

## Comparison of complex predicates – *and, or and more*

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Are complex predicates – in particular, negated (e.g., *not expensive*), conjunctive (e.g., *expensive and time consuming*) and disjunctive predicates (e.g., *tall or bald*) – associated with a graded structure, namely a mapping of entities to degrees? On the one hand, most up to date semantic theories of gradability and comparison in natural language disregard this question. On the other hand, contemporary fuzzy logical theories provide compositional rules to construct a degree function for a complex expression based on the degree functions of its constituents. These composition rules have been found useful for a variety of practical applications. The question is then whether these rules can correctly represent the interpretation of complex natural language expressions and its relation to the interpretation of their constituents. The relevance of this question is enhanced by recent findings from a variety of studies (Ripley, 2010; Serchuk et al., 2010; Alxatib and Pelletier 2010), according to which high percentages of subjects count contradictory predicates such as *tall and not tall* as true of borderline cases (neither short nor tall entities). While these findings stand in sharp contrast to predictions of vagueness-based theories of adjectives, they are in accord with the predictions of a fuzzy analysis, as extensively argued by Kamp and Partee (1995). Given these new findings, then, the fact that fuzzy analyses allow for non-zero truth values to contradictions can no longer count against them (for a more detailed discussion see Sauerland, this volume). It is therefore increasingly important to test other predictions of applications of fuzzy analyses to natural language conjunctions and disjunctions. To this end, this paper discusses preliminary results based on a questionnaire eliciting judgments from 35 Hebrew speakers. The results suggest that, counter the predictions of fuzzy analyses, comparative and equative morphemes cannot apply to conjunctions and disjunctions of gradable adjectives.

### 1. Introduction

Part 1 of this paper briefly surveys prominent semantic analyses of natural language comparison constructions (section 1.1) and coordination constructions (section 1.2). Part 2 presents an empirical study of the two constructions, focusing on comparisons of conjunctions and comparisons of disjunctions. Part 3 concludes with the implications this study has concerning the theoretical debates presented below.

#### 1.1 The comparison morpheme

Linguistic theories of gradability and comparison can be divided into two main approaches, ‘ordinal’ and ‘numerical’ (cf. Sassoon 2010a). The ordinal approach attempts to reduce the interpretation of comparative morphemes such as *more* and *as* to ordering relations between individuals or between their ordinal degrees in predicates P, i.e.,  $>P$  and  $\geq P$ , respectively (cf. Sapir 1944; Creswell 1977; Moltmann 2006; Bale 2008). In particular, in vagueness based

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gradability theories the ordering relations  $>P$  and  $\geq P$  are derived based on facts pertaining to membership in  $P$ 's denotation, rather than based on fine grained numerical measurements. On these theories, an entity is *more P* than other entities iff it falls under  $P$  relative to more delineations (possible boundary specifications for vague predicates  $P$ ; cf. Lewis 1970, 1979; Kamp 1975; Fine 1975; Klein 1980; Landman 1991; van Rooij 2009).

In opposition, the numerical approach provides a unified analysis of comparative morphemes with and without numerical modifiers. This approach characterizes gradable adjectives as associated with numerical degree functions, i.e. mapping of entities  $x \in D_x$  to a set of degrees isomorphic to the real numbers  $r \in \mathfrak{R}$  (Russell 1905; Bartsch and Venneman 1972; Klein 1991; Kamp and Partee 1995; Kennedy 1999; Heim 2000; Schwarzschild and Wilkinson 2002; Landman 2005; Sassoon 2010). Assuming a  $\lambda$ -categorial language in the style of Heim and Kratzer (1998), with basic types  $x$  for individuals,  $t$  for truth values, and  $r$  for numerical degrees, and basic semantic domains  $D_x$ ,  $D_t$ , and  $D_r = \mathfrak{R}$  (sets of individuals, truth values, and numerical degrees, respectively). Gradable adjectives are interpreted as follows:

- (1) Let  $T_c$  stand for a set of indices, the worlds (or completions) consistent with a background context  $c$  (cf. Stalnaker, 1978; Kamp 1975).
- (2) For any context  $c$ , for any  $t \in T_c$  and any gradable adjective  $P$ :
  - a.  $f_{P,t} \in \mathfrak{R}^{D_x}$  is the degree function of  $P$  in  $t$  (a function from entities  $x$  in the domain  $D_x$  to real numbers  $r$  in  $\mathfrak{R}$ )
  - b.  $P$  holds true of an object  $x \in D_x$  in  $t$  iff  $x$ 's value exceeds  $P$ 's cutoff point:  $f_{P,t}(x) > \text{cutoff}(P,t)$  (Kennedy, 1999).

In particular, theories in this approach tend to assume that gradable predicates map arguments to degrees for which *plus*, *difference* and *ratio* operations are applicable (von Stechow 1984a,b). The numerical approach is prevalent in the literature, as it provides straightforward semantic accounts of expressions whose interpretation is mediated by the application of operations on numbers (identity, multiplication, difference, etc.), such as, e.g., numerical modifiers like *2 meters tall*, ratio predicates like *twice as happy as Sam*, and difference predicates like *2 meters shorter than Sam*. In these theories, the interpretation of *more* and *as* involves the application of a difference operation, as demonstrated in (3), with  $r_{m,t}$  being the degree of height of the meter in  $t$  (von Stechow 1984a; Schwarzschild and Wilkinson 2002; Kennedy and McNally 2005; Schwarzschild 2005; Kennedy and Levin 2007:17; Sassoon 2010a).

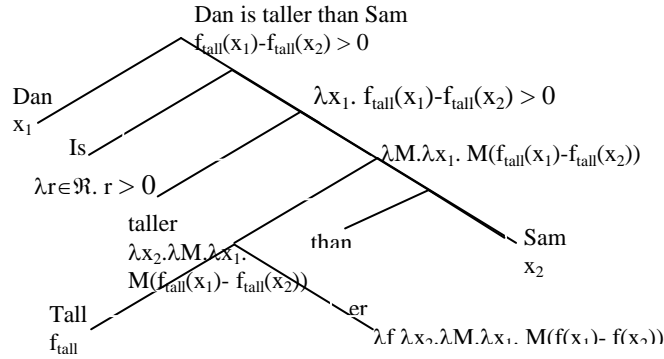
- (3) a.  $\llbracket \text{Dan is 2 meters taller than Sam} \rrbracket_t = 1$  iff  $f_{\text{tall},t}(\llbracket \text{Dan} \rrbracket_t) - f_{\text{tall},t}(\llbracket \text{Sam} \rrbracket_t) = 2r_{m,t}$
- b.  $\llbracket \text{Dan is happier than Sam} \rrbracket_t = 1$  iff  $\exists r > 0: f_{\text{happy},t}(\llbracket \text{Dan} \rrbracket_t) - f_{\text{happy},t}(\llbracket \text{Sam} \rrbracket_t) = r$ .

Thus, despite differences in detail between analyses, all in all, a widely employed view is that, e.g., *Dan is taller than Sam (by 2 meters)* holds true in an index  $t$  iff the difference between Dan and Sam's degrees in  $t$  is a positive real number (twice the degree of a meter unit object in  $t$ ).

The basic interpretation of phrasal *er* as a difference operation is, then, roughly,  $\lambda r_2 \in \mathfrak{R}. \lambda M_{\langle r, \triangleright \rangle}. \lambda r_1 \in \mathfrak{R}. M(r_1 - r_2)$ , where the variable  $M$  has to be saturated by a degree predicate like *two inches* (Schwarzschild and Wilkinson 2002; Landman 2005; Sassoon 2010b) and the interpretation of *as* is  $\lambda r_2 \in \mathfrak{R}. \lambda r_1 \in \mathfrak{R}. \llbracket er \rrbracket(r_2, \lambda r. r \geq 0, r_1)$ , which reduces to:  $\lambda r_2. \lambda r_1. r_1 - r_2 \geq 0$ . Given the latter, the use of the comparative, rather than the equative, normally excludes the possibility that  $M$  is  $\lambda r. r \geq 0$ ; i.e., in the absence of an overt numerical degree modification,  $M$  is thought to be saturated by the predicate  $\lambda r \in \mathfrak{R}. r > 0$  ('somewhat'), implying that  $r_1 - r_2 > 0$  (cf. Schwarzschild and Wilkinson 2002; Landman 2005; Sassoon 2010b), as illustrated below. Finally, when *er* combines with an adjective as in *taller*, interpretation type shifts to an individual

level:  $\lambda f \in \mathfrak{R}^{D_x}. \lambda x_2 \in D_x. \lambda M. \lambda x_1 \in D_x. \llbracket er \rrbracket (f(x_2))(M)(f(x_1)),$  which reduces to  $\lambda f \in \mathfrak{R}^{D_x}. \lambda x_2 \in D_x. \lambda M. \lambda x_1 \in D_x. M(f(x_1) - f(x_2)).$

(4)



On both the vagueness-based and numerical approach, a comparative or equative morpheme applies to one predicate at a time, e.g. *The table is taller than the chair is wide* is a comparison of the table’s status (degree or denotation membership) with respect to a single predicate (*tall*), and the chair’s status with respect to a single predicate (*wide*). An open question, then, regards the interpretation of combinations of comparison morphemes with complex predicates, as in, for instance, *more honest and intelligent* and *equally expensive and time consuming*. How are such statements interpreted? The next section discusses two different possible answers, a fuzzy and a Boolean one. Section 2 presents an empirical investigation whose goal is to decide between the two. Implications to the numerical versus vagueness-based debate are addressed.

## 1.2 The interpretation of coordination constructions

Sentences with conjunctions and modified-nouns in predicate position usually entail the sentences resulting from dropping some of the constituents or changing the constituent ordering. For example, (5a) entails (5b,c) and (6a) entails (6b,c). In addition, (5a) and (6a) are equivalent to (5d) and (6d) respectively. Such entailment-patterns form the basis for the intersective analysis of modified-nouns and conjunctions, whereby they denote the intersection of their constituents’ denotations, as stated and illustrated in (7) (Kamp and Partee 1995; Landman 2000; Heim and Kratzer 1998). The intersection-rule in (7) directly predicts the fact that an item is classified as, for instance, *a four legged animal* or *an animal which is four legged* iff it is classified as *an animal* and it is classified as *four legged*.

