A uniform semantics for declarative and interrogative complements

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Abstract
This paper proposes a semantics for declarative and interrogative complements and for verbs like know, believe and wonder, which embed either kind or both kinds of complements. Following Groenendijk and Stokhof (1984), we pursue a uniform account, in the sense that we take both kinds of complements to be of the same semantic type and we assume a single lexical entry for verbs like know that embed both declarative and interrogative complements. This approach avoids a number of problems for non-uniform theories, such as the reductive theories of Karttunen (1977); Heim (1994); Lahiri (2002); Spector and Egré (2015), among others, and the twin relations theory of George (2011).

On the other hand, our account also addresses the main limitation of Groenendijk and Stokhof’s proposal, which is that it is primarily designed to derive strongly exhaustive readings for interrogative complements. Our account is more flexible in that it straightforwardly derives mention-some and intermediate exhaustive readings as well.

Finally, the proposed semantics also accounts for the fact that verbs like believe only license declarative complements and verbs like wonder only license interrogative complements. These selectional restrictions are shown to follow from the lexical semantics of the embedding verbs and the assumed semantic properties of declarative and interrogative complements.

1 Introduction
This paper provides an account of declarative and interrogative complement clauses, and verbs that take either kind or both kinds of complement as their argument, exemplified in (1)-(3).

(1) a. Mary knows/forgot that John left.
   b. Mary knows/forgot who left.

(2) a. Bill believes/hopes that John left.
   b. *Bill believes/hopes who left.

(3) a. *Bill wonders/investigated that John left.
   b. Bill wonders/investigated who left.

Following Lahiri (2002) we will refer to verbs that license both kinds of complements as responsive verbs, and to verbs that only license interrogative complements as rogative verbs. In the same vein, we will refer to verbs that only license declarative complements as anti-rogative verbs.

*Acknowledgements to be added after the reviewing process.

1We will restrict our attention in this paper to simple polar interrogatives and wh-interrogatives. The theory we will develop needs to be further refined in order to deal, for instance, with disjunctive or conditional interrogatives. We believe such a refinement is feasible, but would involve issues that are orthogonal to our main concerns here.
It is often assumed, at least since Karttunen (1977), that declarative and interrogative complements differ in semantic type: declaratives are taken to denote propositions while interrogatives are often taken to denote sets of propositions. Prima facie, it is unexpected under this view that there are verbs like know and forget which take both declarative and interrogative complements as their argument. Various ways to resolve this tension have been proposed.

Most authors assume that responsive verbs want a proposition as their input—not a set of propositions. This means that if the complement of the verb is interrogative, a type mismatch arises. Heim (1994), Dayal (1996), and Beck and Rullmann (1999), among others, propose that this type mismatch is resolved by a type-shifting answer operator, which compresses the set of propositions generated by the interrogative clause into a single proposition and then feeds this proposition to the verb. Lahiri (2002) proposes that the type mismatch is resolved by raising the interrogative clause to a higher position in the syntactic structure, leaving a proposition-type variable in the verb’s argument slot.

A different strategy, briefly suggested by Karttunen (1977) and elaborated in detail by Spector and Egré (2015), is to assume two lexical entries for every responsive verb, one for each kind of complement. For instance, for know we would have two lexical entries, know_d and know_i, taking declarative and interrogative complements, respectively. Spector and Egré then formulate a general meaning postulate which, given the declarative entry V_d of a verb V, determines the corresponding interrogative entry V_i.

These different strategies all have one thing in common. Namely, they take the declarative-embedding interpretation of responsive verbs to be basic, and reduce the interrogative-embedding interpretation of any given verb in one way or another to its declarative-embedding interpretation. For this reason, all these strategies can be seen as particular instances of one general approach, which is referred to as the reductive approach.

George (2011, 2013) identifies a problem for the reductive approach. Briefly—the problem will be discussed in more detail in Section 2.1 below—George observes that whether an individual stands in the knowledge relation to a certain interrogative does sometimes not only depend on her true propositional knowledge, but also on whether she believes any false answers to that interrogative. This false answer sensitivity implies that interrogative knowledge cannot generally be reduced to true propositional/declarative knowledge.

In reaction to this, George (2011) considers two alternative approaches. The first, which he calls the inverse reductive approach, reduces the declarative-embedding interpretation of responsive verbs to their interrogative-embedding interpretation, rather than the other way around. The second alternative, which George calls the twin relations approach, derives both the declarative-embedding interpretation and the interrogative-embedding interpretation of responsive verbs from a common lexical core. George (2011) spells out a concrete twin relations theory, and also briefly sketches a concrete inverse reductive theory. The latter has been developed in greater detail by Uegaki (2015).

A phenomenon that allows us to tease apart the twin relations theory from the inverse reductive approach (as well as the standard reductive approach) is discussed by Elliott et al. (2016). Briefly—the phenomenon will be considered in more detail in Section 2.2—Elliott et al. observe that so-called predicates of relevance, such as care and matter, carry a certain presupposition when taking a declarative complement which is absent when the complement is interrogative. They argue that this is problematic for the standard reductive approach. Uegaki (2016) shows that it is also problematic for George’s twin relations theory. On the other hand, it can easily be accounted for on the inverse reductive approach. Thus, the latter seems to be the most attractive option among those approaches that share the basic assumption that declarative and interrogative complements differ in semantic type.

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2A similar argument was made by Groenendijk and Stokhof (1984, p.94) against the reductive theory of Karttunen (1977). Elliott et al.’s argument, however, is more explicit and targets the reductive approach in general rather than only Karttunen’s specific theory.
However, as depicted in Figure 1, there is also one prominent approach which does not adhere to this basic assumption, i.e., which treats declarative and interrogative complements uniformly, as being of the same semantic type. This is the approach of Groenendijk and Stokhof (1984). They treat both declarative and interrogative complements as denoting propositions. A declarative complement denotes the same proposition in every world, namely the one expressed by the corresponding declarative root clause. On the other hand, the proposition denoted by an interrogative complement in a world $w$ encodes the true exhaustive answer in $w$ to the issue expressed by the interrogative clause. Since these exhaustive answers together form a partition of the set of all possible worlds, Groenendijk and Stokhof’s theory is referred to as the partition theory.

On the partition theory, the existence of responsive verbs like know, which license both types of complements, does not come as a surprise and needs no particular explanation. Predicates of relevance like care and matter can also be handled straightforwardly, unlike on the reductive approach and on the twin relations theory. However, certain issues remain.

First, partition theory is, so to speak, still too close to the reductive approach to provide an account of George’s (2011) observations concerning false answer sensitivity. That is, it still wrongly predicts that if two individuals have the same propositional knowledge, they must stand in the knowledge relation to exactly the same interrogative complements.

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3The idea to treat declaratives and interrogatives uniformly actually goes further back, at least to Hamblin (1973, p.48). However, Hamblin was exclusively concerned with root clauses; he did not explicitly consider declarative and interrogative complements, and the repercussions of a uniform treatment for the analysis of verbs that take such complements as their argument, which is our main concern here.

4In view of this prediction, George actually classifies partition theory as a reductive theory. This classification, however, blurs the fact that Groenendijk and Stokhof (1984, p.93-94) themselves very explicitly argued against the reductive approach—understood as one that takes declarative and interrogative complements to be of different types and derives the interrogative-embedding interpretation of responsive verbs from their declarative-embedding interpretation. Instead, they chose to pursue a uniform approach. Our own classification is a refinement of Groenendijk and Stokhof’s (distinguishing various non-uniform approaches that did not exist yet in 1984).
Second, a much discussed limitation of partition theory is its lack of flexibility. That is, while it is mainly designed to derive so-called strongly exhaustive (SE) readings for interrogative complements, it needs to invoke additional machinery to derive non-exhaustive/mention-some (MS) readings, and it does not derive weakly exhaustive (WE) or intermediate exhaustive (IE) readings at all. Groenendijk and Stokhof argued that this is in fact a desirable feature of their theory, but other authors have disagreed (e.g., Heim, 1994; Beck and Rullmann, 1999; Spector, 2005; Klinedinst and Rothschild, 2011) and recent experimental results seem to show that intermediate exhaustive readings indeed exist (Cremers and Chemla, 2016b). We will return to this issue in more detail in Sections 2.3 and 2.4.

Third, if declarative and interrogative complements are taken to be of the same semantic type, then the fact that rogative verbs like wonder only license interrogative complements and anti-rogative verbs like believe only license declarative complements cannot be accounted for in terms of a type mismatch, something that could in principle be done, at least to some extent, on approaches that assume a type distinction between the two kinds of complement. We will argue in Section 2.5 that such a type-based account can in fact only cover one of the two cases (either rogative or anti-rogative verbs but not both at once). Moreover, such an account is of course not really explanatory as long as the type distinction is not independently motivated. Thus, non-uniform theories do not have an inherent advantage in this respect over partition theory or other conceivable uniform approaches. That said, Uegaki (2015) develops a rather detailed account of the selectional restrictions of rogative and anti-rogative verbs in the context of his inverse reductive theory. Partition theory, as well as most other existing theories, leave the problem open.

Finally, George (2011) and Spector and Egré (2015) discuss a fourth possible concern for partition theory, which is that it does not make any direct predictions as to how the interrogative-embedding interpretations of responsive verbs are related to their declarative-embedding interpretations. For instance, as Spector and Egré (2015) put it, it is in principle possible to formulate a lexical entry for the fictitious verb shknow in partition theory such that, when taking a declarative complement, the verb behaves just like know, while when taking an interrogative complement, it behaves just like wonder. After all, declarative complements are taken to denote the same proposition in every world, while interrogative complements denote different propositions across different possible worlds. The lexical entry for shknow could be made sensitive to this distinction: it could effectively test whether its complement is declarative or interrogative. Thus, the theory does not predict that verbs like shknow do not exist in natural languages, which is something that Spector and Egré find a theory should predict. Extrapolating from this example, it is clear that the theory does not directly predict the existence of any general constraints on the space of possible responsive verb meanings. This concern also applies to the inverse reductive approach; indeed, the issue was initially raised by George (2011) for this approach, rather than for partition theory. On the other hand, it does not apply to Spector and Egré’s reductive theory or George’s twin relations theory, which do predict the existence of general constraints on responsive verb meanings.

5Another manifestation of the restricted flexibility of partition theory is that it cannot straightforwardly deal with question types which, in some worlds, can be truthfully resolved in multiple, logically independent ways. Such question types include conditional questions and certain kinds of disjunctive questions (see Ciardelli et al., 2015, for discussion).

6It is actually difficult to test empirically whether this prediction is indeed correct. Clearly, a verb like shknow does not exist in English, and the same seems to hold for closely related languages. But we cannot be sure that this generalizes to all languages without checking all of them. However, Spector and Egré do have a principled reason to expect that verbs like shknow do not exist cross-linguistically: based on English and a number of other languages, they argue that there is in fact a very general constraint on the space of possible responsive verb meanings, of which the impossibility of verbs like shknow is just one particular consequence. We will return to this in detail in Section 2.6.
would play a role in fixing the quantificational domain of wh-phrases.

Section 4 lays out some general assumptions which concern both declarative and interrogative complements, these propositions do not encode what the true exhaustive answer to the interrogative is in any given world \( w \), but rather what its *truthful resolutions* are in \( w \). Such truthful resolutions need not be exhaustive, and need not even be true in \( w \); they just need to be *truthful*, which means that they should not imply any false information that is directly relevant w.r.t. the issue expressed by the interrogative. For instance, if Mary is currently watching the sunset, then the proposition that she is watching the sunset drinking a martini is a truthful resolution of the interrogative *whether Mary is watching the sunset* even if she is in fact not drinking a martini.

This switch from true exhaustive answers to truthful resolutions will allow us to provide a general account of the false answer sensitivity observed by George (2011), and to derive not only strongly exhaustive readings but also mention-some and intermediate exhaustive readings in a straightforward way. Moreover (independently from the move to truthful resolutions), we will offer an account of the selectional restrictions of rogative and anti-rogative verbs, and we will show that it is possible within a uniform approach to capture general constraints on the space of possible responsive verb meanings, of the kind identified by Spector and Egré (2015). Finally, we will compare our proposal in some detail with the inverse reductive theory of Uegaki (2015), which, even though it does not assume uniformity, is very close in spirit and empirical reach.

The paper is structured as follows. Section 2 discusses the empirical and theoretical desiderata that our account should satisfy in more detail, and Section 3 briefly reviews the main terminology and notational conventions of inquisitive semantics. Then we turn to the account proper. First, Section 4 lays out some general assumptions which concern both declarative and interrogative complements. Then, Section 5 zooms in on interrogative complements, Section 6 on declarative complements, and Section 7 on embedding verbs. Section 8 concludes, and Appendix A compares our proposal to that of Uegaki (2015).

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Table 1: Pros and cons of existing approaches.

The achievements and limitations of the approaches discussed here are summarized in Table 1. The theory developed in the present paper is like Groenendijk and Stokhof's partition theory in that it treats declarative and interrogative complements uniformly. However, building on recent work in inquisitive semantics (e.g., Ciardelli et al., 2015, 2016), it also differs in crucial respects from partition theory, overcoming its main limitations. Most fundamentally, it does not treat declarative and interrogative complements as denoting propositions, but as denoting sets of propositions. In the case of interrogative complements, these propositions do not encode what the true exhaustive answer to the interrogative is in any given world \( w \), but rather what its *truthful resolutions* are in \( w \). Such truthful resolutions need not be exhaustive, and need not even be true in \( w \); they just need to be *truthful*, which means that they should not imply any false information that is directly relevant w.r.t. the issue expressed by the interrogative. For instance, if Mary is currently watching the sunset, then the proposition that she is watching the sunset drinking a martini is a truthful resolution of the interrogative *whether Mary is watching the sunset* even if she is in fact not drinking a martini.

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7We restrict our attention here to *propositional* theories of interrogatives, leaving out so-called ‘categorial’ theories (e.g., von Stechow, 1991; Križka, 2001) as well as theories couched in other frameworks, such as situation semantics (e.g., Ginzburg, 1995). Categorial theories are not considered here because their main focus is on root interrogatives rather than embedded ones. Various phenomena involving root interrogatives require a more fine-grained notion of question meaning than the one provided by the standard propositional framework. These phenomena, however, can also be explained in extensions of the standard framework that takes dynamic aspects of meaning, i.e., the discourse referents that sentences introduce, into consideration (e.g., Aloni et al., 2007; Roelofsen and Farkas, 2015).

Ginzburg (1995) specifically addresses embedded interrogatives but his focus is on phenomena of context-sensitivity which are arguably orthogonal to the issues addressed in this article. To explain these cases, Ginzburg proposes to parametrise the ‘resolvedness’ relation holding between an interrogative and a piece of information with a number of contextual factors including the goals of the conversational participants. Most of the cases of context-sensitivity discussed by Ginzburg can be explained in an extension of the present approach, adopting for example a conceptual cover style of quantification along the lines of Aloni (2001). On this approach, speakers’ goals and other contextual factors, rather than being parameters of the resolvedness relation, would play a role in fixing the quantificational domain of wh-phrases.
2 Empirical and theoretical desiderata

2.1 False answer sensitivity

The problem identified by George (2011) for traditional reductive theories, as well as for Groenendijk and Stokhof’s uniform partition theory, can be described as follows. Consider a scenario in which there are two stores, Newstopia and Paperworld, of which only one, namely Newstopia, sells Italian newspapers. Janna knows, true to fact, that Newstopia sells Italian newspapers and does not have any beliefs concerning the availability of such newspapers elsewhere. Rupert, on the other hand, while also knowing that one can buy an Italian newspaper at Newstopia, falsely believes that Paperworld sells such newspapers as well. George (2011) observes that there is a salient reading under which sentence (4a) is judged true in this scenario, while (4b) is judged false.

(4) a. Janna knows where one can buy an Italian newspaper.
   b. Rupert knows where one can buy an Italian newspaper.

Crucially, Janna and Rupert know exactly the same set of relevant propositions in the given scenario. Indeed, the only relevant proposition that they both know is the proposition that Newstopia sells Italian newspapers. Rupert additionally believes the proposition that Paperworld sells Italian newspapers, but he doesn’t know this proposition, simply because it is in fact false. This is problematic for the reductive approach, because it shows that ‘interrogative knowledge’ is not always reducible to ‘declarative knowledge.’ It reveals that interrogative knowledge may not only depend on true declarative knowledge but also on beliefs that constitute false answers to the interrogative at hand.

Xiang (2015) observes that such false answers need not be completely resolving answers: the assessment of interrogative knowledge ascriptions is even sensitive to beliefs concerning false partial answers. To see this, consider the same kind of scenario as above but now with three stores, Newstopia, Paperworld, and Celluloid City, of which only Newstopia sells Italian newspapers. Suppose Rupert knows that Newstopia sells Italian newspapers and additionally wrongly believes that either of Paperworld and Celluloid City sells them as well, although he isn’t certain which of the two. Xiang (2015) observes that (4b) is still judged false in this scenario. Thus, Rupert’s belief in the false partial answer ‘that either Paperworld or Celluloid City sells Italian newspapers’ is sufficient to block interrogative knowledge.

2.2 Predicates of relevance

The problem that Elliott et al. (2016) identify for the standard reductive approach, and which Uegaki (2016) shows to be problematic for the twin relations theory as well, concerns predicates of relevance, like care, matter, and be relevant. Consider the two sentences in (5):

(5) a. John cares that Mary left.
    b. John cares which girl left.

Elliott et al. observe that (5a) presupposes that Mary left and that John knows this. On the other hand, (5b) does not presuppose that John believes any answer to the embedded interrogative. This is problematic for standard reductive theories, because they predict that a sentence like (5b) is true if and only if it matters to John that $p$, where $p$ is a proposition that counts as an answer to the interrogative in (5b). Thus, (5b) is wrongly predicted to presuppose that John believes $p$, for some answer $p$. We refer to Uegaki (2016) for a demonstration that the problem arises on George’s twin relations theory as well.
2.3 Flexibility: three levels of exhaustivity

Traditionally, three kinds of readings are distinguished for interrogative complements: strongly exhaustive (SE) readings, weakly exhaustive (WE) readings, and non-exhaustive readings. The latter are also often referred to as mention-some (MS) readings, and we will follow this custom. Partition theory is mainly designed to derive SE readings. It can also derive MS readings, but only invoking substantial additional machinery. WE readings are not derivable at all. We want our account to be able to derive all three kinds of readings in an equally straightforward way (although the specific make-up of some complement clauses and the lexical semantics of some verbs will restrict the range of derivable readings in some cases).

Moreover, following Spector (2005), George (2011), Klinedinst and Rothschild (2011), Xiang (2015), among others, we will assume a particular amendment of the traditional characterization of WE and MS readings, incorporating false answer sensitivity. To explain the needed amendment, let us illustrate the three readings as traditionally described, based on the example in (6), where the embedding verb is know (there has been much discussion as to whether know really admits all three readings; we will turn to this issue in Section 2.4).

(6) John knows who called.

The three traditional readings of (6) come apart in the amount of knowledge ascribed to John. To make this more explicit, assume a domain of discourse $D$, assume that John knows that the domain of discourse is $D$, and assume that the set of answers to the question who called is $A = \{d \text{ called} \mid d \in D\}$. Under the SE reading, (6) is true just in case both of the following conditions hold: for any true answer $a \in A$, John knows that $a$ is true, and moreover, for any false answer $a \in A$, John knows that $a$ is false. The WE reading is less demanding: for the truth of (6) it suffices that, for any true answer $a \in A$, John knows that $a$ is true; it does not matter under this reading what John believes about false answers. Under the MS reading, (6) is true exactly if, for at least one true answer $a \in A$, John knows that $a$ is true. Again, John's beliefs about false answers do not make a difference for MS readings. Thus, traditionally, false answers only play a role for SE readings.

However, Spector (2005), George (2011), Klinedinst and Rothschild (2011), and Xiang (2015) have argued that false answers are relevant for weaker readings as well. Spector (2005) and Klinedinst and Rothschild (2011) point out their relevance for WE readings, based on sentences like (7).

(7) John told Mary who passed the exam.

Suppose that only Anna and Bill passed the exam. Then, under what seems to be the most salient reading of (7), it is judged true if John told Mary that Anna and Bill passed the exam and he didn't tell her anything else. On the other hand, it is judged false if John additionally told Mary, erroneously, that Chris and Daniel passed the exam as well.

As already mentioned above, George (2011) similarly points out the relevance of false answers for MS readings, based on examples like (4) above, and Xiang (2015) notes that false partial answers are relevant for these readings as well.

These observations suggest that the traditional characterization of WE and MS readings needs to be adapted, taking false answer sensitivity (FA sensitivity for short) into account. For example (6), this would yield the following truth conditions, assuming that $A^\vee := \{a_1 \lor \ldots \lor a_n \mid a_i \in A\}$ is the set of partial answers to the question who called:

(8) a. Strongly exhaustive reading, as before:

- for any true answer $a \in A$, John knows that $a$ is true, and
b. FA sensitive weakly exhaustive reading:
   - for any true answer \( a \in A \), John knows that \( a \) is true
   - for any false (partial) answer \( a \in A^\dagger \), John does not believe that \( a \) is true

c. FA sensitive mention-some reading:
   - for at least one true answer \( a \in A \), John knows that \( a \) is true
   - for any false (partial) answer \( a \in A^\dagger \), John does not believe that \( a \) is true

We will follow Klinedinst and Rothschild (2011) in referring to FA sensitive WE readings as intermediate exhaustive (IE) readings. When it comes to deriving these different readings, some theories focus exclusively on FA sensitive MS readings (George, 2011) while others focus on IE readings (Spector, 2005; Klinedinst and Rothschild, 2011; Uegaki, 2015; Spector and Egré, 2015; Cremers, 2016). Like Xiang (2015), we will aim to give a general account of FA sensitivity that applies uniformly across the different levels of exhaustive strength.

Finally, we note that not all interrogative-embedding verbs exhibit FA sensitivity. One example of a verb that doesn’t is be certain:

(9) Rupert is certain where one can buy an Italian newspaper.

In contrast to (4b), where the same complement was embedded under know, (9) is saliently judged true in George’s Italian newspaper scenario. We want our account to predict this lack of FA sensitivity for verbs like be certain.

2.4 Knowledge ascription and introspection

As already alluded to, there has been much discussion in the literature as to whether certain verbs are incompatible with some of the readings for interrogative complements distinguished above. We will focus here on the case of know, which is particularly controversial.

The controversy concerns the availability of the IE reading. On the one hand, Groenendijk and Stokhof (1982, p.180) argued that know does not license IE readings:

“Suppose that John knows of everyone who walks that he/she does; that of no one who doesn’t walk, he believes that he/she does; but that of some individual that actually doesn’t walk, he doubts whether he/she walks or not. In such a situation, John would not say of himself that he knows who walks. We see no reason to override his judgement and to claim that in this situation, John does know who walks. This seems to suggest that for John to know who walks, he should not only know of everyone who walks that he/she does, but also of everyone who doesn’t that he/she doesn’t.”

Many authors have found this argument convincing and have therefore assumed, with Groenendijk and Stokhof, that know only allows for SE and MS readings.

