017).

(ii) \(\text{agree}' = \lambda f(x,T) \cdot \lambda y, \lambda x. \lambda p, \forall w \in p : \forall v \in \text{DOX}_r^w : \text{DOX}_r^w \in f(v)\)

Recall from footnote 10 that we understand FA sensitivity not as sensitivity to answers that are false in the world of evaluation but rather as sensitivity to answers that are false in some relevant world. Under this perspective, both agree and be certain must be classified as FA sensitive: the former is sensitive to answers that are false according to the object’s information state, the latter to those that are false according to the subject’s information state. As we just saw, the FA sensitivity of agree does give rise to FA sensitivity effects, while that of be certain never manifests itself empirically since \(\text{DOX}_r^w\) cannot entail any answer that is false according to \(\text{DOX}_r^w\).
John isn’t right that Mary called. 
\[ \therefore \text{John believes that Mary called.} \]

To capture this, we take \textbf{be right} to presuppose that the subject’s information state $\text{DOX}_x^w$ coincides with a truthful resolution of the complement in all worlds that she considers possible, and to assert that $\text{DOX}_x^w$ coincides with a truthful resolution in the world of evaluation $w$. The assertive component of \textbf{be right} is hence the same as that of \textbf{know}.

\[ \text{be right}' = \lambda f_{(s,T)}, \lambda x. \lambda p. \forall w : \forall v \in \text{DOX}_x^w : \text{DOX}_x^w \in f(v), \forall w \in p : \text{DOX}_x^w \in f(w) \]

Now let us turn to \textbf{be wrong}. We first observe that this verb is non-veridical, both w.r.t. declarative complements and w.r.t. interrogative complements, as witnessed by the invalid inferences in (58) and (59).

(58) John is wrong that Mary called. 
\[ \not\exists M \text{ called.} \]

(59) John is wrong about where Mary was born. 
 Mary was born in Paris. 
\[ \not\exists \text{ John is wrong that Mary was born in Paris.} \]

In fact, \textbf{be wrong} is what we may call \textit{anti}-veridical w.r.t. declarative complements:

(60) John is wrong that Mary called. 
\[ \therefore \text{ Mary didn’t call.} \]

The anti-veridicality implication of \textbf{be wrong} is an entailment, not a presupposition, just like the declarative veridicality implication of \textbf{be right}:

(61) John isn’t wrong that Mary called. 
\[ \not\exists \text{ Mary didn’t call.} \]

Both (60) and (61) do presuppose that John \textit{believes} that Mary called, again just as in the case of \textbf{be right}. Thus, we arrive at the following entry:

(62) \textbf{be wrong}' = \lambda f, \lambda x. \lambda p. \forall w : \exists v \in \text{DOX}_x^w : \text{DOX}_x^w \in f(v), \forall w \in p : \text{DOX}_x^w \not\in f(w) \]

The only difference between \textbf{be right} and \textbf{be wrong} is that the former requires $\text{DOX}_x^w \in f(w)$, whereas the latter requires the opposite, $\text{DOX}_x^w \not\in f(w)$. This captures all the entailment patterns exemplified above.

5.2.3 \textbf{be surprised}

Emotive factives like \textbf{be surprised} show that veridicality is not a sufficient condition for \textit{FA} sensitivity effects. To see this, first note that \textbf{be surprised} is veridical w.r.t. both declarative and interrogative complements:

(63) Mary is surprised that John was born in Paris. 
\[ \therefore \text{ John was born in Paris.} \]

(64) Mary is surprised at where John was born. 
 John was born in Paris. 
\[ \therefore \text{ Mary is surprised that John was born in Paris.} \]

Turning to \textit{FA}-sensitivity effects, however, consider the following sentence:

(65) Rupert is surprised at where one can buy an Italian newspaper.
For (65) to be true on an MS reading, there has to be at least one store \(x\) such that Rupert correctly believes but did not expect that \(x\) sells Italian newspapers. What Rupert believes or expected about stores that do not sell Italian newspapers seems immaterial. So, \textbf{be surprised} does not exhibit FA-sensitivity effects. A simple lexical entry that would achieve this is given in (66), where we write \(\text{EXP}_x^w\) for the set of worlds compatible with \(x\)’s previous expectations at \(w\).

\[(66) \text{ be surprised}' = \lambda f. \lambda x. \lambda p. \forall w \in p : \exists q \in \text{alt}(f(w)) : \text{DOX}_x^w \subseteq q \land \text{EXP}_x^w \subseteq q\]

Note what happens here: the entry makes specific reference to the set of truthful resolutions of the complement in the world of evaluation, \(f(w)\), but then only the maximal elements of the set, \(\text{alt}(f(w))\), are taken into account. It is required that there exists an alternative \(q\) such that \(x\) believes \(q\) in \(w\), \(\text{DOX}_x^w \subseteq q\), but \(q\) is incompatible with \(x\)’s previous expectations in \(w\), \(\text{EXP}_x^w \subseteq q\). So, explicit reference is made to the set of truthful resolutions in the world of evaluation (which captures the veridical nature of the verb), but then exactly the part of this set that would be needed to generate FA-sensitivity effects is disregarded.

5.2.4 \textit{care}

As mentioned in Section 1, Elliott et al. (2017) argue that predicates of relevance, such as \textit{care, matter} and \textit{be relevant} pose a problem for reductive theories of question embedding. They observe that what is presupposed by these predicates depends on whether they take a declarative or an interrogative complement. With a declarative complement, they presuppose that the complement is true and that the subject knows this. E.g., (66a) presupposes that Mary left and that John knows this. With an interrogative complement, on the other hand, predicates of relevance don’t carry an analogous presupposition. E.g., (66b) doesn’t presuppose that John believes any answer to the embedded interrogative.

\[(67)\]

\[\begin{align*}
\text{a. } & \text{John cares that Mary left.} \\
\text{b. } & \text{John cares which girl left.}
\end{align*}\]

This is problematic for standard reductive theories, because they predict that a sentence like (67b) is true if and only if it matters to John that \(p\), where \(p\) is a proposition that counts as an answer to the interrogative in (67b). Thus, (67b) is wrongly predicted to presuppose that John believes \(p\), for some answer \(p\). Uegaki (2017) shows that this problem arises on George’s twin relations theory as well.

Predicates of relevance are also interesting for another reason. Namely, they are veridical w.r.t. declarative complements but not w.r.t. interrogative complements. That is, inferences like (68) are valid, but ones like (69) are not.

\[(68)\]

\[\text{Mary cares that John was born in Paris. } \therefore \text{John was born in Paris.}\]

\[(69)\]

\[\text{Mary cares where John was born. } \quad \text{John was born in Paris. } \quad \text{Mary cares that John was born in Paris.}\]

\[30\text{The entry given here is merely meant to illustrate that it is possible in our framework to deal with verbs that are veridical but not sensitive to false answers. It is not meant as a fully realistic treatment of \textbf{be surprised}, which involves several complexities that are orthogonal to our present concerns. In particular, our entry does not account for the fact that \textbf{be surprised} and other emotive factives do not license polar and disjunctive interrogative complements. We refer to Guerzoni and Sharvit (2007), Szebeh (2007), Nicolae (2013), Spector and Egré (2015), Romero (2015b) for recent work in this area. See in particular Roelofsen (2017) for an approach that is compatible with the present proposal.}\]

\[31\text{See Xiang (2016a) for another possible account of the fact that emotive factives like \textbf{be surprised} do not exhibit FA sensitivity effects. A detailed comparison between the two accounts must be left for another occasion.}\]
In particular, even if both premises of the inference in (69) are true, the conclusion may still not be true due to presupposition failure, i.e., Mary might not know that John was born in Paris. This is problematic for the proposal of Spector and Egré (2015), which specifically aims to derive the empirical generalization that verbs which are veridical w.r.t. declarative complements are also veridical w.r.t. interrogative complements.\footnote{This generalization will be discussed in more detail in Section 6.}

On our account, it is straightforward to define a lexical entry for care that captures the above observations. In particular, both the differences in presuppositions and those in veridicality naturally fall out from the semantic properties of declarative and interrogative complements. We propose the following lexical entry, where $\beta^w_x$ is the bouletic state of $x$ in $w$, i.e., the set of all those worlds that are compatible with what $x$ desires in $w$.

\begin{equation}
\text{care}' := \lambda f. \lambda x. \lambda p. \forall w \in p : (f(w) \neq \emptyset \land \forall v \in \text{DOX}^w_x : f(v) \neq \emptyset).
\end{equation}

In words, it is presupposed that the set of truthful resolutions is non-empty in the world of evaluation and that it is non-empty in all worlds in the subject’s information state. It is asserted that, among the minimally informative possible resolutions of the complement, there is at least one which the subject either desires to be true or to be false.