- (5) a. Tweety is brown and big  
 b. Tweety is brown  
 c. Tweety is big  
 d. Tweety is big and brown

- (6) a. Tweety is a four legged animal  
 b. Tweety is four legged  
 c. Tweety is an animal  
 d. Tweety is an animal and is four legged

- (7)  $\forall t \in T: \llbracket [P \text{ (and) } Q] \rrbracket_t = \llbracket [P] \rrbracket_t \cap \llbracket [Q] \rrbracket_t$   
 a.  $\llbracket [\text{brown and big}] \rrbracket_t = \lambda x \in D_x. \text{brown}(x) \wedge \text{big}(x) = \llbracket [\text{brown}] \rrbracket_t \cap \llbracket [\text{big}] \rrbracket_t$   
 b.  $\llbracket [\text{brown apple}] \rrbracket_t = \lambda x \in D_x. \text{brown}(x) \wedge \text{apple}(x) = \llbracket [\text{brown}] \rrbracket_t \cap \llbracket [\text{apple}] \rrbracket_t$

The same basic facts hold in the verbal domain, too. For example, the entailments from (8a) to (8b-e) are instances of intersective inference patterns in modified verbs. These additional facts form the basis for the Davidsonian intersective analysis of modified verbs ((9); Landman 2000).

- (8) a. Dan ate quickly with a knife  
 b. Dan ate with a knife  
 c. Dan ate quickly  
 d. Dan ate  
 e. Dan ate with a knife quickly
- (9)  $\exists e \in E: \llbracket \text{eating} \rrbracket_t(e) \wedge \text{Agent}(e) = \llbracket \text{Dan} \rrbracket_t \wedge \exists x \in D_x, \text{Instrument}(e) = x \wedge \llbracket \text{knife} \rrbracket_t(x) \dots$

While using the intersection rule, most up to date semantic theories of gradability and comparison in natural language do not associate complex predicates (e.g., negated, conjunctive and disjunctive ones) with graded structures (say, a mapping of entities to numerical degrees). However, fuzzy logical theories can be used to do precisely that (cf. Hajek, 2009). Fuzzy logic is a form of multi-valued logic, whereby propositions may have as a truth value any number in the real interval [0,1]. The disjunction, conjunction and negation operators of Boolean logic exist in fuzzy logic and are usually defined as the maximum, minimum, and complement, respectively (Zadeh 1965); when they are defined this way, they are called the *Zadeh operators*. So for the fuzzy propositions  $P(x)$  and  $Q(y)$ :

- (10) a.  $\llbracket \neg P(x) \rrbracket_t = 1 - \llbracket P(x) \rrbracket_t$   
 b.  $\llbracket P(x) \wedge Q(y) \rrbracket_t = \min(\llbracket P(x) \rrbracket_t, \llbracket Q(y) \rrbracket_t)$   
 c.  $\llbracket P(x) \vee Q(y) \rrbracket_t = \max(\llbracket P(x) \rrbracket_t, \llbracket Q(y) \rrbracket_t)$

Other definitions exist for conjunctive and disjunctive expressions that are not based merely on a selection of one of the constituents' degrees; rather, these definitions make use of functions  $t_{\text{and}}$  and  $t_{\text{or}}$  (often called *t norms* and *t-conorms* for conjunctions and disjunctions, respectively) to compute a value for the complex expression based on the values of both of its constituents, as follows:

- (11) a.  $\llbracket \neg P(x) \rrbracket_t = t_{\text{not},t}(\llbracket P(x) \rrbracket_t)$   
 b.  $\llbracket P(x) \wedge Q(y) \rrbracket_t = t_{\text{and},t}(\llbracket P(x) \rrbracket_t, \llbracket Q(y) \rrbracket_t)$   
 c.  $\llbracket P(x) \vee Q(y) \rrbracket_t = t_{\text{or},t}(\llbracket P(x) \rrbracket_t, \llbracket Q(y) \rrbracket_t)$

There are multiple choices for the fuzzy conjunction and disjunction operators. A common choice is the algebraic product for fuzzy conjunction and algebraic sum for fuzzy disjunction, but there are an infinite number of other choices (Yen, 1999; Hajek, 2009). Rather than the definitions of (or axioms constraining) the t-functions, the very possibility that such functions may be relevant to natural language coordination constructions is the focus of interest of this paper. Recall that *more* is analyzed as denoting a difference modifier, e.g., *Dan is taller than Sam* is true in  $c$  iff  $f_{\text{tall},c}(\llbracket \text{Dan} \rrbracket_c) - f_{\text{tall},c}(\llbracket \text{Sam} \rrbracket_c) > 0$  (cf. section 1.1). According to this analysis, *more* cannot apply to the interpretations of two predicates – two degree functions – simultaneously. However, if natural language semantics is ‘fuzzy’, i.e. conjunctive and disjunctive predicates are systematically associated with composed degree functions, *more* should be capable of accessing these functions and operating on them. Let  $f_{P\text{-and-}Q,t}$  be the function  $\lambda x \in D_x. t_{\text{and}}(f_{P,t}(x), f_{Q,t}(x))$  and  $f_{P\text{-or-}Q,t}$  be the function  $\lambda x \in D_x. t_{\text{or}}(f_{P,t}(x), f_{Q,t}(x))$ . A fuzzy natural language semantic theory predicts the following interpretations for expressions of the form *more P and Q* and *more P or Q*.

(12) A fuzzy natural language semantic theory:

- a.  $[[\text{more P and Q}]]_t = \lambda_{x_2} \in D_x. \lambda_{x_1} \in D_x. f_{P\text{-and-}Q,t}(x_1) - f_{P\text{-and-}Q,t}(x_2) > 0$
- b.  $[[\text{more P or Q}]]_t = \lambda_{x_2} \in D_x. \lambda_{x_1} \in D_x. f_{P\text{-or-}Q,t}(x_1) - f_{P\text{-or-}Q,t}(x_2) > 0$

On the one hand, psychological findings pertaining to modified nouns seem to support a fuzzy semantic analysis of complex natural language expressions. Psychological theories associate concepts (typically, nominal ones) with functions corresponding to the mean of entities in a variety of dimensions. Hampton (1987; 1988a; 1997a,b) has analyzed ratings of goodness of example (typicality) of a list of entities in modified-nouns of the form 'Ps which are Qs' (such as, for instance, *pets which are birds*) and in their constituents (e.g. *pets* and *birds*). The following patterns emerged.

First, for any item  $x$ , it is possible to predict  $x$ 's typicality rating in a modified-noun,  $f_{P\text{-and-}Q}(x)$ , from  $x$ 's ratings in the constituents,  $f_P(x)$  and  $f_Q(x)$ , by an equation like (13a).  $W_P$  and  $W_Q$  represent the constituents' weights and  $W_{P \times Q}$  represents the weight of the constituents' interaction (the product  $f_P(x) \times f_Q(x)$ ). For example, the values for *pets which are birds* were:  $W_{\text{pets}} = .30$ ,  $W_{\text{birds}} = .78$ , and  $W_{\text{pets} \times \text{birds}} = .10$ .

Second, the typicality ratings in modified-nouns with negated constituents,  $f_{P\text{-and-not-}Q}(x)$ , are predicted by adding a negative sign to the weight of the negated constituent ( $-W_Q$ ). The interaction term is also negative when significant (13b). For example, for *pets which are not birds* the weights were:  $W_P = .32$ ,  $W_Q = -.75$ , and  $W_{P \times Q} = -.11$ . Why? Because the better an item is as an example of  $Q$ , the worse it is as an example of not- $Q$ .

Third, given the logical connections between disjunctions, conjunctions and negations ( $P \vee Q = \neg(\neg P \wedge \neg Q)$ ), and the fact that negation affects the equation by changing the coefficient sign, Hampton predicted that the typicality ratings in disjunctions like *hobbies or games*,  $\text{deg } f_{P\text{-or-}Q}(x)$ , would be given by adding a negative sign to the interaction term ( $-W_{P \times Q}$ ). Why? The value  $f_{P\text{-or-}Q}(x)$  ought to be identical to  $f_{\neg(\neg P \wedge \neg Q)}(x)$ , which, in turn, should be given by an equation in which a negative sign is added to the weight of each negated-constituent (namely by the equation:  $-( -W_P f_P(x) - W_Q f_Q(x) + W_{P \times Q}(f_P(x) \times f_Q(x)) )$ ). After the elimination of double negative-signs, this equation reduces to the one in (13c), with the negative interaction-weight. And indeed, using (13c), Hampton (1988b) could predict the typicality ratings in disjunctions from the ratings in the disjuncts.

$$\begin{aligned}
 (13) \quad & \text{a. } f_{P\text{-and-}Q}(x) &= & W_P f_P(x) + W_Q f_Q(x) + W_{P \times Q}(f_P(x) \times f_Q(x)). \\
 & \text{b. } f_{P\text{-and-not-}Q}(x) &= & W_P f_P(x) - W_Q f_Q(x) - W_{P \times Q}(f_P(x) \times f_Q(x)). \\
 & \text{c. } f_{P\text{-or-}Q}(x) &= & W_P f_P(x) + W_Q f_Q(x) - W_{P \times Q}(f_P(x) \times f_Q(x)).
 \end{aligned}$$

On the other hand, the constituent-based equations in (13) seem to be too coarse-grained. Negated constituents, for instance, sometimes have a decreased weight, because some dimensions are treated as characterizing both the predicate and its negation. For example, *animate* often characterizes both birds and entities that are not birds, and *bird-hood* characterizes both robins and non-robins. In general, the typicality ratings in modified-nouns are better fitted by a composite-prototype representation, wherein the weight of each dimension is adjusted by a special function.