However, recent experimental work by Cremers and Chemla (2016b) seems to show quite clearly that know does license IE readings. Cremers and Chemla asked the participants in their experiment to consider the following context:

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-This characterization of strong exhaustivity, which has been chosen here for expository purposes, does not correspond completely to that given in Groenendijk and Stokhof (1984). Under the latter, John would be required to know what the extension of the predicate call is. The two notions do coincide if we assume that John is fully informed about which individuals constitute the domain of discourse.
There is a set of cards, each consisting of four squares. Each square can be blue (B), green (G) or red (R). John is playing a game with these cards: he uncovers a card, looks at it briefly and tries to remember which of the squares on the card were blue. In the first round, the card he looked at was the left one in Figure 2. Now, consider two different scenarios: in scenario A, John’s beliefs about the card he looked at are as represented by the second picture in Figure 2; in scenario B, John’s beliefs about the card he looked at are as represented by the third picture in Figure 2.

Now consider the following sentence:

(10) John knew which squares were blue.

Cremers and Chemla found that (10) was saliently judged false in scenario A, while it was saliently judged true in scenario B. This can only be the case if the complement in (10) received an IE reading. Under an SE reading the sentence would have been judged false in both scenarios.\(^9\)

How could this experimental result be reconciled with the widely held view, rooted in Groenendijk and Stokhof’s argument, that know does not license IE readings? What is crucial, we believe,\(^{10}\) is to recognize that knowledge ascriptions are multiply ambiguous: besides the different readings of the complement, the verb itself also allows for two different interpretations. Groenendijk and Stokhof only considered one of these interpretations, namely the one that requires a strong form of introspection on the part of the individual to whom knowledge is ascribed. For Groenendijk and Stokhof it is unwarranted to claim that John knows who walks in a situation in which John would not say of himself that he knows who walks. Another interpretation, however, seems to be made particularly salient in the experimental setting of Cremers and Chemla. Here, it is not really at stake whether John would say of himself that he knew which squares were blue; rather, what is at stake is whether we, as external observers, find that there is a sufficient match between John’s beliefs (the second/third picture) and actuality (the first picture).

Thus, Groenendijk and Stokhof assumed an internal interpretation of knowledge ascriptions, requiring a strong form of introspection, whereas Cremers and Chemla’s experimental setting lends particular salience to an external interpretation of knowledge ascriptions, which does not come with the relevant introspection requirement.

Our aim will be twofold. First, we want to capture the difference between these two interpretations, i.e., the pertinent notion of introspection. And second, we want our theory to derive that the external interpretation is indeed compatible with IE readings, while the internal interpretation (Groenendijk and Stokhof’s) is incompatible with such readings.

\(^9\)An MS reading is in general unavailable for plural which-interrogatives with a distributive predicate, such as the one in (10).

\(^{10}\)We are much indebted to Jeroen Groenendijk for discussion of this issue.
2.5 Selectional restrictions

We have seen above that rogative verbs like wonder only license interrogative complements, while anti-rogative verbs like believe only license declarative complements. At first sight, it may seem attractive to account for these selectional restrictions by assuming that (i) declarative and interrogative complements differ in semantic type, (ii) rogative verbs like wonder only take complements of one type, and (iii) anti-rogative verbs like believe only take complements of the other type. However, as long as the type distinction is not independently motivated, this is of course merely a reformulation of the observed pattern in terms of semantic types; it has no explanatory value.

What is more, as pointed out by Groenendijk and Stokhof (1984, p.93-94) in arguing for a uniform approach, the different kinds of complements can be conjoined.

(11) a. John knows that Mary arrived and whether she brought the wine.
    b. John knows that Mary arrived and what she brought.

To handle such conjunctions, any non-uniform account of declarative and interrogative complements needs to assume a type-shifting operation that can shift the semantic type of one of the complements to that of the other. Indeed, such a type-shifting operation is explicitly assumed by Heim (1994), Dayal (1996), Beck and Rullmann (1999), and Uegaki (2015), among others. However, as soon as we admit such an operation, we either lose the account of the selectional restrictions of verbs like believe, or of verbs like wonder. After all, if the assumed type-shifting operator adapts the type of declarative complements to that of interrogative complements, then there is no reason anymore why verbs like wonder would not accept declarative complements. And vice versa, if the operator adapts the type of interrogative complements to that of declarative complements, then there is no reason anymore why verbs like believe would not accept interrogative complements.

Thus, a type-based account of the observed selectional restrictions cannot capture the full range of facts, and needs to be supplemented by independent evidence for the assumed semantic type distinction in order to have any explanatory value. Therefore, non-uniform theories do not have an inherent advantage in comparison with uniform theories when it comes to accounting for selectional restrictions. Even if the different kinds of complements are taken to have the same semantic type, they may still differ in more specific semantic properties, and those properties may clash with those of certain verbs. This is the approach that we will take in accounting for the selectional restrictions of rogative and anti-rogative verbs.

2.6 Constraints on responsive verb meanings

As briefly discussed in the introduction, George (2011) and Spector and Egré (2015) argue that a theory of clause-embedding verbs should be able to account for general constraints on responsive verb mean-
Spector and Egré give empirical arguments for one particular such constraint, which involves the distinction between veridical and non-veridical verbs.

The notion of veridicality comes in two flavors, pertaining to declarative and interrogative complements, respectively. A verb is called veridical w.r.t. declarative complements if, when taking a declarative complement, it gives rise to the implication that this complement is true. We will call this a declarative veridicality implication. For instance, know is veridical w.r.t. declarative complements, as illustrated in (12), while be certain is not, as illustrated in (13).

(12) John knows that it is raining. 
\[\therefore \text{It is raining.}\]

(13) John is certain that it is raining. 
\[\lnot \therefore \text{It is raining.}\]

The notion of veridicality w.r.t. interrogative complements is not so straightforward. Spector and Egré (2015, footnote 7) provide the following characterization: a responsive verb \(V\) is veridical w.r.t. interrogative complements just in case for every interrogative complement \(Q\), every individual \(x\), and every world \(w\), \(V(Q)(x)\) is true in \(w\) exactly if \(V(P)(x)\) is true in \(w\), where \(P\) is a declarative complement expressing the true complete answer to \(Q\) in \(w\). However, this characterization needs some further elaboration, because whether \(V(Q)(x)\) is true in \(w\) generally depends on whether \(Q\) receives an SE, IE, or MS interpretation. Moreover, what constitutes a true complete answer to a given interrogative varies between different theories; for instance, for Groenendijk and Stokhof (1984) it is not the same as for Karttunen (1977).

This leads to unintended results. For instance, Spector and Egré intend to classify know as a veridical verb, but their formal characterization doesn’t. After all, as we have already seen, (14) below can very well be true (on an MS reading) even if Rupert doesn’t know the true complete answer (either in Karttunen’s sense or in Groenendijk and Stokhof’s sense) to the question where one can buy an Italian newspaper.

(14) Rupert knows where one can buy an Italian newspaper.

To avoid this and other unintended results, we will make an assumption that already seems implicit in Spector and Egré’s characterization, namely that in testing whether a verb is veridical w.r.t. interrogative complements, one only needs to consider interrogative complements whose SE, IE, and MS interpretation coincide. We will call such complements exhaustivity-neutral. There are two kinds of exhaustivity-neutral interrogative complements: polar interrogatives such as whether it is raining, and wh-interrogatives such as who won the race which involve a property that, in any possible world, applies to a unique individual. For any verb \(V\), individual \(x\), exhaustivity-neutral complement \(Q\), and world \(w\), it is unmistakable whether \(V(x, Q)\) is true in \(w\)—this doesn’t depend on the reading that \(Q\) receives. Similarly, if \(Q\) is exhaustivity-neutral it is indisputable what the true complete answer is to \(Q\) in \(w\)—Karttunen’s notion and Groenendijk and Stokhof’s notion coincide in this case. The complete answers to an exhaustivity-neutral complement always form a partition of the set of all possible worlds.

Using the notion of exhaustivity-neutral complements, we propose the following variant of Spector and Egré’s definition of veridicality w.r.t. interrogative complements. We say that \(V\) is veridical w.r.t. interrogative complements just in case for every individual \(x\), every world \(w\), every exhaustivity-neutral interrogative complement \(Q\), and every declarative complement \(P\) expressing a complete answer to \(Q\):

\[V(Q)(x) \text{ is true in } w \implies [V(P)(x) \text{ is true in } w \iff P \text{ is true in } w]\]

Under this definition, know is classified as veridical w.r.t. interrogative complements, as intended, because inferences like (15a-b) are valid.
(15)  a. Mary knows where John was born.
    John was born in Paris.
    \[\therefore\] Mary knows that John was born in Paris.

    b. Mary knows where John was born.
    It is not the case that John was born in Paris.
    \[\therefore\] It is not the case that Mary knows that John was born in Paris.

On the other hand, be certain is classified as non-veridical w.r.t. interrogative complements, because the inferences in (16a-b) are invalid (note that even if just one of these inferences were invalid, this would already be sufficient to classify be certain as non-veridical w.r.t. interrogative complements).

(16)  a. Mary is certain where John was born.
    John was born in Paris.
    \[\therefore\] Mary is certain that John was born in Paris.

    b. Mary is certain where John was born.
    It is not the case that John was born in Paris.
    \[\therefore\] It is not the case that Mary is certain that John was born in Paris.

Now we can turn to the general constraint on responsive verb meanings that Spector and Egré propose. Namely, they hold that a responsive verb is veridical w.r.t. interrogative complements if and only if it is veridical w.r.t. declarative complements. We will refer to this as the veridicality generalization. Note that the absence of verbs like shknow, discussed in the introduction, would be a particular consequence of this generalization, because shknow would be veridical w.r.t. declarative complements, just like know, but not w.r.t. interrogative complements.

Many previous authors have actually rejected the veridicality generalization (see, e.g., Groenendijk and Stokhof, 1984; Berman, 1991; Higginbotham, 1996; Lahiri, 2002) based on Karttunen’s (1977, p.11) observation that communication verbs like tell appear to be veridical w.r.t. interrogative complements, but non-veridical w.r.t. declarative complements. That is, inferences like (17a-b) appear to be valid, while inferences like (18) seem invalid.

(17)  a. Mary told Alice where John was born.
    John was born in Paris.
    \[\therefore\] Mary told Alice that John was born in Paris.

    b. Mary told Alice where John was born.
    It is not the case that John was born in Paris.
    \[\therefore\] It is not the case that Mary told Alice that John was born in Paris.

(18) John told Mary that it was raining.

\[\therefore\] It was raining.

However, these judgements have been challenged by Tsohatzidis (1993) and more elaborately by Spector and Egré (2015), who point out that, with communication verbs, inferences like (17a) are in fact defeasible.

(19) Mary told Alice where John was born.
    John was born in Paris.
    But Mary confused John with his brother Peter, who was born in Bordeaux.
    \[\therefore\] Mary told Alice that John was born in Paris.
Every day, the meteorologists tell the population where it will rain the following day, but they are often wrong. (Spector and Egré, 2015, p.1737)

This clearly contrasts with the behavior exhibited by verbs like know, whose veridicality implications are indefeasible:

Mary knew where John was born, but she turned out to be wrong.

On the other hand, while tell is typically interpreted as non-veridical w.r.t. declarative complements, Spector and Egré hold that there are also cases where it receives a veridical interpretation. For instance, they hold that (22a-c) all presuppose, and thus imply, that Mary is pregnant.

a. Sue told someone that she is pregnant.
   b. Sue didn’t tell anyone that she is pregnant.
   c. Did Sue tell anyone that she is pregnant?

Based on these empirical observations, Spector and Egré (2015) suggest that communication verbs like tell are ambiguous. On one reading they are veridical w.r.t. both declarative and interrogative complements; on another reading they are not veridical w.r.t. either type of complement. Under this assumption, communication verbs no longer form a counterexample to the veridicality generalization. Spector and Egré thus conclude that the generalization is valid after all.

We would like to suggest, however, another kind of counterexample to the generalization. Namely, it seems that predicates of relevance like care and matter, already discussed above for different purposes, are veridical w.r.t. declarative complements but not w.r.t. interrogative complements. That is, inferences like (23) seem valid, but ones like (24) not.

It matters to Mary that John was born in Paris.

\[ \therefore \text{John was born in Paris.} \]

It matters to Mary where John was born.

\[ \therefore \text{John was born in Paris.} \]

\[ \therefore \text{It matters to Mary that John was born in Paris.} \]

In particular, even if both premises of the inference in (24) are true, the conclusion may still not be true due to presupposition failure, i.e., Mary might not know that John was born in Paris.

Thus, even if the veridicality generalization can be reconciled with communication verbs, it does not appear to hold in full generality. This is problematic for Spector and Egré’s reductive theory, where the assumed one-to-one connection between declarative and interrogative veridicality is a direct and inescapable consequence of the meaning postulates that connect the interrogative-embedding interpretation of responsive verbs to their declarative-embedding interpretation.

Still, it seems fair to say that the vast majority of responsive verbs complies with the generalization, at least in English and related languages. Our aim will be to show how this tendency can be accounted for within a uniform theory of clause-embedding, without ruling out occasional counterexamples such as care and matter.

In sum, we have laid out six concrete desiderata for our account: (i) address George’s observation concerning false answer sensitivity, which is problematic for partition theory as well as reductive approaches, (ii) address Elliott et al.’s observation concerning predicates of relevance, which is problematic for reductive approaches as well as the twin relations theory, (iii) overcome the restricted flexibility of partition theory:

Uegaki (2015, p.158f) provides a possible explanation of why the veridical reading of communication verbs is preferred with interrogative, but not with declarative complements.
derive SE, IE, and MS readings without invoking additional machinery and capture FA sensitivity in a uniform way across all three levels of exhaustive strength, (iv) capture the difference between internal and external interpretations of knowledge ascriptions, (v) capture the selectional restrictions of roguative and anti-rogative verbs without relying on a difference in semantic types between declarative and interrogative complements, and (vi) account for the tendency of responsive verbs to comply with Spector and Égré’s veridicality generalization, in a way that leaves room for occasional counterexamples.

3 Semantic framework

Our account will be couched in inquisitive semantics (Ciardelli, Groenendijk, and Roelofsen, 2015). More specifically, we will adopt the type-theoretic inquisitive semantics framework developed in Ciardelli, Roelofsen, and Theiler (2016). This framework is particularly suitable for our purposes here, because it offers a natural way to treat declarative and interrogative sentences in a uniform way (cf., Farkas and Roelofsen, 2016). In this section, we briefly review the basic features of the framework and introduce some notational conventions that will be useful in later sections.

3.1 Sentence meanings in inquisitive semantics

Traditionally, the meaning of a sentence is construed as a proposition, i.e., a set of possible worlds. Intuitively, a proposition carves out a region in the space of all possible worlds \( W \), and in asserting a sentence, a speaker is taken to provide the information that the actual world is located within this region. In this way, the proposition expressed by a sentence captures the informative content of the sentence.

Inquisitive semantics generalizes this notion of meaning to capture not just informative, but also inquisitive content, i.e., the issue raised in uttering a sentence. To achieve this, the meaning of a sentence is construed as a set of propositions, namely the set of all those propositions that resolve the issue raised by the sentence. When a speaker utters a sentence with meaning \( P \), she is taken to raise an issue whose resolution requires establishing one of the propositions in \( P \), while at the same time providing the information that at least one of these propositions must be true, i.e., that the actual world is contained in \( \bigcup P \).

It is assumed that if a certain proposition \( p \) resolves a given issue, then any stronger proposition \( q \subset p \) will also resolve that issue. This means that sentence meanings are downward closed: if \( p \in P \) and \( q \subset p \), then \( q \in P \) as well. Finally, it is assumed that the inconsistent proposition, \( \emptyset \), resolves any issue. This means that any sentence meaning has \( \emptyset \) as an element and is therefore non-empty. These considerations lead to the following characterization of sentence meanings:

**Definition 1** (Sentence meanings in inquisitive semantics).
A sentence meaning in inquisitive semantics is a non-empty, downward closed set of propositions.

The maximal elements of \( P \) are referred to as the alternatives in \( P \). We will write \( \text{alt}(P) \) for the set of alternatives in \( P \). In depicting the meaning of a sentence, we will generally only depict the alternatives that it contains. We will further write \( \text{alt}_w(P) \) for the set of alternatives in \( \text{alt}(P) \) that are true in \( w \). In anticipation of our treatment of FA sensitivity, we also define \( \text{alt}^*_{w}(P) \) as the set of all alternatives in \( \text{alt}(P) \), as well as unions of such alternatives, that are false in \( w \). Intuitively, \( \text{alt}^*_{w}(P) \) can be thought of as the set of (possibly partial) answers to \( P \) that are false in \( w \). Finally, \( \bigcup P \) is referred to as the informative content of \( P \), denoted as \( \text{info}(P) \), and a sentence with meaning \( P \) is said to be true in a world \( w \) just in case \( w \in \text{info}(P) \).

**Definition 2** (Alternatives, informative content, and truth).
For any sentence meaning \( P \) and any world \( w \):

- \( \text{alt}(P) := \{ p \in P \mid \text{there is no } q \in P \text{ such that } p \subset q \} \)
• \( \text{alt}_w(P) := \{ p \in \text{alt}(P) \mid w \in p \} \)

• \( \text{alt}^*_w(P) := \{ \bigcup Q \mid Q \subseteq \text{alt}(P) \text{ and } w \notin \bigcup Q \} \)

• \( \text{info}(P) := \bigcup P \)

• A sentence with meaning \( P \) is true in \( w \) just in case \( w \in \text{info}(P) \).

To illustrate these notions, consider the following two sentences.

(25)  
(a) Did Amy leave?  
(b) Amy left.

The polar interrogative in (25a) is taken to have the meaning in Figure 3(a), where \( w_1 \) and \( w_2 \) are worlds where Amy left, and \( w_3 \) and \( w_4 \) are worlds where she didn’t leave. The shaded rectangles are the alternatives contained in the given meanings. By downward closure, all propositions contained in one of these alternatives are also included in the meanings of the sentences. The meaning assigned to (25a) captures the fact that, in uttering this sentence, a speaker (i) provides the trivial information that the actual world must be \( w_1, w_2, w_3, \text{ or } w_4 \) (all options are open) and (ii) raises an issue whose resolution requires establishing either that Amy left, or that she didn’t leave. Since \( \text{info}([\text{Did Amy leave?!}]) = \{w_1, w_2, w_3, w_4\} \), this sentence is true in all of \( w_1, w_2, w_3, \text{ and } w_4 \). More generally, since the informative content of a non-presuppositional interrogative sentence always covers the entire logical space, such a sentence is always true in all worlds.

The declarative in (25b) is assigned the meaning in Figure 3(b), which captures the fact that the sentence (i) provides the information that the actual world must be either \( w_1 \) or \( w_2 \), i.e., one where Amy left, and (ii) raises an issue whose resolution requires establishing that Amy left. In this case, the information provided by the speaker is already sufficient to resolve the issue that is raised; no further information is needed from other conversational participants. Furthermore, as expected, Amy left is true in worlds \( w_1 \) and \( w_2 \).

3.2 Informative and inquisitive sentences

In the case of the interrogative Did Amy leave? the information that is provided is trivial in the sense that it does not exclude any candidate for the actual world. Such sentences are called non-informative. Conversely, a sentence with meaning \( P \) is called informative just in case it does exclude at least one candidate for the actual world, i.e., iff \( \text{info}(P) \neq W \).

On the other hand, in the case of the declarative Amy left, the inquisitive content of the sentence is trivial, in the sense that the issue that is raised in uttering the sentence is already resolved by the information provided; no further information is required. Such sentences are called non-inquisitive. Conversely,
a sentence with meaning \( P \) is called \textit{inquisitive} just in case resolving the issue that it expresses requires more than the information that it provides, i.e., iff \( \text{info}(P) \not\subseteq P \).

Given a picture of the meaning of a sentence, it is easy to see whether the sentence is inquisitive or not. This is because a sentence is inquisitive just in case its meaning contains at least two alternatives.\footnote{Strictly speaking, this generalization only holds under the assumption that there are finitely many possible worlds—but this is a safe assumption to make for all the examples to be considered in this paper.} For instance, the meaning in Figure 3(a) contains two alternatives, which means that the polar interrogative \textbf{Did Amy leave?} is inquisitive, while the meaning in Figure 3(b) contains only one alternative, which means that the declarative \textbf{Amy left} is not inquisitive. Following Ciardelli et al. (2015); Farkas and Roelofsen (2016), we will assume that declarative sentences are never inquisitive, i.e., that their meaning always contains a single alternative.

### 3.3 Composing meanings

We adopt a standard two-step approach for composing the meaning of a sentence, summarized in Figure 4. In the first step, we translate a natural language expression into a type-theoretic language, by translating every lexical item into a certain type-theoretic expression and deriving the translation of complex constituents by means of function application and abstraction. We write \((\alpha)\)' for the translation of a natural language expression \(\alpha\). In the second step, type-theoretic expressions are interpreted relative to a model \(M\) and an assignment \(g\).

The type theory we assume is two-sorted, with basic types \(e\), \(s\), and \(t\), for individuals, worlds, and truth values, respectively. Since sentence meanings are construed as sets of propositions, sentences are taken to be of type \(\langle\langle s, t\rangle, t\rangle\), which we abbreviate as \(T\). From this, one can reverse engineer the types that should be assigned to various kinds of sub-sentential expressions:

\begin{align*}
\text{(26)} & \quad \text{John} : e \\
& \quad \text{walks} : \langle e, T \rangle \\
& \quad \text{likes} : \langle e, \langle e, T \rangle \rangle \\
& \quad \text{not} : \langle T, T \rangle \\
& \quad \text{and} : \langle T, \langle T, T \rangle \rangle \\
& \quad \text{somebody} : \langle \langle e, T \rangle, T \rangle \\
\end{align*}

For instance, we take the meaning of a sentence like \textbf{John walks} to be the set of propositions \(p\) such that \textbf{John walks} in every world \(w \in p:\)

\begin{align*}
\text{(27)} & \quad (\text{John walks})' = \lambda p. \forall w \in p : W(j)(w)
\end{align*}

This set of propositions is downward closed since, if \(p\) is a proposition such that \textbf{John walks} in every world \(w \in p\), then the same goes for any \(q \subseteq p\). To obtain the above sentence meaning, the verb \textbf{walks} should express a function that takes an individual \(x\) and yields the set of propositions \(p\) such that \textbf{x walks} in every \(w \in p\):

\begin{align*}
\text{(28)} & \quad \text{walks} = \lambda x. \lambda p. \forall w \in p : W(x)(w)
\end{align*}

This is all we need to know about type-theoretical inquisitive semantics to give a compositional account of the constructions we are interested in here. For a more systematic introduction to this framework, we refer to Ciardelli et al. (2016).
4 General assumptions about complement constructions

We now lay out our account of clausal complements, starting with some high-level syntactic and semantic assumptions about complement constructions.

We assume that complement constructions have the global structure in Figure 5. For reasons that we will return to in Section 5.4, declarative and interrogative complements both involve an embedding operator $E$, which adjoins to a clause that we will call the nucleus of the complement. The nucleus has the same semantic properties—for our current purposes—as a declarative respectively interrogative root clause. So, in particular, the semantic type of the nucleus of a complement is $T$.

The $E$ operator takes the nucleus as its input and returns a function from worlds to sets of propositions. Thus, it is of type $\langle T, \langle s, T \rangle \rangle$. As we will see, this function maps every world $w$ to the set of propositions that can be thought of as truthful resolutions, in $w$, of the issue expressed by the nucleus.