If care takes an interrogative complement, its presupposition is trivially satisfied since the meaning of an interrogative nucleus covers the entire logical space and $f(w)$ will therefore be non-empty for all worlds $w$. In contrast, with declarative complements, there are usually worlds $w$ such that $f(w)$ is empty and, in that case, the presupposition of care is non-trivial. This pattern is already familiar from our discussion of know in Section 5.1.

Now assume that care takes a declarative complement and let $q$ be the unique alternative in the nucleus meaning. Then, the second conjunct of the presupposition amounts to $q$ being true in all worlds in the subject’s information state. Combined with the first conjunct, which requires that $q$ is true in the world of evaluation, this amounts to demanding that the subject knows $q$.

Turning to veridicality, we find that by virtue of the factivity presupposition of care, declarative veridicality inferences like (68) indeed come out as valid. In contrast, interrogative veridicality inferences are not predicted to go through. In order for the conclusion in, e.g., (69) to hold, it would have to be the case that Mary knows that John was born in Paris—this, however, is not guaranteed by the given premises.\footnote{Two side notes. First, our lexical entry predicts that the SE and IE reading of care coincide. More precisely, for any nucleus $\alpha$, if $f_{SE} = E_{(\semp \alpha)}(\alpha_{+\text{exh}})$ and $f_{IE} = E_{(\semp \alpha)}(\alpha_{-\text{exh}})$, then $\text{care}(f_{SE}) = \text{care}(f_{IE})$. Secondly, our analysis predicts that when care takes an interrogative complement, the MS reading entails the IE/SE reading. Thus, when care takes an interrogative complement, we only find two distinct readings, namely MS and SE/IE, and of these two the former entails the latter. It appears that with non-negated statements like (ia), there is a preference for the MS interpretation, while with negated statements like (ib) the IE/SE interpretation is preferred.}

5.3 Some predicted connections between embedding verbs

Many of the lexical entries we introduced in the preceding sections are built up from similar ingredients. For instance, know\textsuperscript{ext}, and be right\textsuperscript{'} have the same assertive component, and know\textsuperscript{ext} is built up from know\textsuperscript{ext} and an additional introspection requirement. Taking these similarities into account, it is not surprising that we can identify multiple connections that obtain between the embedding verbs. Figure 8(a) and 8(b) display an interesting subset of those connections. 

(i) a. John cares who left.
   b. John doesn’t care who left.

Since this means that in either case the stronger one of the two readings is favored, this pattern could be explained by appealing to the strongest meaning hypothesis (Dalrymple et al., 1998).
Figure 8: Predicted connections between responsive verbs.

The former shows the relations that obtain between the verbs on their declarative-embedding use and the latter those that obtain between the verbs on their interrogative-embedding use.

The solid black arrows are to be understood as implications. For instance, in both figures, we have an arrow from know_{int} to be certain, meaning that, if an individual x stands in a know'_{int} relation to some complement meaning f, then x is predicted to also stand in a be certain' relation to f. Also note that the visualisation does not distinguish between whether an implication holds due to the asserted meaning components of the lexical entries or due to a presupposition. For example, on its declarative-embedding use, be wrong implies be certain, but this is only the case because, whenever be wrong'(f)(x) is true, the definedness restriction of be wrong' is satisfied, and this definedness restriction amounts to be certain(f)(x).

The dashed red double arrows, labelled with not, are to be read as true iff not true. For instance, in Figure 8(b), be wrong and know_{int} are connected with such an arrow because, whenever be wrong'(f)(x) holds, know'_{int}(f)(x) doesn’t hold and vice versa. Note, however, that this does not indicate that know_{int} simply amounts to not be wrong. Rather, know'_{int}(f)(x) can fail to hold because x is wrong or because x doesn’t satisfy the resolution introspection requirement. Furthermore, just as the solid arrows, the not-arrows don’t distinguish between asserted or presuppositional content. For example, if be wrong'(f)(x) is true, know'_{int}'(f)(x) cannot be true because of presupposition failure.

A couple of observations are worth making here. Firstly, once we restrict our attention to declarative complements, as in Figure 8(a), the meanings of many verbs are Strawson-equivalent, e.g., that of know_{int}, know_{ext} and be right. On the other hand, moving to interrogative complements, not all of these Strawson-equivalences obtain anymore. Instead, Figure 8(b) nicely reflects the distinction between internal, i.e., resolution-introspective, and external verbs. The external verbs know_{ext} and be right are still equivalent. Furthermore, by combining internal be certain and external be right, we obtain a meaning that is equivalent to know_{int}: be certain contributes the resolution introspection condition of know_{int}, and be right contributes its “truthfulness condition”.

6 Constraints on responsive verb meanings

In the previous section we have seen that our framework is flexible enough to formulate lexical entries for a variety of verbs. In particular, we saw that we have a great amount of freedom when it comes to capturing the different properties that verbs may have: declarative veridicality, interrogative veridicality, factivity, and FA sensitivity. In this section, we take a more critical
perspective, asking whether the flexibility we have is really only a virtue, or whether it has a
downside as well. We do this in light of arguments by George (2011) and Spector and Egré (2015)
that a comprehensive theory of clause-embedding should predict certain general constraints on
responsive verb meanings. Spector and Egré give empirical arguments for one particular such
constraint, which involves the distinction between veridical and non-veridical verbs. We discuss
the first part of this constraint in Section 6.1 and propose an account of it in Section 6.2. In
Section 6.3 and 6.4, we do the same for the second part of the constraint.

6.1 Spector and Egré’s interrogative veridicality generalization

Spector and Egré (2015) hold that a responsive verb is veridical w.r.t. declarative complements
exactly if it is veridical w.r.t. interrogative complements. We will refer to the “⇒”-direction
of this generalization as the interrogative veridicality generalization, and to the “⇐”-direction
as the declarative veridicality generalization. In this subsection, we focus on the former; in
Section 6.3 we will turn to the latter.

First off, we would like to suggest a counterexample to the interrogative veridicality generalization.
It seems that predicates of relevance like care and matter, as already discussed above,
are veridical w.r.t. declarative complements but not w.r.t. interrogative complements. That is,
inferences like (71) seem valid, but ones like (72) don’t.

(71) It matters to Mary that John was born in Paris.
      ∴ John was born in Paris.

(72) It matters to Mary where John was born.
      John was born in Paris.
      ¬ It matters to Mary that John was born in Paris.

Even if both premises of the inference in (72) are true, the conclusion may still not be true due
to presupposition failure, i.e., Mary might not know that John was born in Paris. Thus, the
veridicality generalization does not hold in full generality. This is problematic for Spector and
Egré’s reductive theory, where the assumed connection between declarative and interrogative
veridicality is a direct and inescapable consequence of the meaning postulates that connect
the interrogative-embedding interpretation of responsive verbs to their declarative-embedding
interpretation.

Still, it seems fair to say that the vast majority of responsive verbs comsses with the
generalization, at least in English and related languages. Our aim will be to show how this
tendency can be accounted for within a uniform theory of clause-embedding, without ruling out
occasional counterexamples such as care and matter.

6.2 Accounting for the interrogative veridicality generalization

Why would only a subset of the space of possible responsive verb meanings be lexicalized in
natural languages? Before coming to address this question, let us note that the same question
has arisen in other empirical domains as well, and in some cases it has already received a lot of
attention. Consider the case of determiners. In the standard generalized quantifier framework,
there is a huge space of meanings that determiners could in principle have, but in practice there
seem to be certain constraints on which of these possible meanings are actually lexicalized in
natural languages. For instance, a well-known empirical generalization in this area is that all
natural language determiners are conservative (Barwise and Cooper, 1981; Keenan and Stavi,
1986). A determiner $D$ is conservative if for every two set-denoting expressions $A$ and $B$,
$D(A, B)$ is equivalent with $D(A, A \cap B)$. Only a small portion of the entire space of possible
determiner meanings is conservative. For instance, in a universe with just two objects, there

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are $2^{16} = 65,536$ possible determiner meanings, but only $2^9 = 512$ of these are conservative (Keenan and Stavi, 1986).

Whether the conservativity generalization is a strict universal or rather a ‘soft constraint’ with occasional counterexamples is a matter of ongoing debate. For instance, Cohen (2001) argues that *many* and *few* have a reading under which they are not conservative, but Romero (2015a) suggests a decompositional analysis of these determiners under which their ‘core semantics’ is conservative.