More importantly, these findings may not extend to adjectival conjunctions and disjunctions. The graded structure of adjectival predicates is rather different in nature from that of nominal ones. The interpretation of adjectives like *tall* and *expensive*, for example, directly relate to conventional measurements of height and cost, rather than to a weighted mean in a set of dimensions. For one, these adjectival degree functions are unbound from above. In addition, while adjectives can combine with *more* (or *er*) to create within-predicate comparisons (as in,

e.g., *two meters taller*), nouns and noun phrases do not combine with *more*, as illustrated by the infelicity of, for example, \**Tweety is more a bird/birder than Tan*, \**x is (a) more midget giant than y* and \**x is (a) more fat bald man than y*. When licensed, the comparative morpheme either associates with the modifier most adjacent to it alone, as in *x is a fatter bald man than y*, or has to be modified by *of*, as in *x is more of a midget giant than y*. The latter statement is interpreted as if the noun phrase is modified with *typical*, as in *x is more typical of a midget giant than y*, where the adjective *typical* forms the argument of *more* and the noun phrase only provides typicality dimensions for *typical* to bind. In opposition, combinations of *more* with conjunctions or disjunctions of adjectives are fine. For example, *x is more expensive and elegant than y* is perfectly grammatical, and so is ‘*x yoter shamen ve kerea x*’ (‘*x is more fat and bald than y*’) in Hebrew and similar languages, whereby the comparative morpheme always surfaces as an independent word (*yoter*; ‘more’) and never as a dependent morpheme similar to English *er*.

Moreover, rather than to relate to a unique graded structure of the conjunctive concepts in question, intuitively, these phrases seem to convey ‘**more** expensive and **more** elegant’ and ‘**fatter** and **balder**’, respectively. The same phenomenon pertains also to conjunctive multi-dimensional adjectives, like *typical with respect to flying and singing* or *healthy with respect to blood pressure and pulse*.

Thus, gradability is different in nouns and adjectives. While nominal functions may be correctly described using some sort of fuzzy semantics, it is questionable whether adjectival functions can be so described. Perhaps adjectival conjunctions and disjunctions, such as *tall and/or fat*, are not systematically associated with degree functions at all. If so, *more* should not be capable of combining with conjunctive or disjunctive predicates directly; rather, in constructions of the form *more P and Q* or *more P or Q*, the Boolean operators *and* and *or*, respectively, should take wide scope with respect to *more*, so that *more* would modify each conjunct/ disjunct separately, operating on one basic degree function at a time (Sassoon 2007; Bale 2007). Such a non-fuzzy, classically ‘Boolean’ natural language semantic theory predicts the following interpretations for expressions of the form *more P and Q* and *more P or Q*.<sup>1</sup>

(14) A Boolean natural language semantic theory:

- a.  $[[\text{more } P \text{ and } Q]]_t = [[\text{more } P \text{ and more } Q]]_t$   
 $= \lambda x_2 \in D_x. \lambda x_1 \in D_x. (f_{P,t}(x_1) - f_{P,t}(x_2) > 0) \wedge$   
 $(f_{Q,t}(x_1) - f_{Q,t}(x_2) > 0)$
- b.  $[[\text{more } P \text{ or } Q]]_t = [[\text{more } P \text{ or more } Q]]_t$   
 $= \lambda x_2 \in D_x. \lambda x_1 \in D_x. (f_{P,t}(x_1) - f_{P,t}(x_2) > 0) \vee$   
 $(f_{Q,t}(x_1) - f_{Q,t}(x_2) > 0)$

While linguists belonging to the numerical degree approach (cf. section 1.1 above) such as Kennedy (1997) sometimes argue that this approach resembles fuzzy logic, they never, to the best of my knowledge, actually study the question of whether conjunctions and disjunctions of morphologically gradable adjectives are also morphologically gradable, i.e. felicitously licensing *more*. A basic way to test whether the interpretation of expressions of the form *more P and/or Q* is fuzzy (cf. (12)) or non-fuzzy (cf. (14)), is by presenting subjects with pairs of entities differing along one conjunct/ disjunct (say – P) but otherwise identical (equally Q), and asking them

<sup>1</sup> The requirement for a unique dimension in the use of a comparative morpheme can only be abandoned in between-predicate comparisons. Conjunctive and disjunctive concepts seem to be felicitous and to receive interpretations with the connective in narrow scope in such comparisons (cf. (i)-(iii)), though more systematic future research needs to carve out the precise set of interpretations that may be assigned to such statements.

- i. This is more a kitchen utensil than an electronic device.  
 ii. This is more a piece of furniture and a game than a kitchen utensil or an electronic device.  
 iii. Dan is more fat, bald and unhappy than good-looking, energetic and funny.

whether these pairs stand in the relations (i) *more P and Q*, (ii) *less P and Q* and/or (iii) *equally P and Q* and (iv) *more P or Q*, (v) *less P or Q* and/or (vi) *equally P or Q*. A non-fuzzy analysis predicts that such pairs stand in none of the three conjunctive relations (i)-(iii) (because they stand in none of the relations *more P and more Q*, *less P and less Q* and *equally P and equally Q*) and in the two disjunctive relation (iv) and (vi) (because they stand in the relations *more P or more Q* and *equally P or equally Q*). In sharp contrast, a fuzzy analysis does not allow for these possibilities; pairs of entities ought to stand in one and only one conjunctive relation and disjunctive relation, depending on their composed  $t_{P\text{-and-}Q}$  and  $t_{P\text{-or-}Q}$  degrees, respectively. Entity pairs standing in no relation or in more than one relation are at least not straightforwardly accounted for by a fuzzy analysis.

A small questionnaire was designed to examine what the facts actually are, i.e. whether they are more easily fitted by a fuzzy analysis such as the one provided in (12) or by a Boolean analysis such as the one in (14).

## 2. *And, or and more, a general judgments questionnaire*

### 2.1 Method

#### *Subjects*

The subjects were 35 native speakers of Hebrew, 21 females and 14 males, in the age range 20-40 with three exceptions of ages 41, 44 and 59 (average age 31) and with academic education of at least one year (17 graduate students).

#### *Design and Material*

The subjects received a written questionnaire. An opening paragraph included general instructions. This opening paragraph, translated from Hebrew to English for the purpose of presentation in this paper, is as follows:

The goal of this questionnaire is to understand the way people think and the way they use certain words. Hence, there are no right and wrong answers. For each question provide the answer which on your opinion is the most reasonable and accurate. A slot for comments follows each section. We will be happy with any comment pertaining to the reasons for which you choose to answer the way you do or to uncertainty you might have. Filling in the comment slot is not obligatory. While some sections look alike, it is very important that you relate to each one of them separately. While the questions in this questionnaire are given in masculine forms, they are addressed to both genders. Many thanks in advance for your patience in filling in the questionnaire.

On every section, draw a circle around the answer you select.

The questionnaire included 13 sections, 5 of which are relevant for the present research. Each section included a short paragraph with a brief description of two characters consisting of their values or relative status in two gradable properties. The paragraph was followed by either yes-no questions (followed by Yes/No), or two-valued questions, followed by two names (for instance, Dan/Sam). The yes/no questions asked whether the two characters stand in a certain comparison relation with respect to a given predicate; the two valued questions asked which entity ranks higher in asymmetric comparison relations. The questions can be divided to various different conditions depending on the type of *comparison relation* and the type of *predicate*.

Predicate types included basic gradable adjectives such as *tall*, conjunctions and disjunctions of gradable adjectives such as *bald and/or tall*, and nominal constructions modified by gradable adjectives, such as *bald tall [one]*. Notice that adjectives stand alone (with no overt noun phrase to modify) significantly more easily in Hebrew than in English, as is apparent from the translations of some of the questions below. Therefore, some material that did not occur in the original questionnaire has been added in square brackets to enhance clarity of the English translation (e.g. *[one]* in the nominal example given above).

Comparison types included simple comparisons (as in, e.g., *taller*, *less tall*, and *equally tall*) and complex comparisons, mainly of ease of classification (as in *easier/ less easy / equally easy to determine that Moshe is tall than that Danny is tall*) and difficulty of classification (*harder*, *less hard / equally hard to determine...*), but also, on few sections, comparisons of *typicality* (as in *more typical of a tall person*), *fitness* (as in *fits more to be a subject in a scientific experiment studying properties of tall people*) and *certainty* (as in *if Danny fits, Moshe definitely fits*). Asking questions with different forms of comparison can reveal whether different ways to relate to the relative ordering of entities along dimensions like height produce similar or different answers. At the same time they may serve to test reliability. Notice that comparative adjectives like *taller* in Hebrew are construed of two separate words: *yoter gavoha* ('more tall'; 'taller'), thus the data below is presented in the Hebrew way, e.g. using *more fat* rather than *fatter*, even where English speakers would prefer the latter.

For example, on section 1 the subjects read the following description of two characters called Moshe and Danny: "Assume Moshe weighs 100kg and Danny weighs 90kg and they are alike in other things (for instance, height)." The questions following this paragraph include simple comparison, as well as comparison of ease and difficulty of classification, but only in relation to a basic adjective (*fat*), for this section was introductory in nature, with the goal to check whether subjects understand the general logic of the questions in the questionnaire:

(15) Section 1, the questions:

**The basic condition, simple comparison**

a. Is Moshe more fat than Danny? Yes/No

**The basic condition, complex comparison**

b. Is it easier to determine that Moshe is fat than that Danny is fat? Yes/No

c. Is it harder to determine that Danny is fat than that Moshe is fat? Yes/No

On section 3, the subjects read the following description: "Assume Moshe weighs 100kg and he is 1.95 cm tall, and Danny weighs 70 kg and is he is 1.95 cm tall." (i.e., Moshe is fatter than Danny, but they are equally tall). The questions included both basic (*fat*, *tall*) and conjunctive adjectives (*fat and tall*), as well as simple and complex comparisons. If natural language semantics is generally fuzzy, subjects should regard *fat* and *tall* as assigning Moshe and Danny degrees in a bound interval isomorphic to the real interval [0,1]. Then, based on these degrees, subjects should try to compute Moshe and Danny's degrees in *fat and tall*, thereby judging either Moshe or Danny as *more fat and tall*.

In opposition, if natural language semantics is not generally fuzzy (the 'Boolean' hypothesis), subjects would not have access to degrees in complex predicates like *fat and tall*; rather they will interpret *more fat and tall* with *and* scoping over *more*; i.e. they will try to determine whether it is Moshe or Danny that is *more fat and more tall* (i.e. *fatter and taller*). Since neither one is *fatter and taller*, nor are they *equally fat and tall* in the sense of being *equally fat and equally tall*, subjects are expected to say that (i) Moshe is **not** *more "fat and bald"*; (ii) Danny is **not** *more "fat and bald"* and (iii) They are **not** *equally "fat and bald"*.