Typically a verb and its complement together form a verb phrase that has the same semantic type as an intransitive verb like walk, i.e., $\langle e, T \rangle$, expressing a function from individuals to sets of propositions. To achieve this, a verb that takes clausal complements has to be of type $\langle\langle s; T \rangle; \langle e; T \rangle\rangle$. It takes as its input a function from worlds to sets of propositions, generated by its complement, and it yields as its output a function from individuals to sets of propositions.

These are all our high-level assumptions about complement constructions. We will now zoom in on the two different cases, first on interrogative and then on declarative complements.

5 Interrogative complements

In this section, we will formulate a semantics for interrogative complements, starting at the level of the nucleus (Section 5.1), then moving on to the level of the complement, where the $E$ operator and the notion of truthful resolutions will be introduced (Section 5.2), and finally proceeding to the level of the verb phrase, where our treatment of the verb know will be spelled out (Section 5.3). Once all these ingredients are in place, we will show how the proposed account captures FA sensitivity across different levels of exhaustivity (Section 5.3.2). The section ends with some remarks on the division of labor we assume between the $E$ operator and the verb, and, at a more general level, between semantics and pragmatics (Section 5.4).
5.1 Interrogative nuclei

We assume that a root wh-interrogative like (29), and thus also the nucleus of the corresponding interrogative complement, has two possible readings, an exhaustive and a non-exhaustive one.

(29) Who left?

On the exhaustive reading, the sentence raises an issue whose resolution requires establishing exactly who left and who didn’t. Let’s assume that there are two individuals in the domain, Amy and Bob. Then, in order to resolve the issue raised by (29), one would have to specify for both Amy and Bill whether they left. On the non-exhaustive reading, on the other hand, the issue can be resolved by establishing that Amy left, that Bill left, or that neither of them left. The meaning we take (29) to have on the exhaustive reading is depicted in Figure 6(a) and the one we take it to have on the non-exhaustive reading in Figure 6(b). As before, \(w_1\) and \(w_2\) in the diagrams are worlds where Amy left, while \(w_3\) and \(w_4\) are worlds where she didn’t leave. Furthermore, \(w_1\) and \(w_3\) are worlds where Bill left, and \(w_2\) and \(w_4\) are worlds where he didn’t leave.

Depending on the precise nature of the nucleus, either the exhaustive or the non-exhaustive interpretation may be blocked. For instance, the Dutch example in (30) only has an exhaustive interpretation, due to the presence of the exhaustivity marker allemaal, while the example in (31), which contains the non-exhaustivity marker zoal, only has a non-exhaustive interpretation (Beck and Rullmann, 1999). The existence of such explicit (non-)exhaustivity markers motivates a particular view on the division of labor between semantics and pragmatics, which will be discussed in Section 5.4.1.

(30) Welke acteurs zijn er allemaal genomineerd voor een Oscar dit jaar?
which actors are there +EXH nominated for a Oscar this year
‘Which actors are nominated for an Oscar this year?’ (exhaustive)

(31) Welke acteurs zijn er zoal genomineerd voor een Oscar dit jaar?
which actors are there −EXH nominated for a Oscar this year
‘Which actors are nominated for an Oscar this year?’ (non-exhaustive)

We will focus here on embedding and will treat the compositional derivation of the nucleus meaning as a blackbox, referring to Champollion et al. (2015) for a concrete compositional semantics that is compatible with the account developed here. Our account of embedding is also compatible with other treatments of interrogative nuclei that derive both exhaustive and non-exhaustive interpretations (e.g., Nicolae, 2013; Theiler, 2014).\(^{14}\)

\(^{14}\)We will largely leave presuppositional interrogative nuclei out of consideration in this paper. For instance, a singular which-question like Which student left? is often taken to presuppose that exactly one student left. The issue of how such presuppositions should be modeled and derived is complicated, and orthogonal to our main concerns here. We believe that our account can be
5.2 The $E$ operator: from resolutions to truthful resolutions

As we saw in Section 3, in order to count as a resolution of some issue, a proposition has to provide enough information to resolve this issue. Naturally, if a proposition $p$ resolves an issue $P$, then any more informative proposition $q \subset p$ will resolve $P$ as well. This is the reason why sentence meanings in inquisitive semantics, which are taken to be sets of resolutions, are downward-closed.

However, unlike the meaning of an interrogative nucleus, the meaning of an interrogative complement is not represented as a plain set of resolutions, but rather as a function from worlds to sets of truthful resolutions. Truthful resolutions are still resolutions, but in addition they have to fulfill two requirements: (i) they need to be consistent, and (ii) they must not provide false information w.r.t. the given issue.

Formally, the latter condition amounts to the following. Let $P$ be a sentence meaning, and recall that $\text{alt}^*_w(P)$ consists of alternatives in $\text{alt}(P)$, as well as unions of such alternatives, that are false in $w$. A resolution $p \in P$ provides false information w.r.t. $P$ in $w$ iff it entails some proposition in $\text{alt}^*_w(P)$. Conversely, $p$ provides no false information w.r.t. $P$ in $w$ iff it does not entail any proposition in $\text{alt}^*_w(P)$. For instance, assume that Amy and Bill are the only individuals in the domain and that only Amy left (in the diagrams in Figure 6, this means the actual world is $w_2$). Consider the sentence meaning $P = [\text{who left} \lnot \text{exh}]$, depicted in Figure 6(b), which contains one alternative that is true in $w_2$ (Amy left) and two alternatives that are false in $w_2$ (Bill left; neither Amy nor Bill left). Let $p$ be the proposition that Amy and Bill left. Observe that $p$ entails the alternative that Bill left, $q$. Since $q$ is false in $w_2$ and $q$ is an alternative in $P$, $q \in \text{alt}^*_w(P)$. Hence, $p$ provides false information w.r.t. $P$. As another example, let $p'$ be the proposition that Amy didn’t leave, i.e., $p' = \{w_3, w_4\}$. Now consider the alternatives $\{w_1, w_3\}$ and $\{w_4\}$, both of which are false in $w_2$. So, the union of these alternatives, $\{w_1, w_3, w_4\}$, is an element of $\text{alt}^*_w(P)$. Since this union is furthermore entailed by $p'$, we find that $p'$ provides false information w.r.t. $P$ as well.

We hence arrive at the following definition of truthful resolutions. In what follows, we will occasionally make reference to the crucial third clause in this definition as the no false alternatives (NFA) condition.\footnote{This NFA condition is stronger than the one we assumed in Théliner et al. (2016), in that it does not only exclude propositions which entail false alternatives, but also those that entail false unions of alternatives. See Section 2.1 for the motivation behind this stronger formulation, due to Xiang (2015).}

**Definition 3** (Truthful resolution). Let $P$ be a sentence meaning and $w$ a possible world. A proposition $p$ is a truthful resolution of $P$ in $w$ iff:

(i) $p$ is a resolution of $P$ ($p \in P$),

(ii) $p$ is consistent ($p \neq \emptyset$),

(iii) **NFA condition:** $p$ doesn’t provide information w.r.t. $P$ that is false in $w$ ($\neg \exists q \in \text{alt}^*_w(P) : p \subseteq q$).

We further distinguish between truthful resolutions *simpliciter* and *complete* truthful resolutions, which entail all true alternatives.

**Definition 4** (Complete truthful resolution). Let $P$ be a sentence meaning and $w$ a possible world. A proposition $p$ is a complete truthful resolution of $P$ in $w$ iff:

(i) $p$ is a truthful resolution of $P$ in $w$,

(ii) $p$ entails all alternatives in $\text{alt}^*_w(P)$.
To exemplify, consider a scenario in which there are three people, Amy, Bill and Clara. Assume that in world $w$ Amy and Bill left, but Clara didn’t. Again, let $P = [\text{who left}]_{\text{exh}}$. Then, the proposition that Amy left, the proposition that Bill left, and the proposition that both of them left are all truthful resolutions of $P$ in $w$. The proposition that both of them left is additionally a complete truthful resolution of $P$ in $w$. The proposition that Amy, Bill and Clara left, on the other hand, is not a truthful resolution of $P$ in $w$. Observe that, once we make the step from resolutions to truthful resolutions, we are not dealing with downward-closed sets anymore: although the proposition that Amy, Bill and Clara left is a subset of the proposition that Amy left, only the latter is a truthful resolution of $P$ in $w$.

Crucially, although a truthful resolution $p$ has to entail a true alternative, $p$ itself need not be true. For instance, assume that in the above scenario it is Monday. Consider the proposition $p$ that Amy left and that it is Tuesday. Clearly, $p$ is false. Nonetheless, it counts as a truthful resolution because it only provides true information w.r.t. the issue of who left; the false information that it provides—namely that it is Tuesday—is not relevant w.r.t. the issue of who left. It is in this sense that we may say truthful resolutions embody a notion of truth radically relativized to a given issue: they must not provide any false information w.r.t. to that issue.

To get a more visual understanding of this concept, consider Figure 7. Let $p$ be the proposition that Italian newspapers are sold at Newstopia and $q$ the proposition that they are sold at Paperworld. The actual world, $w_0$, is located in $p$, but not in $q$ since only Newstopia stocks Italian newspapers. So, $q \in \text{alt}^*_w(P)$. Let us reflect on which propositions in the picture count as truthful resolutions of where can one get an Italian newspaper in $w_0$. Clearly, $p$ is a truthful resolution, or, to be more precise, the minimally informative truthful resolution. More interesting, however, is the question which subsets of $p$ are truthful resolutions and which are not. To begin with, all true propositions entailing $p$ are automatically truthful resolutions because they are consistent resolutions and cannot entail any proposition in $\text{alt}^*_w(P)$. On the other hand, with false propositions that entail $p$, we have to distinguish two cases. First, let $r$ be the proposition that both Newstopia and Paperworld sell Italian newspapers (the crossed-out proposition in the diagram). Since $r$ entails $q$, it does not count as a truthful resolution. Second, let $r'$ be some other consistent proposition such that $r' \subseteq p$, but $r' \not\subseteq q$ (the one with a tick mark in the diagram). There is no proposition in $\text{alt}^*_w(P)$ that is entailed by $r'$. Hence, although both $r$ and $r'$ are false, $r'$ counts as a truthful resolution in $w_0$ whereas $r$ doesn’t.

We now turn to defining the $E$ operator. When applied to a nucleus meaning $P$, this operator yields a function mapping every world $w$ to the set of (complete) truthful resolutions of $P$ in $w$. Formally, we can characterize $E$, which comes in a complete and a non-complete variant, as follows.

**Definition 5 (The $E$ operator).**

$$E_{[-\text{cmp}]} := \lambda P_T. \lambda w. \lambda p. \left( p \in P \land p \neq \emptyset \land \exists q \in \text{alt}^*_w(P) : p \subseteq q \right)$$
$$E_{[+\text{cmp}]} := \lambda P_T. \lambda w. \lambda p. \left( p \in P \land p \neq \emptyset \land \neg \exists q \in \text{alt}^+_{w}(P) : p \subseteq q \land \forall q \in \text{alt}_{w}(P) : p \subseteq q \right)$$

For an illustration of the functions that $E$ yields, consider the examples below, which show the result of applying this operator to typical interrogative nucleus meanings.\textsuperscript{16}

\begin{equation}
E_{[-\text{cmp}]}(\begin{array}{c}
\text{w1} \\
\text{w2} \\
\text{w3} \\
\text{w4}
\end{array}) = \left\{ \begin{array}{c}
\{ \circ \circ, \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ 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I know who called a third person in the latter case (third person perspective, not introspective). One prediction we make is that the difference and external readings is that the observer is the agent herself in the former case (first person perspective, introspective), and by looking at the information state of the relevant agent (and at how it relates with the facts). The difference between internal are based on a ‘mentalistic’ (hence arguably internalistic) notion of justification. Whether one knows is determined in both cases between internal vs. external readings only applies to interrogative knowledge. Secondly, both our internal and external readings in epistemology. Firstly, in epistemology the discussion is centred around cases of declarative knowledge, while our distinction are discussed in Section 5.3.1 Internal and external interpretation of know

5.3 Embedding verbs: the case of know

So far, we laid out an account of interrogative complements. This account, however, only yields concrete predictions when combined with an analysis of the verbs that take such complements as their argument. Instead of diving right into the full range of verbs, we will first zoom in on the case of know. As discussed in Section 2, we expect our treatment of this verb to achieve two things. First, it should capture the difference between the internal and external interpretation of know and relate this difference to the availability of IE readings. This objective will be addressed in Section 5.3.1. Second, the FA sensitivity of know should receive a uniform analysis across all levels of exhaustive strength. This objective will be addressed in Section 5.3.2.

5.3.1 Internal and external interpretation of know

In Section 2.4, we distinguished an internal and an external interpretation of knowledge ascriptions. The internal interpretation, which we take to be the interpretation that was assumed in Groenendijk and Stokhof (1982) and much subsequent literature, is the one under which the subject would say of herself that she knows—hence it requires introspection. The external interpretation, on the other hand, which we argued was made particularly salient in Cremers and Chemla’s (2016b) experimental setting, is typically not introspective. We will start by giving a lexical entry for the external interpretation and later strengthen it to capture the internal interpretation.

**External interpretation.** We will formally characterize the meaning of know in terms of the subject x’s information state in a world w, which we understand to be the set of worlds compatible with what x takes to be the case in w. We will write $\sigma^w_x$ for this set. Crucially, an individual’s information state in w does not have to contain w itself, i.e., it does not necessarily hold that $w \in \sigma^w_x$. As is commonplace in doxastic

<table>
<thead>
<tr>
<th>$E_{[-cmp]}$</th>
<th>$E_{[+cmp]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mention-some</td>
<td>strongly exhaustive</td>
</tr>
<tr>
<td>intermediate exhaustive</td>
<td>strongly exhaustive</td>
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</tbody>
</table>

We end this subsection by noting that, although at first glance the non-complete and complete variant of E might appear to differ only minimally, formally they actually come apart in a fundamental way. The computation carried out by $E_{[-cmp]}$ is an innocent type-shift: if we have a function $f = E_{[-cmp]}(P)$, we can retrieve the set P from f, since $P = \bigcup_{w \in W} f(w) \cup \{\emptyset\}$. In contrast, the computation carried out by $E_{[+cmp]}$ is not an innocent type-shift. To see this, consider the following two sets of resolutions:

$P_1 = \{\circ \circ, \circ \circ\}$ and $P_2 = \{\circ \circ, \circ \circ, \circ \circ, \circ \circ\}$, where the downarrow (↓) indicates closure under subsets. Applying $E_{[+cmp]}$ to either $P_1$ or $P_2$ yields the same function $f = \{w_1 \mapsto \{\circ \circ\}, w_2 \mapsto \{\circ \circ\}, w_3 \mapsto \{\circ \circ\}, w_4 \mapsto \{\circ \circ\}\}$. So, in general it is impossible to retrieve P from $E_{[+cmp]}(P)$.

---

17 This distinction relates to, but differs from the distinction between internalist and externalist conceptions of justification in epistemology. Firstly, in epistemology the discussion is centred around cases of declarative knowledge, while our distinction between internal vs. external readings only applies to interrogative knowledge. Secondly, both our internal and external readings are based on a ‘mentalistic’ (hence arguably internalistic) notion of justification. Whether one knows is determined in both cases by looking at the information state of the relevant agent (and at how it relates with the facts). The difference between internal and external readings is that the observer is the agent herself in the former case (first person perspective, introspective), and a third person in the latter case (third person perspective, not introspective). One prediction we make is that the difference disappears for first person ascriptions: I know who called only has an internal interpretation.
logic, we do assume that $\sigma^w_x$ is always consistent (i.e., non-empty) and that $x$ always knows what her own information state is (i.e., $\sigma^w_x = \sigma^w_x$ for all $v \in \sigma^w_x$).

We assume the following basic entry for know, capturing its external interpretation.\(^{18}\)

\[(35) \quad \text{know} := \lambda f : \{s, T\}, \lambda x : \lambda p. \forall w \in p : \sigma^w_x \in f(w)\]

In words, know takes the complement meaning $f$ and the subject $x$ as arguments, and yields a set of propositions. Recall that $f$ is a function mapping each world to the set of truthful resolutions of the complement in that world. Hence, the set that know yields contains only propositions $p$ such that for every world $w \in p$ the information state of $x$ in $w$ exactly matches one of the truthful resolutions in $f(w)$.

This entry for know differs from classical accounts in two respects. Firstly, on classical accounts, $f(w)$ has a fixed exhaustive strength; i.e., it is the true WE answer in $w$ (Karttunen, 1977) or the true SE answer in $w$ (Groenendijk and Stokhof, 1984). In comparison, our account is more flexible. Depending on the interpretation of the nucleus of the complement (exhaustive or non-exhaustive) and the $E$ operator (complete or non-complete), $f(w)$ will consist of MS, IE or SE truthful resolutions.

The second difference concerns the relation between $\sigma^w_x$ and $f(w)$. Standardly, it is required that $\sigma^w_x$ is a subset of $f(w)$, whereas for us $\sigma^w_x$ has to be an element of $f(w)$—we will see in Section 5.3.2 that this is instrumental in accounting for FA sensitivity.

**Internal interpretation.** We will now strengthen the entry in (35) to capture the internal interpretation of know. This will be done by requiring a certain kind of introspection on the part of the subject, which goes beyond just knowing what her own information state is. There are at least two natural ways to spell out this introspection condition. We will first introduce what we dub resolution introspection, then what we call Heim introspection, and finally compare the two, concluding in favor of the former.

The idea of resolution introspection is very simple: besides requiring that $\sigma^w_x \in f(w)$, i.e., that $x$’s information state in $w$ matches one of the truthful resolutions of the complement in $w$, we also require that $x$ is fully aware of this match, i.e., that every world she considers possible is one where her information state matches one of the truthful resolutions of the complement in that world. Formally: $\forall v \in \sigma^w_x : \sigma^v_x \in f(v)$. In other words, under the internal introspection it is not sufficient if $x$’s information state just happens to coincide with a truthful resolution in the world of evaluation; $x$ also has to take herself to know that this is the case. Incorporating this requirement results in the following entry:\(^{19}\)

\[(36) \quad \text{know}_\text{int} := \lambda f : \{s, T\}, \lambda x, \lambda p. \forall w \in p : (\sigma^w_x \in f(w) \land \forall v \in \sigma^w_x : \sigma^v_x \in f(v))\]

Recall that for all $v \in \sigma^w_x$, $\sigma^v_x = \sigma^v_x$. Thus, know\(_\text{int}\) can also be formulated as follows, without making reference to $\sigma^v_x$:

---

\(^{18}\)This is a refinement of the treatment of the knowledge modality in inquisitive epistemic logic (IEL) (Ciardelli and Roelofsen, 2015). In IEL, $\sigma^w_x$ is simply required to coincide with a resolution of the complement. Our entry, on the other hand, requires that $\sigma^w_x$ coincides with a truthful resolution of the complement in the world of evaluation. As we will see, it is precisely this refinement that allows us to capture FA sensitivity.

\(^{19}\)One way to understand the role of this introspection requirement is to compare our system to standard doxastic logic. There, the notion of knowledge is limited to declarative knowledge, and the condition that $\sigma^w_x = \sigma^w_x$ for every $v \in \sigma^w_x$—let’s call this condition information state introspection—guarantees the validity of the positive and negative introspection principles: $K\varphi \rightarrow KK\varphi$ and $\neg K\varphi \rightarrow K\neg K\varphi$ for all declarative complements $\varphi$. By contrast, our account additionally models interrogative knowledge, and while information state introspection still guarantees the validity of the introspection principles w.r.t. declarative complements, it does not guarantee their validity w.r.t. interrogative complements. Once we add resolution introspection, however, the principles do become generally valid. In other words, know\(_\text{int}\) guarantees full introspection w.r.t. declarative and interrogative complements, whereas know only guarantees introspection w.r.t. declarative complements.
We now turn to another way of spelling out the introspection condition in the entry for know\textsubscript{int}. Namely, instead of requiring, as we just did, that the subject has to take herself to know a truthful resolution, we could also proceed along the lines of Heim (1994) and demand that the subject has to take herself to know what the set of truthful resolutions is in the world of evaluation.\footnote{It should be noted that Heim (1994) is not concerned with formulating an introspection condition, in fact, but with deriving SE answers from complete answers. To do so, she defines two different notions of answers. Given a question \( Q \) and a world \( w \), her answer\textsubscript{1}(\( Q \))(\( w \)) is the true complete answer of \( Q \) in \( w \); her answer\textsubscript{2}(\( Q \))(\( w \)) is the set of all worlds \( v \) such that answer\textsubscript{1}(\( Q \))(\( v \)) is the same as answer\textsubscript{1}(\( Q \))(\( w \)). Hence, if you take yourself to know answer\textsubscript{2}(\( Q \))(\( w \)), you take yourself to know what answer\textsubscript{1}(\( Q \))(\( w \)) is. Translated into our framework, this amounts to taking yourself to know what the set of truthful resolutions in \( w \) is.} We will refer to this requirement as Heim introspection. Put more loosely, the relevant difference is between taking yourself to know \( \text{that you have a truthful resolution} \) (resolution introspection) and taking yourself to know \( \text{what the truthful resolutions are} \) (Heim introspection). Given a world of evaluation \( w \), Heim introspection amounts to \( \forall v \in \sigma_x^w : f(v) = f(w) \). Adding this to our basic entry for know, we arrive at the following entry:

\[
\begin{align*}
\text{know}_{\text{int}} & = \lambda f(\langle \text{s,T} \rangle, \ldots). \forall x. \lambda p. \forall w \in p : (\sigma_x^w \in f(w) \land \forall v \in \sigma_x^w : \sigma_x^w \in f(v)) \\
\text{Heim introspection} & = \lambda f(\langle \text{s,T} \rangle, \ldots). \forall x. \lambda p. \forall w \in p : (\sigma_x^w \in f(w) \land \forall v \in \sigma_x^w : f(v) = f(w))
\end{align*}
\]

In terms of empirical predictions, know\textsubscript{Heim} and know\textsubscript{int} only come apart when taking an interrogative complement with an MS reading. To see this, consider (39) in George’s scenario, which was discussed above.

(39) Janna knows where one can buy an Italian newspaper.

It seems to us that Janna, since she takes herself to know that one can buy an Italian newspaper at Newstopia, would say of herself that she knows where one can buy an Italian newspaper—although she is not certain whether Paperworld sells such newspapers as well. Accordingly, we think (39) should come out true under an internal interpretation of know and an MS reading of the complement.

Let us check which predictions the two introspection requirements make. If we assume an MS interpretation of the complement, the complement meaning is \( f(w) \). Furthermore, we get the following sets of truthful resolutions: \( f(w_1) = \{0 \cdot 0, \cdot 0 \cdot \cdot, \cdot \cdot \cdot \cdot \} \) and \( f(w_2) = \{0 \cdot 0 \cdot, \cdot \cdot \cdot \cdot\} \). Hence, the resolution introspection requirement, \( \forall v \in \sigma_j^w : \sigma_j^w \in f(v) \), is satisfied since \( 0 \cdot 0 \cdot \in f(w_1) \) and \( 0 \cdot 0 \cdot \in f(w_2) \). On the other hand, the Heim introspection requirement, \( \forall v \in \sigma_j^w : f(v) = f(w_2) \), is not satisfied since \( f(w_1) \neq f(w_2) \). This means that know\textsubscript{Heim} predicts (39) to be false in \( w_2 \), contra our intuitions, while know\textsubscript{int} predicts (39) to be true in \( w_2 \), as desired. Thus, we will use resolution introspection rather than Heim introspection in modelling the internal interpretation of know.