It is also an open question why natural language determiners are generally conservative. It seems plausible that such an explanation may be given in computational terms. Indeed, it may well be that the cognitive load of verifying whether a determiner $D$ applies to two sets $A$ and $B$ can be kept relatively low as long as $D$ is conservative, because in this case we only need to consider objects in $A$; we don’t need to look at objects in $B \setminus A$ or in $A \cup B$. Another (not necessarily causally independent) reason may be that the meaning of conservative determiners is easier to learn from examples: it has been shown experimentally that children exposed to a novel conservative determiner show significant understanding of it after having been told the truth value of a number of example sentences in a number of contexts, while children exposed to an imaginary non-conservative determiner do not come to grasp its meaning at all after having received such information (Hunter and Lidz, 2013).

Given that natural language determiners are, or at least tend to be conservative, it is natural to expect that similar properties may be identified in other domains, in particular in the domain of clause-embedding predicates. Below we will formulate such a property of clause-embedding predicates, which we will call *clausal distributivity* (c-distributivity for short), and we will show that all c-distributive responsive verbs must satisfy Spector and Egré’s interrogative veridicality generalization.

Roughly, we will say that a verb $V$ is c-distributive if for any complement meaning $f$ that can be decomposed into a set of simpler complement meanings $\{f_1, f_2, \ldots \}$, and for every individual $x$, $V(f)(x)$ holds if and only if $V(f_i)(x)$ holds for some $f_i$. Informally, c-distributivity says that, whenever $f$ can be decomposed into simpler parts, $V(f)(x)$ is fully determined by those simpler parts, i.e., in computing $V(f)(x)$ we don’t have to look at $f$ as a whole but only at its parts.

To make this more precise, we have to define the relevant notion of *decomposition*. Recall that a complement meaning $f$ is always obtained in our framework by applying $E$ to a nucleus meaning $P$. We define the decomposition of a nucleus meaning as follows.

**Definition 7 (Decomposing nucleus meanings).**

A nucleus meaning $P$ can be decomposed if there is a set of nucleus meanings $\mathcal{D}$ such that:

1. $P = \bigcup \mathcal{D}$
2. Every two distinct elements $P', P'' \in \mathcal{D}$ are mutually inconsistent, i.e., $P' \cap P'' = \{\emptyset\}$
3. Every $P' \in \mathcal{D}$ is non-inquisitive.

For any $P$, there is at most one set $\mathcal{D}$ satisfying these requirements. If there is indeed one, we refer to it as the decomposition of $P$, and denote it as $\text{decomp}(P)$. Otherwise the decomposition of $P$ is undefined.

Notice that the first two requirements should be part, in some form or other, of any reasonable notion of decomposition.\(^{34}\) The first requires that putting the elements of $\text{decomp}(P)$ together leads us back to the original $P$. The second requires that the elements of $\text{decomp}(P)$ have to be

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\(^{34}\)For a concrete example of a notion of decomposition that has precisely these two components, consider the notion of vector decomposition in linear algebra: a decomposition of a vector $v$ is a set of vectors $\text{decomp}(v)$ such that (i) the sum of all vectors in $\text{decomp}(v)$ is $v$ itself, and (ii) the vectors in $\text{decomp}(v)$ are all linearly independent.
mutually independent, which is made precise here in terms of mutual inconsistency.\textsuperscript{35} The third requirement, on the other hand, specifies that the elements of a decomposition must be ‘elementary’ nucleus meanings, which in the present setting means that they must be non-inquisitive. In other words, while $P$ itself may be inquisitive and thus contain multiple alternatives, every element of $\text{decomp}(P)$ must contain a unique alternative. This is a natural requirement for ‘elementary’ nucleus meanings, because it ensures that such nucleus meanings cannot be further decomposed. That is, for any non-inquisitive nucleus meaning $P$, $\text{decomp}(P) = \{ P \}$. Vice versa, $\text{decomp}(P) = \{ P \}$ also implies that $P$ is non-inquisitive. So there is a one-to-one connection between non-inquisitiveness and non-decomposability.

**Fact 3.** A nucleus meaning $P$ is non-inquisitive if and only if $\text{decomp}(P) = \{ P \}$.

Now note that under our notion of decomposition, $\text{decomp}(P)$ is only defined if $P$ does not contain any overlapping alternatives. To see this, suppose that $P$ does contain two alternatives $p$ and $q$ that overlap, i.e., such that $p \cap q \neq \emptyset$. Then, by the first requirement, there must be some $P' \in \text{decomp}(P)$ such that $p \in P'$ and some $P'' \in \text{decomp}(P)$ such that $q \in P''$. But then, since both $P'$ and $P''$ are downward closed, we have that $p \cap q \subseteq P'$ and $p \cap q \subseteq P''$. This means that $P' \cap P'' \neq \emptyset$, in violation of the second requirement.

On the other hand, if $P$ does not contain any overlapping alternatives, then $\text{decomp}(P)$ is always well-defined, and moreover, its elements correspond precisely to the alternatives in $P$: $\text{decomp}(P) = \{ \{ p \}^\dag \mid p \in \text{alt}(P) \}$.

**Fact 4 (Decomposition and alternatives).**

- $\text{decomp}(P)$ is defined if and only if $P$ does not contain any overlapping alternatives.
- Whenever defined, $\text{decomp}(P)$ amounts to $\{ \{ p \}^\dag \mid p \in \text{alt}(P) \}$.

Finally, we note that whenever $\text{decomp}(P)$ is defined, i.e., whenever $P$ does not contain overlapping alternatives, applying $E_{[\text{+cmp]}}$ or $E_{[-\text{cmp}]}$ to $P$ gives exactly the same results. Thus, below, whenever it is assumed that $\text{decomp}(P)$ is defined, we simply write $E(P)$ rather than $E_{[\text{+cmp}]}(P)$ or $E_{[-\text{cmp}]}(P)$.

We can now give a precise formulation of c-distributivity. For expository purposes we formulate the property for predicates that have one clausal and one individual argument slot—it will be clear how it can be generalized to predicates with zero or more than one individual argument slots.

**Definition 8 (C-distributivity).**
A predicate $V$ with one clausal and one individual argument slot is c-distributive if and only if for any individual $x$, any world $w$, and any nucleus meaning $P$ such that $\text{decomp}(P)$ is defined:

$$V(E(P))(x) \text{ is true in } w \text{ iff } V(E(P'))(x) \text{ is true in } w \text{ for some } P' \in \text{decomp}(P)$$

Informally, c-distributivity says that we should be able to evaluate whether the verb applies to a certain complement by checking whether it applies to one of the elements of the complement’s decomposition, in case such a decomposition exists.

Now we are ready to state the main result of this subsection: all c-distributive responsive verbs comply with Spector and Egré’s interrogative veridicality generalization. A proof of this fact is given in Appendix B.2.

\textsuperscript{35}There is another natural way to construe independence in our framework as well. Namely, instead of inconsistency we could also just require non-entailment, i.e., $P' \not\sqsubseteq P''$. Note that this requirement is weaker than inconsistency. Thus, we could refer to decompositions as defined in Definition 7 as strict decompositions, and to ones that only require non-entailment between components as soft decompositions. The fact that strict decompositions are of primary interest to us here will become clear below, see in particular footnote 36.
**Fact 5.** A c-distributive responsive verb that is veridical w.r.t. declarative complements is also veridical w.r.t. interrogative complements.

Most responsive verbs in English are c-distributive. Indeed, the only exceptions that we have been able to identify are predicates of relevance like *care* and *matter*, which are exactly the ones which violate Spector and Egré’s interrogative veridicality generalization. The fact that so many responsive verbs are c-distributive may possibly be explained in computational terms, just like the fact that determiners are generally conservative. That is, it seems reasonable to hypothesize that the cognitive load of verifying whether a verb applies to a certain complement can be kept relatively low as long as it is guaranteed that this can be done by verifying whether the verb applies to the elements of the decomposition of the given complement, in case such a decomposition exists.

What is most important for our present purposes is that we have seen how general constraints on responsive verb meanings, such as Spector and Egré’s interrogative veridicality generalization, can be captured within a uniform theory of clause-embedding. This addresses the main concern that George (2011) and Spector and Egré (2015) raised for the uniform approach, as well as the inverse reductive approach. The particular way in which we have proposed to capture Spector and Egré’s interrogative veridicality generalization also seems to have some advantages over their own account. First, it allows for counterexamples, which is needed to accommodate verbs like *care* and *matter*. And second, it allows us to draw parallels with other domains—e.g., that of determiners—and paves the way for possible explanations of the generalization in terms of minimizing cognitive processing load.

### 6.3 Spector and Egré’s declarative veridicality generalization

Recall that Spector and Egré do not only hold that every responsive verb which is veridical w.r.t. declaratives is also veridical w.r.t. interrogatives, but also the reverse, i.e., that every responsive verb which is veridical w.r.t. interrogatives is veridical w.r.t. declaratives as well. We called this the declarative veridicality generalization.