Three measures were taken in order to bias subjects **against** such 'Boolean' answers with wide scope for *and/or* with respect to *more*, and towards 'fuzzy' answers, whereby degrees and ordering relations are construed for complex predicates. First, the following introductory comment preceded the questions of section 3:

(16) Section 3, introductory comment:

**"An important comment regarding sections in the questionnaire of the form:**

(1) Is Moshe more tall and fat than Danny?

(2) Is Moshe more tall or fat than Danny?

The intention is not to ask whether Moshe is more tall and/or whether Moshe is more fat; rather, the intention in (1) is to ask whether Moshe exemplifies better the complex property fat and tall. The intention in (2) is to ask whether Moshe exemplifies better the complex property fat or tall.”

Second, in each and every section, the adjectival conjunctions and disjunctions were underlined in all the questions under concern, so that they will be processed as relating to a single unified property.

Third, section 9 presented the two figures as equally fat and one balder than the other (i.e. the same pattern as in section 3); however, this section begins with 9a directly asking whether Moshe is both more fat and more bald than Danny is (an unambiguously wide-scope *and/or* question) and immediately continued by asking whether Moshe is more fat and Bald than Danny is (9b). On this setup, subjects are expected to try to interpret 9b as asking for something different than 9a, thereby interpreting *and* within the scope of *more*. Likewise, sections 3 and 4 begin by asking who is *more fat* and who is *more tall/bald*, except in two separate subsections (3a,c and 4a,b) rather than in a conjoined question.

Finally, section 4 presented characters in inverse relations (Aharon *fatter* and Danny *balder*) and section 7 involved a conjunctive typicality adjective referencing two typicality features; this adjective translates roughly to *typical of a flying and calling creature*. A complete presentation of the relevant sections (1,3,4, 7 and 9) is found in the appendix (see also the tables in the result section below).

#### *Procedure:*

The subjects have received the questionnaire by email. They were asked to fill it in themselves and not to consult with anyone but me if they have questions. When they were undecided, they were encouraged to nonetheless select the answer that fits best their opinion. They have received as much time as they needed to fill in answers and were encouraged to add comments on each section.

## **2.2 Results**

The results of sections 1, 3 and 4 are presented first, divided to results of basic conditions (atomic predicates), followed by conjunctive, disjunctive and modifier-position conditions.<sup>2</sup> Only then are the results of sections 9 and 7 presented, which appear to reflect mainly the effect of repetition of judgments on all conditions.

### **2.2.1 Judgments for atomic ('basic') predicates**

#### *Simple comparison*

The answers pertaining to basic predicates confirmed expectations – on sections 1 and 3 Moshe was generally judged fatter, with 89% and 94% agreement when his 100kgs were compared to Danny's 90k and 70kg, respectively. Apparently, the bigger weight difference in 3a vs. 1a explains the higher percentage of agreement in 3a.

The judgments of the few subjects that did not agree to say that Moshe is fatter (even in the 30kg difference condition) are probably explained by the fact that (as often mentioned in the comment sections throughout the questionnaire) ordering judgments in adjectives like *fat* (as well as *tall*) may be based on both weight and height as well as on general look (since mere weight may reflect muscles rather than fat).

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<sup>2</sup>The results for complex comparisons are basically the same as the results for simple ones, except somewhat weaker. However, as discussed below, they appear to have been affected by a methodological problem. Thus, these results are presented separately from those for simple comparisons; readers that are only interested in the main question the paper asks can skip the paragraphs pertaining to complex comparisons and still capture the main findings.

On 3c subjects generally did not agree that any one of the 1.95cm tall characters is taller than the other (6% agreement). On section 4, subjects' answers to 4a,b unequivocally indicate that they agree that Aharon is fatter (100%) and Danny is balder (97%), as expected given that Aharon is 30kg fatter than Danny, but is not bald, respectively.

| <b>Section 1</b>  |           |              |               |
|---|-----------|--------------|---------------|
| Moshe -100kg; Danny -90 kg  | % 'Yes'   | 'No' answers | 'Yes' answers |
| a. Is Moshe more <u>fat</u> than Danny?   | 89%       | 4            | 31            |
| b. Is it easier to determine that Moshe is <u>fat</u> than that Danny is <u>fat</u> ? | 69%       | 11           | 24            |
| c. Is it harder to determine that Danny is <u>fat</u> than that Moshe is <u>fat</u> ? | 60%       | 14           | 21            |
| <b>Section 3, a-d</b>   |           |              |               |
| Moshe -100kg; Danny -70 kg;<br>both -1.95 cm tall                                     | % 'Yes'   | 'No' answers | 'Yes' answers |
| a. is Moshe more <u>fat</u> than Danny?   | 94%       | 2            | 33            |
| b. Is it easier to determine that Moshe is <u>fat</u> than that Danny is <u>fat</u> ? | 89%       | 4            | 31            |
| c. is one of them more <u>tall</u> than the other?                                    | 6%        | 30           | 2             |
| d. Is it easier to determine that he is <u>tall</u> ?                                 | 40%       | 6            | 4             |
| <b>Section 4a,b</b>   |           |              |               |
| Aharon -100kg; Danny -70 kg;<br>Aharon –not bald; Danny – bald                        | % 'Danny' | 'Aharon'     | 'Danny'       |
| a. Who is more fat  | 0%        | 33           | 0             |
| b. Who is more bald   | 97%       | 1            | 32            |

Table 1: The basic conditions

(i) Given his higher weight, Moshe is generally judged fatter (sections 1,3; same with Aharon in 4); (ii) given Moshe and Danny's equal heights, none is judged taller (sections 1 and 3) and (iii) given their inverse classification as bald and not bald, Danny is judged to be balder (section 4).

### Complex comparison

The same pattern is found with comparison of ease of classification (e.g., *easier to determine that x is fat than that y is*), but with smaller percentages of agreement (69% vs. 89% in 1b and 3b, respectively). Using comparison of difficulty of classification in 1c (e.g., *harder to determine that y is fat than that x is*), yields even smaller percentage (60%). One comment regarding 1b indicates that it is easy to determine that both are fat. Apparently, this yields the use of *easier to determine* less appropriate than the use of *fatter*; it yields the use of *harder to determine* even less appropriate.

Also, given that subjects' negative answer to (3c) implies that the characters are equally tall, most (25) subjects ignored question 3d concerning whether it is easier to determine that one of them is tall; however, four of the 35 subjects (12%) did agree that tallness is easier to determine for one than for the other, justifying their answers by assigning a role to their very different weights.

## 2.2.2 Judgments for conjunctive predicates

### Simple comparison

The answers to the conjunctive questions in sections 3 and 4 are generally supportive of the Boolean theory. Recall that on section 3, the characters were equally tall but Moshe was 30kg fatter. Despite this difference on one of the conjuncts, Moshe was generally judged neither *more fat and bald* nor *less fat and bald* than Danny, with 6% and 0% agreement in 3e and 3f, respectively. Obviously, the two characters are not equally fat and bald (cf. the results for question 4e below). Thus, this result suggests that the questions tend to be interpreted with *and* scoping over *more*, i.e.

subjects try to determine whether Moshe is *more fat and more tall* in 3e and whether he is *less fat and less tall* in 3f. The latter is clearly not the case (testified by across-the board disagreement); the former is not the case because the characters are equally tall. Counter the prediction of a fuzzy semantic theory, then, subjects did not judge Moshe *more fat and tall*, presumably because they did not compose a degree function for the conjunctive concept *fat and tall*.

Concerning section 4, recall that subjects' answers to 4a,b unequivocally indicate that they agree that Aharon is fatter (100%) and Danny is balder (97%). Probably precisely because of that, counter the prediction of a fuzzy theory, subjects did not appear to compute degrees for the conjunctive concept *fat and bald*; rather, in line with the Boolean hypothesis, the answers to 4a-f unequivocally indicate that subjects agree to say neither that any of the two characters is more fat and bald (0% agreement to 4c), nor that they are equally fat and bald (9% agreement to 4e). This suggests that these questions are interpreted with *and* scoping over *more*; subjects were trying to determine whether Aharon is *more fat and more bald* or whether he is *less fat and less bald* in 4a (both are clearly not the case) and whether Aharon and Danny are equally fat and equally bald in 4c (which again is clearly not the case).

In sum, counter the prediction of a fuzzy theory, subjects did not judge any character to be *more fat and bald*, and at the same time judged them not to be *equally fat and bald*. This seems to indicate that they did not compose a degree function for the conjunctive concept *fat and bald*.<sup>3</sup>

| <b>Section 3, e-f</b>  |         |      |       |
|--|---------|------|-------|
| Moshe -100kg; Danny -70 kg;<br>both -1.95 cm tall                                  | % 'Yes' | 'No' | 'Yes' |
| e. Is Moshe more <u>fat and tall</u> than Danny?                                   | 6%      | 33   | 2     |
| f. Is Moshe less <u>fat and tall</u> than Danny?                                   | 0%      | 32   | 0     |
| g. Is it easier to determine that Moshe is <u>fat and tall</u> than that Danny is? | 38%     | 21   | 13    |
| h. Is it harder to determine that Moshe is <u>fat and tall</u> than that Danny is? | 12%     | 30   | 4     |

Table 2: The first conjunction condition:

Despite a 30kg weight difference (all other things being equal), Moshe is generally not judged *more fat and tall*.

| <b>Section 4c-f</b>  |         |      |       |
|--|---------|------|-------|
| Aharon -100kg; Danny -70 kg;<br>Aharon –not bald; Danny – bald                               | % 'Yes' | 'No' | 'Yes' |
| c. Is any of them more <u>fat and bald</u> than the other?                                   | 0%      | 34   | 0     |
| d. Is it easier to determine that one of them is <u>fat and bald</u> than that the other is? | 3%      | 34   | 1     |
| e. Are they equally <u>fat and bald</u> ?  | 9%      | 31   | 3     |
| f. Is it equally easy to determine that they are <u>fat and bald</u> ?                       | 6%      | 31   | 2     |

Table 3: The second conjunction condition:

When Aharon is *fatter* and Danny *balder*, generally, neither is judged to be *more fat and bald*.