**Availability of IE readings for know.** Whether a sentence like John knows who called is true depends on two factors on our account: whether the verb receives an internal or an external interpretation, and whether the complement gets an MS, IE, or SE reading. Interestingly, however, these two factors interact in such a way that only three distinct readings are predicted (rather than six, as one would expect prima facie). More specifically, as depicted in Table 2, we can establish the following two Facts, the proofs of which are given in Appendix B.1.
Fact 1. If the complement receives an MS or an SE interpretation, then the external and the internal interpretation of the verb yield exactly the same reading for the sentence as a whole.

Fact 2. If the verb receives an internal interpretation, then the IE and the SE interpretation of the complement yield exactly the same reading for the sentence as a whole.

In view of Fact 1, we will from now on always assume our basic entry for know when the complement receives an MS or SE interpretation.

Fact 2 says that, under an internal interpretation of the verb, what is required for John knows who called to be true on an IE reading is exactly the same as what is required on an SE reading. Namely, of all people who called, John needs to know that they called, and moreover he needs to know that nobody else called. Thus, Groenendijk and Stokhof’s claim that know does not allow for an IE reading is salvaged, though only under an internal interpretation of the verb, the interpretation that they seem to have had in mind.

On the other hand, under an external interpretation of the verb, IE readings are predicted to exist independently of SE ones. This accounts for the findings of Cremers and Chemla (2016b), whose experimental setting arguably made the external interpretation of the verb particularly salient.

As Table 2 shows, the three readings that we predict for John knows who called can all be derived with our basic entry for the verb, know, which was intended to capture the external interpretation. Our second entry, know\textsubscript{int}, does not yield any additional readings, i.e., it does not overgenerate. Still, these two entries, and the underlying distinction between the internal and the external interpretation of know, make it possible to reconcile Groenendijk and Stokhof’s argument with Cremers and Chemla’s experimental findings.

An additional prediction is that when we consider self-ascriptions of knowledge by speakers who can be assumed to comply with the Gricean maxims of cooperative conversational behavior, then the IE reading will coincide with the SE reading even under an external interpretation of the verb. To see why, consider the following example.

(40) I know who called.

Assume an external interpretation of the verb and an IE interpretation of the complement. Then the sentence is true in \( w \) just in case the speaker’s information state in \( w \) coincides with an IE truthful resolution in \( w \), i.e., just in case \( \sigma_{x}^{w} \in f_{IE}(w) \), where \( x \) is the speaker and \( f_{IE}(w) \) the set of IE truthful resolutions of the complement in \( w \). Now, we can assume that the speaker is complying with the Gricean maxims, in particular with Quality. This means that she should believe that her information state coincides with
an IE truthful resolution of the complement, i.e., every world \( v \in \sigma^w_x \) should be such that \( \sigma^w_x \in f_{IE}(v) \). From here we can derive, as is done in the proof of Fact 2 in Appendix B.1, that it must be the case that \( \sigma^w_x \in f_{SE}(w) \), where \( f_{SE}(w) \) the set of SE truthful resolutions of the complement in \( w \), i.e., that the sentence is true under an SE reading.

### 5.3.2 False answer sensitivity across levels of exhaustivity

We argued in Section 2.3 that an account which successfully deals with embedded questions will have to implement FA sensitivity across all levels of exhaustive strength. On our account, FA sensitivity is captured by the NFA condition in the definition of truthful resolutions (Definition 3), which says that a proposition \( p \) is only a truthful resolution of a sentence meaning \( P \) in a world \( w \) if it does not entail any proposition in \( \text{alt}^w_r(P) \). Let us see what the consequences of this condition are across the different levels of exhaustive strength.

We begin with George’s scenario, involving an MS example. Recall that in the actual world \( w_0 \) only Newstopia sells Italian newspapers. Janna and Rupert know that Newstopia sells Italian newspapers. Additionally, Rupert falsely believes that also Paperworld sells such newspapers. Janna has no beliefs about Paperworld. Then, under an MS reading, (41) is judged true, while (42) is judged false.

\[
\begin{align*}
(41) & \quad \text{Janna knows where one can buy an Italian newspaper.} \\
(42) & \quad \text{Rupert knows where one can buy an Italian newspaper.}
\end{align*}
\]

This is indeed what we predict. To see why, assume the above complements each involve \( E_{[-\text{cmp}]} \) and the nuclei receive a \([-\text{exh}]\) interpretation, resulting in MS readings. Let \( P \) be the nucleus meaning. Observe that \( P \) contains one true alternative, namely the proposition that one can buy an Italian newspaper at Newstopia, and two false alternatives, namely the proposition that one can buy an Italian newspaper at Paperworld and the proposition that one can buy such a newspaper at another place. Janna’s information state \( \sigma^w_{jw_0} \) is a truthful resolution of the complement since it is consistent, it coincides with one of the resolutions in \( P \) and it does not entail any proposition in \( \text{alt}^w_0(P) \), while Rupert’s information state \( \sigma^w_{rw_0} \) is not a truthful resolution of the complement since it does entail a proposition in \( \text{alt}^w_0(P) \), namely the proposition that one can buy a newspaper at Paperworld. Thus, (41) comes out as true because \( \sigma^w_{jw_0} \in E_{[-\text{cmp}]}(P)(w_0) \), while (42) comes out as false because \( \sigma^w_{rw_0} \notin E_{[-\text{cmp}]}(P)(w_0) \).

In the case of IE readings, FA sensitivity arises from exactly the same mechanism. Consider Cremers and Chemla’s (2016b) example:

\[
\begin{align*}
(43) & \quad \text{John knew which squares were blue.}
\end{align*}
\]

\[\text{Xiang (2015) argues that there are two different kinds of false answers relevant in the context of MS readings, namely over-affirming and over-denying false answers. George (2011) only takes the former into account. To see what the difference is, consider a modified Italian-newspaper scenario. As before, Newstopia sells Italian newspapers, while Paperworld doesn’t. In addition, however, there is a third store, Celluloid City, which also sells Italian newspapers. Suppose that Janna believes one can get an Italian newspaper at Newstopia and Paperworld. Since this is a falsely positive belief, Xiang classifies it as over-affirming. Now suppose that Janna correctly believes one can buy an Italian newspaper at Newstopia, that she doesn’t have any beliefs about Paperworld, and she wrongly believes one cannot buy an Italian newspaper at Celluloid City. Since this is a falsely negative belief, Xiang classifies it as over-denying.}

According to Xiang’s experimental results, in a scenario like the one above, sentences like (41) are judged false by a significant number of participants if Janna believes an over-denying answer. This could either be accounted for by assuming that the respective participants are accessing a mention-all reading instead of an MS reading or by making it part of the truth-conditions of the MS reading that over-denying beliefs are not permitted. Xiang pursues the latter strategy. Our account, as presented here, takes the former route: while over-affirming propositions are excluded from the set of truthful resolutions by virtue of the NFA condition, over-denying propositions are included. It would be easy, however, to expand the NFA condition in Definition 3 in such a way that it also rules out over-denying propositions; all we would have to demand is that a truthful resolution in \( w \) is consistent with every alternative in \( \text{alt}^w_w(P) \).
Recall that in the actual world $w_0$ the two upper squares were blue and the others weren’t. An IE reading arises if the complement is headed by $E_{+[\text{cmp}]}$, the nucleus receives a $[-\text{exh}]$ interpretation, and the verb receives an external interpretation. We predict that the following is required for (43) to be true in $w_0$ under this configuration. John’s information state in $w_0$, $\sigma_j^{w_0}$, has to be an element of $E_{+[\text{cmp}]}(P)(w_0)$, where $P$ is the nucleus meaning. This means that (i) $\sigma_j^{w_0}$ has to be consistent, (ii) it has to entail all true alternatives in $P$, i.e., it has to entail that the two upper squares are blue, and (iii) in view of the NFA condition, it should not entail any proposition in $\text{alt}_{w_0}(P)$, i.e., it should not entail that at least one of the two squares was blue. This precisely amounts to the IE reading.

Finally, for an SE reading of (43), under an external interpretation of the verb, the nucleus must receive a $[+\text{exh}]$ interpretation. This means that the alternatives in the nucleus meaning $P$ form a partition of the logical space such that all worlds in any given partition cell agree on which squares were blue and which weren’t. Now suppose that the complement is headed by $E_{[-\text{cmp}]}$. In this case we predict that for (43) to be true in $w_0$ it is required that (i) $\sigma_j^{w_0}$ is consistent, (ii) $\sigma_j^{w_0}$ is a resolution of $P$, which means that it entails at least one alternative in $P$, and (iii) in view of the NFA condition, it should not entail any proposition in $\text{alt}_{w_0}(P)$. Taken together, requirements (ii) and (iii) imply that $\sigma_j^{w_0}$ has to entail a true alternative in $P$. There is only one true alternative in $P$, which is the proposition that the two upper squares were blue and the other squares were not blue. It is required, then, that John’s information state entails this proposition, which is again precisely what we expect under an SE reading.

If we assume that the complement is headed by $E_{+[\text{cmp}]}$ rather than $E_{[-\text{cmp}]}$ we get an additional completeness requirement, namely, that $\sigma_j^{w_0}$ should entail all true alternatives in $P$. However, since $P$ forms a partition here, we know that it contains only one true alternative. So the completeness requirement is vacuous in this case, and the end result is exactly the same as with $E_{[-\text{cmp}]}$.

Thus, we have seen that our notion of truthful resolutions, in particular the NFA condition, captures FA sensitivity in a uniform way across the different levels of exhaustivity.

5.4 Division of labor

We end this section with some remarks concerning the division of labor (i) between semantics and pragmatics in determining whether the preferred reading for a given sentence is MS, IE, or SE (Section 5.4.1), and (ii) between the $E$ operator and the verb in distinguishing between complete and non-complete truthful resolutions (Section 5.4.2).

5.4.1 Semantics versus pragmatics

Generating the different levels of exhaustive strength is a purely semantic affair on our account. What, then, is the role of pragmatics? The way we suggest our analysis could be integrated into a broader theory of interpretation is the following. We assume that the semantic component makes a range of possible readings available, and the pragmatic component selects from this range that reading which was most likely intended by the speaker, given the particular context of utterance. One reason for assuming this division of labor is that the range of permissible readings of an interrogative clause can be constrained by a number of conventional means. For instance, as was illustrated in Section 5.1, some languages arguably have explicit markers for exhaustivity (see Li, 1995, for Mandarin and German) or for non-exhaustivity (see Beck and Rullmann, 1999, for German, Dutch and English). It is difficult to envision how pragmatics, operating at the matrix clause level, could be sensitive to these subsentential, conventional ways of marking exhaustive strength. In light of this, it seems that the range of possible readings should be captured semantically.
5.4.2 The $E$ operator versus the verb

On our account the $[\pm cmp]$ ambiguity is situated at the level of the $E$ operator. One may wonder whether this ambiguity could not be incorporated into the meaning of the embedding verb instead. However, coordination data suggest that this would be problematic. Consider sentence (44c) below, in which two interrogative complements are embedded under know. The first of these comes from Cremers and Chemla (2016b), whose experimental setting was discussed above. Recall that participants were asked to consider two different scenarios: in scenario A, John’s recollection of the card he looked at was as depicted in the second picture in Figure 2 on page 9; in scenario B, John’s recollection of the card he looked at was as depicted in the third picture in Figure 2. Sentence (44a) was saliently judged false in scenario A, while it was saliently judged true in scenario B. This can only be the case if the complement in (44a) receives an IE reading. The second complement in (44c), on the other hand, is a typical MS example, as witnessed by the fact that sentence (44b) is judged true even if John only knew of one relevant store that sells Italian newspapers.

(44) a. John knew which squares were blue.
   b. John knew where he could get an Italian newspaper.
   c. John knew which squares were blue and where he could get an Italian newspaper.

If the two complements are coordinated, as in (44c), these judgements seem to be retained: assuming that John knew at least one store that sells Italian newspapers, (44c) is perceived as false in scenario A, but true in scenario B. This means that the first complement needs to receive an IE reading—and hence bear a $[+cmp]$ feature—while the second complement needs to receive an MS reading—and hence bear a $[–cmp]$ feature. If completeness were taken to be a feature of the verb, this reading would be impossible to derive since the complex complement clause could only be interpreted $[+cmp]$ as a whole or $[–cmp]$ as a whole.22

This concludes our treatment of interrogative complements embedded under know. We have proposed an account that captures FA sensitivity in a uniform way across different levels of exhaustive strength. Moreover, our treatment of know, involving a distinction between an internal and an external interpretation, has shed light on the controversy concerning the existence of IE readings in interrogative knowledge ascriptions. In Section 6 we turn to declarative complements, and in Section 7 we will consider a broader range of verbs.

6 Declarative complements

Even though we focused on interrogative complements so far, our account has been set up in such a way that it can directly be applied to declarative complements as well. Here, we will focus on three specific predictions: (i) any truthful resolution of a declarative complement is complete, (ii) declarative complements do not exhibit FA sensitivity effects because their truthful resolution sets are fully downward closed, and (iii) declarative complements embedded under know trigger veridicality implications.

6.1 All truthful resolutions are complete

In order to see what happens when the $E$ operator applies to a declarative nucleus, let us look at a concrete example:

22Thanks to Lucas Champollion (p.c.) for pointing this out. The line of reasoning is borrowed from an argument that has been made in connection with distributivity, cf. Dowty (1987).
In (45), $E_{[-\text{cmp}]}(\varnothing) = E_{[+\text{cmp}]}(\varnothing) = \left\{ \begin{array}{c} w_1 \mapsto \{\varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing\} \\ w_2 \mapsto \{\varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing\} \\ w_3 \mapsto \emptyset \\ w_4 \mapsto \emptyset \end{array} \right\}$

As a consequence, while with interrogative complements our account generates multiple readings (MS, IE, and SE), in the case of declarative complements it always generates just one reading.

6.2 No false answer sensitivity effects

As we have seen in Section 5.2, when $E$ applies to a non-exhaustive interrogative nucleus, the resulting set of truthful resolutions only exhibits a restricted form of downward closedness. If $E$ applies to a declarative nucleus, however, the resulting set of truthful resolutions is always fully downward closed. To see why, consider an arbitrary declarative nucleus meaning $P$ and let $q$ be the unique alternative in $P$. Then, if $w \in q$, we have that $E(P)(w) = \{q\}$, while if $w \notin q$, we have that $E(P)(w) = \emptyset$. As a consequence, the set of truthful resolutions is always fully downward closed.

This has repercussions for the predictions that our analysis makes concerning FA sensitivity. To see this, consider the following example:

(46) Rupert knows that one can buy an Italian newspaper at Newstopia.

Let $p$ be the proposition that Newstopia sells Italian newspapers, and $r$ the proposition that both Newstopia and Paperworld sell Italian newspapers. Now, since in the case of a declarative complement, the set of truthful resolutions is downward closed, both $p$ and $r$ are truthful resolutions. This is why it is correctly predicted that (46) is true even if Rupert wrongly believes $r$.

6.3 Veridicality

As discussed in Section 2.6, know is veridical w.r.t. declarative complements. This means that, when taking a declarative complement, it triggers a declarative veridicality implication.

(47) John knows that Mary called.

\[ \therefore \] Mary called.

Our account already captures this fact. To see why, suppose that in $w$ Mary didn’t call, and let $P$ be the meaning of the declarative nucleus in (47), that Mary called. This is the set of propositions which consist exclusively of worlds where Mary called. Thus, $P$ contains exactly one alternative, namely the set $q$ of all worlds where Mary called. Since Mary didn’t call in $w$, we find that $\text{alt}^w_\text{cmp}(P) = \{q\}$. This means, however, that $E_{[\pm\text{cmp}]}(P)(w)$ is empty, since for a proposition $p$ to be included in $E_{[\pm\text{cmp}]}(P)(w)$, it would have to hold that $p \in P$, i.e., $p \subseteq q$, and $p \not\subseteq q$, which is impossible. Hence, John’s information state cannot
be an element of $E_{[\leq \text{cmp}]}(P)(w)$, and John knows that Mary called comes out as false. Conversely, John knows that Mary called can only be true in $w$ if Mary called is also true in $w$.

Now note that, in the case of know, the observed veridicality implication is actually a presupposition. As illustrated in (48), it projects under negation. Such veridicality implications are referred to as factivity presuppositions, and the verbs that trigger them as factive verbs.

(48) John doesn’t know that Mary called.
    \[ \not\text{Mary called.} \]

On our account veridicality implications arise from the interaction between the verb and the $E$ operator. Now, if the implication is presuppositional in nature, as in the case of know, should this presuppositional nature be determined by the $E$ operator, or rather by the embedding verb? We opt for the latter, for the following reason. If we were to let $E$ earmark veridicality implications as presuppositions, then we would be predicting, at least in the absence of any further stipulations, that all verbs which are veridical w.r.t. declarative complements trigger a presuppositional veridicality implication and are therefore factive. This prediction, as pointed out by Uegaki (2015) based on Egré (2008), is too strong. There are a number of verbs that trigger veridicality implications that are not presuppositional. As illustrated by (49), be right is a case in point. Sentence (49a) implies that Mary called, but this implication clearly doesn’t project under the negation in (49b).

(49) a. John is right that Mary called.
    \[ \text{Mary called.} \]

b. John isn’t right that Mary called.
    \[ \not\text{Mary called.} \]

We will give a lexical entry for be right in Section 7.1.2. For now, we conclude that it shouldn’t fall to the $E$ operator to earmark veridicality implications as presuppositions. Instead, the nature of this implication only gets determined at the level of the embedding verb. For know, this can be implemented by means of a definedness restriction in the lexical entry of the verb, as is done in (50).

(50) $\text{know}^\prime = \lambda f_{(s,T)}. \lambda x. \lambda p. \forall w \in p. f(w) \neq \emptyset. \forall w \in p : \sigma^w \in f(w)$

Recall that $f(w) = E(P)(w)$ is non-empty if and only if $w$ is contained in at least one alternative in $\text{alt}(P)$. In the case of a declarative complement, $\text{alt}(P)$ consists of a unique alternative $q$. As a result, $\text{know}^\prime(f)(x)(p)$ is only defined for those propositions $p$ such that $p \subseteq q$. On the other hand, in the case of an interrogative complement, the alternatives in $\text{alt}(P)$ cover the set of all possible worlds, so there will never be a world $w$ such that $f(w) = \emptyset$. Hence, in this case, the definedness restriction of $\text{know}^\prime$ is trivially satisfied.24

---

23This could be done via a definedness restriction, i.e., by postulating that $E_{[\leq \text{cmp}]}(P)(w)$ is only defined if $\text{alt}(P) \neq \emptyset$. This is indeed what we proposed in a preliminary presentation of the present paper (Roelofsen et al., 2014). We thank Wataru Uegaki for bringing to our attention that this is problematic in view of verbs like be right, whose veridicality implications are not presuppositional. Incidentally, as we will discuss in detail in the Appendix, Uegaki’s (2015) account involves an operator which is very close to our $E$ operator, but which does earmark the veridicality implications of verbs like know as presuppositions. We show in Appendix A that this raises a number of issues.

24We only specify the basic entry for know here, capturing its external interpretation. To capture the internal reading of the verb, we could add the requirement that $\forall v \in \sigma^w : \sigma^v \in f(v)$, as discussed in Section 5.3. However, if the complement of know is declarative, this requirement will automatically be satisfied and adding it explicitly will have no effect. The proof of this is similar to that of Fact 1 in Appendix B.1.

25In the case of a presuppositional interrogative nucleus it would also hold that $f(w)$ is never empty, although there may be worlds where $f(w)$ is undefined. For instance, in the case of which student called, $f(w)$ would only be defined if exactly one student called in $w$. As a consequence, John knows which student called (p) would only be defined if $p$ consisted exclusively of worlds where exactly one student called. This way the existence and uniqueness presuppositions of the nucleus would be
Next, we observe that know is also predicted to be veridical w.r.t. interrogative complements, i.e., to license inferences like those in (51a-b).\textsuperscript{26}

(51)  
\begin{enumerate}
\item[51a.] Mary knows where John was born.
John was born in Paris.
\therefore Mary knows that John was born in Paris.
\item[51b.] Mary knows where John was born.
It is not the case that John was born in Paris.
\therefore It is not the case that Mary knows that John was born in Paris.
\end{enumerate}

To see this, assume that Mary knows where John was born is true in w. On our account, this means that \(\sigma^w_m \in E(\text{where John was born})(w)\) (whether the E operator is complete or non-complete does not make a difference here because the complement is exhaustivity-neutral). Now, further assume that, in w, John was born in Paris. It follows, then, that \(E(\text{where John was born})(w) = E(\text{that John was born in Paris})(w)\). Thus, we find that \(\sigma^w_m \in E(\text{that John was born in Paris})(w)\), which means that Mary knows that John was born in Paris is true in w. Similarly, if we assume that, in w, John was not born in Paris, it follows that \(E(\text{where John was born})(w) \cap E(\text{that John was born in Paris})(w) = \emptyset\). Thus, we get that \(\sigma^w_m \not\in E(\text{that John was born in Paris})(w)\), which means that Mary knows that John was born in Paris is not true in w.

Thus, we now have a uniform account of declarative and interrogative complements embedded under know. The effects of the proposed semantics depend on whether the complement is interrogative or declarative: with interrogative complements, it predicts interrogative veridicality and FA sensitivity effects, while with declarative complements, it predicts declarative veridicality but no FA sensitivity effects.

7 Embedding verbs

We have so far restricted our attention to only one verb, know. Now it’s time for a more systematic and comprehensive account of the verbs that take declarative and/or interrogative complements as their argument. In Section 7.1 we provide an analysis of a number of responsive verbs which differ in interesting ways from know, and in Sections 7.2 and 7.3 we do the same for rogative and anti-rogative verbs, respectively, showing in particular that the selectional restrictions of these verbs can be derived from their lexical semantics. In Section 7.4 we point out a number of entailment patterns that are predicted to hold between the various verbs discussed, and finally, in Section 7.5 we propose an account of the fact that veridicality w.r.t. declarative complements and veridicality w.r.t. interrogative complements tend to go hand in hand (Spector and Egré, 2015).

7.1 Responsive verbs

We have seen that know is veridical w.r.t. both declarative and interrogative complements, and that it exhibits FA sensitivity effects when taking an interrogative complement. Below, we will consider be certain (which is non-veridical), be right and be wrong (which are veridical but not factive), be surprised (which is veridical but does not exhibit FA sensitivity effects), and care (which is veridical w.r.t. declarative complements but not w.r.t. interrogative complements).