As discussed by Spector and Egré, many previous authors have rejected this generalization (e.g., Groenendijk and Stokhof, 1984; Berman, 1991; Higginbotham, 1996; Lahiri, 2002) based on Karttunen’s (1977, p.11) observation that communication verbs like *tell* appear to be veridical w.r.t. interrogative complements, but non-veridical w.r.t. declarative complements. That is, inferences like (73) appear to be valid, while inferences like (74) seem invalid.

\[(73)\] Mary told Alice where John was born.  
John was born in Paris.  
\[\vdash\] Mary told Alice that John was born in Paris.

\[(74)\] John told Mary that it was raining.  
\[\vdash\] It was raining.

However, these judgements have been challenged by Tsohatzidis (1993) and more elaborately

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36Recall from footnote 35 that the strict notion of decomposition that we assume here has a natural alternative, which is ‘softer’ in that it does not require the components of a decomposition to be mutually inconsistent, but just that they do not entail one another. Under this notion of decomposition, the class of c-distributive responsive verbs would be much smaller. For instance, *know* would not be c-distributive, exactly because it exhibits FA sensitivity effects when the nucleus of its complement is not a partition. This is why the strict notion of decomposition is most relevant for our purposes.

37As far as we can tell, the strategy we have taken here could also be adopted on the inverse reductive approach.

38We conjecture that, when formulated at a sufficiently abstract level, a property similar to c-distributivity may actually hold not just of most predicates that take declarative and/or interrogative clauses as their argument, but of most predicates in general, also ones that take atomic and/or plural individuals as their argument. However, we must leave the formulation of such a more general property for further work.
by Spector and Egré (2015), who point out that, with communication verbs, inferences like (73) are in fact defeasible.

(75) Old John told us whom he saw in the fog, but it turned out that he was mistaken (the person he saw was Mr. Smith, not Mr. Brown).  
(Tsotahzidis, 1993, p.272)

(76) Every day, the meteorologists tell the population where it will rain the following day, but they are often wrong.  
(Spector and Egré, 2015, p.1737)

This clearly contrasts with the behavior exhibited by verbs like know, whose veridicality implications are indefeasible:

(77) #Mary knew where John was born, but she turned out to be wrong.

On the other hand, while tell is typically interpreted as non-veridical w.r.t. declarative complements, Spector and Egré hold that there are also cases where it receives a veridical interpretation. For instance, they hold that (78a-b) both presuppose, and thus imply, that Mary is pregnant.39

(78) a. Sue didn’t tell anyone that she is pregnant.
    b. Did Sue tell anyone that she is pregnant?

Based on these empirical observations, Spector and Egré suggest that communication verbs like tell are ambiguous. On one reading they are veridical w.r.t. both declarative and interrogative complements; on another reading they are not veridical w.r.t. either type of complement.40

Under this assumption, communication verbs no longer form a counterexample to the declarative veridicality generalization. Spector and Egré thus conclude that the generalization is valid after all.

6.4 Accounting for the declarative veridicality generalization

Spector and Egré are mainly driven by the interrogative veridicality generalization. They don’t explicitly show that their account derives the declarative veridicality generalization, and it seems to us that this is in fact impossible. That is, the account does not rule out responsive verbs that are veridical w.r.t. interrogative complements but not w.r.t. declarative complements. Consider, for instance, a fictitious verb trueifbelieve with the following basic ‘declarative’ lexical

39We should note that the veridical readings of declarative-embedding communication verbs seem to surface only when these verbs are embedded under entailment-canceling operators. This observation isn’t explained by assuming a lexical ambiguity as Spector and Egré do. Rather, one might take it to indicate that some independent mechanism is at work here. One possible line of explanation can be found in recent QUD-based approaches to presupposition projection. Drawing on Beaver (2010), Tonhauser (2016) and Simons et al. (2017) propose that the projection of factivity presuppositions depends on the placement of focus. More specifically, they maintain that if the embedding verb is focused, the presupposition projects, while if some element in the embedded clause is focused, the presupposition doesn’t project. The former is the case in (ia), the latter in (ib). Indeed, we can infer from (ia) but not from (ib) that Sue is pregnant.

(i) a. Sue hasn’t TOLD anyone that she is pregnant. She was going to wait until Christmas.  \( \sim P(s) \)
    b. Sue hasn’t told anyone that she is PREGNANT. She only said that they are TRYING.  \( \not\sim P(s) \)

Crucially, Simons et al.’s reasoning does not only apply to factive verbs but also to notoriously non-factive verbs like believe: from (ii), we can infer that the car is parked in the parking garage.

(ii) Polly: Why is it taking Phil so long to get here?  
    Amy: He didn’t believe that the car’s parked in the parking garage.  
    (Simons et al., 2017, p. 203)

If this account is correct, there seems to be no reason why it should not apply to non-veridical communication verbs. But then there would be no reason either to assume that declarative-embedding communication verbs are lexically ambiguous.

40Uegaki (2015, p.158f) provides a possible explanation of why the veridical reading of communication verbs is preferred with interrogative, but not with declarative complements.
entry (recall that on Spector and Egré’s account every responsive verb has a basic declarative entry which applies when the verb combines with a declarative complement, and an interrogative entry which is derived from this basic declarative entry via existential quantification; moreover, note that Spector and Egré take a declarative complement to denote a single proposition and an interrogative complement to denote a set of propositions).

(79) \text{trueifbelieve}'_{\text{decl}} := \lambda p. \lambda x. \lambda w. (\text{dox}_{x}^w \subseteq p \to w \in p)

This entry says that when the verb trueifbelieve combines with a proposition \( p \) and an individual \( x \), it returns the set of worlds \( w \) such that, when \( x \) believes \( p \) in \( w \), then \( p \) is true in \( w \). Using Spector and Egré’s general recipe for deriving the interrogative entry of a responsive verb from its declarative entry, we get the following interrogative entry for trueifbelieve:

(80) \text{trueifbelieve}'_{\text{int}} := \lambda Q_{\langle s,t,t \rangle}. \lambda x. \lambda w. \exists p \in Q : (\text{dox}_{x}^w \subseteq p \to w \in p)

Now, let us consider whether this verb is veridical w.r.t. declarative and interrogative complements. First, consider the inference in (81):

(81) Mary trueifbelieves where John was born.
    John was born in Paris.
    ;: Mary trueifbelieves that John was born in Paris.

Such inferences are valid, because the second premise alone already ensures that the conclusion holds, irrespectively of Mary’s information state. Thus, trueifbelieve is veridical w.r.t. interrogative complements.

Now consider the following inference, which is invalid:

(82) John trueifbelieves that it is raining.
    ;: It is raining.

The inference is invalid because the premise can be true even if it is not raining, as long as John does not believe that it is raining. Thus, trueifbelieve is not veridical w.r.t. declarative complements. This means that Spector and Egré’s theory does not rule out verbs that fail to comply with the declarative veridicality generalization.

In our own framework, it can be proven that all c-distributive predicates which have a certain natural property that we will call the choice property comply with the generalization. Informally, we say that a predicate has the choice property just in case, for any two declarative nucleus meanings \( P \) and \( P' \) such that \( \text{info}(P') \cap \text{info}(P) = \emptyset \), and any world \( w \), \( V(E(P))(x) \) and \( V(E(P'))(x) \) cannot both be true at \( w \).

Definition 9 (Choice property).
We say that a declarative-embedding verb \( V \) has the choice property just in case for any two declarative nucleus meanings \( P \) and \( P' \) such that \( \text{info}(P') \cap \text{info}(P) = \emptyset \), and any world \( w \), \( V(E(P))(x) \) and \( V(E(P'))(x) \) cannot both be true at \( w \).

This property applies to a large class of verbs, which in particular contains all factive verbs. After all, if \( V \) is factive and \( V(E(P))(x) \) and \( V(E(P'))(x) \) are both true at a world \( w \), then \( P \) and \( P' \) must also both be true at \( w \), which means that \( \text{info}(P') \cap \text{info}(P) \) cannot be empty.

Any c-distributive responsive verb with the choice property complies with the declarative veridicality generalization. A proof of this fact is given in Appendix B.3.