### Complex comparison

The pattern of results with comparisons of ease of classification (e.g., *easier to determine that x is fat and tall*) is similar to the pattern found with direct comparison (e.g. *more fat and bald*), except for larger percentage (almost 40%) of agreement to say, on section 3g, that it is easier to

<sup>3</sup> To the best of my understanding, product based t-norms and sum based t-conorms cannot account for these data, except perhaps by virtue of a residua given an assumption of interaction between, e.g., *fat* and *bald*. But such an assumption is not justified, for *fat* and *bald* are (intuitively) independent.

determine that Moshe is more fat and tall. At any rate, most subjects still disagree to rank Moshe higher.

Two points reveal that the results concerning ease of classification should be taken with a grain of salt.

First, subjects commented that Danny is obviously not fat and tall (because he is by no means fat), rendering this comparison inappropriate; again, we see evidence supporting the hypothesis that comparisons of the form *easier to determine that x is P than that y is P* are only appropriate if it is possible to determine that both x and y are P, but it isn't too easy to do so or to reject doing so (P-hood is somewhat uncertain or dubious, but is definitely a live option). But if classification as, P, not P or undetermined is the main issue at stake in ease-of-classification comparisons, their acceptance is compatible with a Boolean theory; hence, these comparisons are not ideal means to distinguish between fuzzy and boolean interpretations.

Second, probably subjects would have been more willing to say that it is easier to determine *whether* Moshe is fat and tall than whether Danny is. This issue is left for future research to resolve. Notice, however, that reinterpretation of *that* as *whether* may explain the unexpected 12% (rather than 0%) agreement to 3h (it is harder to determine that [ $\cong$  whether] Moshe is fat and tall), since it is very easy to determine that Danny falls outside this conjunction (because he is by no means fat), while Moshe's classification is uncertain. Indeed, some subjects commented that they could only have answered the questions positively if *easier to determine that* would have been substituted for *easier to determine whether* and others said that they have answered *as if* the question included a *whether-*, rather than a *that-complementizer*.

At any rate, the results pertaining to section 4 are pretty clear. Subjects agree to say neither that for any of the two characters it is easier to determine that he is fat and bald (3% agreement to 4c), nor that for both this is equally easy to determine (6% agreement to 4f). This suggests that these questions are interpreted with *and* scoping over *more*; subjects were trying to determine whether, e.g. for Aharon, it is *easier to determine that he is fat and easier to determine that he is bald* or whether it is *less easy to determine that he is fat and less easy to determine that he is bald* in 4b (both are clearly not the case) and whether for Aharon and Danny it is *equally easy to determine that they are fat and equally easy to determine that they are bald* in 4d (which again is clearly not the case).

### 2.2.3 Judgments for disjunctive predicates

#### *Simple comparison*

The answers to the disjunctive questions confirmed the Boolean theory's predictions, although to a lesser extent.

Notice that if subjects were to construct a degree function for *fat or bald*, their answers to 4e-g should have been that none is *more fat or bald*, for both characters have a high degree in one disjunct and a low degree in the other disjunct, rendering their degrees in fat or bald more or less equal. However, first, we have already seen that they are not judged equally fat and bald (cf. Table 3, 4e,f). Second, the answers to 4g indicate that  $\frac{3}{4}$  of the subjects agree that one of the two characters is more fat or bald (74% agreement in 4g). Third, importantly, less than half of the subjects (46%) were willing to provide a single name indicating who is more fat or bald, Danny or Aharon (only 16/35 answers for 4h).

Thus, first and foremost, the 19 subjects (54%) not providing an answer to this question indicate that they interpreted *more fat or bald* as *more fat or more bald*, classifying both characters as such. In other words, these subjects interpreted *or* as scoping over *more*, trying to determine whether Aharon is *more fat or more bald* and whether Danny is *more fat or more bald*; both are clearly the case as Aharon is fatter and Danny balder, rendering 4g true and at the same time making it impossible to choose one answer to 4h.

Second, of the 16 answers, the majority (10, which make 63%) selected Danny as their candidate, explaining that Danny is clearly bald and hence fat or bald, while Aharon is not clearly fat, thus not clearly fat or bald. Thus, these subjects were using a Boolean union rule for classification under disjunctions. The remaining 6 subjects did not indicate why they selected Aharon.

Third, one could argue that the 16 subjects answering 4h were using fuzzy disjunctive degrees; however, had this been the case, these 16 subjects would have agreed to consider one of the characters as *more fat and bald* in the second conjunctive condition, but they did not (cf. 0% and 3% agreement to 4c,d, Table 3). Thus, these results do not indicate fuzzy reasoning.<sup>4</sup>

| <b>Section 4g-j</b>   |                     |                     |                    |
|---|---------------------|---------------------|--------------------|
| Aharon -100kg; Danny -70 kg;<br>Aharon –not bald; Danny – bald                              | %‘Danny’/<br>%‘Yes’ | ‘ Aharon’ /<br>‘No’ | ‘Danny’ /<br>‘Yes’ |
| g. Is one of them more <u>fat or bald</u> than the other?                                   | 74%                 | 9                   | 25                 |
| h. Who is more <u>fat or bald</u> ?   | 63%                 | 6                   | 10                 |
| i. Is it easier to determine that one of them is <u>fat or bald</u> than that the other is? | 68%                 | 11                  | 23                 |
| j. For whom is it easier to determine that?   | 77%                 | 5                   | 17                 |

Table 4: The first disjunction condition:

When Aharon is *fatter* and Danny *balder*, generally, both are judged to be *more fat or tall*.

### Complex comparison

Similar patterns are found in the ease of classification questions. Most subjects agree to say that for one of the two characters it is easier to determine that he is fat or bald (68% agreement in 4i), and 77% of the 22 subjects that provided an answer to 4j have selected Danny. Danny is clearly bald and hence fat or bald, while Aharon is not clearly fat, thus not clearly fat or bald. For this reason, it is easier to determine that Danny is fat or bald, assuming a union rule for classification under disjunctions.

Moreover, 13 subjects did not give an answer to this question, indicating that they probably classified both as ones for whom is *easier to determine fat or bald*, as predicted by the Boolean theory. They have probably interpreted these questions with *or* scoping over *more*, trying to determine whether for Aharon it is *easier to determine that he is fat or easier to determine that he is bald* or whether for Danny it is *easier to determine that he is fat or easier to determine that he is bald*; both are clearly the case (as Aharon is fatter and Danny balder), making it impossible to choose one answer.

Also, the results of 4s-w with fitness comparisons and certainty comparisons support reliability. On 4s, 92% of the subjects agree that Danny fits more to an experiment for which fat or bald subjects are needed. Likewise, on 4t, 82% refused to rank Aharon higher (to say that if Danny fits, Aharon definitely fits), while on 4w, 61% agreed to rank Danny higher (if Aharon fits, Danny definitely does).

<sup>4</sup>The use of product and sum functions for *and* and *or* cannot account for these data, as it predicts, for any pair of entities standing in the relation “equally P and Q”, that in order to also stand in the relation “more P or Q”, their P values should both be greater than both their Q values (or vice versa.) This condition is not satisfied in the present scenario, since Danny’s degree in *bald* exceeds Aharon’s degree in *fat* (Danny is definitely bald, but 100kg is not definitely fat), but Aharon’s degree in *bald* doesn’t (Aharon is ‘maybe not bald ‘). Similarly, Aharon’s degree in *fat* exceeds his degree in *bald*, but not Danny’s.

| <b>Section 4s-w</b><br>Aharon -100kg; Danny -70 kg;<br>Aharon –not bald; Danny – bald                            | %‘Danny’/<br>%‘Yes’ | ‘Aharon’ /<br>‘No’ | ‘Danny’ /<br>‘Yes’ |
|--|---------------------|--------------------|--------------------|
| s. <u>Fat or bald</u> subjects are needed to fill in a questionnaire for a scientific experiment. Who fits more? | 92%                 | 2                  | 22                 |
| t. It is true that if Danny fits Aharon definitely fits?   | 18%                 | 27                 | 6                  |
| w. It is true that if Aharon fits Danny definitely fits?   | 61%                 | 13                 | 20                 |

Table 5: The second disjunction condition:

When Aharon is *fatter* (100kg vs. 70kg) and Danny is *balder*, generally, Danny ‘fits more’ and is ‘more definitely classified in’ the category *fat or tall*.

## 2.2.4 Judgments for predicates in modifier position

On this condition the adjectives occur with no overt coordination marker; rather, they are in a modifier position of a null noun (perhaps the rightmost adjective functions as a noun in this construction). On 4o-4r, it is *bald* that is more adjacent to the empty noun head (or perhaps itself functions as a noun), whereas in 4k-4n *fat* is in the more ‘nominal’ position. At any rate, subjects could not take *more* in the modified noun condition to refer only to the adjective not adjacent to (or not itself functioning as) the head noun (trying to determine whether any of the characters is a *fatter bald man* in 4k-n or a *balder fat man* in 4o-r), because for this interpretation to occur in Hebrew the comparative morpheme and the adjective it modifies should follow the noun they modify rather than precede it.

The results on this condition are similar to the results on the conjunction condition, though a bit weaker. The majority of the subjects refused to agree that any of the characters is more *fat bald [person]* (13% agreement in 4k) and *bald fat [person]* (23% agreement in 4o), with less than a third of the subjects – 9 and 11 answering the questions ‘which one is’ in 4l and 4p, of which 89% and 91% respectively selected Danny. This suggests that most subjects interpreted these questions with a Boolean conjunctive operator scoping over *more*, as explained above for *more bald and fat*.

The results for the ease of classification comparisons are similar but slightly weaker, as in previous sections, suggesting that the subjects answered reliably. Most subjects refused to say for any of the two characters that it is easier to determine that he is fat bald (27% agreement in 4m) or that he is bald fat (25% agreement in 4q), with about a third of the subjects – 11 and 14 answering the questions ‘which one is’ in 4n and 4r, of which 82% and 100% respectively selected Danny to be better in the modified construction.