\textsuperscript{26}Note that this could not be established yet in the previous section, where we dealt with interrogative complements, because veridicality w.r.t. interrogative complements is defined in terms of inference patterns that involve both interrogative and declarative complements.
7.1.1 be certain

Clearly, be certain is close in meaning to know. However, we propose that there are two differences between the verbs. First, we take be certain to be sensitive to truthful resolutions of the complement in all worlds that the subject considers possible, not necessarily in the world of evaluation (only if this happens to be a world that the subject considers possible). For instance, John is certain who called can be true in a world \(w\) even if John’s information state in \(w\) does not coincide with a truthful resolution of who called in \(w\); as long as it does coincide with a truthful resolution of who called in some world that John considers possible. This is captured by the preliminary entry for be certain in (52). For comparison, we repeat the non-presuppositional version of our basic (external) entry for know in (53).

\[
\text{(52) be certain'} = \lambda f_{(s,T)} \cdot \lambda x. \lambda p. \forall w \in p: \exists v \in \sigma^w_x: \sigma^w_x \in f(v) \quad \text{(preliminary)}
\]

\[
\text{(53) know'} = \lambda f_{(s,T)} \cdot \lambda x. \lambda p. \forall w \in p: \sigma^w_x \in f(w)
\]

Just like know, our preliminary entry for be certain takes a function \(f\) from worlds to sets of propositions as its first argument, an individual \(x\) as its second argument, and yields a set of propositions. Different from know, however, there is a layer of existential quantification over worlds in \(x\)’s information state \(\sigma^w_x\), and \(f\) is fed worlds \(v \in \sigma^w_x\), rather than the world of evaluation \(w\) itself.

Notice the subtle, but crucial change that this world shift brings: in order to determine whether John is certain who called is true in \(w\), we don’t have to compute the set of truthful resolutions of who called in \(w\) itself, but rather in worlds \(v \in \sigma^w_x\). We will see below that, as a consequence be certain is not veridical and doesn’t exhibit FA sensitivity.

We now turn to a second difference between know and be certain. Recall that we argued that know has both an internal interpretation, which requires resolution introspection, and an external interpretation, which does not require such introspection. We propose that be certain only has an internal interpretation, requiring resolution introspection. In order for John is certain who called to be true in \(w\), it is not sufficient if John’s information state in \(w\) just happens to match a truthful resolution of who called in some world that John considers possible; rather, in any world that is compatible with John’s information state such a match must obtain.\(^\text{27}\)

Our preliminary entry for be certain needs to be strengthened in order to ensure resolution introspection. This can be done in the same way as we did earlier with our basic entry for know.

\[
\text{(54) be certain'} = \lambda f_{(s,T)} \cdot \lambda x. \lambda p. \forall w \in p: (\exists v \in \sigma^w_x: \sigma^w_x \in f(v) \land \forall v \in \sigma^w_x: \sigma^w_x \in f(v))
\]

Now, since \(\sigma^w_x\) is assumed to be consistent, i.e., non-empty, the first conjunct is implied by the second. So we can simplify, and our final entry for be certain is the following:

\[
\text{(55) be certain'} = \lambda f_{(s,T)} \cdot \lambda x. \lambda p. \forall w \in p: \forall v \in \sigma^w_x: \sigma^w_x \in f(v)
\]

Our entry for be certain is very similar to that proposed by Uegaki (2015) (though the latter is formulated in a different framework). Uegaki makes two observations in support of his proposal. First, it predicts that be certain does not give rise to IE readings, unlike know. Our treatment also makes this prediction. Moreover, going one step beyond Uegaki’s proposal, it also offers an intuitive explanation for what is responsible for this contrast between be certain and know. Namely, be certain lacks an external interpretation: it is only true to say that an individual \(x\) is certain of something if \(x\) would say of herself that she is

\(^{27}\)Thus, in Cremers and Chemla’s experimental setting, if John’s beliefs about the card he saw are as depicted in the third picture in Figure 2, we would say that the sentence John is certain which squares were blue is false.
certain. On an internal interpretation, both *be certain* and *know* require resolution introspection, which is incompatible with IE readings. It is only on the *external* interpretation of *know* that it does not require resolution introspection and therefore permits IE readings.\(^{28}\)

Uegaki's second observation is that his entry makes desirable predictions about presupposition projection. For instance, *John is certain which student left* is predicted to presuppose that John believes that exactly one student left. A detailed account of presupposition projection in our framework is beyond the scope of the present paper, but it seems that under reasonable assumptions, Uegaki's result would carry over to our treatment.\(^{29}\)

Finally, as expected, *be certain* doesn't exhibit FA sensitivity. For instance, (56) is judged true on an MS reading even if Rupert mistakenly believes that one can buy an Italian newspaper at Paperworld.

\[(56) \quad \text{Rupert is certain where one can buy an Italian newspaper.}\]

To see that this is correctly predicted on our account, suppose that the complement clause in (56) contains an \(E_{\text{cmp}}\) operator and that the nucleus receives a [–exh] interpretation, which gives us an MS reading. Then (56) comes out as true even if Rupert mistakenly believes that both Newstopia and Paperworld sell Italian newspapers. This is the case because all the worlds in Rupert's information state are ones where both Newstopia and Paperworld indeed sell Italian newspapers. This means that in all of these worlds, \(\sigma^w_r\) is a truthful resolution of the complement.

### 7.1.2 *be right* and *be wrong*

As briefly mentioned in Section 6.3, *be right* is veridical w.r.t. declarative complements. This is illustrated in (57). It is also veridical w.r.t. interrogative complements, as illustrated in (58a-b).

\[(57) \quad \text{John is right that Mary called.} \quad \therefore \quad \text{Mary called.}\]

\[(58) \quad \begin{align*}
\quad & \text{a. John is right about where Mary was born.} \\
& \text{Mary was born in Paris.} \\
& \therefore \quad \text{John is right that Mary was born in Paris.}
\end{align*}\]

\[(58) \quad \begin{align*}
\quad & \text{b. John is right about where Mary was born.} \\
& \text{It is not the case that Mary was born in Paris.} \\
& \therefore \quad \text{It is not the case that John is right that Mary was born in Paris.}
\end{align*}\]

We observed in Section 6.3 that the declarative veridicality implication of *be right* is not a presupposition, unlike in the case of *know*.

\[(59) \quad \text{John isn't right that Mary called.} \quad \therefore \quad \text{Mary called.}\]

What *is* presupposed by both (57) and (59) is that John *believes* that Mary called.

\[(60) \quad \text{John is right that Mary called.} \quad \therefore \quad \text{John believes that Mary called.}\]

\(^{28}\)Another advantage of our treatment of *be certain* in comparison with Uegaki's is that, even though it blocks IE readings, it does allow us to derive FA sensitive MS readings as well as SE readings in a uniform way. On Uegaki's account, SE readings are readily obtained, but deriving MS readings requires additional assumptions. We refer to the Appendix for a more general and more detailed comparison between our account and Uegaki's.

\(^{29}\)The main assumption that we would have to make is that \(E(\text{which student left})(w)\) is undefined whenever there is not a unique student who left in \(w\). See also footnote 14.
(61) John isn’t right that Mary called.
    \[ \therefore \text{John believes that Mary called.} \]

To capture this, we take be right to presuppose that the subject’s information state $\sigma^w_x$ coincides with a truthful resolution of the complement in at least one world that she considers possible, and to assert that $\sigma^w_x$ coincides with a truthful resolution in the world of evaluation $w$. The assertive component of be right is hence the same as that of know.

(62) be right$'$ = $\lambda f_{(s,T)} . \lambda x . \lambda p . \forall w \in p : \exists v \in \sigma^w_y : \sigma^v_x \in f(v). \forall w \in p : \sigma^w_x \in f(w)$

Now let us turn to be wrong. We first observe that this verb is non-veridical, both w.r.t. declarative complements and w.r.t. interrogative complements, as witnessed by the invalid inferences in (63) and (64).

(63) John is wrong that Mary called.
    \[ \lnot \text{M called.} \]
(64) John is wrong about where Mary was born.
    Mary was born in Paris.
    \[ \lnot \text{John is wrong that Mary was born in Paris.} \]

In fact, be wrong is what we may call anti-veridical w.r.t. declarative complements:

(65) John is wrong that Mary called.
    \[ \lnot \text{Mary didn’t call.} \]

The anti-veridicality implication of be wrong is an entailment, not a presupposition, just like the declarative veridicality implication of be right:

(66) John isn’t wrong that Mary called.
    \[ \lnot \text{Mary didn’t call.} \]

Both (65) and (66) do presuppose that John believes that Mary called, again just as in the case of be right. Thus, we arrive at the following entry:

(67) be wrong$'$ = $\lambda f . \lambda x . \lambda p . \forall w \in p : \exists v \in \sigma^w_y : \sigma^v_x \in f(v). \forall w \in p : \sigma^w_x \not\in f(w)$

Notice that the only difference between be right and be wrong is that the former requires $\sigma^w_x \in f(w)$, whereas the latter requires the opposite, $\sigma^w_x \not\in f(w)$. This captures all the entailment patterns exemplified above.

### 7.1.3 be surprised

The verbs we have seen so far suggest a certain pattern: those that are veridical, e.g., know and be right, are also FA-sensitive, while those that are non-veridical, e.g., be certain, are not FA-sensitive. However, we believe that not all embedding verbs follow this pattern. Emotive factives like be surprised are a case in point. To begin with, be surprised is veridical w.r.t. both declarative and interrogative complements:

(68) Mary is surprised that John was born in Paris.
    \[ \therefore \text{John was born in Paris.} \]

(69) a. Mary is surprised at where John was born.
    John was born in Paris.
    \[ \therefore \text{Mary is surprised that John was born in Paris.} \]
b. Mary is surprised at where John was born.
   It is not the case that John was born in Paris.
   \[\therefore\] It is not the case that Mary is surprised that John was born in Paris.

Turning to FA-sensitivity, however, consider the following sentence:

(70) Rupert is surprised at where one can buy an Italian newspaper.

For (70) to be true on an MS reading, there has to be at least one store \( x \) such that Rupert correctly believes but did not expect that \( x \) sells Italian newspapers. What Rupert believes or expected about stores that do not sell Italian newspapers seems immaterial. So, be surprised is not FA-sensitive. A simple lexical entry that would achieve this is given in (71), where we write \( \epsilon^w_x \) for the set of worlds compatible with \( x \)'s previous expectations at \( w \).

(71) \( \text{be surprised}' = \lambda f, \lambda x, \lambda p, \forall w : p : \exists q \in \text{alt}(f(w)) : \sigma^w_x \subseteq q \land \epsilon^w_x \subseteq \overline{q} \)

Note what happens here: the entry makes specific reference to the set of truthful resolutions of the complement in the world of evaluation, \( f(w) \), but then only the maximal elements of the set, \( \text{alt}(f(w)) \), are taken into account. It is required that there exists an alternative \( q \) such that \( x \) believes \( q \) in \( w \), \( \sigma^w_x \subseteq q \), but \( q \) is incompatible with \( x \)'s previous expectations in \( w \), \( \epsilon^w_x \subseteq \overline{q} \). So, explicit reference is made to the set of truthful resolutions in the world of evaluation (which captures the veridical nature of the verb), but then exactly the part of this set that would be needed to generate FA-sensitivity is disregarded.

7.1.4 care

Predicates of relevance, such as care, have been identified as problematic for existing theories on different grounds. Firstly, as discussed in Section 2.2, what is presupposed by care depends on whether the verb takes a declarative or an interrogative complement. With a declarative complement, care presupposes that the complement is true and that the subject knows this. For instance, (72a) presupposes that Mary left and John knows that Mary left. With an interrogative complement, on the other hand, care doesn’t carry analogous presuppositions. In particular, (72b) doesn’t presuppose that John believes some answer to the question of which girl left.

(72) a. John cares that Mary left.
    b. John cares which girl left.

Secondly, it was argued in Section 2.6 that care is veridical w.r.t. declarative but not w.r.t. interrogative complements:

(73) Mary cares that John was born in Paris.
    \[\therefore\] John was born in Paris.
(74) Mary cares where John was born.
    John was born in Paris.
    \[\not\therefore\] Mary cares that John was born in Paris.

\[\text{30} \text{The entry given here is merely meant to illustrate that it is possible in our framework to deal with verbs that are veridical but not FA-sensitive. It is not meant as a fully realistic treatment of be surprised, which involves several complexities that are orthogonal to our present concerns. In particular, our entry does not account for the fact that be surprised and other emotive factives do not license polar and disjunctive interrogative complements. We refer to Guerzoni and Sharvit (2007), Sæbø (2007), Nicolae (2013), Spector and Egré (2015), Romero (2015b) for recent work in this area. See in particular Roelofsen et al. (2016) for an approach that is compatible with the present proposal.}\]
On our account, it is straightforward to define a lexical entry for care that delivers on these criteria. In particular, both the differences in presuppositions and those in veridicality naturally fall out from the semantic properties of declarative and interrogative complements. We propose the following lexical entry, where $\beta^w_x$ is the bouletic state of $x$ in $w$, i.e., the set of all those worlds that are compatible with what $x$ desires in $w$.

$$\text{care} := \lambda f. \lambda x. \lambda p. \forall w \in p: (f(w) \neq \emptyset \land \forall v \in \sigma^w_x : f(v) \neq \emptyset),$$

$$\forall w \in p: \exists v \in W : \exists q \in \text{alt}(f(v)) : (\beta^w_x \subseteq q \lor \beta^w_x \cap q = \emptyset)$$

In words, it is presupposed that the set of truthful resolutions is non-empty in the world of evaluation and that it is non-empty in all worlds in the subject’s information state. It is asserted that, among the minimally informative possible resolutions of the complement, there is at least one which the subject either desires to be true or to be false.

If care takes an interrogative complement, its presupposition is trivially satisfied since the meaning of an interrogative nucleus covers the entire logical space and $f(w)$ will therefore be non-empty for all worlds $w$. In contrast, with declarative complements, there are usually worlds $w$ such that $f(w)$ is empty and, in that case, the presupposition of care is non-trivial. This pattern is already familiar from our discussion of know in Section 6.3, and we will encounter it again once we turn to the selectional restrictions of anti-rogative verbs like be true.

Now assume that care takes a declarative complement and let $q$ be the unique alternative in the nucleus meaning. Then, the second conjunct of the presupposition amounts to $q$ being true in all worlds in the subject’s information state. Combined with the first conjunct, which requires that $q$ is true in the world of evaluation, this amounts to demanding that the subject knows $q$.

Turning to veridicality, we find that by virtue of the factivity presupposition of care, declarative veridicality inferences like (73) indeed come out as valid. In contrast, interrogative veridicality inferences are not predicted to go through. In order for the conclusion in, e.g., (74) to hold, it would have to be the case that Mary knows that John was born in Paris—but this is not guaranteed by the premises in that argument.

Two final observations are due. Firstly, the above lexical entry for care predicts that its SE and IE reading coincide. More precisely, if $f_{SE} = E_{[+]cmp}(\text{nucleus}_{[+]exh})$ and $f_{IE} = E_{[+]cmp}(\text{nucleus}_{[-]exh})$, then care$(f_{SE}) = care(f_{IE})$. Secondly, our analysis predicts that when care takes an interrogative complement, the MS reading entails the IE/SE reading. Thus, when care takes an interrogative complement, we only find two distinct readings, namely MS and SE/IE, and of these two the former entails the latter. It appears that with non-negated statements like (76a), there is a preference for the MS interpretation, while with negated statements like (76b) the IE/SE interpretation is preferred. Since this means that in either case the stronger one of the two readings is favored, this pattern could be explained by appealing to the strongest meaning hypothesis (Dalrymple et al., 1998).

$$\text{(76)} \quad \begin{array}{l}
\text{a. John cares who left.} \\
\text{b. John doesn’t care who left.}
\end{array}$$

7.2 Rogative verbs

We now turn to rogative verbs, providing a detailed analysis of two representative members of this class, wonder and depend on. We will show that it follows from their lexical semantics that they don’t license declarative complements. Finally, we will also suggest an explanation for the fact that rogative speech act verbs like ask and inquire do not license declarative complements.
7.2.1 wonder

The main features of our analysis of wonder are adopted from Ciardelli and Roelofsen (2015), though the treatment of interrogative complements is more fine-grained in our framework, and the entry for wonder therefore somewhat more involved.

In order to formally capture what it means for an individual to wonder about something, it is not enough to model her information state, i.e., what she takes herself to know; instead, we also need a notion of the issues she is entertaining, i.e., what she would like to know. We will model this as the individual’s inquisitive state. The inquisitive state of an individual \( x \) in a world \( w \), which we will write as \( \Sigma_w^x \), is a downward closed set of consistent information states which together cover \( \sigma^w_x \), i.e., \( \bigcup \Sigma_w^x = \sigma^w_x \). The information states in \( \Sigma_w^x \) are exactly those where the issues that \( x \) is entertaining are resolved—in other words, the information states that \( x \) would like to reach.

Informally, \( x \) wonders about a question, e.g., about who called, just in case (i) \( x \) isn’t certain who called, and (ii) she wants to learn who called. This is the case exactly if (i) \( x \)’s current information state does not coincide with a truthful resolution of the question in every world that she considers possible; and (ii) every information state that \( x \) wants to reach is one in which \( x \) has a set of propositions that amounts to the powerset of \( \sigma^w_x \), i.e., what she takes herself to know; instead, we also need a notion of the issues that \( x \) is entertained.

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In words, wonder takes a function \( f \) from worlds to sets of propositions and an individual \( x \), and yields a set of propositions. Since \( f \) is a function mapping a world to the set of truthful resolutions of the complement in that world, the propositions that wonder yields only contain worlds \( w \) such that the following holds: (i) there are worlds \( v \in \sigma^w_x \) such that \( \sigma^w_x \) does not coincide with a truthful resolution in \( v \), i.e., given \( x \)’s current information state, she isn’t certain; and (ii) every state \( \sigma \) in \( \Sigma_w^x \) does coincide with a truthful resolution at all worlds \( v \in \sigma \), i.e., every information state \( x \) wants to reach is one in which she is certain.

Now let us turn to the fact that wonder does not license declarative complements:

(78) *John wonders that Mary called.

We will show that this fact can be derived from the lexical entry of the verb and the semantic properties of declarative complements (cf. Ciardelli and Roelofsen, 2015, who give an analogous explanation for a simpler version of the lexical entry). We will use the fact that the inquisitive state of \( x \) in \( w \), \( \Sigma_w^x \), is a set of information states which together form a cover of \( \sigma^w_x \), i.e., \( \bigcup \Sigma_w^x = \sigma^w_x \). Recall that, by assumption, \( \sigma^w_x \) is always non-empty. Therefore, \( \Sigma_w^x \) always has to be non-empty as well. Finally, it will be crucial that the meaning of a declarative nucleus is a set of propositions \( P \) that contains a single alternative \( q \). Since nucleus meanings are downward closed, this means that \( P \) amounts to the powerset of \( q \), \( \wp(q) \).

Now, if we apply \( E \) to \( P \), we get a function \( f \) from worlds to sets of propositions, such that \( f(w) = \wp(q) \setminus \{\emptyset\} \) if \( w \in q \) and \( f(w) = \emptyset \) otherwise. Suppose that the first conjunct in the entry for wonder holds, i.e., \( \forall v \in \sigma^w_x : \sigma^w_x \subseteq f(v) \). This can only be the case if there exists some world \( v^* \in \sigma^w_x \) such that \( v^* \notin q \). For, if all worlds in \( \sigma^w_x \) were contained in \( q \), then we would get that \( \sigma^w_x \subseteq \wp(q) \setminus \{\emptyset\} \), which would mean that \( \sigma^w_x \subseteq f(v) \) for all \( v \in \sigma^w_x \), which would falsify the first conjunct. Now, since \( \bigcup \Sigma_w^x = \sigma^w_x \), it follows that there exists some \( \sigma \in \Sigma_w^x \) such that \( v^* \in \sigma \). Because \( v^* \notin q \), we find that \( f(v^*) = \emptyset \). Thus, we know that there exists a \( \sigma \in \Sigma_w^x \) such that, for some \( v \in \sigma \), \( \sigma \notin f(v) \). This falsifies the second conjunct.

Hence, whenever wonder takes a declarative complement, we get a contradiction, no matter what the specific content of the complement is. This type of systematic contradictoriness explains why won-
der does not license declarative complements (see Gajewski, 2002, for a general discussion of explaining ungrammaticality in terms of systematic contradictoriness or triviality).

A similar explanation can be given for other rogative verbs such as investigate and be curious. For instance, the lexical semantics of investigate may be taken to have the same two components as that of wonder, one conveying that the subject’s information state does not resolve the issue expressed by the complement, and the other conveying that the subject would like to get to a state which does resolve this issue. In the case of investigate, unlike in the case of wonder, the verb also conveys that the subject is taking specific actions to reach such a state. In both cases alike, however, the two components will yield a contradiction when applied to the meaning of a declarative complement. And similarly for be curious.

7.2.2 depend on

Our entry for depend on is essentially the one proposed by Karttunen (1977, footnote 6), adapted to our present framework and made a bit more explicit in some ways. First of all, we assume that dependency statements are modal statements (see also Ciardelli, 2016, Section 6.5). We can only sensibly say that one complement meaning depends on another complement meaning relative to some specific range of relevant possible worlds, i.e., a modal base. This modal base can either be explicitly given, as in (79), or it can be inferred from the context, as in (80), where, roughly, it is construed as given the laws of nature and the layout of the electric circuit under discussion.

(79) According to the law, the fine depends on whether the speed limit.

(80) Whether the light is on depends on whether the switch if up.

Informally, a complement meaning \( f(s,T) \) depends on another complement meaning \( f'(s,T) \) relative to a modal base \( \sigma_w \) if the values that \( f \) takes across worlds in \( \sigma_w \) are determined, at least partly, by the values that \( f' \) takes in these worlds. More formally, this obtains when there is a function \( g(s,T) \) of the same type as \( f \) and \( f' \), embodying other factors that may influence the values that \( f \) takes in worlds in \( \sigma_w \), with the following two properties:

1. there is no function \( \mathcal{D}_{(T,T)} \) s.t. for all \( v \in \sigma_w \), \( \mathcal{D}(g(v)) = f(v) \), but
2. there is a function \( \mathcal{D}'_{(T,(T,T))} \) s.t. for all \( v \in \sigma_w \), \( \mathcal{D}'(g(v), f'(v)) = f(v) \).

In short, \( g \) does not by itself determine the value of \( f \), but together with \( f' \) it does. We add one more ingredient, which is the requirement that for every world \( v \) in \( \sigma_w \), both \( f(v) \) and \( f'(v) \) have to be non-empty. This is analogous to the definedness restriction of know, and it is needed to ensure, for instance, that the dependency described in (80) cannot hold relative to a modal base which contains worlds where there is no light, or no switch. Thus, we propose the following entry:

(81) \[ \text{depend on'} = \lambda f'_{(s,T)} \lambda f_{(s,T)} \lambda p. \forall w \in p. \forall v \in \sigma_w : f(v) \neq \emptyset \land f'(v) \neq \emptyset. \]

\[ \forall w \in p. \exists g(s,T) \left( \forall v \in \sigma_w : \mathcal{D}(g(v)) = f(v) \land \mathcal{D}'_{(T,(T,T))}(v, g(v), f'(v)) = f(v) \right) \]

Let’s try to get a better grasp of how this entry works. Consider the example in (82), understood as a statement about the electric circuit in Figure 8. We expect (82) to come out as true, since the position of the left switch does indeed play a role in determining whether the light is on: the light is on only if both switches are up or both are down.

(82) Whether the light is on depends on whether the left switch if up or down.
Now, let \( f \) be the meaning of the question \textbf{Is the light on}, \( f' \) the meaning of the question \textbf{Is the left switch up or down}, and \( g \) the meaning of the question \textbf{Is the right switch up or down}. Further, let \( w_1 \) be a world in which both switches are up (this is the actual world), \( w_2 \) one in which only the left switch is up, \( w_3 \) one in which only the right switch is up, and \( w_4 \) one in which neither switch is up. This situation is summarized in Table 3. Our modal base \( \sigma_{w_1} \) consists of all worlds in which the laws of nature are the same as in the actual world and in which the circuit is exactly as given in Figure 8 modulo the position of the switches. That is, our modal base is \( \sigma_{w_1} = \{ w_1, w_2, w_3, w_4 \} \).