Fact 6. Any c-distributive responsive verb that has the choice property and is veridical w.r.t. interrogative complements must also be veridical w.r.t. declarative complements.
7 Conclusion

We have proposed a uniform account of declarative and interrogative complements, and the verbs that take both kinds of complement as their argument. The account overcomes a problem for the reductive approach concerning false answer sensitivity (pointed out by George, 2011), as well as a problem for reductive theories and the twin relations theory concerning predicates of relevance (pointed out by Elliott et al., 2017; Uegaki, 2017). It also addresses the limitations of Groenendijk and Stokhof’s uniform partition theory; in particular, it straightforwardly derives MS and IE readings as well as SE ones. Finally, it addresses a concern raised by George (2011) and Spector and Egré (2015) for uniform and inverse reductive theories, showing that it is possible to capture general constraints on responsive verb meanings within a uniform framework. In Appendix A our approach is compared in some detail with the inverse reductive theory of Uegaki (2015).

A Comparison with Uegaki (2015)

In Section 1 we situated the present proposal w.r.t. a range of previous approaches in rather general terms. Here, we will offer a more detailed comparison with one of these approaches, namely that of Uegaki (2015), which in our view provides the most comprehensive previous account of the issues that we have been concerned with in this paper (see Table 1 on page 3). The core of our proposal and that of Uegaki were developed independently, as witnessed by preliminary presentations of the two accounts (Theler, 2014; Roelofsen et al., 2014; Uegaki, 2014). However, in further developing our initial proposal we have crucially benefited from some of the insights in Uegaki’s work. The two resulting proposals are very much in the same spirit, but there are also some significant differences, which we will highlight below.

The discussion will center on two main themes: variability in the exhaustive strength of interrogative complements (Section A.1), and the connection between veridicality, factivity, and extensionality (Section A.2). These themes correspond to two of the core chapters in Uegaki (2015). The third core chapter in Uegaki (2015) is concerned with the selectional restrictions of clause-embedding verbs. As mentioned earlier (see p.4), our own account of these selectional restrictions is presented in a separate paper. There, it is also compared with Uegaki’s proposal.

A.1 Variability in exhaustive strength

As discussed in Section 3.1, sentences with interrogative complements usually exhibit variability in exhaustive strength. With certain verbs, however, this variability is restricted. For instance, be certain is incompatible with an IE reading, and our theory accounts for this fact. Uegaki aims to predict more generally which readings are available for any responsive verb and to derive these predictions from the lexical properties of the verb. It has to be noted that such predictions—while clearly desirable—will only be explanatory to the extent that the involved mechanisms are independently motivated. This means in particular that it should be possible to provide reasons for assuming the relevant properties of the embedding verbs that are not connected to deriving the observed levels of exhaustive strength.

41 To be sure, neither our own proposal nor that of Uegaki (2015) covers all the issues that have been discussed in previous work on declarative and interrogative complements and the verbs that embed them. For instance, quantificational variability effects (e.g., Berman, 1991; Lahiri, 2002; Beck and Sharvit, 2002; Cremers, 2016), the de re/de dicto ambiguity (e.g., Groenendijk and Stokhof, 1984; Sharvit, 2002), the licensing of negative polarity items (e.g., Guerzoni and Sharvit, 2007, 2014; Nicolae, 2013), homogeneity effects (Kriz, 2015; Cremers, 2016), and perspective sensitivity (e.g., Aloni, 2005) are not explicitly accounted for. We are hopeful that our proposal can be extended to do so, incorporating insights from the work cited here, but a full implementation must be left for future work.
We will first consider the general architecture that Uegaki assumes and the distinction between extensional and intensional verbs that is relevant for his account. We then turn to the ‘baseline’ readings that are predicted for different embedding verbs and finally to the non-baseline readings, which are obtained by additional semantic operations.

**General architecture and extensional/intensional responsive verbs.** Uegaki decomposes every responsive verb \( V \) into a core predicate \( R_V \), which is the proposition-taking counterpart of \( V \), plus an answer operator. The answer operator, \( \text{Ans}_d \), has the denotation in (83).

It takes a world \( w \) and a question denotation \( Q \) as arguments and delivers the true WE answer to \( Q \) in \( w \).

\[
\begin{align*}
\text{Ans}_d = \lambda w'. \lambda Q_{(s,t), t}. \left\{ \\
&\quad \forall p \in Q. \left( p(w') \land \forall p' \in Q, \langle p'(w') \rightarrow p \subseteq p' \rangle \right) \quad \text{if } \exists! p \in Q. \\
&\quad \langle p(w') \land \forall p' \in Q, \langle p'(w') \rightarrow p \subseteq p' \rangle \rangle \quad \text{otherwise}
\end{align*}
\]

The difference between extensional and intensional responsive verbs is the following on Uegaki’s account. The world argument of \( \text{Ans}_d \) can get bound either by the core predicate \( R_V \) or by an exhaustification operator, \( X \). Intensional verbs like \textit{be certain} or \textit{tell} have a core predicate that binds the world argument of \( \text{Ans}_d \), as in (84a), while extensional verbs like \textit{know} do not bind this argument themselves, but leave it to be bound by the exhaustification operator, as in (84b).

(83) \( \text{Ans}_d = \lambda w'. \lambda Q_{(s,t), t}. \left\{ \\
&\quad \forall p \in Q. \left( p(w') \land \forall p' \in Q, \langle p'(w') \rightarrow p \subseteq p' \rangle \right) \quad \text{if } \exists! p \in Q. \\
&\quad \langle p(w') \land \forall p' \in Q, \langle p'(w') \rightarrow p \subseteq p' \rangle \rangle \quad \text{otherwise}
\end{align*}\)

This contrast between extensional and intensional verbs can also be observed from the lexical entries in (85): the intensional verbs in (85a) and (85b) take a world-sensitive argument \( P \), which they evaluate in some possible world (in the case of \textit{tell}) or all possible worlds that are compatible with the subject’s beliefs (in the case of \textit{be certain}). The extensional verb in (85c) on the other hand takes a simple proposition \( p \) as its argument.

(84) a. intensional verb:

\[
\begin{array}{c}
\text{John} \\
R_{\text{certain}} \\
1 \\
\text{Ans}_d \quad w_1 \quad \text{who} \quad \text{called}
\end{array}
\]

b. extensional verb:

\[
\begin{array}{c}
\text{John} \\
R_{\text{know}} \\
\text{Ans}_d \quad w_1 \quad \text{who} \quad \text{called}
\end{array}
\]

This contrast between extensional and intensional verbs can also be observed from the lexical entries in (85): the intensional verbs in (85a) and (85b) take a world-sensitive argument \( P \), which they evaluate in some possible world (in the case of \textit{tell}) or all possible worlds that are compatible with the subject’s beliefs (in the case of \textit{be certain}). The extensional verb in (85c) on the other hand takes a simple proposition \( p \) as its argument.

(85) a. \[ [R_{\text{certain}}]_w = \lambda P_{(s,t), t}. \lambda x. \forall w' \in \text{DOX}_x^w : \text{DOX}_x^w \subseteq P(w') \]

Following Spector and Egré (2015), Uegaki assumes that \textit{tell} is ambiguous between a veridical and a non-veridical interpretation. We use \textit{tell}_{\text{[non-ver]}} to denote the non-veridical version of the verb.
b. \[ R_{\text{tell}}(\cdot, \cdot, \cdot) \] 
\[ (86) \quad [X]^{w} = \lambda \mathcal{P}(s, (s, t)). \mathcal{P}(w) \land \forall v : \{ w' | \mathcal{P}(w')(v) \} \subset \{ w' | \mathcal{P}(w')(v) \} \rightarrow \neg \mathcal{P}(w)(v) \]

Baseline readings. In Uegaki’s system, each verb comes with a baseline reading, which is the reading it has in the absence of further semantic operations such as exhaustification. In the case of extensional verbs like know, the baseline reading is WE. To see why, consider again the lexical entry in (85c) for the knowledge relation \( R_{\text{know}} \) and the entry in (83) for the answer operator \( \text{Ans}_d \). As can be seen from the structure in (84b), the propositional argument \( p \) that \( R_{\text{know}} \) takes in the semantic derivation is delivered by \( \text{Ans}_d \). Since, given a world \( w \) and a question meaning \( Q \), \( \text{Ans}_d(w)(Q) \) is the true WE answer to \( Q \) in \( w \), we find that for a subject \( x \) to know \( Q \) in \( w \), \( x \)’s belief state in \( w \), \( \text{DOX}_x^w \), has to be a subset of the true WE answer in \( w \). This amounts to a WE reading.