The selection of Danny rather than Aharon by subjects in 4l,n,p,r may be merely due to Danny’s description as bald and Aharon’s description as maybe not bald, as opposed to the absence of direct descriptions of their status in *fat*. Still, it seems puzzling that 70kg counts as fat here. Thus, the order of conditions may have affected the results. The present condition directly followed the disjunction condition, whereby Danny ranked higher for obvious reasons (cf. 2.2.3).

| <b>Section 4k-r</b>  |                     |                     |                    |
|--|---------------------|---------------------|--------------------|
| Aharon -100kg; Danny -70 kg;<br>Aharon –not bald; Danny – bald                           | %‘Danny’/<br>%‘Yes’ | ‘ Aharon’ /<br>‘No’ | ‘Danny’ /<br>‘Yes’ |
| k. Is any of them more <u>fat bald</u> than the other?                                   | 13%                 | 27                  | 4                  |
| l. Who is more fat bald?   | 89%                 | 1                   | 8                  |
| m. Is it easier to determine that one of them is <u>fat bald</u> than that the other is? | 27%                 | 24                  | 9                  |
| n. For whom is it easier to determine that?  | 82%                 | 2                   | 9                  |
| o. Is any of them more <u>bald fat</u> than the other?                                   | 23%                 | 23                  | 7                  |
| p. Who is more <u>bald fat</u> ?   | 91%                 | 1                   | 10                 |
| q. Is it easier to determine that one of them is <u>bald fat</u> than that the other is? | 25%                 | 24                  | 8                  |
| r. For whom is it easier to determine that?  | 100%                | 0                   | 14                 |

Table 6: The modifier-position condition:

When Aharon is *fatter* (100kg vs. 70kg) and Danny is *balder*, generally, neither is more [of a] fat bald [person] and neither is more [of a] bald fat [person]; but if one is selected, Danny is.

We see that, as in previous sections, the results support a Boolean theory rather than a fuzzy theory for complex concepts, over and above the fact that complex concepts were underlined and an explicit comment asked subjects to consider the whole concept rather than each constituent separately (cf. section 2.1). We can, therefore, conclude that the results for sections 1,3 and 4 support the existence of a Boolean bias, whereby Boolean operators (conjunction or disjunction) are interpreted in wide scope with respect to comparative morphemes (or phrases).

### 2.2.5 Judgments for section 9 - the ‘repetition’ conditions

To overcome this Boolean bias, section 9 begins by directly asking whether Moshe is both more fat and more bald (9a) versus whether Moshe is more fat and Bald than Danny (9b). The idea is that on this setup, subjects will try to interpret 9b as asking for something different than 9a, thereby interpreting *and* within the scope of *more*. Recall that on section 9, the characters were equally fat (100kg) but Moshe was balder. This setup is, then, very similar to the one in section 3. The results, however, are different.

Subjects generally disagreed to answer 9a positively (9% agreement); however, while most subjects still disagreed to answer 9b positively, much fewer did (43% agreement). These results indicate that an alternative fuzzy interpretation may be accessed, although it is not the preferred or natural interpretation for contexts such as those given in the questionnaire.

Even more striking is the 91% and 79% agreement to it being easier to determine that Moshe is fat and bald (9c) and bald and fat (9d) than that Danny is, respectively. As in previous sections, replacing direct comparison with comparisons of ease of classification facilitates agreement to rank Moshe higher, but this time facilitation is extreme.<sup>5</sup>

Nonetheless, two problems in the experimental design make it impossible to conclude that under the conditions specified above (when sentences such as 9b are adjacent to sentences such as 9a) speakers generally tend to interpret *and* within the scope of *more*, i.e. to compute a graded structure – degrees and/or ordering – for conjunctions, in line with a fuzzy semantic theory.

The first problem is that the order of presentation of sections and questions was one and the same for all subjects. Thus, the high percentages in 9b-d may also result from an effect of repetition of the same sort of questions, together with a desire on the part of the subjects to be cooperative, i.e., to answer positively, a desire that was repressed by virtue of the Boolean bias.

<sup>5</sup> Recall that on section 3 Moshe was generally judged neither *more fat and tall* nor *less fat and tall* than Danny, with 6% and 0% agreement respectively, and that less than 40% agreed to say that *it is easier to determine that Moshe is more fat and tall*.

But having answered more than 20 conjunctive/disjunctive questions before getting to section 9 (4 questions on section 3 plus 19 questions on section 4), subjects' tendency to disagree that *and/or* can scope under *more* may have slowly diminished. Perhaps, then, repetition in itself facilitates the composition of a unified degree function or ordering for the complex category.

The second problem regards an additional difference between section 3 and section 9. If both characters of section 9 are considered possibly fat (100kgs) and either definitely or possibly bald, this may have facilitated positive answers. Recall that on section 3 subjects commented that Danny is *obviously not* fat and tall, and that, for this reason, comparison of ease/difficulty of classification under these concepts is inappropriate; it may well be the case, then, that on question 9 more subjects were willing to accept the use of *easier to determine that* because no character was *obviously not* fat and *obviously not* bald. Furthermore, it may well be that more subjects answered *as if* the relevant questions included *whether* instead of *that* on section 9 than on section 3, due to the fact that 9 occurred after 3. But if it is only the status of entities as *bald* or *not bald* (or *undetermined*) that matters, the interpretation, in both cases, could be Boolean (supplemented by a representation of epistemic ignorance and/or vagueness, cf. part 3), with wide-scope for *and* with respect to the comparison phrase, as discussed earlier.

The same considerations apply to the modified noun conditions in 9e-h and even more so to the disjunction conditions, as seen in 9i-o.

| <b>Section 9a-d</b>   |             |           |           |
|---|-------------|-----------|-----------|
| Moshe is bald;  |             |           |           |
| Danny is less bald (maybe not bald);  | % 'Danny' / | 'Moshe' / | 'Danny' / |
| Both-100kg  | % 'Yes'     | 'No'      | 'Yes'     |
| a. Is Moshe both <u>more fat</u> and <u>more bald</u> ?   | 9%          | 32        | 3         |
| b. Is Moshe more <u>fat</u> and <u>Bald</u> than Danny?   | 43%         | 20        | 15        |
| c. Is it easier to determine that Moshe is <u>fat</u> and <u>bald</u> than that Dan is <u>fat</u> and <u>bald</u> ? | 91%         | 3         | 32        |
| d. Is it easier to determine that Moshe is <u>bald</u> and <u>fat</u> than that Dan is <u>bald</u> and <u>fat</u> ? | 79%         | 7         | 27        |
| Table 7a: The 'repetition' condition - conjunction  |             |           |           |
| e. Is Moshe more <u>fat bald</u> than Danny is?   | 55%         | 15        | 18        |
| f. Is it easier to determine that Moshe is <u>fat bald</u> than that Danny is <u>fat bald</u> ?                     | 71%         | 10        | 25        |
| g. Is Moshe more <u>bald fat</u> than Danny is?   | 54%         | 16        | 19        |
| h. Is it easier to determine that Moshe is <u>bald fat</u> than that Danny is <u>bald fat</u> ?                     | 70%         | 10        | 23        |
| Table 7b: The 'repetition' condition – modifier-position  |             |           |           |
| i. Is Moshe more <u>fat or Bald</u> than Danny?   | 100%        | 0         | 35        |
| j. Is it easier to determine that Moshe is <u>fat or bald</u> than that Danny is?                                   | 91%         | 3         | 31        |
| k. Is Moshe more <u>Bald or fat</u> than Danny?   | 100%        | 0         | 35        |
| l. Is it easier to determine that Moshe is <u>bald or fat</u> than that Dan is?                                     | 97%         | 1         | 32        |
| m. <u>Fat or bald</u> subjects are needed to fill a questionnaire for a scientific experiment. Who fits more?       | 7%          | 28        | 2         |
| n. It is true that if Danny fits Moshe definitely fits?   | 77%         | 8         | 27        |
| o. It is true that if Moshe fits Danny definitely fits  | 15%         | 29        | 5         |

Table 7c: The 'repetition' condition - disjunction

The answers to section 7 were basically Boolean in nature. Subjects were presented with two creatures not satisfying the first conjunct (*flying*) and differing along the second (*calling*), the latter being more typical of a calling creature than the former (non-calling creature). In this case, both creatures obviously do not satisfy the conjunction and the former obviously does not satisfy the disjunction. Most subjects disagreed about the latter being more typical of the modified noun *flying calling* [creature] (29% agreement for 7a) and conjunction *flying and calling* [creature] (37% agreement for 7b). Conversely, most subjects agreed about it being more typical of the disjunction *flying or calling* [creature] (80% agreement for 7c).

The results on section 7 are somewhere in between those of section 3 and those of section 9. This supports the hypothesis that the results on section 9 are explained by the two above mentioned problems, i.e. an effect of repetition, together with characters not obviously violating any of the constituents of the complex predicates in section 9.

| <b>Section 7</b>   |         |      |       |
|--|---------|------|-------|
| Two non-flying creatures; The first one doesn't call; The second one is more typical of a calling creature |         |      |       |
|  | % 'Yes' | 'No' | 'Yes' |
| a. Is the second more typical of a <u>flying calling</u> [creature]?                                       | 29%     | 25   | 10    |
| b. Is the second more typical of a <u>flying and calling</u> [creature]?                                   | 37%     | 22   | 13    |
| c. Is the second more typical of a <u>flying or calling</u> [creature]?                                    | 80%     | 7    | 28    |

Table 7d: The 'repetition' condition - general

### 3. General discussion

We have seen that Hebrew speakers tend to dislike ordering entities under conjunctive or disjunctive concepts when the two entities do not stand in one and the same ordering relation (e.g. *more*) in all the constituents. The subjects have taken care to make this point clear in their comments, stating that questions that ask them to do so are impossible to answer, are inappropriate, that the complex (conjunctive or other) comparison will not be used by a person sensitive to the language in such circumstances and so on. One subject has written that these questions are of the same type as questions such as *is a cucumber more long or more green*. We cannot tell. In fact answering the latter is impossible for precisely the same reasons; two properties that are not easily comparable are involved. They are not easily comparable precisely because they do not consist of mappings of entities to degrees on a bound interval isomorphic to the real interval between 0 and 1.

In accordance with these comments, the results presented in part 2 show that a truly compositional interpretation, i.e. a degree function (or ordering) of a conjunction/disjunction of properties, composed based on the degree functions (or ordering relations) of the constituents, is hardly ever occurring naturally in the absence of a particularly encouraging context. Evidence for this is, for example, the fact that in section 3, the two characters are described as *equally tall* (1.95m), but Moshe is 30kg *fatter*, and still most subjects refuse to say that either Moshe or Danny is *more fat and tall* (3e-h). These results suggest that speakers are, more often than not, unwilling to interpret *and* inside the scope of *more*, i.e. *more P and Q* is interpreted as *more P and more Q*, not as *More (P and Q)*. The same holds for *or*. These results are in line with a Boolean analysis more than with a fuzzy one. However, these preliminary findings are restricted to contexts of the nature the questionnaire examined. Other types of contexts should be investigated in the future, as well as the effect of repetition on interpretation. In addition, other languages should be studied.<sup>6</sup>

<sup>6</sup> Novel findings (currently in process) suggest that English is similar to Hebrew in that conjunctive and disjunctive predicates such as *expensive and time consuming* appear incompatible with gradable morphology and interpretation.