Then, we find that there exists no function \( D_{\langle T,T \rangle} \) such that, for all \( v \in \sigma_{w_1} \), \( D(g(v)) = f(v) \). Assume for contradiction that there was such a function. It holds that \( g(w_1) = g(w_3) \), since both are the set of all propositions that consist exclusively of worlds in which the right switch is up. Moreover, \( f(w_1) \) is a set of propositions that consist exclusively of worlds in which the light is on, whereas \( f(w_3) \) is a set of propositions that consist exclusively of worlds in which the light is off. Hence, \( f(w_1) \neq f(w_3) \). But this means that we have \( D(g(w_1)) = f(w_1) \neq D(g(w_3)) = f(w_3) \) although \( g(w_1) = g(w_3) \). Contradiction.

We can however easily find a function \( D_{\langle T,T,T \rangle}' \) such that \( D'(g(v), f'(v)) = f(v) \) for all \( v \in \sigma_{w_1} \), namely, trivially, the function \( D' \) such that \( D'(g(w_i), f'(w_i)) = f(w_i) \) for all \( i \in \{1, \ldots, 4\} \). Intuitively, this is the function \( D' \) such that \( D'(g(v), f'(v)) \) conveys that the light is on just in case both switches are up in \( v \) or both switches are down in \( v \); otherwise \( D'(g(v), f'(v)) \) conveys that the light is off. In conclusion, we find that \((82)\) is predicted to be true, as expected.

Now, let us see how our lexical entry can account for the selectional restrictions of \textbf{depend on}. What happens if either the first or the second argument of this verb is a declarative complement? First consider the following case:

\begin{equation}
\text{That the light is on depends on whether the switch is up.}
\end{equation}

<table>
<thead>
<tr>
<th>world</th>
<th>light? (( f ))</th>
<th>left switch? (( f' ))</th>
<th>right switch? (( g ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_1 )</td>
<td>on</td>
<td>up</td>
<td>up</td>
</tr>
<tr>
<td>( w_2 )</td>
<td>off</td>
<td>up</td>
<td>down</td>
</tr>
<tr>
<td>( w_3 )</td>
<td>off</td>
<td>down</td>
<td>up</td>
</tr>
<tr>
<td>( w_4 )</td>
<td>on</td>
<td>down</td>
<td>down</td>
</tr>
</tbody>
</table>

Table 3: Circuit configuration per world.
The meaning of this sentence is a set of propositions \( p \) such that every world \( w \in p \) has the following properties. First of all, for every \( v \in \sigma_w \), \( f(v) \) and \( f'(v) \) have to be non-empty, which means that in every \( v \in \sigma_w \) the light has to be on, and there has to be a switch. But this means that the value of \( f \) is the same in every \( v \in \sigma_w \), namely the set of propositions \( P \) consisting exclusively of worlds where the light is on. But then for any \( g(s,T) \) there is a function \( D(T,T) \) such that for every \( v \in \sigma_w : D(g(v)) = f(v) \), namely the function \( D \) that maps any set of propositions to \( P \). Thus, the first conjunct in the entry for depend on can never be satisfied. It follows that (83), and any other sentence where depend on takes a declarative complement in subject position, is bound to be contradictory. This explains the ungrammaticality of such sentences.

Now consider a case where the declarative complement is in object position.

(84) Whether the light is on depends on that the switch is up.

The meaning of this sentence is a set of propositions \( p \) such that every world \( w \in p \) has the following properties. First, for every \( v \in \sigma_w \), \( f(v) \) and \( f'(v) \) have to be non-empty, which means that in every \( v \in \sigma_w \) there has to be a light, and the switch has to be up. But this means that the value of \( f' \) is the same in every \( v \in \sigma_w \), namely the set of propositions \( P \) consisting exclusively of worlds where the switch is up. Now suppose that \( g(s,T) \) is such that there is no \( D(T,T) \) such that \( D(g(v)) = f(v) \) for all \( v \in \sigma_w \). Then there can also not be a \( D'(T,T,T) \) such that \( D'(g(v),f'(v)) = f(v) \) for all \( v \in \sigma_w \). After all, \( f'(v) \) is the same for all \( v \in \sigma_w \), so the output of \( D' \) will be fully determined by \( g(v) \), and by assumption \( g(v) \) alone does not fully determine the value of \( f(v) \). So the two conjuncts in the entry for depend on cannot both be satisfied. Again, it follows that (84), and any other sentence where depend on takes a declarative complement in object position, is bound to be contradictory, which explains the ungrammaticality of such sentences.

A similar explanation may be given for the fact that verbs closely related to depend on, like be determined by, do not license declarative complements either.

7.2.3 Rogative speech act verbs

Besides verbs like wonder and depend on, there is one other class of verbs that prohibit declarative complements. This class consists of speech act verbs such as ask and inquire. The incompatibility of these verbs with declarative complements can be given an explanation as well. For instance, consider a sentence of the form \( x \) asked \( \varphi \). It is natural to assume that part of what such a sentence conveys is that \( x \) uttered a sentence \( \varphi \) which was inquisitive w.r.t. the common ground in the context of utterance (something that seems to be an inherent aspect of the speech act of asking, and similarly for inquiring). This is impossible if \( \varphi \) is a declarative, because then it is bound to be non-inquisitive.

To sum up our findings in Section 7.2, we have seen that it is possible to account for the fact that rogative verbs do not license declarative complements, even though we have assumed that declarative and interrogative complements are of the same semantic type. Indeed, the account we have given is based on lexical properties that the verbs in question can naturally be assumed to have, and it is therefore more explanatory than an account that is based on a stipulated difference in semantic type between declarative and interrogative complements.
7.3 Anti-rogative verbs

We now turn to anti-rogative verbs like believe, think, feel, expect, suggest, seem, be likely, want, be true, and be false. Such verbs license declarative but not interrogative complements. The account that we will propose is based on an empirical observation made by Zuber (1982), namely that many of the relevant verbs, if not all, are neg-raising verbs. This is illustrated in (85) and (86) for believe and be true. Similar examples can be constructed for the other verbs listed above.

(85) John does not believe that Mary left.
     \[ \therefore \text{John believes that Mary did not leave.} \]

(86) It is not true that Mary left.
     \[ \therefore \text{It is true that Mary did not leave.} \]

We will show that the incompatibility of anti-rogative verbs with interrogative complements follows from their neg-raising property. For concreteness, we will focus on believe, be true, and be false, but the account could be extended to other anti-rogative neg-raising verbs.

7.3.1 believe

We start with a preliminary entry for believe, which is identical to the one for be certain. This entry will be refined immediately below.

(87) \( \text{believe}^\prime_{\text{prelim}} := \text{be certain}^\prime = \lambda f(s,T). \lambda x. \lambda p. \forall w \in p : \forall v \in \sigma^w_x : \sigma^w_x \in f(v) \)

Now let us consider how this preliminary entry should be refined in order to account for neg-raising effects. There is a large body of work in which such effects have been accounted for by assuming that the relevant verbs come with a so-called excluded middle presupposition (see, e.g., Bartsch, 1973; Gajewski, 2007). That is, a sentence of the form \( x \) believes that \( p \) is taken to come with a presupposition that \( x \) either believes that \( p \) or that \( \neg p \). Since presuppositions survive under negation, \( x \) does not believe that \( p \) still has the same presupposition. In the case of the positive sentence, the asserted content is stronger than the presupposed content, but in the case of the negative sentence, the asserted content and the presupposed content are logically independent, and together they imply that \( x \) believes that \( \neg p \). This accounts for the neg-raising effect.

Incorporating the excluded middle presupposition yields the following refined entry for believe in our setting:

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31 See Egré (2008) for a comprehensive and critical discussion of the literature on this phenomenon. Egré also formulates a constraint on interrogative embedding which accounts for the fact that the relevant verbs do not license interrogative complements. This constraint, however, is not independently motivated, which means that the account, as Egré himself acknowledges, remains rather stipulative.

32 Egré (2008) mentions a potential counterexample to the claim that all anti-rogative verbs are neg-raising, namely the verb be convinced. We think, however, that be convinced is not anti-rogative. This is witnessed by the following examples, taken from the web, where the verb appears with an interrogative complement:

(i) At best, we can do this for EasyBuild v3.0, and even then, I'm not sure I'm convinced whether pros outweigh cons on this one.

(ii) Not convinced who should get your vote? Check out what Henderson has to say on the matter.

We should note that Zuber (1982) did not claim that all anti-rogative verbs are neg-raising, but rather the other way around, that all neg-raising verbs are anti-rogative. This latter generalization is also the one that is most relevant for the account we will present here. If we were to find a verb that is anti-rogative but not neg-raising, this would not really refute the account, though it would show that it does not explain all cases of anti-rogativity. What would really be fatal for the account is if we were to find a verb that is neg-raising but not anti-rogative, contra Zuber's generalization.
(88) \( \text{believe}' := \lambda f. \lambda x. \lambda p. (\forall w \in p : \forall v \in \sigma^w_x : \sigma^v_x \in f(v)) \lor (\forall w \in p : \forall v \in \sigma^w_x : \sigma^v_x \notin \neg f(v)) \). \\
\forall w \in p : \forall v \in \sigma^w_x : \sigma^v_x \in f(v)

where \( \neg \) is the standard inquisitive negation operator, which when applied to a set of propositions \( P \) returns the set of those propositions that are inconsistent with every member of \( P \):

(89) \( \neg := \lambda P_T. \lambda p(s,t). \forall q \in P : p \cap q = \emptyset \)

The refined entry for \text{believe} only differs from our preliminary entry in that it involves a definedness restriction capturing the excluded middle presupposition. Namely, it says that \text{believe}'(f)(x)(p) is only defined if every world \( w \in p \) is one such that either (i) \( \sigma^w_x \) coincides with a truthful resolution of the complement in \( v \) in all worlds \( v \in \sigma^w_x \), or (ii) \( \sigma^w_x \) is inconsistent with all truthful resolutions of the complement in \( v \) in all worlds \( v \in \sigma^w_x \). If \( f \) is the meaning of a declarative complement, \( f(v) \) contains only one alternative \( p \). Then, the first disjunct in the presupposition amounts to \( x \) being certain that \( p \) is true, while the second disjunct amounts to \( x \) being certain that \( p \) is false. This is the inquisitive rendering of the excluded middle presupposition, which indeed accounts for the fact that \text{believe} exhibits neg-raising effects when taking a declarative complement.

Interestingly, without making any further assumptions, the entry also enables us to account for the fact that \text{believe} does not license interrogative complements. Informally, the explanation of this fact is as follows. Since it is impossible to believe the negation of an interrogative complement, the second disjunct in the definedness restriction of \text{believe} can never be true. Thus, if \text{believe} takes an interrogative complement, its semantics reduces to the following:

(90) \( \lambda f. \lambda x. \lambda p. (\forall w \in p : \forall v \in \sigma^w_x : \sigma^v_x \in f(v)) \lor \neg \lambda f. \lambda x. \lambda p. (\forall w \in p : \forall v \in \sigma^w_x : \sigma^v_x \notin \neg f(v)) \).

But this means that whenever \text{believe}'(f)(x)(p) is defined, it is true. In other words, when the verb takes an interrogative complement, its assertive component is trivial, given its presuppositional component. This systematic triviality explains why combining \text{believe} with interrogative complements results in ungrammaticality (we again refer to Gajewski (2002) for a general discussion of explaining ungrammaticality in terms of systematic contradictoriness and triviality).

An analogous explanation could be given for many other anti-rogative verbs. For instance, \text{think} and \text{feel} on their clause-embedding use are essentially synonymous with \text{believe}. Other predicates could be given a semantics that is structurally very similar to that of \text{believe}. For example, instead of demanding that the subject’s information state is a truthful resolution, we might require, roughly, that the set of worlds in line with the subject’s expectations is a truthful resolution (\text{expect}), that the subject’s bouletic state is a truthful resolution (\text{want}), or that, even more roughly, the set of worlds in accordance with the contextually available information is a truthful resolution (\text{see} and perhaps \text{be likely}). The key to deriving the selectional restrictions of all these verbs would be an excluded-middle presupposition, which, just as for \text{believe}, would systematically trivialize the assertive component of sentences in which those verbs combine with an interrogative complement.

\(^{33}\text{To see in more formal detail why this holds, suppose that \text{believe} takes an interrogative complement with meaning } f = E(P), \text{ and suppose for simplicity that this complement is not presuppositional (the argument that follows also goes through if the complement is presuppositional, though in that case it is a bit more involved). Let } w \in W, x \in D_x \text{ and } v \in \sigma^w_x. \text{ Consider the singleton set } \{v\}. \text{ Since the complement is interrogative, we know that } \{v\} \in P. \text{ Furthermore, } \{v\} \text{ necessarily entails every alternative in } \text{alt}_x(P) \text{ and cannot entail any element of } \text{alt}'_x(P). \text{ Therefore, } \{v\} \in f(v). \text{ No matter whether } E \text{ is } [+\text{cmp}] \text{ or } [-\text{cmp}]. \text{ This means that any proposition } p \in \neg f(v) \text{ has to be disjoint from } \{v\}. \text{ However, } \sigma^w_x \text{ is not disjoint from } \{v\} \text{ since } v \in \sigma^w_x. \text{ So, } \sigma^w_x \notin \neg f(v). \text{ Since } w, x \text{ and } v \text{ were chosen arbitrarily, we thus find that } \forall w : \forall x : \forall v \in \sigma^w_x : \sigma^v_x \notin \neg f(v). \text{ Hence, for interrogative complements, there exists no information state } \sigma^w_x \text{ that would make the second disjunct in the definedness restriction true.} \)
7.3.2 **be true and be false**

For **be true** and **be false** the situation is even simpler. We propose the following lexical entries:

\[(91) \quad \text{be true} = \lambda f_{s,T}.\lambda p.\forall w \in p : f(w) \neq \emptyset \]

\[(92) \quad \text{be false} = \lambda f_{s,T}.\lambda p.\forall w \in p : f(w) = \emptyset \]

When combined with a declarative complement, the entries give the expected results. In particular, they correctly predict that **be true** is veridical w.r.t. declarative complements, while **be false** is anti-veridical.

\[(93) \quad \text{It is true that Mary left.} \quad \because \quad \text{Mary left.} \]

\[(94) \quad \text{It is false that Mary left.} \quad \because \quad \text{Mary didn't leave.} \]

Furthermore, without adding an explicit excluded-middle presupposition, they capture the fact that **be true** and **be false** are neg-raising. To see this, consider the following example.

\[(95) \quad \text{It is not true that Mary left.} \]

Let \( f = E((\text{Mary left})') \). Then (95) is translated as \( \neg \text{be true}'(f) \), which is the set of all propositions \( p \) that are inconsistent with all propositions in \( \text{be true}'(f) \). This can only be the case if any such \( p \) consists exclusively of worlds \( w \) such that \( f(w) = \emptyset \), i.e., worlds in which Mary didn’t leave. But this means that \( \neg \text{be true}'(f) \) is exactly the same set of propositions as \( (\text{Mary didn’t leave})' \) and hence also as \( \text{be true}'(E((\text{Mary didn’t leave})')) \), which is the translation of \( \text{It is true that Mary didn’t leave} \).

Now let us see why the entry in (91) accounts for the fact that **be true** does not license interrogative complements (a similar argument applies to the entry in (92) for **be false**). Suppose that **be true** takes such a complement. Then there exists no world \( w \) such that \( f(w) = \emptyset \), i.e., worlds in which Mary didn’t leave. But this means that \( \neg \text{be true}'(f) \) is exactly the set of all propositions \( p \), i.e., the tautology. So, just like in the case of **believe**, when the verb takes an interrogative complement, it inescapably results in triviality, and we take this to explain that **be true** does not license interrogative complements.

Hence, as in the case of rogative verbs, we have seen that the selectional restrictions of anti-rogative verbs can also be explained based on independently motivated features of the lexical semantics of these verbs. It is not necessary to assume a difference in semantic type between declarative and interrogative complements.

### 7.4 Some predicted connections between embedding verbs

Many of the lexical entries we introduced in the preceding sections are built up from similar ingredients. For instance, **know\textsubscript{ext}** and **be right** have the same assertive component, and **know\textsubscript{int}** is built up from **know\textsubscript{ext}** and an additional introspection requirement. Taking these similarities into account, it is not surprising that we can identify multiple connections that obtain between the embedding verbs. Figure 9 and 10 display an interesting subset of those connections. The former shows the relations that obtain between the verbs on their declarative-embedding use and the latter those that obtain between the verbs on their interrogative-embedding use.

The solid black arrows are to be understood as implications. For instance, in both figures, we have an arrow from **know\textsubscript{int}** to **be certain**, meaning that, if an individual \( x \) stands in a **know\textsubscript{int}** relation to some complement meaning \( f \), then \( x \) is predicted to also stand in a **be certain** relation to \( f \). Also note that the visualisation does not distinguish between whether an implication holds due to the asserted meaning.
Figure 9: Connections between verbs on their declarative-embedding use.

components of the lexical entries or due to a presupposition. For example, on its declarative-embedding use, be wrong implies believe, but this is only the case because, whenever be wrong′(f)(x) is true, the definedness restriction of be wrong is satisfied, and this definedness restriction amounts to believe(f)(x).

The dashed red double arrows, labelled with not, are to be read as true iff not true. For instance, in Figure 10, wonder and be certain are connected with such an arrow because, whenever wonder′(f)(x) holds, be certain′(f)(x) doesn’t hold and vice versa. Note, however, that this does not indicate that wonder simply amounts to not be certain. Rather, wonder′(f)(x) can fail to hold because x isn’t certain or because x does not want to know. Were there a single lexical item in English that meant want to know, it would appear in the graph as well and would be connected to wonder with a not-arrow. Furthermore, just as the solid arrows, the not-arrows don’t distinguish between asserted or presuppositional content. For example, if be wrong′(f)(x) is true, know_int′(f)(x) cannot be true because of presupposition failure.

Figure 10: Connections between verbs on their interrogative-embedding use.
A couple of observations are worth making here. Firstly, once we restrict our attention to declarative complements, as in Figure 9, the meanings of many verbs become equivalent, e.g., that of believe, be certain, know_{int}, know_{ext} and be right. On the other hand, moving to interrogative complements, not all of these equivalences obtain anymore. Instead, Figure 10 nicely reflects the distinction between internal, i.e., resolution-introspective, and external verbs. The external verbs know_{ext} and be right are still equivalent. Furthermore, by combining internal be certain and external be right, we obtain a meaning that is equivalent to know_{int}: be certain contributes the resolution introspection condition of know_{int}, and be right contributes its “truthfulness condition”.

### 7.5 Spector and Egré’s veridicality generalization

Recall Spector and Egré’s veridicality generalization, discussed in Section 2.6: a responsive verb is veridical w.r.t. interrogative complements if and only if it is veridical w.r.t. declarative complements. We have pointed out that there are counterexamples to this generalization. In particular, predicates of relevance like care and matter are veridical w.r.t. declarative complements but not w.r.t. interrogative complements.

Nevertheless, it seems that most responsive verbs do comply with the generalization, and our theory so far does not account for this tendency. Our framework allows us to formulate lexical entries for verbs that are only veridical w.r.t. declarative complements, or only w.r.t. interrogative complements. This is needed to deal with verbs like care and matter. But why would there be so few verbs of this kind, at least in English and related languages?

Note that the situation is similar to that in other empirical domains. Consider the case of determiners. In the standard generalized quantifier framework, there is a huge space of meanings that determiners could in principle have, but in practice there seem to be certain constraints on which of these possible meanings are actually lexicalized in natural languages. For instance, a well-known empirical generalization in this area is that all natural language determiners are conservative (Barwise and Cooper, 1981; Keenan and Stavi, 1986). A determiner D is conservative if for every two set-denoting expressions A and B, D(A, B) is equivalent with D(A, A \ B). Only a small portion of the entire space of possible determiner meanings is conservative. For instance, in a universe with just two objects, there are $2^{16} = 65,536$ possible determiner meanings, but only $2^9 = 512$ of these are conservative (Keenan and Stavi, 1986).

Whether the conservativity generalization is a strict universal or rather a ‘soft constraint’ with occasional counterexamples is a matter of ongoing debate. For instance, Cohen (2001) argues that many and few have a reading under which they are not conservative, but Romero (2015a) suggests a decompositional analysis of these determiners under which their ‘core semantics’ is conservative.

It is also an open question why natural language determiners are generally conservative. It seems plausible that such an explanation may be given in computational terms. Indeed, it may well be that the cognitive load of verifying whether a determiner D applies to two sets A and B can be kept relatively low as long as D is conservative, because in this case we only need to consider objects in A; we don’t need to look at objects in B \ A or in A \ B. Another (not necessarily causally independent) reason may be that the meaning of conservative determiners is easier to learn from examples: it has been shown experimentally that children exposed to a novel conservative determiner show significant understanding of it after having been told the truth value of a number of example sentences in a number of contexts, while children exposed to an imaginary non-conservative determiner do not come to grasp its meaning at all after having received such information (Hunter and Lidz, 2013).

Given that natural language determiners are, or at least tend to be conservative, it is natural to assume that a similar property may be identified in the domain of clause-embedding verbs. Below we will formulate such a property, called clausal distributivity (c-distributivity for short), and show that all c-distributive responsive verbs satisfy Spector and Egré’s veridicality generalization. Roughly, we will say that a verb V is c-distributive if for any complement meaning f that can be decomposed into a set of simpler com-
plement meanings \(\{f_1, f_2, \ldots\}\), and for every individual \(x\), \(V(f)(x)\) holds if and only if \(V(f_j)(x)\) holds for some \(f_j\). Informally, c-distributivity says that, whenever \(f\) can be decomposed into simpler parts, \(V(f)(x)\) is fully determined by those simpler parts, i.e., in computing \(V(f)(x)\) we don’t have to look at \(f\) as a whole but only at its parts.

To make this more precise, we have to define the relevant notion of decomposition. Recall that a complement meaning \(f\) is always obtained in our framework by applying \(E\) to a nucleus meaning \(P\). We define the decomposition of a nucleus meaning as follows.

**Definition 6** (Decomposing nucleus meanings).
A nucleus meaning \(P\) can be decomposed if there is a unique set of nucleus meanings \(\text{decomp}(P)\) such that:

1. \(P = \bigcup \text{decomp}(P)\)
2. Every two distinct elements \(P', P'' \in \text{decomp}(P)\) are mutually inconsistent, i.e., \(P' \cap P'' = \emptyset\)
3. Every \(P' \in \text{decomp}(P)\) is non-inquisitive.

In this case, \(\text{decomp}(P)\) is called the decomposition of \(P\). Otherwise the decomposition of \(P\) is undefined.