The semantics of intensional verbs, on the other hand, is expressed in terms of quantification over possible worlds, and the strength of their baseline reading depends on the kind of quantification that is used. For instance, as can be seen from the entries in (85a) and (85b), a universal semantics is assumed for be certain, while an existential semantics is assumed for tell\(_{-\text{ver}}\), the non-veridical variant of tell.\(^{43}\)

The propositional concept \( \mathcal{P} \) that these verbs take as an argument is a function mapping every world to the true WE answer at that world. If be certain is applied to a propositional concept \( \mathcal{P} \) that has been computed from the meaning \( Q \) of an interrogative complement, then the semantics in (85a) amounts to requiring that the subject’s belief state \( \text{DOX}_x^w \) is homogeneous with respect to every answer to \( Q \)—or, in other words, that the subject has to believe an SE answer to \( Q \). In comparison, the unrestricted existential quantification in the semantics of tell\(_{-\text{ver}}\) is much weaker: it is only required that the subject stands in a tell-relation to the true WE answer at some world \( w' \). However, since for every proposition \( p \) that is an MS answer at \( w' \), there also exists a world \( w'' \) such that \( p \) is a WE answer at \( w'' \), this condition boils down to requiring that the subject stands in a tell-relation to some possible MS answer. Hence, the baseline reading for be certain is an SE reading, whereas that for tell\(_{-\text{ver}}\) is an MS reading. As far as we can see, however, the choice for universal as opposed to existential quantification in the case of be certain does not receive an independent motivation and is therefore tantamount to hardcoding the baseline exhaustive strength into the lexical entry. In comparison, on our account, the unavailability of IE readings for be certain follows from the assumption that this verb only permits an internal, i.e., resolution-introspective interpretation.

Intermediate exhaustivity. Turning to intermediate exhaustivity, for extensional verbs this reading is derived by applying the exhaustification operator \( X \) at the root level, i.e., above the embedding verb.\(^{44}\) Roughly, if this operator applies to a sentence, it asserts the proposition expressed by the sentence and negates all strictly stronger alternatives of this proposition.

\[ [X]^{w} = \lambda \mathcal{P}(s, (s, t)). \mathcal{P}(w) \land \forall w' : \{ w' | \mathcal{P}(w')(v) \} \subset \{ w' | \mathcal{P}(w')(v) \} \rightarrow \neg \mathcal{P}(w)(v) \]

As can be seen from the structure in (84b), the propositional concept \( \mathcal{P} \) that \( X \) takes as its argument is the result of first computing the meaning of the sentence to which \( X \) applies, and then lambda-abstracting over the world argument of the answer operator in that sentence. For example, in the semantic derivation of (87a), \( X \) applies to the propositional concept in (87b).

\(^{43}\)The entry in (85b) is only a preliminary version; Uegaki’s final entry for tell has an excluded-middle presupposition, which we will turn to below.

\(^{44}\)This operator may be regarded as a refinement of the EXH operator in Klinedinst and Rothschild (2011). However, in contrast to Uegaki, Klinedinst and Rothschild are only concerned with deriving IE readings of non-factive verbs. Their account fails to derive such readings for factive verbs—which to us seem to be the prime case of FA sensitivity effects.
To see that this gives $X$ access to the relevant set of alternatives, assume, e.g., that in the world of evaluation $w$ Ann and Bill called, but Chris didn’t, whereas in $v$ all of Ann, Bill and Chris called. Then, $\{w' \mid \mathcal{P}(w')(v)\}$ is the set of all those worlds in which Mary told John the true WE answer to who called? in $v$, i.e., she told him that Ann, Bill and Chris called. In contrast, $\{w' \mid \mathcal{P}(w')(w)\}$ is the set of all those worlds in which Mary told John the true WE answer to who called? in $w$, i.e., she told him that Ann and Bill called. Observe that $\{w' \mid \mathcal{P}(w')(v)\} \subset \{w' \mid \mathcal{P}(w')(w)\}$. Hence, what $X$ asserts is that Mary told John that Ann and Bill called and she didn’t tell him that Ann, Bill and Chris called. This is exactly the IE reading of (87).

This is the way in which IE readings can in principle be derived on Uegaki’s account. When it comes to restricting their availability, a certain feature of the exhaustivity operator becomes crucial: this operator interacts with the monotonicity properties of the embedding verb in such a way that, if the verb is upward-monotonic, IE is derived, whereas, if the verb is not upward-monotonic, the contribution of $X$ is vacuous. To see why, consider again the definition of $X$ in (86). In the case of a predicate that is not upward-monotonic, the implication in the second conjunct is vacuously satisfied because it will never be the case that $\{w' \mid \mathcal{P}(w')(v)\} \subset \{w' \mid \mathcal{P}(w')(w)\}$. For upward-monotonic predicates, on the other hand, $\{w' \mid \mathcal{P}(w')(v)\} \subset \{w' \mid \mathcal{P}(w')(w)\}$ can become true; it holds for all worlds $v$ and $w$ such that $\text{Ans}_d(v)(Q) \subset \text{Ans}_d(w)(Q)$. Hence, for these verbs the second part in the definition of $X$ applies non-vacuously.

Uegaki’s account thus establishes a connection between the monotonicity properties of embedding verbs and the availability of IE readings. More specifically, by assuming that emotive factives like be happy and be surprised are non-monotonic, these verbs are predicted to lack IE readings. The only predicted reading for emotive factives is their baseline reading, i.e., WE. On the other hand, epistemic attitude verbs like know and the veridical variants of communication verbs like tell, which are assumed to be upward monotonic, are predicted to have IE readings.

While we find the approach ingenious, we have four reservations. First, as also noted by Uegaki himself, the non-monotonicity of emotive factives, on which his analysis crucially relies, is debatable (von Fintel, 1999; Crnič, 2011). Second, the assumed unavailability of SE readings for emotive factives has been contradicted by recent experimental work (Cremers and Chemla, 2017). Third, as argued by Xiang (2016b), it seems difficult to extend the account to derive FA-sensitive MS readings. Fourth, Uegaki’s account only derives a connection between monotonicity properties and exhaustive strength for extensional verbs. For intensional verbs, essentially, exhaustive strength must still be encoded in the individual lexical entries. In sum, this means that Uegaki’s theory of exhaustive strength is explanatory only for extensional verbs and, for these verbs, only to the extent that their monotonicity properties indeed correlate with the presence/absence of IE readings; more experimental work is needed to determine conclusively whether such a correlation exists (cf., Cremers and Chemla, 2017).

Strong exhaustivity. An SE reading can be derived in two different ways on Uegaki’s account, depending on whether the verb is extensional or intensional. In either case, however, SE arises from an excluded-middle assumption, which is encoded in the lexical entry of the embedding verb as a soft presupposition (Gajewski, 2007; Abusch, 2010).

To begin with, consider an intensional verb like tell, whose lexical entry including the relevant presupposition is given in (88). According to Uegaki, tell comes with an excluded-middle assumption stating that, for every possible answer $p$ to the embedded interrogative, the subject $x$ has either told the addressee $y$ that $p$ or told her that $\neg p$. It is easy to see that this condition gives rise to an SE reading. Uegaki thus predicts that for intensional verbs like tell the SE reading can directly arise from their excluded-middle presupposition. Under this view,
in order for the SE reading not to arise, on the other hand, the presupposition needs to be suspended.

\[ \langle R_{\text{tell}} \rangle^w = \lambda P_{(s,(s,t))} \lambda y \lambda x. \]

\[ \exists w'. \text{tell}(x,y,P(w'),w) \quad \text{if} \forall w'. \begin{cases} \text{tell}(x,y,P(w'),w) \lor \\ \text{tell}(x,y,\neg P(w'),w) \end{cases} \quad \text{undefined otherwise} \]

Turning to extensional verbs, the situation becomes slightly more complex. Here, it is not the verb itself that is relevant for the excluded-middle presupposition, but its non-factive counterpart. For example, in the case of know, Uegaki encodes the excluded-middle presupposition in terms of believe, i.e., in terms of the subject’s doxastic state:

\[ \langle R_{\text{know}} \rangle^w = \lambda P_{(s,t)} \lambda y \lambda x. \]

\[ \text{DOX}_w^x \subseteq p \quad \text{if} \quad p(w) \land \begin{cases} \text{DOX}_w^x \subseteq p \lor \\ \text{DOX}_w^x \subseteq \neg p \end{cases} \quad \text{undefined otherwise} \]

Note that, in contrast to the intensional verb, here the excluded-middle presupposition is not formulated with respect to every possible answer; instead it only concerns a specific proposition \( p \). If \( p \) is the true WE answer in the world of evaluation, for example, the excluded-middle presupposition in (89) does not itself amount to an SE reading. However, as soon as this presupposition is combined with an IE reading, SE follows. To see why, recall that an IE reading, derived by applying the \( X \) operator, would assert that for every answer \( p \), if \( p \) is true then the subject \( x \) believes \( p \), and if \( p \) is false then \( x \) does not believe \( p \). Now, since the excluded-middle assumption tells us that for every \( p \), \( x \) either believes \( p \) or believes \( \neg p \), it follows from the IE reading that, for every false answer \( p \), \( x \) believes \( \neg p \). This gives us an SE reading.