The results are also consistent with the hypothesis that a use of a truly compositional degree function (or ordering) may be facilitated by repetition of discussion and processing of the interpretation of complex concepts such as *fat and bald* (cf., the results of section 9). Also, mentioning a conjunctive comparison, e.g. *more fat and bald*, following a conjunction of comparisons, e.g. *more fat and more bald*, may trigger such an interpretation. Therefore, it may be useful to test such contexts in the future, separating the two variables (repetition and conjunction of comparisons) from one another.<sup>7</sup>

Another variable affecting the results is whether the ranked entities obviously violate one of the atomic concepts in question or not. That membership or non-membership in the categories in question plays an important role in judgments of ordering is in line with vagueness based accounts of gradability in natural language. According to theories in this approach, an entity pair  $\langle x, y \rangle$  is classified as *more P* iff, roughly, either  $x$  is  $P$  but  $y$  not necessarily  $P$ , or alternatively  $y$  is not  $P$  but  $x$  is not necessarily not  $P$  (Kamp 1975; Fine 1975; Klein 1980; van Rooij 2009). The present study shows that when subjects are forced to give a comparative judgment about the degrees of instances with respect to conjunctions or disjunctions, judgments seem to be determined by the characters' likelihood of classification in the positive or negative denotations of the conjunction or disjunction. For example, when Moshe weighed 100 kg and Danny only 70 kg (Moshe was *fatter*), but Danny was *bald* and Moshe was *not bald* (Danny was *balder*), according to the few subjects that selected one character to be *more fat and/or bald* than the other, Danny scored much better than Moshe relative to *fat or bald* (questions 4), because it is easier to determine that Danny is *fat or bald* (Danny is *bald*, while Moshe is not *bald* and is not necessarily *fat*); also, Danny scored much better than Moshe relative to *fat bald / bald fat*, because it is easier to determine that Moshe is not: *fat and bald* (because Moshe is not bald), while Danny might still be *fat and bald*. A similar pattern occurred in other scenarios as well. When the two characters were *equally tall* (1.95m), but Moshe was *fatter* (100 kg as opposed to Danny's 70 kg), among those who answered the question, Moshe was regarded as *more fat and tall* than Danny (section 3). When the characters were *fat*, and *equally fat* (100 kg), but Moshe was *balder*, 90%-100% of the subject agreed that Moshe is *more fat or bald* than Danny (section 9). Future research may, then, profit from asking what happens when knowledge about denotation membership is more uncertain.

A different issue raising many questions yet to be examined regards different sorts of ways to refer to ordering relations with adjectives (e.g., *more* versus *less* comparatives, as well as simple versus complex comparison). Different ways to refer seem to differ in a variety of respects, one of which is the extent to which they raise the expectation that the ordered entities are predicate members; this expectation may also be stronger for the subject or for the object (Sassoon 2007). These issues call for further empirical investigation.

The phenomenon this paper investigates pertains also to conjunctive multi-dimensional adjectives, like *typical with respect to flying and singing* or *healthy with respect to blood pressure and pulse*, when these are combined with *more*. Our results suggest that, by and large, *and* takes wide scope, and *more* combines with a single function at a time (per a conjunct); e.g., we understand *healthier with respect to blood pressure and pulse* to mean *healthier with respect to blood pressure and healthier with respect to pulse*.

An important open question is to what extent ordering in conjunctive predicates can be based on the entities' weighted mean in the constituents, in context in which information about weights is made available. If, for example, I compare a patient with cancer to a patient with the flu, I may weigh the cancer as more important, and judge the former patient as having a higher degree of sickness with respect to the conjunction of these dimensions. When context does not tell us how

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<sup>7</sup>Novel findings (currently in process) strongly support the assumption that it is repetition, rather than conjunction of comparisons, that facilitates fuzzy interpretations.

the conjuncts are to be weighed (for instance, if I compare a patient with cancer to a patient with serious heart problems), this strategy might fail. When it fails, we are left with a wide-scope interpretation like *healthier with respect to cancer and healthier with respect to the heart*, and we are likely to be reluctant to say about any of the patients that he is healthier (in the given conjunction of respects). Thus, another open question for the future is to examine the effect of constituents with variable weights.

An alternative strategy for construing an ordering relation for conjunctive and disjunctive adjectives is by using the mean in the typicality dimensions of the modified nouns corresponding to them. Any adjective can modify a trivial noun such as *object, individual, one*. Like other noun phrases, the noun phrase *healthy entity* is linked with a set of typicality dimensions such that entities whose mean degree in these dimensions is high are classified as members in the denotation (cf. Murphy 2002's review of data concerning basic nouns and Hampton's 1997a,b review of data concerning nouns modified by a nominal relative clause, such as *birds which are pet*); examples of dimensions typical of *healthy people* include *calm, does not smoke, does not drink, does not eat fat, eats fruit and vegetables, is regularly involved in sport activities*, etc. Such dimensions may be directly linked to modified nouns such as *healthy person*, but only indirectly related to the adjective *healthy* itself, for otherwise we would expect ordering judgments in conjunctions and disjunctions of adjectives to be as easy as the typicality judgments are in nouns modified by a nominal relative clause or in noun-noun compounds. The study presented in this paper suggests that this is not the case.

In sum, compositionality, in the case of the ordering in conjunctions and disjunctions, more often than not, fails. In many cases (for many pairs of entities) their ordering in a conjunction (or in a disjunction) cannot be predicted from their ordering in the constituents. Thus, while a fuzzy analysis could have been considered a natural extension of the numerical approach (cf. Kennedy 1999), this no longer seems a viable option. Rather, ordering relations for conjunctions and disjunctions need to be learnt directly based on whether entities are classified in their intersective and union-based denotations or not, as well as, perhaps, on context-dependent information regarding the constituents' weights or their typicality features. However, knowledge concerning these issues is often not available. Thus, conjunctions and disjunctions tend not to license gradable morphology.

Last but not least, what are the implications for the numerical versus vagueness-based debate? On the one hand, the fact that conjunctive and disjunctive predicates do not appear gradable is surprising given a vagueness-based approach, for if we use denotations (rather than numerical measurements) to systematically build ordering relations, why aren't we able to systematically use the denotations of conjunctive and disjunctive concepts to do so? After all, these predicates are at least as vague as their constituents are. The numerical approach fairs better over here; since *and* and *or* are merely Boolean, conjunctive and disjunctive predicates denote entity sets, not degree functions, which explains why they are non-gradable. On the other hand, we have seen that when we do make gradable judgments in relation to complex predicates, they do seem to go along the line suggested by vagueness based theories. This is probably the case because no other option (no unified numerical degree function) is available.

Hence, the general moral to draw from all the above must be in favor of a combined approach. Both measurement-based degree functions and vagueness-based ordering relations play a role in the semantics of natural language expressions (for a discussion and a detailed model see Sassoon 2007). In addition, different predicate types may be associated with different types of degree functions (e.g., numerical versus ordinal; cf. Sassoon 2010a).

## References

- Alxatib, Sam. and Pelletier, Jeff. (2010). The psychology of vagueness: Borderline cases and contradictions. Forthcoming in *Mind and Language*.
- Bale, Alan. C. (2007). Boolean AND and the Semantic Correlates of Gradable Adjectives. *International Conference on Adjectives*. Lille (France), 13 – 15, September, 2007.
- Bale, Alan. C. (2008). A universal scale of comparison. *Linguistics and Philosophy*, 31(1), 1–55.
- Bartsch, Renate., & Venneman, Theo. (1972). *Semantic structures*: 47–86. Frankfurt: Anthenäum Verlag.
- Cresswell, Maxwell John. (1977). The semantics of degree. In Barbara Partee (Ed.), *Montague grammar* : 261–292. New York: Academic Press.
- Fine, Kit. (1975). Truth, vagueness and logics. *Synthese*, 30, 265–300.
- Hampton, James. (1987), Inheritance of Attributes in Natural concept conjunction. *Memory and Cognition* 15(1): 55-71.
- Hampton, James. (1988a) Overextension of conjunctive concepts: Evidence for a unitary model of concept typicality and class diagnosticity. *Journal of Experimental Psychology: Learning, Memory & Cognition*, 14, 12-32.
- Hampton, James. (1988b), Disjunction of natural concepts, *Memory & Cognition*, 16(6): 579-591.
- Hampton, James. (1997a), Conceptual combination: Conjunction and negation of natural concepts, *Memory & Cognition*, 25 (6): 888-909.
- Hampton, James. (1997b), Conceptual Combination. In Koen Lamberts and David Shanks (eds.), *Knowledge, Concepts and Concepts*. The MIT Press. Cambridge, MA: 135-162.
- Hajek, Petr. (2009). Fuzzy logic. In Edward N. Zalta (eds.), *The Stanford Encyclopedia of Philosophy*. Spring 2009 edition.
- Heim, Irene. (2000). Degree operators and scope. *Proceedings of SALT X*. Ithaca, NY: CLC Publications.
- Kamp, Hans. (1975). Two theories about adjectives. In Edward Keenan (ed.), *Formal semantics for natural language* (pp. 123–155). Cambridge: Cambridge University Press.
- Kamp, Hans, & Partee, Barbara. (1995). Prototype theory and compositionality. *Cognition*, 57, 129–191.
- Kennedy, Christopher. (1999). *Projecting the adjective: The syntax and semantics of gradability and comparison*. New York: Garland (1997 UCSC Doctoral dissertation).
- Kennedy, Christopher. (2001). Polar opposition and the ontology of degrees. *Linguistics and Philosophy*, 24(1), 33–70.
- Kennedy, Christopher, & Levin, Beth. (2007). Measure of change: The adjectival core of degree achievements. In Louise McNally & Christopher Kennedy (eds.), *Adjectives and adverbs: Syntax, semantics and discourse*. Oxford: Oxford University Press.
- Kennedy, Christopher, & McNally, Louise. (2005). Scale structure and the semantic typology of gradable predicates. *Language*, 81, 345–381.
- Klein, Ewan. (1980). A semantics for positive and comparative adjectives. *Linguistics and Philosophy*, 4, 1–45.
- Klein, Ewan. (1991). Comparatives. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantik/semantics, an international handbook of contemporary research* (pp. 673–691). Berlin, NY: Walter de Gruyter.
- Landman, F. (1991). *Structures for semantics*. Dordrecht: Kluwer.
- Landman, Fred. (2000). *Events and Plurality*, Kluwer, Dordrecht
- Landman, F. (2005). An almost (but not quite) naïve theory of measures, manuscript, Tel Aviv University.
- Lewis, David. (1970). General semantics. *Synthese*, 22, 18–67. Reprinted In David K. Lewis (1983). *Philosophical Papers*, 1, 189–229. New York: Oxford University Press.