Notice that the first two requirements should be part, in some form or other, of any reasonable notion of decomposition.\(^{34}\) The first requires that putting the elements of \(\text{decomp}(P)\) together leads us back to the original \(P\). The second requires that the elements of \(\text{decomp}(P)\) have to be mutually independent, which is made precise here in terms of mutual inconsistency.\(^{35}\) The third requirement, on the other hand, specifies that the elements of a decomposition must be ‘basic’ nucleus meanings, in that they must be non-inquisitive. In other words, while \(P\) itself may be inquisitive and thus contain multiple alternatives, every element of \(\text{decomp}(P)\) must contain a unique alternative. This is a natural requirement for ‘basic’ nucleus meanings, because it ensures that such nucleus meanings cannot be further decomposed. That is, for any non-inquisitive nucleus meaning \(P\), \(\text{decomp}(P) = \{P\}\). Vice versa, \(\text{decomp}(P) = \{P\}\) also implies that \(P\) is non-inquisitive. So there is a one-to-one connection between non-inquisitiveness and non-decomposability.

**Fact 3.** A nucleus meaning \(P\) is non-inquisitive if and only if \(\text{decomp}(P) = \{P\}\).

Now note that under our notion of decomposition, \(\text{decomp}(P)\) is only defined if \(P\) does not contain any overlapping alternatives. To see this, suppose that \(P\) does contain two alternatives \(p\) and \(q\) that overlap, i.e., such that \(p \cap q \neq \emptyset\). Then, by the first requirement, there must be some \(P' \in \text{decomp}(P)\) such that \(p \in P'\) and some \(P'' \in \text{decomp}(P)\) such that \(q \in P''\). But then, since both \(P'\) and \(P''\) are downward closed, we have that \(p \cap q \in P'\) and \(p \cap q \in P''\). This means that \(P' \cap P'' \neq \emptyset\), in violation of the second requirement.

On the other hand, if \(P\) does not contain any overlapping alternatives, then \(\text{decomp}(P)\) is always well-defined, and moreover, its elements correspond precisely to the alternatives in \(P\): \(\text{decomp}(P) = \{\{p\} \mid p \in \text{alt}(P)\}\).

**Fact 4** (Decomposition and alternatives).
- \(\text{decomp}(P)\) is defined if and only if \(P\) does not contain any overlapping alternatives.

\(^{34}\)For a concrete example of a notion of decomposition that has precisely these two components, consider the notion of vector decomposition in linear algebra.

\(^{35}\)It is also possible to construe independence in different ways in our framework. For instance, instead of inconsistency we could also just require non-entailment, i.e., \(P' \nsubseteq P''\). Under this construal of independence, which is strictly weaker than the one assumed here, c-distributivity would not imply compliance with Spector and Egré’s generalization.
Whenever defined, \( \text{decomp}(P) \) amounts to \( \{\{p\}^+ \mid p \in \text{alt}(P)\} \).

Finally, we note that whenever \( \text{decomp}(P) \) is defined, i.e., whenever \( P \) does not contain overlapping alternatives, applying \( E[+\text{cmp}] \) or \( E[-\text{cmp}] \) to \( P \) gives exactly the same results. Thus, below, whenever it is assumed that \( \text{decomp}(P) \) is defined, we simply write \( E(P) \) rather than \( E[+\text{cmp}](P) \) or \( E[-\text{cmp}](P) \).

We can now give a precise formulation of \( \text{c-distributivity} \). For expository purposes we formulate the property for verbs that have one individual argument slot—it will be clear how it can be generalized to verbs with zero or more than one individual argument slots.

**Definition 7 (\( \text{c-distributivity} \)).**

A verb meaning \( V \) is \( \text{c-distributive} \) if and only if for any individual \( x \), any world \( w \), and any nucleus meaning \( P \) such that \( \text{decomp}(P) \) is defined:

\[
V(E(P))(x) \text{ is true in } w \text{ iff } V(E(P'))(x) \text{ is true in } w \text{ for some } P' \in \text{decomp}(P)
\]

Informally, \( \text{c-distributivity} \) says that we should be able to evaluate whether the verb applies to a certain complement by checking whether it applies to one of the elements of the complement’s decomposition, at least as long as such a decomposition exists.

Now we are ready to state the main result of this subsection: all \( \text{c-distributive} \) responsive verbs comply with Spector and Egré’s \( \text{veridicality generalization} \). A proof of this fact is given in Appendix B.2.

**Fact 5.** A \( \text{c-distributive} \) responsive verb is \( \text{veridical w.r.t.} \) declarative complements if and only if it is \( \text{veridical w.r.t.} \) interrogative complements.

If we consider responsive verbs in English we find that most of them are \( \text{c-distributive} \). Indeed, the only exceptions that we have been able to identify are predicates of relevance like \textit{care} and \textit{matter}. The fact that so many responsive verbs are \( \text{c-distributive} \) may possibly be explained in computational terms, just like the fact that determiners are generally \( \text{conservative} \). That is, it seems reasonable to hypothesize that the cognitive load of verifying whether a verb applies to a certain complement can be kept relatively low as long as it is guaranteed that this can be done by verifying whether the verb applies to the elements of the decomposition of the given complement.

What is most important for our present purposes is that we have seen how general constraints on responsive verb meanings, such as Spector and Egré’s \( \text{veridicality generalization} \), can be captured within a uniform theory of clause-embedding. This addresses a concern that George (2011) and Spector and Egré (2015) raised for the uniform approach, as well as the inverse reductive approach, as discussed in Section 2.6.\footnote{As far as we can tell, the strategy we have taken here could also be adopted on the inverse reductive approach.} The particular way in which we have proposed to capture Spector and Egré’s \( \text{veridicality generalization} \) also seems to have some advantages over their own account. First, it allows for counterexamples, which is needed to accommodate verbs like \textit{care} and \textit{matter}. And second, it allows us to draw parallels with other domains—e.g., that of determiners—and paves the way for possible explanations of the generalization in terms of minimizing cognitive processing load.\footnote{We conjecture that, when formulated at a sufficiently abstract level, a property similar to \( \text{c-distributivity} \) may actually hold not just of most predicates that take declarative and/or interrogative clauses as their argument, but of most predicates in general, also ones that take atomic and/or plural individuals as their argument. However, we must leave the formulation of such a more general property for further work.}

### 8 Conclusion

We have proposed a uniform account of declarative and interrogative complements, and the verbs that take either or both kinds of complement as their argument. The account overcomes problems for the reductive
approach as well as the twin relations theory, recently discussed by George (2011), Elliott et al. (2016), and Uegaki (2016). It also addresses the limitations of Groenendijk and Stokhof’s uniform partition theory; in particular, it straightforwardly derives MS and IE readings as well as SE ones, and it accounts for the selectional restrictions of rogative and anti-rogative verbs. Finally, it addresses a concern raised by George (2011) and Spector and Egré (2015) for uniform and inverse reductive theories, showing that it is possible to capture general constraints on responsive verb meanings within a uniform framework. In Appendix A our approach is compared in some detail with the inverse reductive theory of Uegaki (2015).

A Comparison with Uegaki (2015)

In Section 1 we situated the present proposal w.r.t. a range of previous approaches in rather general terms. Here, we will offer a more detailed comparison with one of these approaches, namely that of Uegaki (2015), which in our view provides the most comprehensive previous account of the issues that we have been concerned with in this paper (see Table 1 on page 5).38 The core of our proposal and that of Uegaki were developed independently, as witnessed by preliminary presentations of the two accounts (Theiler, 2014; Roelofsen et al., 2014; Uegaki, 2014). However, in further developing our initial proposal we have crucially benefited from some of the insights in Uegaki’s work. The two resulting proposals are very much in the same spirit, but there are also some significant differences, which we will highlight below.

The discussion will center on three main themes: selectional restrictions of embedding verbs (Section A.1), variability in the exhaustive strength of interrogative complements (Section A.2), and veridicality (Section A.3).39

A.1 Selectional restrictions

We have shown that the selectional restrictions of rogative and anti-rogative verbs can be derived from the lexical semantics of these verbs, and we argued that such an account is to be preferred over one that relies on a difference in semantic type between declarative and interrogative complements, at least as long as such a difference in type is not independently motivated.

Uegaki (2015) assumes that declarative complements denote propositions, that interrogative complements denote sets of propositions, and that there is a type shifting operation that transforms single propositions into sets of propositions if needed to avoid a type mismatch. This type shifting operation, denoted $\text{Id}$, simply turns any proposition $p$ into the corresponding singleton set $\{p\}$.40

\[(96) \quad [\text{Id}]_w = \lambda p.\{p\}\]

When a declarative and an interrogative complement are conjoined, it is not the semantic value of the interrogative clause that is shifted, as on the standard reductive approach, but rather that of the declarative clause. Similarly, type shifting is not needed when a responsive verb like know takes an interrogative complement, but it is needed when such a verb takes a declarative complement. For instance, John knows that Mary left is rendered as follows:

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38 To be sure, neither our own proposal nor that of Uegaki (2015) covers all the issues that have been discussed in previous work on declarative and interrogative complements and the verbs that embed them. For instance, quantificational variability effects (e.g., Berman, 1991; Lahiri, 2002; Beck and Sharvit, 2002; Cremers, 2016), the de re/de dicto ambiguity (e.g., Groenendijk and Stokhof, 1984; Sharvit, 2002), the licensing of negative polarity items (e.g., Guerzoni and Sharvit, 2007, 2014; Nicolae, 2013), homogeneity effects (Kriz, 2015; Cremers, 2016), and perspective sensitivity (e.g., Aloni, 2005) are not explicitly accounted for. We believe that our proposal can be extended to do so, incorporating insights from the work cited here, but a full implementation must be left for future work.

39 These three themes correspond precisely to the three core chapters of Uegaki (2015).

40 In discussing Uegaki’s proposal we adopt his convention to specify the denotation of each expression $\alpha$ at a specific world $w$, $[\alpha]_w$, rather than the full meaning of the expression, $[\alpha]$, which would be the function $\lambda w.[\alpha]_w$.
(97) John knows [Id [that Mary left]]

In this setup, the selectional restrictions of anti-rogative verbs like believe can be seen as resulting from a type mismatch, under the assumption that such verbs require a single proposition as their input. On the other hand, the selectional restrictions of rogative verbs like wonder have to be given a different kind of explanation, because in terms of semantic type they do not differ from responsive verbs like know. Uegaki provides such an explanation, as well as independent motivation for the assumed type distinction between anti-rogative verbs on the one hand and responsive and rogative verbs on the other. We will consider these aspects of Uegaki’s proposal in Section A.1.1 and A.1.2, respectively, in each case drawing comparisons with our own approach.

A.1.1 Rogative verbs

Summary of Uegaki’s account. The fact that wonder does not license declarative complements is accounted for by Uegaki (2015, Section 2.3.3) in a way that is quite close in spirit to our account, but different in implementation and empirical predictions. Uegaki proposes to decompose wonder into want to know and to derive the incompatibility with declarative complements from independently motivated assumptions about the lexical semantics of want. In particular, in line with earlier work on want, Uegaki takes $x$ wants $p$ to presuppose (i) that $x$ believes that the presuppositions of $p$ are satisfied, and (ii) that $x$ does not believe that $p$ is true.

(98) $<x \text{ wants } p> (w)$ is defined only if:

(i) $\sigma^w_x \subseteq \{w' \mid p(w') = 1\} \cup \{w' \mid p(w') = 0\}$

(ii) $\sigma^w_x \not\subseteq \{w' \mid p(w') = 1\}$

Now consider a case where wonder takes a declarative complement.

(99) "John wonders that Mary left.

If wonder is analyzed as want to know, then the truth value of (99) is only defined if (i) John believes that the presuppositions of John knows that Mary left are satisfied, i.e., he believes that Mary left, and (ii) John does not believe that John knows that Mary left is true. Assuming that $x$ believes $p$ generally entails $x$ believes that $x$ knows $p$, these two conditions are contradictory. Thus, it is predicted that the presuppositions of (99) can never be satisfied. This explains the fact that wonder does not license declarative complements, and Uegaki suggests that the account can be extended to other rogative verbs as well, assuming that all these verbs have want to know as a core component.

Problems and comparison. We see two problems for this proposal, one concerning the treatment of wonder itself, and one concerning the extension to other rogative verbs. Let us first consider the predictions of the account for a case where wonder takes an interrogative complement:

(100) John wonders whether Mary left.

It is predicted that this sentence presupposes that John does not believe that John knows whether Mary left is true. Assuming that John is introspective, this is just to say that the sentence presupposes that John doesn’t know whether Mary left. Since presuppositions survive under negation, it is therefore also predicted that (101) presupposes that John doesn’t know whether Mary left.

(101) John does not wonder whether Mary left.
This is a problematic prediction, because (101) can very well be true in a situation in which John already knows whether Mary left. We take this to show that the ‘ignorance component’ of wonder is an entailment rather than a presupposition, and this is indeed how it is modeled on our account. As a result, we do not predict that (101) implies that John is ignorant as to whether Mary left.

Now let us turn to the possibility of extending Uegaki’s account of wonder to other rogative verbs. It is indeed natural to assume that investigate and be curious are, just like wonder, very close in meaning to want to know. However, we do not think that this assumption is justifiable for rogative verbs like ask and depend on (recall that we provided an explanation for the fact that these verbs do not license declarative complements in Section 7.2).

There are certainly many contexts in which asking a question pragmatically implicates not knowing the answer to that question. But the assumption that asking a question generally, i.e., independently of the context of utterance, presupposes not knowing the answer to that question is too strong. For instance, (102) below does not imply that the math teacher doesn’t know the square root of 169.

(102) Our math teacher asked us today what the square root of 169 is.

As for depend on, it is clear that a sentence like (103) does not make reference to any agent’s knowledge or desires, and can therefore not be paraphrased in terms of want to know.

(103) Whether the light is on depends on whether the switch is up.

Thus, we think that the present proposal improves on Uegaki’s account both in its treatment of wonder and in covering a broader range of verbs.

A.1.2 Anti-rogative verbs

Summary of Uegaki’s account. As mentioned above, Uegaki assumes that anti-rogative verbs like believe require a single proposition as their input, while responsive and rogative verbs require sets of propositions. Moreover, he assumes that a declarative complement denotes a single proposition, while an interrogative complement denotes a set of propositions. This immediately accounts for the fact that anti-rogative verbs cannot take interrogative complements. Further assuming that a single proposition can be transformed into a set of propositions using the type-shifter Id, it is also predicted that responsive verbs can take both declarative and interrogative complements.

Uegaki motivates the assumption that anti-rogative verbs like believe and responsive verbs like know require different types of input, based on a contrast that arises when these two types of verbs are combined with so-called ‘content DPs’, like the rumor that Mary left. The contrast, first noted by Vendler (1972) and also discussed by Ginzburg (1995), King (2002), and Moltmann (2013), is illustrated in (104).

(104) a. John believes the rumor that Mary left.
   → John believes that Mary left.
   \(\not\) John knows the rumor that Mary left.
   \(\not\) John knows that Mary left.

In general, \(x\) believes the rumor that \(p\) entails \(x\) believes that \(p\), whereas \(x\) knows the rumor that \(p\) does not entail \(x\) knows that \(p\), and the same is true if rumor is replaced by story, claim, hypothesis, et cetera.

Now, Uegaki claims that all anti-rogative verbs behave just like believe in this respect, while all responsive verbs behave just like know. He then provides an account of the contrast in (104) which relies on the assumption that believe requires a single proposition as its input, while know requires a set of propositions. Thus, to the extent that the account makes correct predictions for other anti-rogative and
responsive verbs as well, it indeed provides independent motivation for the type distinction that Uegaki assumes to account for the selectional restrictions of anti-rogative verbs.

**Problem and comparison.** The problem for this approach is that there are counterexamples to the claim that all anti-rogative verbs behave like \textit{believe} when combined with content DPs, and that all responsive verbs behave like \textit{know} in this respect. First, there are many anti-rogative verbs which, unlike \textit{believe}, cannot be combined with content DPs at all.

(105) *John thinks/wants/feels/supposes the rumor that Mary left.*

While this does not directly counter Uegaki’s account of the fact that \textit{believe} is anti-rogative, it does show that the scope of the account is restricted; it certainly does not cover the full range of anti-rogative verbs.

A more drastic problem is that there are also responsive verbs that do not behave like \textit{know} when combined with content DPs. Such verbs include \textit{hear} and \textit{prove}, as illustrated in (106)-(107).\footnote{Uegaki (2015, p.49,61) remarks that certain responsive verbs allow for a so-called entity-relating reading (such as the acquaintance reading of \textit{know}), and that his theory leaves open the possibility that under this reading, these verbs do license inferences like those in (106)-(107). However, to the extent that such readings exist for \textit{hear} and \textit{prove}, they don’t seem to be necessary for the inferences in (106)-(107) to go through.}

(106) John heard the rumor that Mary left.

\[\therefore \text{John heard that Mary left.}\]

(107) John proved the hypothesis that every positive integer has a unique prime factorization.

\[\therefore \text{John proved that every positive integer has a unique prime factorization.}\]

On Uegaki’s account these verbs are thus predicted to be anti-rogative, just like \textit{believe}, contrary to fact. This means that the independent motivation that Uegaki provides for his account of the selectional restrictions of anti-rogative verbs in terms of a type mismatch collapses. As a result, the account loses its explanatory force.

In comparison, our account derives the selectional restriction of anti-rogative verbs from an independent property. Namely, anti-rogative verbs license neg-raising inferences.

### A.2 Variability in exhaustive strength

As discussed in Section 2.3, sentences with interrogative complements usually exhibit variability in exhaustive strength. With certain verbs, however, this variability is restricted. For instance, \textit{be certain} is incompatible with an IE reading, and our theory accounts for this fact. Uegaki aims to predict more generally which readings are available for any responsive verb and to derive these predictions from the lexical properties of the verb. It has to be noted that such predictions—while clearly desirable—will only be explanatory to the extent that the involved mechanisms are independently motivated. This means in particular that it should be possible to provide reasons for assuming the relevant properties of the embedding verbs that are not connected to deriving the observed levels of exhaustive strength.

We will first consider the general architecture Uegaki assumes and the distinction between extensional and intensional verbs that is relevant for his account. We then turn to the ‘baseline’ readings that are predicted for different embedding verbs and finally to the non-baseline readings, which are obtained by additional semantic operations.

#### General architecture and extensional/intensional responsive verbs.

Uegaki decomposes every responsive verb \(V\) into a \textit{core predicate} \(R_V\), which is the proposition-taking counterpart of \(V\), plus an answer
The difference between extensional and intensional responsive verbs is the following on Uegaki’s account. The world argument of Ans$_d$ can get bound either by the core predicate $R_V$ or by an exhaustification operator, $X$. Intensional verbs like be certain or tell$_{-ver}$ have a core predicate that binds the world argument of Ans$_d$, as in (109a), while extensional verbs do not bind this argument themselves, but leave it to the exhaustification operator to bind it, as in (109b).

(109) a. intensional verb:

```
        John
         |
         R$_{certain}$
         1
         |
        Ans$_d$

w$_1$
who
called

b. extensional verb:

```

        X
         |
         1
         |
        John
         |
         R$_{know}$

w$_1$
who
called

This contrast between extensional and intensional verbs can also be observed from the lexical entries in (110): the intensional verbs in (110a) and (110b) take a world-sensitive argument $P$, which they evaluate in some possible world (in the case of tell$_{-ver}$) or all possible worlds that are compatible with the subject’s beliefs (in the case of be certain). The extensional verb in (110c) on the other hand takes a simple proposition $p$ as its argument.

(110) a. $[R$_{certain}^w] = \lambda (s,(s,t),t). \lambda x. \forall w'. \exists w'' : \sigma^w_x \subseteq P(w')$

b. $[R$_{tell$_{-ver}}^w] = \lambda (s,(s,t),t). \lambda x. \lambda y. \exists w'. \text{tell}(x, y, P(w'), w)$

c. $[R$_{know}^w] = \lambda (s,(s,t),t). \lambda x. \sigma^w_x \subseteq p$

Baseline readings. In Uegaki’s system, each verb comes with a baseline reading, which is the reading it has in the absence of further semantic operations such as exhaustification. In the case of extensional verbs like know, the baseline reading is WE. To see why, consider again the lexical entry in (110c) for the knowledge relation $R$_{know} and the entry in (108) for the answer operator Ans$_d$. As can be seen from the structure in (109b), the propositional argument $p$ that $R$_{know} takes in the semantic derivation is delivered.
by \text{Ans}_d. Since, given a world \( w \) and a question meaning \( Q \), \( \text{Ans}_d(w)(Q) \) is the true WE answer to \( Q \) in \( w \), we find that for a subject \( x \) to \text{know} \( Q \) in \( w \), \( x \)'s belief state in \( w \), \( \sigma_x^w \), has to be a subset of the true WE answer in \( w \). This amounts to a WE reading.

The semantics of intensional verbs, on the other hand, is expressed in terms of quantification over possible worlds, and the strength of their baseline reading depends on the kind of quantification that is used. For instance, as can be seen from the entries in (110a) and (110b), a universal semantics is assumed for \text{be} \text{ certain}, while an existential semantics is assumed for \text{tell}[^{-}\text{ver}], the non-veridical variant of \text{tell}.\footnote{The entry in (110b) is only a preliminary version; Uegaki’s final entry for \text{tell} has an excluded-middle presupposition, which we will turn to below.}

The propositional concept \( P \) that these verbs take as an argument is a function mapping every world to the true WE answer at that world. If \text{be} \text{ certain} is applied to a propositional concept \( P \) that has been computed from the meaning \( Q \) of an interrogative complement, then the semantics in (110a) amounts to requiring that the subject’s belief state \( \sigma_x^w \) is homogeneous with respect to every answer to \( Q \)—or, in other words, that the subject has to believe an SE answer to \( Q \). In comparison, the unrestricted existential quantification in the semantics of \text{tell}[^{-}\text{ver}] is much weaker: it is only required that the subject stands in a \text{tell}-relation to the true WE answer at some world \( w' \). However, since for every proposition \( p \) that is an MS answer at \( w' \), there also exists a world \( w'' \) such that \( p \) is a WE answer at \( w'' \), this condition boils down to requiring that the subject stands in a \text{tell}-relation to some possible MS answer. Hence, the baseline reading for \text{be} \text{ certain} is an SE reading, whereas that for \text{tell}[^{-}\text{ver}] is an MS reading. As far as we can see, however, the choice for universal as opposed to existential quantification in the case of \text{be} \text{ certain} does not receive an independent motivation and is therefore tantamount to hardcoding the baseline exhaustive strength into the lexical entry. In comparison, on our account, the unavailability of IE readings for \text{be} \text{ certain} follows from the assumption that this verb only permits an internal, i.e., resolution-introspective interpretation.