In the case of extensional verbs, SE readings are thus parasitic on IE readings on Uegaki’s account. In particular, this means that emotive factives, which are predicted to lack IE readings, are predicted not to have SE readings either. For extensional verbs that do permit IE readings on the other hand, and for intensional verbs, the availability of SE readings depends on whether the verb comes with an excluded-middle presupposition, which Uegaki assumes to be the case exactly if it licenses neg-raising.

As far as we can see, the main problem for this analysis is that, in order to derive SE readings, excluded-middle presuppositions need to be assumed even for verbs for which it is very debatable whether such presuppositions exist. For instance, if Uegaki wants to derive SE readings for intensional communication verbs like \text{tell}[-\text{ver}]—and it seems that he does (p.156)—then an excluded-middle presupposition needs to be assumed for such verbs, although they do not readily seem to license neg-raising:\footnote{Uegaki distinguishes between a literal and a deductive reading of communication verbs (Theiler, 2014). This distinction, however, does not appear to have a bearing on the licensing of excluded-middle inferences: \text{tell} seems to license such inferences neither on the literal nor on the deductive reading.}

(90) Ann didn’t tell Bill that it is raining.

\[ \neg \text{Ann told Bill that it is not raining.} \]

A further problem arises in connection with \text{know}. Since \( R_{\text{know}} \) carries an excluded-middle presupposition that is formulated in terms of the subject’s doxastic state, the inference in (91) is predicted to go through. This ‘pseudo neg-raising’ effect is clearly undesirable.

(91) Ann doesn’t know that it is raining.

\[ \neg \text{Ann believes that it is not raining.} \]
A.2 Veridicality, factivity, and extensionality

Uegaki’s account differs from ours in that it predicts a one-to-one connection between factivity and extensionality. We will see that, as a result, non-factive veridical verbs like be right have to be treated as intensional verbs—which has a number of undesirable consequences.

In Uegaki’s system, there are two reasons why a verb can come out as veridical: either because the verb is extensional, i.e., the answer operator is evaluated in the root evaluation world, or because the verb meaning is decomposed into an inherently veridical core predicate plus an answer operator. Let us consider both in turn.

First, recall that extensional verbs do not bind the world argument of Ans_d themselves, but leave it to the exhaustivity operator X to bind this argument. This operator, whose definition is repeated in (92) below, takes a propositional concept $P$ and, among other things, asserts that the result of evaluating $P$ at the root world of evaluation $w$ holds at $w$. Since $X$ obligatorily applies in the case of extensional verbs, for these verbs Ans_d thus gets evaluated in $w$. This means that, if extensional verbs take an interrogative complement, Ans_d delivers the true WE answer in the world of evaluation. If they take a declarative complement, whose meaning Uegaki represents as a singleton set containing the proposition that the complement is standardly taken to express, the definedness restriction of Ans_d amounts to a factivity presupposition. Crucially, for Uegaki, extensionality thus always entails both veridicality with respect to interrogative complements and factivity with respect to declarative complements.

\begin{equation}
[X]_w = \lambda P_{(s,(s,t))}. P(w)(w) \land \forall v : \{ w' \mid P(w'(v)) \} \subset \{ w' \mid P(w')(w) \} \rightarrow \neg P(w)(w)
\end{equation}

The second way in which a verb can be veridical is by virtue of the inherent veridicality of its core predicate. For example, Uegaki assumes the following core predicate for prove.

\begin{equation}
[R_{prove}]_w = \lambda P_{(s,(s,t))}. \lambda x. \exists w'. \text{prove}(x, P(w'), w)
\end{equation}

The format of this predicate does not by itself differ from that of other intensional verbs with an existential semantics such as tell[\textunderscore ver]. However, Uegaki additionally assumes by way of a meaning postulate that the implication in (94) holds.

\begin{equation}
\forall p. \forall x. \forall w. (\text{prove}(x, p, w) \rightarrow p(w))
\end{equation}

This means that prove comes out as veridical with respect to both interrogative and declarative complements: if $P$ is the meaning of an interrogative complement, $R_{prove}(P)(x)$ is only true in $w$ if there exists a $w'$ such that $P(w')$ is a true answer in $w$ and prove$(x, P(w'), w)$. If $P$ is the meaning of a declarative and it holds in $w$ that $R_{prove}(P)(x)$, then it follows that the information conveyed by the declarative complement is true in $w$.

Note that, different from those verbs for which veridicality arises from extensionality, intensional verbs like prove, for which veridicality results from the inherent veridicality of their core predicate, are not predicted to be factive. This means that the only way in which a verb can be factive on Uegaki’s account is by virtue of its extensionality. Hence, under this analysis, there is a one-to-one connection between extensionality and factivity.

This connection has consequences for which verbs get classified as extensional and which as intensional on Uegaki’s approach. In particular, verbs like prove and be right have to be treated as intensional since they don’t give rise to factivity presuppositions. Different from garden-variety intensionals, however, such verbs are veridical. As we will see below, their treatment as intensionals has a number of undesirable consequences: they are predicted to exhibit no FA sensitivity effects, to have no WE/IE readings, and to have SE readings only in so far as they trigger an excluded-middle presupposition. Let us examine these predictions in some more detail.

Recall that intensional verbs, unlike extensional ones, bind the world argument of the answer operator. Hence, while in the case of extensions this argument remains free and can be bound
by the exhaustivity operator $X$, in the case of intensionals $X$ does not have a world to bind. As a consequence, $X$ cannot apply to sentences with intensional embedding verbs. However, in Uegaki’s system the $X$ operator is used to derive FA sensitivity effects. This means that intensional verbs are predicted not to exhibit FA sensitivity effects. While this indeed seems to be a correct prediction for prima-facie intensional verbs like be certain or tell [−ver], it appears to be wrong for veridical verbs like prove and be right, as illustrated by the following example.

Assume that Ann and Bill, but not Chris were at a party. Mary believes that Ann and Bill were there, but has no beliefs about Chris’ presence at the party. In this scenario, (95) can be judged true. This means that readings other than the SE reading need to be available for (95), because under the SE reading (95) is false. Now assume instead that Mary believes Ann, Bill and Chris were at the party. In this scenario, it seems that (95) would be judged false. This means that an IE reading, i.e., an FA sensitive reading is needed for (95)—but this reading is unavailable for be right on Uegaki’s account.

(95) Mary is right about who was at the party.

On the other hand, whether intensional verbs have SE readings on Uegaki’s account depends on whether they carry an excluded-middle presupposition. For be right and prove, however, this does not seem to be the case, as illustrated by (96) and (97). Hence, be right and prove would come out as lacking SE readings.

(96) Ann isn’t right that it is raining.
   \not\exists w \text{ Ann is right that it’s not raining.}

(97) Ann didn’t prove that 3 is prime.
   \not\exists w \text{ Ann proved that 3 is not prime.}

In terms of exhaustive strength, Uegaki’s analysis thus predicts intensional verbs to be limited to their baseline reading (unless they carry excluded-middle presuppositions, in which case also the SE reading is available). As discussed above using the example of tell [−ver], if an existential semantics is assumed, the baseline reading is an MS reading. Since Uegaki proposes an existential semantics for be right and prove, the only reading these verbs are predicted to have is a non-FA-sensitive MS reading.

To sum up, Uegaki’s account makes a number of problematic predictions for non-factive veridical verbs like be right and prove, which arise from the treatment of such verbs as intensionals. This treatment, however, is unavoidable for Uegaki since on his account extensionality and factivity cannot be teared apart. In comparison, on our account these problems do not arise since there is no comparable connection between extensionality and factivity.

B  Formal proofs

B.1  Internal/external interpretation of know

Here, we provide proofs of Fact 1 on page 22 and of Fact 2 on page 23, both repeated below.

Fact 1. If the complement receives an MS or a SE interpretation, then the external and the internal interpretation of the verb yield exactly the same reading for the sentence as a whole.