- Lewis, David. (1979). Scorekeeping in a language game. *Journal of Philosophical Logic*, 8, 339–359. (Reprinted In *Philosophical Papers*, 1, pp. 233–249, by D. K. Lewis (ed.), 1983, New York: Oxford University Press.
- Moltmann, Friederike. (2006). *Comparatives without degrees. A new approach*. A manuscript for the workshop on scalar meaning, University of Chicago.
- Murphy, Gregory L. (2002). *The big book of concepts*. Cambridge, MA: The MIT Press.
- Ripley, David. (2010). Contradictions at the borders. Forthcoming in: Rick Nouwen, Uli Sauerland, Hans-Christian Schmitz, and Robert van Rooij, editors. *Vagueness in Communication*. Springer, Heidelberg, Germany.
- Russell, Bertrand. (1905). On denoting. *Mind*, 14, 479–493 (Reprinted from *Essays in Analysis*, pp. 103–119, by Russell, B., 1973, London: Allen and Unwin).
- Sapir, Edward. (1944). Grading: A study in semantics. *Philosophy of Science*, 11, 93–116.
- Sassoon, W. Galit. (2007). *Vagueness, gradability and typicality, a comprehensive semantic analysis*. Doctoral Dissertation, Tel Aviv University.
- Sassoon, W. Galit. (2010a). Measurement theory in linguistics, *Synthese* 174(1):151–180
- Sassoon, W. Galit. (2010b). The degree functions of negative adjectives. *Natural language semantics* 18(2), 141-181.
- Sauerland, Uli, (2011), Vagueness in Language: The Case Against Fuzzy Logic Revisited. In: Petr Cintula, Chris Fermüller, Lluís Godó, Petr Hajek (eds.), *Reasoning under Vagueness - Logical, Philosophical, and Linguistic Perspectives*, Studies in Logic series of College Publications.
- Schwarzschild, Roger. (2008). The semantics of comparatives and other degree constructions. *Language and Linguistics Compass*, 2.2, 308–331.
- Schwarzschild, Roger, & Wilkinson, Karina. (2002). Quantifiers in comparatives: A semantics of degree based on intervals. *Natural Language Semantics*, 10, 1–41.
- Serchuk, Phil, Hargreaves, Ian, and Zach, Richard. (2010). Vagueness, logic and use: Four experimental studies on vagueness. Forthcoming in *Mind and Language*.
- Seuren, Peter. (1978). The structure and selection of positive and negative gradable adjectives. In *Papers from the Para-session on the Lexicon. 14th Regional Meeting of the Chicago Linguistic Society* (pp. 336–346). CLS, Chicago, IL.
- Stalnaker, Robert. (1978). Assertion. In Peter Cole, *Syntax and semantics 9: Pragmatics* (pp. 315–332). New York: Academic Press.
- van Fraassen, Bas C. (1969). Presuppositions, supervaluations and free logic. In K. Lambert, *The logical way of doing things* (pp. 67–91). New Haven: Yale University Press.
- van Rooij, Robert. (2009). Vagueness and linguistics. In Giuseppina Ronzitti (ed.), *The vagueness handbook*. Dordrecht: Springer.
- Veltman, Frank. (1984). Data semantics. In Jeroen Groenendijk, Theo Janssen & Martin Stokhof (eds.), *Truth, interpretation and information proceedings of the 3rd Amsterdam colloquium*.
- von Stechow, Arnim. (1984a). Comparing semantic theories of comparison. *Journal of Semantics*, 3, 1–77.
- von Stechow, Arnim. (1984b). My reaction to Cresswell's, Hellan's, Hoeksema's and Seuren's comments. *Journal of Semantics*, 3, 183–199.
- Yen, John (1999). Fuzzy Logic—A Modern Perspective, *Transactions on Knowledge and Data Engineering* 11(1): 153-165.
- Zadeh, Lotfi A. (1965). Fuzzy sets. *Information and Control* 8(3): 338–353.

## Appendix

On section 1 the subjects read the following description of two characters called Moshe and Danny: “Assume Moshe weighs 100kg and Danny weighs 90kg and they are alike in other things (for instance, height).” Here are the questions that followed the paragraph.

(17) Section 1, the questions:

### The basic condition

- a. Is Moshe more fat than Danny? Yes/No
- b. Is it easier to determine that Moshe is fat than that Danny is fat? Yes/No
- c. Is it harder to determine that Danny is fat than that Moshe is fat? Yes/No

On section 3, the subjects read the following description: “Assume Moshe weighs 100kg and he is 1.95 cm tall, and Danny weighs 70 kg and is he is 1.95 cm tall.” (i.e., Moshe is fatter than Danny, but they are equally tall).

(18) Section 3, the questions:

### The basic condition

- a. is Moshe more fat than Danny? Yes/No
- b. Is it easier to determine that Moshe is fat than that Danny is fat? Yes/No
- c. is one of them more tall than the other? Yes/No
- d. Is it easier to determine that he is tall? Yes/No

### The conjunction condition

- e. Is Moshe more fat and tall than Danny? Yes/No
- f. Is Moshe less fat and tall than Danny? Yes/No
- g. Is it easier to determine that Moshe is fat and tall than that Danny is? Yes/No
- h. Is it harder to determine that Moshe is fat and tall than that Danny is? Yes/No
- i. Comments: [three empty lines]

On section 4, the subjects read the following description of characters called Aharon and Danny: “Assume Aharon weighs 100kg and he is not bald, and Danny weighs 70 kg and is bald. They are alike in other respects.” Thus, in this scenario Aharon is *fatter* and Danny is *balder*.

(19) Section 4, the questions:

### The basic condition

- a. Who is more fat? Aharon/ Danny
- b. Who is more bald? Aharon/ Danny

### The conjunction condition

- c. Is any of them more fat and bald than the other? Yes/No
- d. Is it easier to determine that one of them is fat and bald than that the other is? Yes/No
- e. Are they equally fat and bald? Yes/No
- f. Is it equally easy to determine that they are fat and bald? Yes/No

### The disjunction condition

- g. Is one of them more fat or bald than the other? Yes/No
- h. Who is more fat or bald? Aharon/ Danny
- i. Is it easier to determine that one of them is fat or bald than that the other is? Yes/No
- j. For whom is it easier to determine that? Aharon/Danny

### The modifier condition

- k. Is any of them more fat bald than the other? Yes/No
- l. Who is more fat bald? Aharon/ Danny
- m. Is it easier to determine that one of them is fat bald than that the other is? Yes/No
- n. For whom is it easier to determine that? Aharon/Danny
- o. Is any of them more bald fat than the other? Yes/No
- p. Who is more bald fat? Aharon/ Danny
- q. Is it easier to determine that one of them is bald fat than that the other is? Yes/No
- r. For whom is it easier to determine that? Aharon/Danny

**The disjunction condition - continued**

- s. Fat or bald subjects are needed to fill a questionnaire for a scientific experiment.  
Who fits more? Aharon/Danny
- t. It is true that if Danny fits Aharon definitely fits? Yes/No
- w. It is true that if Aharon fits Danny definitely fits Yes/No
- u. Comments: [3 lines]

Section 9 began with the following description: “Assume Moshe and Danny both weigh 100kg and also that Moshe is bald and Danny is less bald (maybe even isn’t bald at all).” Thus, here Moshe and Danny are equally fat, but Moshe is balder (the same pattern as in section 3). They are alike in other respects.”

(20) Section 9, the questions:

**The conjunction condition**

- a. Is Moshe both more fat and more bald? Yes/No
- b. Is Moshe more fat and Bald than Danny? Yes/No
- c. Is it easier to determine that Moshe is fat and bald than that Dan is fat and bald? Yes/No
- d. Is it easier to determine that Moshe is bald and fat than that Dan is bald and fat? Yes/No

**The modifier condition**

- e. Is Moshe more fat bald than Danny is? Yes/No
- f. Is it easier to determine that Moshe is fat bald than that Danny is fat bald? Yes/No
- g. Is Moshe more bald fat than Danny is? Yes/No
- h. Is it easier to determine that Moshe is bald fat than that Danny is bald fat? Yes/No

**The disjunction condition**

- i. Is Moshe more fat or Bald than Danny? Yes/No
- j. Is it easier to determine that Moshe is fat or bald than that Danny is? Yes/No
- k. Is Moshe more Bald or fat than Danny? Yes/No
- l. Is it easier to determine that Moshe is bald or fat than that Dan is? Yes/No
- m. Fat or bald subjects are needed to fill a questionnaire for a scientific experiment.  
Who fits more? Moshe/Danny
- n. It is true that if Danny fits Moshe definitely fits? Yes/No
- o. It is true that if Moshe fits Danny definitely fits Yes/No
- p. Comments: [3 lines]

On section 7 the subjects read a description of two creatures differing only on whether they call or not: “Imagine two creatures that do not fly; also, the first one does not call, while the second is more typical of a calling creature than the first”.

(21) Section 7, the questions:

**The multidimensional, conjunctive adjective condition**

- |  |        |
|--|--------|
| a. Is the second more typical of a <u>flying calling</u> [creature]?     | Yes/No |
| b. Is the second more typical of a <u>flying and calling</u> [creature]? | Yes/No |
| c. Is the second more typical of a <u>flying or calling</u> [creature]?  | Yes/No |
| d. comments [3 lines]  |        |