**Intermediate exhaustivity.** Turning to intermediate exhaustivity, for extensional verbs this reading is derived by applying the exhaustification operator \( X \) at the root level, i.e., above the embedding verb.\footnote{This operator may be regarded as a refinement of the \text{EXH} operator in Klinedinst and Rothschild (2011). However, in contrast to Uegaki, Klinedinst and Rothschild are only concerned with deriving IE readings of non-factive verbs. Their account fails to derive such readings for factive verbs—which to us seem to be the prime case of FA sensitivity.}

Roughly, if this operator applies to a sentence, it asserts the proposition expressed by the sentence and negates all strictly stronger alternatives of this proposition.

\[
(111) \quad [X]^w = \lambda P(\sigma(w, s, t)). P(w) \land \forall v : (\{w' | P(w')(v)\} \subset \{w' | P(w')(w)\} \rightarrow \sim P(w)(v))
\]

As can be seen from the structure in (109b), the propositional concept \( P \) that \( X \) takes as its argument is the result of first computing the meaning of the sentence to which \( X \) applies, and then lambda-abstracting over the world argument of the answer operator in that sentence. For example, in the semantic derivation of (112a), \( X \) applies to the propositional concept in (112b).

\[
(112) \quad \begin{align*}
\text{a.} & \quad \text{Mary told}[\text{[ver]}] \text{ John who called.} \\
\text{b.} & \quad P = \lambda w_1. [\text{Mary [ told[\text{[ver]}] [ John [ [\text{Ans}_d w_1 [ who called ] ] ] ] ] ] ]]
\end{align*}
\]

To see that this gives \( X \) access to the relevant set of alternatives, assume, e.g., that in the world of evaluation \( w \) Ann and Bill called, but Chris didn’t, whereas in \( v \) all of Ann, Bill and Chris called. Then, \( \{w' | P(w')(v)\} \) is the set of all those worlds in which Mary told John the true WE answer to \text{who called?} in \( v \), i.e., she told him that Ann, Bill and Chris called. In contrast, \( \{w' | P(w')(w)\} \) is the set of all those worlds in which Mary told John the true WE answer to \text{who called?} in \( w \), i.e., she told him that Ann and Bill called. Observe that \( \{w' | P(w')(v)\} \subset \{w' | P(w')(w)\} \). Hence, what \( X \) asserts is that Mary told
John that Ann and Bill called and she didn’t tell him that Ann, Bill and Chris called. This is exactly the IE reading of (112).

This is the way in which IE readings can in principle be derived on Uegaki’s account. When it comes to restricting their availability, a certain feature of the exhaustivity operator becomes crucial: this operator interacts with the monotonicity properties of the embedding verb in such a way that, if the verb is upward-monotonic, IE is derived, whereas, if the verb is not upward-monotonic, the contribution of X is vacuous. To see why, consider again the definition of X in (111). In the case of a predicate that is not upward-monotonic, the implication in the second conjunct is vacuously satisfied because it will never be the case that \( \{ w' \mid P(w')(v) \subset \{ w' \mid P(w')(w) \} \). For upward-monotonic predicates, on the other hand, \( \{ w' \mid P(w')(v) \subset \{ w' \mid P(w')(w) \} \) can become true; it holds for all worlds \( v \) and \( w \) such that \( \text{Ans}_d(v)(Q) \subset \text{Ans}_d(w)(Q) \). Hence, for these verbs the second part in the definition of X applies non-vacuously.

Uegaki’s account thus establishes a connection between the monotonicity properties of embedding verbs and the availability of IE readings. More specifically, by assuming that emotive factives like be happy and be surprised are non-monotonic, these verbs are predicted to be lacking IE readings. The only predicted reading for emotive factives is their baseline reading, i.e., WE. On the other hand, epistemic attitude verbs like know and the veridical variants of communication verbs like tell, which are assumed to be upward monotonic, are predicted to have IE readings.

While we find the approach ingenious, we have three reservations. First, as also noted by Uegaki himself, the non-monotonicity of emotive factives, on which his analysis crucially relies, is debatable (von Fintel, 1999; Crnič, 2011). Second, the assumed unavailability of SE readings for emotive factives has been contradicted by recent experimental work (Cremers and Chemla, 2016a). Third, Uegaki’s account only derives a connection between monotonicity properties and exhaustive strength for extensional verbs. For intensional verbs, essentially, exhaustive strength must still be encoded in the individual lexical entries. In sum, this means that Uegaki’s theory of exhaustive strength is explanatory only for extensional verbs and, for these verbs, only to the extent that their monotonicity properties indeed correlate with the presence/absence of IE readings; more experimental work is needed to find out whether such a correlation exists (cf., Cremers and Chemla, 2016a).

**Strong exhaustivity.** An SE reading can be derived in two different ways on Uegaki’s account, depending on whether the verb is extensional or intensional. In either case, however, SE arises from an excluded-middle assumption, which is encoded in the lexical entry of the embedding verb as a soft presupposition (Gajewski, 2007; Abusch, 2010).

To begin with, consider an intensional verb like tell\(_{-\text{ver}}\), whose lexical entry including the relevant presupposition is given in (113). According to Uegaki, tell comes with an excluded-middle assumption stating that, for every possible answer \( p \) to the embedded interrogative, the subject \( x \) has either told the addressee \( y \) that \( p \) or told her that \( \neg p \). It is easy to see that this condition gives rise to an SE reading. Uegaki thus predicts that for intensional verbs like tell the SE reading can directly arise from their excluded-middle presupposition. Under this view, in order for the SE reading not to arise, on the other hand, the presupposition needs to be suspended.

\[
(113) \quad [R_{\text{tell}_{-\text{ver}}}]^w = \lambda s. \lambda t. \lambda y. \lambda x. \\
\{ \exists w'. \text{tell}(x, y, P(w'), w) \quad \text{if} \forall w'. \left( \begin{array}{ll}
\text{tell}(x, y, P(w'), w) \\
\neg \text{tell}(x, y, \neg P(w'), w)
\end{array} \right) \\
\text{undefined} \quad \text{otherwise}
\}
\]

Turning to extensional verbs, the situation becomes slightly more complex. Here, it is not the verb itself that is relevant for the excluded-middle presupposition, but its non-factive counterpart. For example, in
the case of know, Uegaki encodes the excluded-middle presupposition in terms of believe, i.e., in terms of the subject’s doxastic state:

\[
[R_{\text{know}}]w = \lambda p(s,t), \lambda y, \lambda x. \begin{cases} 
\sigma_x^w \subseteq p & \text{if } p(w) \wedge \left( \sigma_x^w \subseteq p \lor \sigma_x^w \subseteq \neg p \right) \\
\text{undefined} & \text{otherwise}
\end{cases}
\]

Note that, in contrast to the intensional verb, here the excluded-middle presupposition is not formulated with respect to every possible answer; instead it only concerns a specific proposition \(p\). If \(p\) is the true WE answer in the world of evaluation, for example, the excluded-middle presupposition in (114) does not itself amount to an SE reading. However, as soon as this presupposition is combined with an IE reading, SE follows. To see why, recall that an IE reading, derived by applying the \(X\) operator, would assert that, for every true answer \(p\), the subject \(x\) believes \(p\), and for every false answer \(p\), \(x\) does not believe \(p\). Now, since the excluded-middle assumption tells us that \(x\) either believes \(p\) or believes \(\neg p\), it follows from the IE reading that, for every false answer \(p\), \(x\) believes \(\neg p\). This gives us an SE reading.

In the case of extensional verbs, SE readings are thus parasitic on IE readings on Uegaki’s account. In particular, this means that emotive factives, which are predicted to lack IE readings, are predicted not to have SE readings either. For extensional verbs that do permit IE readings on the other hand and for intensional verbs, the availability of SE readings depends on whether the verb comes with an excluded-middle presupposition, which Uegaki assumes to be the case exactly if it licenses neg-raising.

As far as we can see, the main problem for this analysis is that, in order to derive SE readings, excluded-middle presuppositions need to be assumed even for verbs for which it is very debatable whether such presuppositions exist. For instance, if Uegaki wants to derive SE readings for intensional communication verbs like tell— and it seems that he does (p.156)— then an excluded-middle presupposition needs to be assumed for such verbs, although they do not readily seem to license neg-raising:

\[(115)\] Ann didn’t tell Bill that it is raining.
\[\neg \text{ Ann told Bill that it is not raining.}\]

A further problem arises in connection with know. Since \(R_{\text{know}}\) carries an excluded-middle presupposition that is formulated in terms of the subject’s doxastic state, the inference in (116) is predicted to go through. This ‘pseudo neg-raising’ effect is clearly undesirable.

\[(116)\] Ann doesn’t know that it is raining.
\[\therefore \text{ Ann believes that it is not raining.}\]

A.3 Veridicality

Although Uegaki’s treatment of veridicality is relatively close in spirit to our approach, we will see that there are some crucial differences. In particular, his account departs from ours in that it predicts a one-to-one connection between factivity and extensionality. We will see that, as a result, non-factive veridical verbs like be right have to be treated as intensional verbs—with all consequences that this entails.

In Uegaki’s system, there are two reasons why a verb can come out as veridical: either because the verb is extensional, i.e., the answer operator is evaluated in the root evaluation world, or because the verb meaning is decomposed into an inherently veridical core predicate plus an answer operator. Let us consider both in turn.

\[44\] To be a little more precise, Uegaki distinguishes between the literal and the deductive reading of communication verbs (Theiler, 2014). This distinction does not appear to have a bearing on the licensing of excluded-middle inferences, however: tell seems to license such inferences neither on the literal nor on the deductive reading.
First, recall that extensional verbs do not bind the world argument of $\text{Ans}_d$ themselves, but leave it to the exhaustivity operator $X$ to bind this argument. This operator, whose definition is repeated in (117) below, takes a propositional concept $P$ and, among other things, asserts that the result of evaluating $P$ at the root world of evaluation $w$ holds at $w$. Since $X$ obligatorily applies in the case of extensional verbs, for these verbs $\text{Ans}_d$ thus gets evaluated in $w$. This means, if extensional verbs take an interrogative complement, $\text{Ans}_d$ delivers the true WE answer in the world of evaluation. If they take a declarative complement, whose meaning Uegaki represents as a singleton set, the definedness restriction of $\text{Ans}_d$ amounts to a factivity presupposition. Crucially, for Uegaki, extensionality thus always entails both veridicality with respect to interrogative embedding and factivity.

\begin{equation}
[X]_w = \lambda \forall v \exists w : (\{w' \mid P(w')(v)\} \subset \{w' \mid P(w')(w)\} \rightarrow \neg P(w)(v))
\end{equation}

The second way in which a verb can be veridical is by virtue of the inherent veridicality of its core predicate. For example, Uegaki assumes the following core predicate for prove.

\begin{equation}
[R_{\text{prove}}]_w = \lambda \forall x . \exists w' . \text{prove}(x, P(w'), w)
\end{equation}

The format of this predicate does not by itself differ from that of other intensional verbs with an existential semantics such as tell[–ver]. However, Uegaki additionally assumes by way of a meaning postulate that the implication in (119) holds.

\begin{equation}
\forall p. \forall x . \forall w . (\text{prove}(x, p, w) \rightarrow p(w))
\end{equation}

This means that prove comes out as veridical with respect to both interrogative and declarative complements: if $P$ is the meaning of an interrogative complement, $R_{\text{prove}}(P)(x)$ is only true in $w$ if there exists a $w'$ such that $P(w')$ is a true answer in $w$ and $\text{prove}(x, P(w'), w)$. If $P$ is the meaning of a declarative and it holds in $w$ that $R_{\text{prove}}(P)(x)$, then it follows that the information conveyed by the declarative complement is true in $w$.

Note that, different from those verbs for which veridicality arises from extensionality, intensional verbs like prove, for which veridicality results from the inherent veridicality of their core predicate, are not predicted to be factive. This means that the only way in which a verb can be factive on Uegaki’s account is by virtue of its extensionality. Hence, under this analysis, there is a one-to-one connection between extensionality and factivity.

It is a problem for Uegaki’s approach that this connection also has consequences for which verbs get classified as extensional and which as intensional. In particular, verbs like prove and be right have to be treated as intensional since they don’t give rise to factivity presuppositions. Different from garden-variety intensionals, however, such verbs are veridical—which is one of the reasons why their treatment as intensionals has a number of undesirable consequences: as we will see, they are predicted to exhibit no FA sensitivity, to have no WE/IE readings, and to have SE readings only in so far as they trigger an excluded-middle presupposition. Let us examine these predictions in some more detail.

Recall that intensional verbs, unlike extensional ones, bind the world argument of the answer operator. Hence, while in the case of extensionals this argument remains free and can be bound by the exhaustivity operator $X$, in the case of intensionals $X$ does not have a world to bind. As a consequence, $X$ cannot apply to sentences with intensional embedding verbs. However, in Uegaki’s system the $X$ operator is used to derive FA sensitivity. This means that intensional verbs are predicted not to exhibit FA sensitivity. While this indeed seems to be a correct prediction for prima-facie intensional verbs like be certain or tell[–ver], it appears to be wrong for veridical verbs like prove and be right, as illustrated by the
following example. Assume that Ann and Bill, but not Chris were at a party. Mary believes that Ann and Bill were there, but has no beliefs about Chris’ presence at the party. In this scenario, (120) would be judged true. This means that readings other than the SE reading need to be available for (120), because under the SE reading (120) would have been judged false. Now assume instead that Mary believes Ann, Bill and Chris were at the party. In this scenario, it seems that (120) would be judged false. This means that an IE reading, i.e., an FA sensitive reading is needed for (120)—but this reading is unavailable for be right on Uegaki’s account.

(120) Mary is right about who was at the party.

On the other hand, whether intensional verbs have SE readings on Uegaki’s account depends on whether they carry an excluded-middle presupposition. For be right and prove, however, this does not seem to be the case, as illustrated by (121) and (122). Hence, be right and prove would come out as lacking SE readings.

(121) Ann isn’t right that it is raining.
\[\neg\] Ann is right that it’s not raining.

(122) Ann didn’t prove that 3 is prime.
\[\neg\] Ann proved that 3 is not prime.

In terms of exhaustive strength, Uegaki’s analysis thus predicts intensional verbs to be limited to their baseline reading (unless they carry excluded-middle presuppositions, in which case also the SE reading is available). As discussed above using the example of tell\_[-verb], if an existential semantics is assumed, the baseline reading is an MS reading. Since Uegaki proposes an existential semantics for be right and prove, the only reading these verbs are predicted to have is a non-FA-sensitive MS reading.

To sum up, Uegaki’s account makes a number of problematic predictions for non-factive veridical verbs like be right and prove, which arise from the treatment of such verbs as intensional. This treatment, however, is unavoidable for Uegaki since on his account extensionality and factivity cannot be teased apart. In comparison, on our account these problems do not arise since there is no comparable connection between extensionality and factivity.

B Formal details

B.1 Internal/external interpretation of know

Here, we provide proofs of Fact 1 on page 24 and of Fact 2 on page 25, both repeated below.

Fact 1. If the complement receives an MS or a SE interpretation, then the external and the internal interpretation of the verb yield exactly the same reading for the sentence as a whole.

Proof. (Sketch) Consider the sentence John knows who called and assume that the complement receives an MS reading. Then, under an external interpretation of the verb, the sentence is true in \( w \) just in case \( \sigma^w_j \in f_{MS}(w) \), where \( f_{MS}(w) \) is the set of MS truthful resolutions of the complement in \( w \). Now, it follows straightforwardly from the definition of MS truthful resolutions that if a proposition \( p \) is an MS truthful resolution in some world \( w \), then it is also an MS truthful resolution in any world in which it is true, i.e., in any \( v \in p \). But this means that \( \forall v \in \sigma^v_j : \sigma^v_j \in f_{MS}(v) \). Thus, the resolution introspection requirement is automatically satisfied, and the sentence is not only true in \( w \) under an external interpretation of the verb but also under an internal interpretation. A similar argument can be given in case the complement receives an SE reading. \( \square \)
Fact 2. If the verb receives an internal interpretation, then the IE and the SE interpretation of the complement yield exactly the same reading for the sentence as a whole.

Proof. Suppose that in a world \( w \), \( x \) knows who called under an internal interpretation of the verb and an IE interpretation of the complement. Then we do not only have that \( \sigma^w_x \in f_{IE}(w) \), where \( f_{IE}(w) \) is the set of IE resolutions of the complement in \( w \), but also, by resolution introspection, that \( \sigma^v_x \in f_{IE}(v) \) for every \( v \in \sigma^w_x \).

Now, towards a contradiction, assume that \( \sigma^w_x \) does not coincide with an SE resolution of the complement in \( w \), i.e., that \( \sigma^w_x \not\in f_{SE}(w) \). Then there must be an individual \( y \) such that \( y \) did not call in \( w \) but \( x \) doesn’t know this, i.e., \( \sigma^w_x \) must contain at least one world \( w_y \) where \( y \) did call. On the other hand, \( \sigma^w_x \) cannot consist exclusively of worlds where \( y \) called, because \( y \) did not call at \( w \) and \( \sigma^w_x \in f_{IE}(w) \). Now, since \( w_y \in \sigma^w_x \), we must have, by the introspection requirement above, that \( \sigma^w_x \in f_{IE}(w_y) \). But all IE truthful resolutions at \( w_y \) must be propositions that establish that \( y \) called. So \( \sigma^w_x \) must establish that \( y \) called as well, i.e., it must consist exclusively of worlds in which \( y \) called. This is in contradiction with what we derived earlier. So we can conclude that \( \sigma^w_x \) must coincide, after all, with an SE resolution of the complement in \( w \), i.e., that \( \sigma^w_x \in f_{SE}(w) \).

It remains to be shown that \( \sigma^w_x \in f_{SE}(v) \) for all \( v \in \sigma^w_x \). Suppose that \( v \in \sigma^w_x \). Then, since we have just established that \( \sigma^w_x \in f_{SE}(w) \), we have that \( v \in f_{SE}(w) \) as well. But then, given the partition-inducing nature of SE resolutions, it follows that \( f_{SE}(v) = f_{SE}(w) \). Thus, we can conclude that \( \sigma^w_x \in f_{SE}(v) \), as desired. \( \square \)

B.2 Veridicality and c-distributivity

Here we provide a proof of Fact 5 on page 47, repeated below.

Fact 5. A c-distributive responsive verb is veridical w.r.t. declarative complements if and only if it is veridical w.r.t. interrogative complements.

In order to prove this connection between c-distributivity and veridicality, we first give fully explicit definitions of veridicality w.r.t. declarative and interrogative complements, respectively. That is, we translate the informal definitions of these notions that were given in Section 2.6 into the formal framework that was developed in later sections.

We start with veridicality w.r.t. declarative complements, which is a straightforward notion.

Definition 8 (Veridicality w.r.t. declarative complements).

A declarative-embedding verb \( V \) is veridical w.r.t. declarative complements if and only if for any individual \( x \), any world \( w \) and any declarative nucleus meaning \( P \):

\[
\text{If } V(E(P))(x) \text{ is true in } w, \text{ then } P \text{ is true in } w.
\]

Veridicality w.r.t. interrogative complements is a more complex notion. In Section 2.6 we said that a responsive verb \( V \) is veridical w.r.t. interrogative complements just in case for every exhaustivity-neutral interrogative complement meaning \( f \), every individual \( x \), and every world \( w \): if \( V(f)(x) \) is true in \( w \) and \( f' \) a declarative complement expressing a complete answer to \( f \) then \( V(f')(x) \) is true in \( w \) if and only if \( f' \) is true in \( w \).

Now, in the framework that we developed an interrogative complement meaning is always obtained by applying \( E \) to an interrogative nucleus meaning \( Q \), and a declarative complement meaning is obtained by applying \( E \) to a declarative nucleus meaning \( P \). Furthermore, \( Q \) is exhaustivity-neutral if and only if the alternatives in \( \text{alt}(Q) \) form a partition of the set of all possible worlds, and \( P \) constitutes a ‘complete answer’ to such an exhaustivity-neutral \( Q \) if and only if the informative content of \( P \) coincides
precisely with one of the cells in the partition induced by $Q$, i.e., $\text{info}(P) \in \text{alt}(Q)$. Thus, veridicality w.r.t. interrogative complements amounts to the following in our framework.

**Definition 9** (Veridicality w.r.t. interrogative complements).
A responsive verb $V$ is veridical w.r.t. interrogative complements if and only if for any individual $x$, any world $w$, any interrogative nucleus meaning $Q$ such that $\text{alt}(Q)$ is a partition of $W$:

If $[V(E(Q))](x)$ is true in $w$ and $P$ a decl. nucleus meaning such that $\text{info}(P) \in \text{alt}(Q)$, then $[V(E(P))](x)$ is true in $w$ if and only if $P$ is true in $w$.

With these definitions in place, we are ready to prove Fact 5.

**Proof of Fact 5.** We first show that any c-distributive responsive verb that is veridical w.r.t. declarative complements is also veridical w.r.t. interrogative complements. Let $V$ be a c-distributive responsive verb that is veridical w.r.t. declarative complements. Towards establishing that $V$ is veridical w.r.t. interrogative complements, let $Q$ be an interrogative nucleus meaning such that $\text{alt}(Q)$ forms a partition of $W$, let $x$ be an individual, and $w$ a world such that $V(E(Q))(x)$ is true in $w$. Furthermore, let $P$ be a declarative nucleus meaning such that $\text{info}(P) \in \text{alt}(Q)$. We have to show that $V(E(P))(x)$ is true in $w$ if and only if $P$ itself is true in $w$ as well.

First assume that $V(E(P))(x)$ is true in $w$. Then, since $V$ is veridical w.r.t. declarative complements, $P$ must be true in $w$ as well. Now, for the other direction, assume that $P$ is true in $w$. We have to show that $V(E(P))(x)$ must be true in $w$ as well. Since $V(E(Q))(x)$ is true in $w$ and $V$ is c-distributive, it must be the case that $V(E(P'))(x)$ is true in $w$ for some $P' \in \text{decomp}(Q)$ (we know that $\text{decomp}(Q)$ exists because $\text{alt}(Q)$ forms a partition of $W$). Now, towards a contradiction, suppose that $P' \neq P$. Then, since both $\text{info}(P')$ and $\text{info}(P)$ are elements of $\text{alt}(Q)$, and $\text{alt}(Q)$ forms a partition of $W$, $\text{info}(P')$ and $\text{info}(P)$ must be disjoint. Since $P$ is true in $w$, it follows that $P'$ cannot be true in $w$. But then, since $V$ is veridical w.r.t. declarative complements, it follows that $V(E(P'))(x)$ cannot be true in $w$ either, contrary to what we assumed. Thus, we can conclude that $P' = P$. It follows that $V(E(P))(x)$ is true in $w$, which is precisely what we needed to show in order to establish that $V$ is veridical w.r.t. interrogative complements.

We now show that any c-distributive responsive verb that is not veridical w.r.t. declarative complements is not veridical w.r.t. interrogative complements either. Let $V$ be a c-distributive responsive verb that is not veridical w.r.t. declarative complements. This means that there is a declarative nucleus meaning $P$, an individual $x$, and a world $w$ such that $V(E(P))(x)$ is true in $w$ but $P$ itself is not true in $w$.

Towards establishing that $V$ cannot be veridical w.r.t. interrogative complements in this case, let $Q$ be the interrogative nucleus meaning $\{\text{info}(P), \text{info}(P')\}$. We can think of $Q$ as the question ‘whether $P$’. Note that $\text{alt}(Q)$ forms a partition of $W$, and $\text{info}(P) \in \text{alt}(Q)$. Now, since $V(E(P))(x)$ is true in $w$ and since $V$ is c-distributive, it follows that $V(E(Q))(x)$ is true in $w$ as well. But this means that, if $V$ were veridical w.r.t. interrogative complements, it should hold for any declarative nucleus meaning $P'$ such that $\text{info}(P') \in \text{alt}(Q)$ that:

$V(E(P'))(x)$ is true in $w$ if and only if $P'$ is true in $w$

$P$ constitutes a counterexample to this, since we have that $\text{info}(P) \in \text{alt}(Q)$, $V(E(P))(x)$ is true in $w$, but $P$ itself is not true in $w$. Thus, $V$ cannot be veridical w.r.t. interrogative complements.

**References**


question embedding. Manuscript, submitted for publication.