Proof. (Sketch) Consider the sentence John knows who called and assume that the complement receives an MS reading. Then, under an external interpretation of the verb, the sentence is true in $w$ just in case $\text{dox}^w_\text{f} \in f_{\text{MS}}(w)$, where $f_{\text{MS}}(w)$ is the set of MS truthful resolutions

\footnote{To be more concrete, the IE reading is derived via application of $X$, and Uegaki also suggests a way to derive FA sensitive MS readings, namely by expressing the verb phrase in terms of a disjunction and then applying $X$ to each of the disjuncts.}
of the complement in \( w \). Now, it follows straightforwardly from the definition of MS truthful resolutions that if a proposition \( p \) is an MS truthful resolution in some world \( w \), then it is also an MS truthful resolution in any world in which it is true, i.e., in any \( v \in p \). But this means that \( \forall v \in \text{DOX}_w: \text{DOX}_v^w \in f_{\text{MS}}(v) \). Thus, the resolution introspection requirement is automatically satisfied, and the sentence is not only true in \( w \) under an external interpretation of the verb but also under an internal interpretation. A similar argument can be given in case the complement receives an SE reading.

\[ \]

**Fact 2.** If the verb receives an internal interpretation, then the IE and the SE interpretation of the complement yield exactly the same reading for the sentence as a whole.

**Proof.** Suppose that in a world \( w \), \( x \) knows who called under an internal interpretation of the verb and an IE interpretation of the complement. Then we do not only have that \( \text{DOX}_w \in f_{\text{IE}}(w) \), where \( f_{\text{IE}}(w) \) is the set of IE resolutions of the complement in \( w \), but also, by resolution introspection, that \( \text{DOX}_w^v \in f_{\text{IE}}(v) \) for every \( v \in \text{DOX}_w^w \).

Now, towards a contradiction, assume that \( \text{DOX}_w^w \) does not coincide with an SE resolution of the complement in \( w \), i.e., that \( \text{DOX}_w^w \notin f_{\text{SE}}(w) \). Then there must be an individual \( y \) such that \( y \) did not call in \( w \) but \( x \) doesn’t know this, i.e., \( \text{DOX}_w^w \) must contain at least one world \( w_y \) where \( y \) did call. On the other hand, \( \text{DOX}_w^w \) cannot consist exclusively of worlds where \( y \) called, because \( y \) did not call at \( w \) and \( \text{DOX}_w^w \in f_{\text{IE}}(w) \). Now, since \( w_y \in \text{DOX}_w^w \), we must have, by the introspection requirement above, that \( \text{DOX}_w^v \in f_{\text{IE}}(v_y) \). But all IE truthful resolutions at \( w_y \) must be propositions that establish that \( y \) called. So \( \text{DOX}_w^w \) must establish that \( y \) called as well, i.e., it must consist exclusively of worlds in which \( y \) called. This is in contradiction with what we derived earlier. So we can conclude that \( \text{DOX}_w^w \) must coincide, after all, with an SE resolution of the complement in \( w \), i.e., that \( \text{DOX}_w^w \in f_{\text{SE}}(w) \).

It remains to be shown that \( \text{DOX}_w^w \in f_{\text{SE}}(v) \) for all \( v \in \text{DOX}_w^w \). Suppose that \( v \in \text{DOX}_w^w \). Then, since we have just established that \( \text{DOX}_w^w \in f_{\text{SE}}(w) \), we have that \( v \in f_{\text{SE}}(w) \) as well. But then, given the partition-inducing nature of SE resolutions, it follows that \( f_{\text{SE}}(v) = f_{\text{SE}}(w) \). Thus, we can conclude that \( \text{DOX}_w^w \in f_{\text{SE}}(v) \), as desired.

**B.2 Veridicality and c-distributivity**

Here we provide a proof of Fact 5 on page 36, repeated below.

**Fact 5.** A c-distributive responsive verb that is veridical w.r.t. declarative complements is also veridical w.r.t. interrogative complements.

In order to prove this connection between c-distributivity and veridicality, we first give fully explicit definitions of veridicality w.r.t. declarative and interrogative complements, respectively. We start with veridicality w.r.t. declarative complements, which is a straightforward notion.

**Definition 10** (Veridicality w.r.t. declarative complements).

A declarative-embedding verb \( V \) is veridical w.r.t. declarative complements if and only if for any individual \( x \), any world \( w \) and any declarative nucleus meaning \( P \):

\[
\text{If } V(E(P))(x) \text{ is true in } w, \text{ then } P \text{ is true in } w.
\]

Veridicality w.r.t. interrogative complements is a more complex notion. In the framework that we have developed an interrogative complement meaning is always obtained by applying \( E \) to an interrogative nucleus meaning \( Q \), and a declarative complement meaning is obtained by applying \( E \) to a declarative nucleus meaning \( P \). Furthermore, \( Q \) is exhaustivity-neutral if and only if the alternatives in \( \text{alt}(Q) \) form a partition of the set of all possible worlds, and \( P \) constitutes a ‘complete answer’ to such an exhaustivity-neutral \( Q \) if and only if the informative content of \( P \) coincides precisely with one of the cells in the partition induced by \( Q \), i.e., \( \text{info}(P) \in \text{alt}(Q) \). Given these notions, veridicality w.r.t. interrogative complements is defined as follows.
Definition 11 (Veridicality w.r.t. interrogative complements).
A responsive verb \( V \) is veridical w.r.t. interrogative complements if and only if for any individual \( x \), any world \( w \), any interrogative nucleus meaning \( Q \) such that \( \text{alt}(Q) \) is a partition of \( W \), and any a declarative nucleus meaning \( P \) such that \( \text{info}(P) \in \text{alt}(Q) \):

If \( V(E(Q))(x) \) is true in \( w \) and \( P \) is true in \( w \), then \( V(E(P))(x) \) is true in \( w \) as well

With these definitions in place, we are ready to prove Fact 5.

Proof of Fact 5. Let \( V \) be a c-distributive responsive verb that is veridical w.r.t. declarative complements. Towards establishing that \( V \) is veridical w.r.t. interrogative complements, let \( Q \) be an interrogative nucleus meaning such that \( \text{alt}(Q) \) forms a partition of \( W \), let \( x \) be an individual, and \( w \) a world such that \( V(E(Q))(x) \) is true in \( w \). Furthermore, let \( P \) be a declarative nucleus meaning such that \( \text{info}(P) \in \text{alt}(Q) \) and such that \( P \) is true in \( w \). We have to show that \( V(E(P))(x) \) is true in \( w \) as well.

Since \( V(E(Q))(x) \) is true in \( w \) and \( V \) is c-distributive, it must be the case that \( V(E(P'))(x) \) is true in \( w \) for some \( P' \in \text{decomp}(Q) \) (we know that \( \text{decomp}(Q) \) exists because \( \text{alt}(Q) \) forms a partition of \( W \)). Now, towards a contradiction, suppose that \( P' \neq P \). Then, since both \( \text{info}(P') \) and \( \text{info}(P) \) are elements of \( \text{alt}(Q) \), and \( \text{alt}(Q) \) forms a partition of \( W \), \( \text{info}(P') \) and \( \text{info}(P) \) must be disjoint. Since \( P \) is true in \( w \), it follows that \( P' \) cannot be true in \( w \). But then, since \( V \) is veridical w.r.t. declarative complements, it follows that \( V(E(P'))(x) \) cannot be true in \( w \) either, contrary to what we assumed. Thus, we can conclude that \( P' = P \). It follows that \( V(E(P))(x) \) is true in \( w \), which is precisely what we needed to show in order to establish that \( V \) is veridical w.r.t. interrogative complements. \( \square \)

B.3 Veridicality and choice property

Here we provide a proof of Fact 6 on page 39, repeated below.

Fact 6. Any c-distributive responsive verb that has the choice property and is veridical w.r.t. interrogative complements must also be veridical w.r.t. declarative complements.

Proof of Fact 6. We will prove that any c-distributive verb that has the choice property and is not veridical w.r.t. declarative complements is not veridical w.r.t. interrogative complements either. Let \( V \) be a c-distributive responsive verb that has the choice property and is not veridical w.r.t. declarative complements. This means that there is a declarative nucleus meaning \( P \), an individual \( x \), and a world \( w \) such that \( V(E(P))(x) \) is true in \( w \) but \( P \) itself is not true in \( w \).

Towards establishing that \( V \) cannot be veridical w.r.t. interrogative complements in this case, let \( Q \) be the interrogative nucleus meaning \( P \cup \neg P \). We can think of \( Q \) as the question ‘whether \( P \)’. Note that \( \text{alt}(Q) \) forms a partition of \( W \), and \( \text{info} (\neg P) \in \text{alt}(Q) \). Now, since \( V(E(P))(x) \) is true in \( w \) and since \( V \) is c-distributive, it follows that \( V(E(Q))(x) \) is true in \( w \) as well. We also have that \( \neg P \) is true in \( w \). This means that, if \( V \) were veridical w.r.t. interrogative complements, it should be the case that \( V(E(\neg P))(x) \) is true in \( w \) as well. This cannot be the case, because \( V(E(P))(x) \) is true in \( w \) and \( V \) has the choice property. So, \( V \) cannot be veridical w.r.t. interrogative complements. \( \square \)

References


